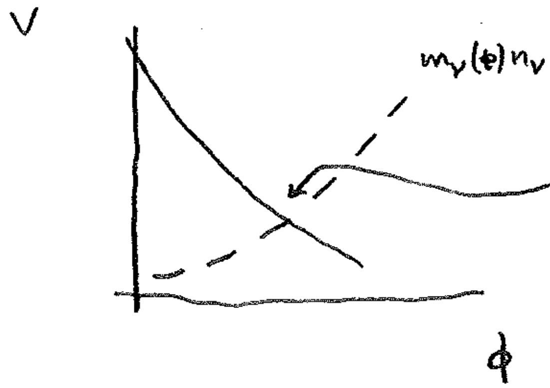
MASS VARYING NEUTRINOS [eg Fardon et al JCAP 0410 005 (2004)]

suppose $m_\nu(\phi)$ depends on a cosmic field

MODEL $\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - V(\phi) - m_\nu(\phi)\bar{\Psi}\Psi$

ignore fermion kinetic term
for non-rel ν's



effective potential has a minimum; can this ΔV provide dark energy?

$$\rho_T = \rho_\phi + \rho_\nu$$

$$\dot\rho_T = \left(\tfrac{1}{2}\dot\phi^2 + V\right)^{\cdot} + \dot\rho_\nu = -3H(\rho_T + p_T)$$

$$\dot\phi\ddot\phi + V'\dot\phi + \dot\rho_\nu = -3H(\dot\phi^2 + \rho_\nu)$$

$$\rho_\nu = m_\nu(\phi)\, n_\nu$$

$$\dot\rho_\nu = \frac{\partial m_\nu}{\partial\phi}\dot\phi\, n_\nu + m_\nu \dot n_\nu$$

$\dot n_\nu = -3H n_\nu$ (# ν's are conserved)

$$\dot\phi\left(\ddot\phi + V' + 3H\dot\phi + \frac{\partial m_\nu}{\partial\phi} n_\nu\right) = 0$$

so $\ddot\phi + 3H\dot\phi + V' = -Q(\phi)\rho_\nu$, $Q = \dfrac{\partial \ln m_\nu}{\partial\phi}$

Now let's suppose ϕ moves along $V_{T,\min}$

$$V_T = V + m_\nu n_\nu$$

$$\frac{\partial V_T}{\partial \phi} = V' + \frac{\partial \ln m_\nu}{\partial \phi} \rho_\nu = 0$$

$$V' = -\frac{\partial \ln m_\nu}{\partial \phi} \rho_\nu \quad \rightarrow \quad \dot{\phi}^2 \ll V_T$$

in which case $\rho_T \simeq V + m_\nu n_\nu$

$$= V - \frac{\partial V}{\partial \ln m_\nu}$$

and $P_T = -V$

The eq'n of state is $w = \frac{P_T}{\rho_T} = \frac{-V}{V + m_\nu n_\nu} = -1 + \frac{\Omega_\nu}{\Omega_\phi + \Omega_\nu}$

ACCELERATES! SUPE.

real mass? $V'' > 0$ easily satisfied ✓

SOUND SPEED? Here is the problem

$$c_a^2 = \frac{\dot{p}}{\dot{\rho}} = \frac{(-V)^{\dot{}}}{(V + m_\nu n_\nu)^{\dot{}}} = \frac{(-V'\dot{\phi})}{(V'\dot{\phi} + \frac{\partial m_\nu}{\partial \phi}\dot{\phi} n_\nu + m_\nu \dot{n}_\nu)}$$

cancel $\qquad -3Hn_\nu$

$$\rightarrow \frac{\frac{\partial m}{\partial \phi} n_\nu \dot{\phi}}{m_\nu(-3Hn_\nu)} = -\frac{1}{3}\frac{\dot{\phi}}{H}\frac{\partial \ln m_\nu}{\partial \phi} \quad \text{GENERALLY NEGATIVE}$$

"adiabatic instability" ruins this scenario*

(see Bean et al. PRD 78 023009 (2008))

Living with adiabatic instability?

If c_a^2 is not too negative

Then growth of perturbations is not too fast.

$\sim$ Compare $c_a^2 k^2$ vs. $\mathcal{H}^2$

See Corasaniti, PRD 78 083538 (2008)

This also leads to "Growing Neutrino" Cosmology

eg Wetterich + Pettorino arxiv: 0905.0715

But here, $\frac{\partial \ln m_\nu}{\partial \phi}$ is large

Large lumps of neutrinos result, so that

non-linear effects ($\delta_\nu > 1$) become important.

COUPLING ϕ / QUINTESSENCE TO STANDARD MODEL?

In most cases, this is ruled out!

$\phi F_{\mu\nu} F^{\mu\nu}$ leads to variation of α_{em}

$\phi \bar{\Psi} \Psi$ long range force between lumps of fermions

Symmetry to prevent most couplings: $\phi \to \phi + c$
that still allows derivative interactions

$$\int d^4x \sqrt{g} \;\; \phi F_{\mu\nu} \tilde{F}^{\mu\nu} = \int d^4x \sqrt{g} \;\; 2\phi \nabla_\mu A_\nu \tilde{F}^{\mu\nu}$$

$$= \int d^4x \sqrt{g}\; 2\Big[\underbrace{\nabla_\mu (\phi A_\nu \tilde{F}^{\mu\nu})}_{\to 0} - \nabla_\mu (\phi \tilde{F}^{\mu\nu}) A_\nu \Big] \qquad \nabla_\mu \tilde{F}^{\mu\nu} = 0$$

$$= -2 \int d^4x \sqrt{g} \;\; A_\nu \nabla_\mu \phi \tilde{F}^{\mu\nu} \qquad \text{respects shift } \checkmark$$

So let's pursue this... COSMIC AXION ELECTRODYNAMICS

$$\mathcal{L}_I = -\frac{1}{4} \frac{\phi}{M} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

recall $\tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\sigma\tau} F_{\sigma\tau}$

MAXWELL + COSMIC FIELD:

$$\nabla_\mu F^{\mu\nu} + J^\nu + \frac{1}{M} \nabla_\mu \phi \tilde{F}^{\mu\nu} = 0 \quad , \quad \nabla_\mu \tilde{F}^{\mu\nu} = 0$$

and $\Box \phi = V' + \frac{1}{4M} F_{\mu\nu} \tilde{F}^{\mu\nu}$

Maxwell's new equations ... in SI units!

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0} - \frac{1}{Mc}\vec{\nabla}\phi\cdot\vec{B} \qquad \vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{B} - \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t} = \mu_0\vec{J} + \frac{1}{Mc^3}\left(\dot{\phi}\vec{B} + \vec{\nabla}\phi\times\vec{E}\right)$$

$$\vec{\nabla}\times\vec{E} + \frac{\partial\vec{B}}{\partial t} = 0 \qquad \frac{\Box\phi}{\hbar c} = V' - \frac{\epsilon_0}{Mc}\vec{E}\cdot\vec{B}$$

Note: $\vec{D} = \epsilon_0\left(\vec{E} + \frac{\phi}{Mc}\vec{B}\right)$

$$\vec{H} = \frac{1}{\mu_0}\left(\vec{B} - \frac{\phi}{Mc^3}\vec{E}\right)$$

restores std form: $\vec{\nabla}\cdot\vec{D} = \rho$ & $\vec{\nabla}\times\vec{H} - \frac{\partial\vec{D}}{\partial t} = \vec{J}$

Interesting behavior!

Let's study EM waves, so set $\epsilon_0 = \mu_0 = c = 1$ for now.

$$\frac{\partial^2}{\partial t^2}\vec{E} - \vec{\nabla}^2\vec{E} = \frac{1}{M}\left[\vec{\nabla}(\vec{\nabla}\phi\cdot\vec{B}) - \frac{\partial}{\partial t}\left(\frac{\partial\phi}{\partial t}\vec{B} + \vec{\nabla}\phi\times\vec{E}\right)\right]$$

$$\frac{\partial^2}{\partial t^2}\vec{B} - \nabla^2\vec{B} = \frac{1}{M}\left[\vec{\nabla}\times\left(\frac{\partial\phi}{\partial t}\vec{B} + \vec{\nabla}\phi\times\vec{E}\right)\right]$$

In an expanding universe

$$\vec{B}\to a^2\vec{B}, \quad \vec{E}\to a^2\vec{E}, \quad \frac{\partial}{\partial t}\to\frac{\partial}{\partial\tau}$$

If ϕ is a cosmic field, then it evolves slowly.

For EM waves with freq, wavenumber ω, k

$$\omega \gg \frac{1}{\phi}\frac{\partial\phi}{\partial t} \qquad k \gg \frac{1}{\phi}\nabla\phi$$

In which case

$$\frac{\partial^2}{\partial t^2}\vec{E} - \nabla^2\vec{E} = \frac{\dot{\phi}}{M}\left(-\frac{\partial\vec{B}}{\partial t}\right) + \frac{1}{M}(\vec{\nabla}\phi\cdot\vec{\nabla})\vec{B}$$

$$\frac{\partial^2}{\partial t^2}\vec{B} - \nabla^2\vec{B} = \frac{\dot{\phi}}{M}\frac{\partial\vec{E}}{\partial t} - \frac{1}{M}(\vec{\nabla}\phi\cdot\vec{\nabla})\vec{E}$$

Consider plane waves: $E, B \propto e^{i\omega t - i\vec{k}\cdot\vec{x}}$

$$\vec{P} \equiv \vec{E} + i\vec{B}, \quad \vec{Q} = \vec{E} - i\vec{B}$$

$$P: \quad \omega^2 = k^2 + \left(\frac{\dot{\phi}}{M}\omega + \frac{\vec{\nabla}\phi}{M}\cdot\vec{k}\right)$$

$$Q: \quad \omega^2 = k^2 - \left(\frac{\dot{\phi}}{M}\omega + \frac{\vec{\nabla}\phi}{M}\cdot\vec{k}\right)$$

Let's write $\vec{k} = k\hat{n}$ where $\hat{n}$ is the direction of propagation

$$\omega^2 = k^2 \pm \left(\frac{\dot{\phi}}{M}\omega + \frac{\vec{\nabla}\phi\cdot\hat{n}}{M}k\right)$$

$$k_\pm = \mp\frac{1}{2}\left(\frac{\dot{\phi}}{M} + \frac{\vec{\nabla}\phi\cdot\hat{n}}{M}\right) + \omega + O\left(\left(\frac{\phi}{M}\right)^2\right)$$

The phase velocity of L & R-circularly polarized waves is different. For a linearly polarized wave

$$\vec{E} = \frac{1}{2}(\vec{P}+\vec{Q}) = \frac{1}{2}E_0 e^{i\omega t}\left(e^{-ik_+ z}[\hat{x}+i\hat{y}] + e^{-ik_- z}[\hat{x}-i\hat{y}]\right)$$

let's write $k_\pm = \omega \mp Q$, $Q = \frac{1}{2}\left(\frac{\dot{\phi}}{M} + \frac{\partial_z \phi}{M}\right)$

$$= \frac{1}{2}E_0 e^{i\omega(t-z/c)}\Big(\left[e^{iQz} + e^{-iQz}\right]\hat{x} + i\left[e^{iQz} - e^{-iQz}\right]\hat{y}\Big)$$

$$= E_0 e^{i\omega(t-z/c)}\left(\cos\theta\,\hat{x} + -\sin\theta\,\hat{y}\right)$$

$\theta = Qz$

But over long distances, $\dot{\phi}, \nabla\phi$ vary so that

$$\theta = \int Q\,dz = \frac{1}{2M}\int(\dot{\phi} + \partial_z\phi)\,dz$$

$$= \frac{1}{2M}\int\left[\frac{d}{dt}\phi(t, z(t))\right]dt$$

$$= \frac{\Delta\phi}{2M} \checkmark$$

In terms of Stokes parameters

$$\Pi = \begin{pmatrix} Q \\ U \end{pmatrix}, \quad \Pi' = \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}\Pi$$

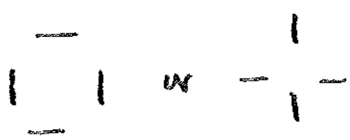
where $Q = |\hat{\epsilon}_1\cdot\vec{E}|^2 - |\hat{\epsilon}_2\cdot\vec{E}|^2$, $U = 2\,\mathrm{Re}[(\hat{\epsilon}_1\cdot\vec{E})^*(\hat{\epsilon}_2\cdot\vec{E})]$

so the rotation angle is $\theta = \frac{\Delta\phi}{2m}$.

This phenomena, known as COSMIC BIREFRINGENCE, is constrained by observations of CMB polarization.

CMB distinguishes two patterns of polarization

"E-type"



headless vectors, the grad of a scalar on the sky.

"B-type"

a swirl: curl of a vector on the sky

The rotation of polarization will turn $E \to B$, $B \to E$.

The non-detection of B-type implies

$$-1.41^\circ < \Delta\theta < 0.91^\circ \quad \text{at } 95\% \text{ CL}$$

$$\text{or} \quad \left|\frac{\Delta\phi}{M}\right| < 0.035$$

See WMAP 7, Komatsu et al, ApJ Supp. 192 18 (2011)

Fluctuations of the scalar field at last scattering can further contribute to a position (on the sky)-dependent rotation of polarization.

CONSIDER QUINTESSENCE ELECTRO/MAGNETO-STATICS

For a cosmic field $\frac{\dot{\phi}}{M}$, $\nabla\phi \frac{1}{M} \sim H$ TINY!

But ϕ defines a special frame, relative to which we move or accelerate. Consider weak gravity in the vicinity of the Earth or Sun

$$\phi \to \phi_0 + \delta\phi(r) \qquad \Box\phi = V'$$

For spherical symmetry

$$\left(\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr}\right)\delta\phi = V_0''\,\delta\phi + 2\Phi V_0'$$

$$\Phi = -GM/r$$ Earth or Sun mass

sol'n: $$\delta\phi(r) = -2\frac{V_0'}{V_0''}\Phi(r)$$

$$\vec{\nabla}\delta\phi(r) = -2\frac{V_0'}{V_0''}\vec{\nabla}\Phi = 2\frac{V_0'}{V_0''}\vec{g}$$

This is small, but perhaps not too small

let's define $\epsilon_Q = 2\frac{V_0'}{V_0''}\frac{1}{Mc^2}$ (~ 1)

coupling mass

then $\hbar\left(\frac{\nabla\delta\phi}{Mc}\right) = \hbar\,\epsilon_Q\, g/c \sim 10^{-32}$ GeV

(MUCH BETTER THAN 10^{-42} GeV!)

Return to Maxwell's Eq'ns

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0} - \frac{\vec{\nabla}\phi}{Mc}\cdot\vec{B} \quad\rightarrow\quad \vec{\nabla}\cdot\vec{E} - \rho/\epsilon_0 = -\frac{\epsilon_Q}{c}\,\vec{g}\cdot\vec{B}$$

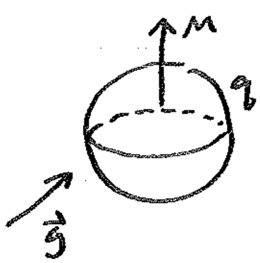
anomalous charge

$$\vec{\nabla}\times\vec{B} - \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t} = \mu_0\vec{J} + \frac{1}{Mc^3}\vec{\nabla}\phi\times\vec{E}$$

$$\rightarrow \vec{\nabla}\times\vec{B} - \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t} - \mu_0\vec{J} = \frac{\epsilon_Q}{c^2}\,\vec{g}\times\vec{E}$$

anomalous current

look at a few cases: charged and magnetized sphere



inside, due to $\vec{E}$ (coulomb) there is an anomalous $\vec{B}$

$$\vec{B} = \frac{\mu_0 q\,\epsilon_Q}{20\pi Rc}\left(5\vec{g} + \frac{r^2}{R^2}\left((\vec{g}\cdot\hat{r})\hat{r} - 2\vec{g}\right)\right)$$

assuming uniform q in radius R

It points in $\vec{g}$ direction, so if $\vec{g}$ is not aligned with the magnetization $\vec{\mu}$, there will be a torque.

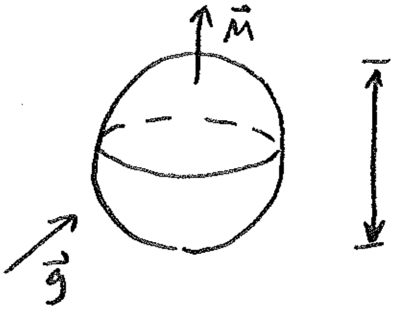
$$\vec{\tau} = \frac{2\mu_0 q\,\epsilon_Q}{5\pi Rc}\,\vec{\mu}\times\vec{g}$$

Spin-flip energy

$$\Delta\mathcal{E} = \frac{2\mu_0 q\,\epsilon_Q}{5\pi Rc}\,\mu g$$

For the proton as a classical object: $\Delta\mathcal{E} \simeq 10^{-34}\,\epsilon_Q$ GeV

Consider a uniformly magnetized sphere

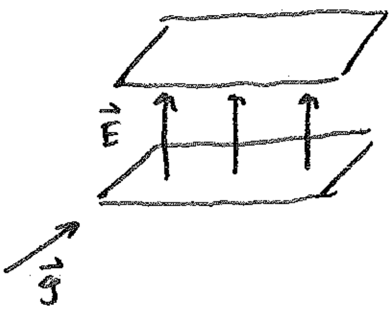


POTENTIAL DIFF.

$$\Delta V = \frac{G_a R^2}{5c}\,\vec{g}\cdot\vec{B}$$

$$\simeq 0.3\, G_a \left(\frac{B}{T}\right)\left(\frac{R}{0.1\,\mathrm{m}}\right)^2 \mathrm{nV}$$

Consider a uniform electric field



Anomalous magnetic field

$$\vec{B} = -\frac{1}{2}\frac{G_a}{c^3}\,\vec{r}\times(\vec{g}\times\vec{E})$$

$$\simeq 10^{-25}\, G_a \left(\frac{\Delta V}{\mathrm{volts}}\right) \mathrm{T}$$

also see: Flambaum et al PRD 80, 105021 (2009)

Bailey + Kostelecky PRD 70, 076006 (2004)

HOW NEGATIVE Eq'N OF State? "PHANTOM"

[RC, PLB 545, 23 (2002)]

$w < -1$? Consistent w/ SNe data

violates energy condition

$\rho > 0$, $p < -\rho$ in cosmic frame means there is a boosted frame in which $\rho' < 0$!

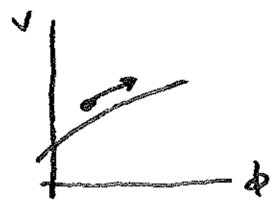
nevertheless... what would it imply?

A simple realization $\mathcal{L}_\phi = +\frac{1}{2}(\partial\phi)^2 - V$

↑ wrong sign leads to instabilities.

EOM $\ddot\phi + 3H\dot\phi - V' = 0$ $\cdot = \frac{d}{dt}$

so field runs up hill!



look at perturbations

STD: $\ddot{\delta\phi} + 3\mathcal{H}\dot{\delta\phi} + (k^2 + V'')\delta\phi = -\frac{1}{2}\dot h\dot\phi$ $\cdot = \frac{\partial}{\partial\tau}$

Here: $\ddot{\delta\phi} + 2\mathcal{H}\dot{\delta\phi} + (k^2 - V'')\delta\phi = -\frac{1}{2}\dot h\dot\phi$

↓ need $V'' < 0$ but not so bad.. $|V''| \lesssim H^2$

↳ NO INSTABILITY DUE TO DRIVING TERM

[see V'' eq'n in terms of $w, \dot w$]

CLASSICALLY, SUCH A FIELD THEORY IS 'OK'

BIG RIP

let's say $w =$ constant, < -1

$$a(t) = \begin{cases} a_m \left(t/t_m \right)^{2/3} & t < t_m \\ a_m \left[-w + (1+w) \dfrac{t}{t_m} \right]^{\frac{2}{3(1+w)}} & t > t_m \end{cases}$$

patch phantom DE onto matter era.

USING $a(t)$

$$H = \frac{2}{3} \left[-w t_m + (t)(1+w) \right]^{-1}$$

$$R = 6 \left(\frac{\ddot{a}}{a} + H^2 \right)$$

$$= \frac{4}{3} \frac{1-3w}{(-w t_m + (1+w)t)^2}$$

OBSERVE $a(t)$, $H(t)$, $R(t)$ all diverge in finite time

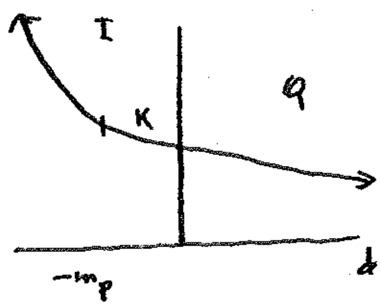
$$t_{BIG\,RIP} = \frac{w}{1+w} t_M$$

EVENTS PRIOR TO $t_{BIG\,RIP}$: EXPANSION "FORCE" DIVERGES, PULLS APART OBJECTS

Further interest: "SUDDEN FUTURE SINGULARITIES" and other exotic behavior resulting from dark energy.

Connect inflation to dark energy?

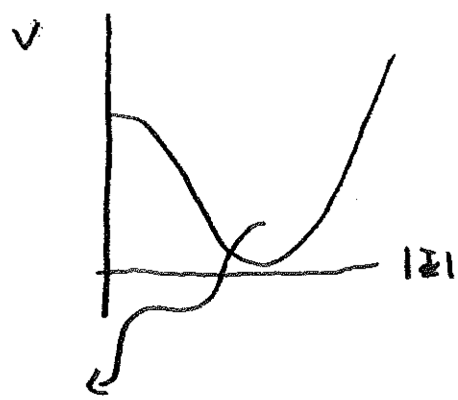
Quintessential Inflation



$$V = \begin{cases} \lambda(\phi^4 + M^4) & \phi < 0 \\ \lambda M^8/(\phi^4 + M^4) & \phi \geq 0 \end{cases}$$

Peebles + Vilenkin PRD59 063505 (1999)

Inflation followed by kinetic phase, after which ϕ is prepped for tracker. (Too much grav. waves, though.)



$$V = \lambda(\Phi^* \Phi - \sigma^2)^2 + M^4(1 + \cos(\arg \Phi))$$

Rosenfeld + Frieman JCAP 0509 003 (2005)

Higgs-Dilaton Inflation-Quintessence

see Garcia-Bellido et al PRD84 123504 (2011)