Dark Gravity

Scalar-Tensor

$$S = \int d^4x \sqrt{g}\left[\frac{f(\phi)R}{16\pi G} - \frac{1}{2}\Theta(\phi)(\partial\phi)^2 - V(\phi) + \mathcal{L}_m\right]$$

Is ϕ responsible for inflation? Dark energy?

Does it improve on inflaton, quintessence?

Novel idea: $\Theta \to 0$

ϕ as a Lagrange multiplier

$$S = \int d^4x \sqrt{g}\left[\frac{f(\phi[R])R}{16\pi G} - V(\phi[R]) + \mathcal{L}_m\right]$$

absorb all R-dependence into a single function: $f(R)$.

New Field Equations [Olmo, gr-qc/0612002]

Ambiguity: Metric variation vs. Palatini variation

(1) $$S = \int d^4x \sqrt{g}\left(\frac{f(R[g])}{16\pi G} + \mathcal{L}_m(g, \Psi_m)\right)$$

Vary S w.r.t. g ✓

(2) $$S = \int d^4x \sqrt{g}\left(\frac{f(R[\Gamma])}{16\pi G} + \mathcal{L}_m(g, \Psi_m)\right)$$

Vary S w.r.t. g, Γ ✗

(1) = (2) IN GR, BUT (2) LEADS TO PROBLEMS IN OTHER THEORIES

f(R) GRAVITY [Amendola & Tsujikawa]

$$S = \int d^4x \sqrt{-g} \left[\frac{f(R)}{16\pi G} + \mathcal{L}_m \right] \qquad F = \frac{\partial f}{\partial R}$$

$$F R_{\mu\nu} - \frac{1}{2} R f g_{\mu\nu} - F_{;\mu\nu} + \Box F g_{\mu\nu} = K^2 T_{\mu\nu}$$

trace: $3\Box F + FR - 2f = K^2 T$

Easy to find accelerating solutions
But what about local behavior?

Consider $F \to F_0 + \delta F$, $T_{\mu\nu} \to T_{\mu\nu} + \delta T_{\mu\nu}$, $g \to g + \delta g$
weak perturbation

static configuration: $\nabla^2 \delta F - M_F^2 \delta F = \frac{K^2}{3F_0} \delta T$

where $M_F^2 = \frac{1}{3} \left(\frac{f_{,R}}{f_{,RR}} - R_0 \right)$ [expect $M_F \propto H_0$!]

sol'n outside mass M_c of radius r_c

$$\delta F = \frac{2GM_c}{3F_0 r} e^{-M_F r}$$

METRIC: $g_{00} = -1 + 2\frac{\tilde{G}M_c}{r}$, $g_{ij} = (1 + 2\frac{\tilde{G}M_c}{r}\gamma)\delta_{ij}$

$$\tilde{G} = G(1 + \tfrac{1}{3} e^{-M_F r})/F_0$$

$$\gamma = \frac{1 - \frac{1}{3} e^{-M_F r}}{1 + \frac{1}{3} e^{-M_F r}} \qquad \text{for } M_F r \gg 1$$

f(R) GRAVITY & CHAMELEON MECHANISM

$$S = \int d^4x \sqrt{-g} \left(\frac{f}{2\kappa^2} + \mathcal{L}_m \right)$$

Make a conformal transformation, $\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$

$$S = \int d^4x \sqrt{-\tilde{g}} \left(\frac{\tilde{R}}{2\kappa^2} - \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - V(\phi) \right) + S_m(g_{\mu\nu}, \Psi_m)$$

$$\Omega^2 = F = \frac{\partial f}{\partial R}$$

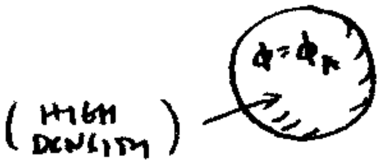
$$\kappa\phi = \sqrt{\frac{3}{2}} \ln F \quad , \quad V = \frac{RF - f}{2\kappa^2 F^2}$$

If scalar ϕ can be trapped at potential minimum

$$V_{eff} = V + e^{-\beta\phi}\rho_m$$

then recover Einstein gravity. The "chameleon" effect hides ϕ in regions of high density (Earth, galaxy) but permits new effects of ϕ to be manifest elsewhere (clusters, cosmology).

$$\nabla^2\phi = V' + (-\beta)\rho e^{-\beta\phi}$$



$$\phi = \phi_B + 2\beta_{eff}\frac{GM_c}{r} \qquad \beta_{eff} \simeq \left| \frac{\phi_B - \phi_A}{\Phi_N(r_c)} \right|$$

(LOW DENSITY)

Force on a particle of coupling β

$$\vec{F} = -\beta\vec{\nabla}\phi \quad \rightarrow \quad |\vec{F}| = 2|\beta\beta_{eff}|\frac{GM_c}{r^2}$$

Look for 5th force!

MASSIVE GRAVITY

$$\nabla^2 \phi = 4\pi G \rho \quad \longrightarrow \quad \nabla^2 \phi - m^2 \phi = 4\pi G \rho$$

suppose ρ is dominated by a constant term.

Then m^2-term can lead to a novel solution

$$\phi = -\frac{4\pi G \rho}{m^2}$$

which nullifies the effects of the constant ρ.

similar to argument by Einstein

Crude picture, but helps to suggest how a mass term in gravity can help "degravitate" a large cosmological const.

GR is a massless theory of a spin-2 particle

but we can try to add a mass...

{ Hinterbichler arxiv: 1105.3735

$$S = \int d^4x \sqrt{g} \left(\frac{R}{16\pi G} + \mathcal{L}_m \right) \qquad g \to g + h \ldots$$

$$= \frac{1}{16\pi G} \int d^4x \Big[-\tfrac{1}{2} \partial_\lambda h_{\mu\nu} \partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda} \partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu} \partial_\nu h$$

$$+ \tfrac{1}{2} \partial_\lambda h \partial^\lambda h + \underbrace{c_1 m^2 h_{\mu\nu} h^{\mu\nu} + c_2 m^2 h^2}_{\text{MASS TERMS}}$$

$$+ 8\pi G h_{\mu\nu} T^{\mu\nu} \Big]$$

Fierz – Pauli : $c_1 = -c_2 \; (= -\tfrac{1}{2})$

Breaks gauge invariance, but avoids ghosts!

Sourceless case $(T^{\mu\nu} = 0)$

$$\frac{\delta S}{\delta h^{\mu\nu}} = 0 \quad \text{yields} \quad (EOM)_{\mu\nu} = 0$$

→ apply $\partial^\mu (EOM)_{\mu\nu} = 0 \longrightarrow \partial^\mu h_{\mu\nu} = \partial_\nu h$

→ into EOM → $\Box h_{\mu\nu} - \partial_\mu \partial_\nu h = m^2 (h_{\mu\nu} - h\,\eta_{\mu\nu})$

→ take trace → $h = 0$

So! $(\Box - m^2) h_{\mu\nu} = 0, \quad \partial^\mu h_{\mu\nu} = 0, \quad h = 0$

5 degrees of freedom ✓

Add a source:

$$(\Box - m^2) h_{\mu\nu} = -\kappa \left(T_{\mu\nu} - \frac{1}{3}\left(\eta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{m^2}\right) T \right)$$

strange factor! bad news!

Sol'n exterior to a spherical mass M_c

$$h_{00} = \frac{8}{3} \frac{GM_c}{r} e^{-mr}$$
$$h_{ij} = \frac{4}{3} \frac{GM_c}{r} e^{-mr} \delta_{ij}$$

so $\gamma = 1/2$

How to add a mass?
Refer to E&M for guidance

Massive E&M

$$S = \int d^4x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} m^2 A_\mu A^\mu + A_\mu J^\mu \right]$$

Proca mass - breaks gauge invariance

Stückelberg mechanism: introduce scalar to maintain G.I.

$$A_\mu \to A_\mu' = A_\mu + \partial_\mu \Lambda$$
$$\phi \to \phi' = \phi - \Lambda$$

$$S = \int d^4x \left[-\frac{1}{4} F^2 - \frac{1}{2} m^2 (A_\mu + \partial_\mu \phi)^2 + A_\mu J^\mu - \phi \partial_\mu J^\mu \right]$$

IBP

Next, rescale $\phi \to \frac{1}{m} \phi$
and assume source is conserved $\partial_\mu J^\mu = 0$

$$S = \int d^4x \left[-\frac{1}{4} F^2 - \frac{1}{2} m^2 A^2 - m A_\mu \partial^\mu \phi - \frac{1}{2} (\partial\phi)^2 + A_\mu J^\mu \right]$$

$$\left. \begin{aligned} A_\mu &\to A_\mu' = A_\mu + \partial_\mu \Lambda \\ \phi &\to \phi' = \phi - m\Lambda \end{aligned} \right\} \text{gauge invariance}$$

Use EOM, replace in action to eliminate ϕ

$$S = \int d^4x \left[-\frac{1}{4} F_{\mu\nu} \left(1 - \frac{m^2}{\Box}\right) F^{\mu\nu} + A_\mu J^\mu \right]$$

Result! $\longrightarrow$ $\left(1 - \frac{m^2}{\Box}\right) \partial_\mu F^{\mu\nu} = -J^\nu$ $\leftarrow$ E&M responds to sources through a HIGH-PASS filter

Stuckelberg scheme for gravity

Introduce compensating scalar, vector to maintain gauge invariance and carry new degrees of freedom, even in the $m \to 0$ limit

long story

"Galileon" is the new scalar degree of freedom

In a weak background

$$S_\pi = \int d^4x \left[\frac{1}{2} \alpha (\partial\pi)^2 + \frac{1}{2} \beta \, \Box\pi (\partial\pi)^2 + \pi T_m \right]$$

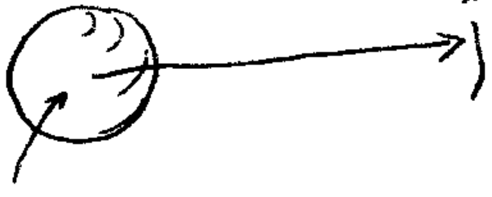
yields static field eq'n

$$\nabla^2 \pi + \lambda^2 \left((\nabla^2 \pi)^2 - (\nabla_i \nabla_j \pi \, \nabla^i \nabla^j \pi) \right) = 4\pi G \rho$$

$$\nabla^2 \phi = 4\pi G \rho$$

$$\vec{a} = -\vec{\nabla}(\phi - \pi)$$

$$r_* = \left(\frac{GM}{c^2} \lambda^2 \right)^{1/3}$$



inside mass M_c and out to r_*

$$|\nabla \pi| \ll |\nabla \phi|$$

Beyond r_*

$\pi \to \phi$

so $\vec{a}$ is suppressed

Modified Gravity, in general

Theories of gravity often display the following behavior

$$\vec{a} = -\vec{\nabla}\psi$$

$$\nabla^2 \phi = 4\pi G \rho$$

$\phi \neq \psi$, but predict a particular relationship

Adopt a phenomenological description

$$\vec{a} = -\vec{\nabla}\psi$$

$$\psi = (1 + \varpi(\tau, k))\,\phi \qquad \tau, k \text{ dependence}$$

$$-k^2 \phi = (1 + \mu(\tau, k))\,\underbrace{4\pi G \rho_m}_{\downarrow}$$

$$4\pi G a^2 \rho_m \Delta_m$$

$$\Delta_m = \delta_m + \frac{3\mathcal{H}}{k^2}\theta_m$$

in conformal-Newtonian gauge

expect ϖ, μ to vanish at early times,
grow to $O(1)$ at present
vanish at $k \gg \mathcal{H}$, $O(1)$ at $k \ll \mathcal{H}$

use cosmological observations to test gravity
or look for signs of modified gravity /
complicated dark energy.