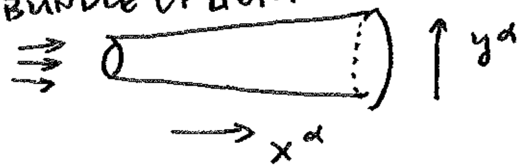
DARK ENERGY ALTERNATIVE

Speculate: "dark energy" is an effect of large scale structure.

Here is how.

BUNDLE OF LIGHT RAYS



$$\frac{dx^\alpha}{d\lambda} = k^\alpha$$

PHOTON $k \cdot k = 0$

DIVERGENCE OF RAYS DESCRIBED BY GEODESIC DEVIATION

$$\frac{d^2}{d\lambda^2} y^\alpha = -R_{\beta\gamma\delta}{}^{\alpha} k^\beta k^\delta y^\gamma$$

$$\downarrow$$

SOLN: $y^\alpha = A^\alpha{}_\beta \, (y_0)_\beta$

TRANSFORMATION MAT. ($A^\alpha{}_\beta$) — INITIAL BUNDLE WIDTH ($(y_0)_\beta$)

$$A = \begin{pmatrix} 1 - \kappa - \gamma_1 & -\gamma_2 \\ -\gamma_2 & 1 - \kappa + \gamma_1 \end{pmatrix}$$

in transverse plane

κ is convergence

γ_1, γ_2 are shear.

ANGULAR DIAMETER DISTANCE: $d_A = \sqrt{A} \,/\, \delta\Omega$

LUMINOSITY DISTANCE: $d_L = (1+z)^2 \sqrt{A} \,/\, \delta\Omega$

Natural to ask: can LSS (via Riemann tensor) produce the same d_A, d_L in $\Omega_m = 1$ (NO DARK ENERGY) UNIVERSE AS IN ΛCDM?

ANSWER: IF BKGD SPACETIME IS $\Omega_m = 1$ RW, THEN "NO"

LSS only weakly perturbs spacetime
CMB tells us so.

[See Ishibashi + Wald, CQG 23 235 (2006)
Hirata + Seljak, PRD 72 083501 (2005)]

Let's look a bit closer.

In RW spacetime $ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right)$

$$R_{\beta\gamma\delta}{}^{\alpha} k^\beta k^\delta = \omega^2 \left(\frac{k}{a^2} + H^2 - \frac{\ddot{a}}{a}\right) \delta^\alpha{}_\gamma$$

where $k \cdot u = -\omega$ observed photon freq.

geodesic deviation:

$$\frac{d^2}{d\lambda^2}\sqrt{A} = -\left(\frac{k}{a^2} + H^2 - \frac{\ddot{a}}{a}\right)\sqrt{A}$$

$$\frac{da}{d\lambda} = \omega_0 a_0 H$$

together yields $\left[\frac{d^2}{da^2} + \frac{H'}{H}\frac{d}{da} + \frac{1}{a^2}\left(\frac{k}{a^2H^2} - a\frac{H'}{H}\right)\right]\sqrt{A} = 0$

w/ B.C. $\sqrt{A}\,\big|_i = 0$ and $(\sqrt{A})'\,\big|_i = \delta\Omega / a_0 H_i$

General solution

$$\sqrt{A} = \frac{a}{a_0} \sin\left(\sqrt{K} \int \frac{da}{a^2 H} \right) S\Omega / \sqrt{K}$$

$$d_L = (1+z) \frac{1}{\sqrt{K}} \sin\left(\sqrt{K} \int_a^{a_0} \frac{da}{a^2 H} \right)$$

where $K = \Omega_K H_0^2$

In a realistic spacetime $ds^2 = -(1+2\Psi)dt^2 + a^2(1-2\phi)d\vec{x}^{\,2}$

$\Psi, \phi \ll 1$

- photon beams are de/magnified by lss — but weakly
- matter is not uniform, but in/around halos and galaxies
- std formula for d_L is valid on average, but the effects of lss skew the distribution.

CHOICE REFS: Frieman, astro-ph/9608068
Holz + Wald, PRD <u>58</u> 063501 (1998)
Y. Wang, ApJ <u>536</u> 531 (2001)

A repeated cry: "backreaction"
That large scale structure is fundamentally inhomogeneous, yet some sort of coarse graining makes the universe look like RW... with an effective dark energy.

Here's the gist. (See Buchert + Rasanen arXiv: 1112.5335)

$\theta = \nabla_m u^m$ volume expansion rate of flow lines

in Raychaudhuri eq'n

$$\dot{\theta} + \frac{1}{3}\theta^2 = -4\pi G\rho - 2\sigma^2 + 2\omega^2$$

$$\& \quad \frac{1}{3}\theta^2 = 8\pi G\rho - \frac{1}{2}\,{}^{(3)}R + \sigma^2 - \omega^2$$

$$\& \quad \dot{\rho} + \theta\rho = 0 \quad \text{(dust)}$$

shear and vorticity (scalars)^2

But $\rho, \sigma, \omega, {}^3R$ vary in space, so we need to use volume-averaged quantities. However, such averaging does not commute with time derivatives, due to cosmic expansion.

Denote volume average $\langle \ldots \rangle_D$

Define $Q_D \equiv \frac{2}{3}\left(\langle \theta^2 \rangle_D - \langle \theta \rangle_D^2 \right) - 2\langle \sigma^2 \rangle_D$

assume no vorticity: $\omega = 0$

This "backreaction variable" Q_D vanishes in RW.

Now fluid eq'ns are

$$3\left(\frac{\dot{a}_D}{a_D}\right)^2 = 8\pi G\langle\rho\rangle_D - \frac{1}{2}\langle {}^{(3)}R\rangle_D - \frac{1}{2}Q_D$$

$$3\,\frac{\ddot{a}_D}{a_D} = -4\pi G\langle\rho\rangle_D + Q_D$$

$$\partial_t\langle\rho\rangle_D + 3\,\frac{\dot{a}_D}{a_D}\langle\rho\rangle_D = 0$$

$$\frac{1}{a_D^2}\frac{\partial}{\partial t}\left(a_D^2\langle {}^{(3)}R\rangle_D\right) + \frac{1}{a_D^6}\frac{\partial}{\partial t}\left(a_D^6 Q_D\right) = 0$$

$Q_D \neq 0$ plays role of dark energy

$$\rho_{DE} = -\frac{Q_D}{16\pi G} - \frac{\langle {}^3R\rangle_D}{16\pi G}$$

$$p_{DE} = -\frac{Q_D}{16\pi G} + \frac{\langle {}^3R\rangle_D}{48\pi G}$$

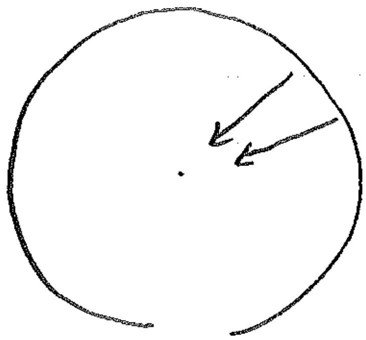
So $w = \frac{p}{\rho} \simeq -1$ means $\boxed{Q_D = -\frac{1}{3}\langle {}^{(3)}R\rangle_D}$

Can this happen? Does it?

Need a strong, non-perturbative departure from RW.

It is hard to see how this will work... and yet

ANTI-COPERNICAN sol'n : dispense w/ radial homogeneity



consider effect of radial profile of mass on bundles of light rays.

EXACT:

$$ds^2 = -dt^2 + \frac{R'(r,t)^2}{1+\beta(r)}dr^2 + R^2 d\Omega^2$$

where $R'(r,t) = \partial_r R(r,t)$

describing universe w/ pressureless matter.

This metric / solution contains three free functions

radial profile of mass density $\rho(t,r)$

radial profile of spatial curvature $\beta(r)$

radial profile of bang time $t_{BB}(r)$

MAKE NO MISTAKE: THIS MODEL SUFFERS FROM FINE-TUNING, TOO.

POSITION - we are at center of sphere

TIME - "acceleration-like effects" relevant now

See Kolb + Lamb 0911.3852

Mustapha et al, MNRAS 292 817 (1997)

Celerier, arXiv: 1108.1373

LTB cont'd

$' = \partial_r \qquad \dot{} = \partial_t$

EXPANSION: $H_{\parallel} = \frac{\dot{R}'}{R'}$ & $H_{\perp} = \frac{\dot{R}}{R}$

Einstein Eq'ns:

$$H_{\perp}^2 + 2H_{\perp}H_{\parallel} - \frac{\beta}{R^2} - \frac{\beta'}{RR'} = 8\pi G \rho(t, r)$$

$$6\frac{\ddot{R}}{R} + 2H_{\perp}^2 - 2\frac{\beta}{R^2} - 2H_{\perp}H_{\parallel} + \frac{\beta'}{RR'} = -8\pi G \rho(t, r)$$

No pressure: No radiation

This is a flaw of this simplified model. If we add ρ_r then is it homogeneous? Does its radial profile vary as ρ_M? Why? There's no big picture to explain all this and more.

MANIPULATE TO OBTAIN

$$\dot{R}(r,t) = \sqrt{\beta(r) + \alpha(r)/R(r,t)}$$

$$8\pi G \rho(r,t) = \alpha'(r) / R^2 R'$$

$$\dot{R}'(r,t) = \frac{\beta' + \alpha'/R - \alpha R'/R^2}{2\dot{R}}$$

These equations fit together as the familiar form

$$R(r,t) = \frac{\alpha(r)}{2\beta(r)}\Big(\cosh\eta(r) - 1\Big)$$

$$t - t_{BB}(r) = \frac{\alpha(r)}{2\beta^{3/2}(r)}\Big(\sinh\eta(r) - \eta(r)\Big)$$

For this model

$$\Theta = 2\frac{\dot{R}}{R} + \frac{\dot{R}'}{R'}$$

$$\sigma^2 = \frac{1}{3}\left(\frac{\dot{R}}{R} - \frac{\dot{R}'}{R'}\right)$$

$$^{(3)}R = -2\frac{\beta}{R^2} - 2\frac{\beta'}{RR'}$$

Indicate geodesic $\hat{t}(r)$ for radial photons

$$\frac{d\hat{t}}{dr} = - \frac{R'(r, \hat{t}(r))}{\sqrt{1+\beta(r)}}$$

$$\frac{dz}{dr} = (1+z) \frac{\dot{R}'(r, \hat{t}(r))}{\sqrt{1+\beta(r)}}$$

in which case the luminosity distance is

$$d_L(z) = (1+z)^2 R(r, \hat{t}(r)) \quad \checkmark$$

So our procedure to BUILD a model is as follows

- given $\alpha(r)$, $\beta(r)$, ~~[illegible]~~
- integrate $\dot{R}(r,t)$ to get $R(r,t)$
- integrate geodesic eq'ns $\hat{t}(r) \to d_L(z)$
- compare with observations

Popularity of this model largely based on ability to explain $d_L(z)$ for SNe. However it has not stood up to, or been subjected to, full scrutiny of CMB, LSS analysis.

[see Moss et al PRD 83 103515 (2011)
Bc + Stebbins, PRL 100, 191302 (2008)
Zhang + Stebbins, PRL 107, 041301 (2011)
Garcia-Bellido + Haugbolle, JCAP 0804, 003 (2008)]

EXAMPLE: (Garfinkle CQG 23 4811 (2006))

$$R(r,t) = a(r,t)\, r$$
$$\beta(r) = k(r)\, r^2$$
$$t_{BB}(r) = 0 \qquad \text{CONSTANT BANG TIME}$$

$$\dot{R} = \dot{a} r = \sqrt{\beta + \alpha/R} = \sqrt{k(r) r^2 + \frac{4}{9}\frac{r^2}{a}}$$

$$\dot{a} = \sqrt{k(r) + \frac{4}{9a}}$$

$$a' = \dot{a} k' I \qquad \dot{a}' = \frac{1}{2} k' \left(\frac{1}{\dot{a}} - \frac{4}{9}\frac{I}{a^2} \right)$$

$$\text{where } I = \frac{1}{2}\int_0^a \left(k + \frac{4}{9u} \right)^{-3/2} du \qquad \text{[evaluate in closed form]}$$

Specific model: try $k(r) = (1 + (cr)^2)^{-1}$
where c = constant.

So $R = ar$, $\dot{R} = \dot{a} r$, $R' = a'r + a$, $\dot{R}' = \dot{a}'r + \dot{a}$

geodesic:
$$\frac{dR}{dz} = \frac{R'}{\dot{R}'} \frac{(\sqrt{1 + kr^2} - \dot{R})}{(1+z)}$$

$$\frac{da}{dz} = \frac{a'\sqrt{1+kr^2} - \dot{a} R'}{\dot{R}'(1+z)}$$

Integrate outwards and back in time. Start at $z = R = r = 0$
when $a_0 = \frac{4}{9}\left(\frac{1}{\Omega} - 1\right)$.

Try $c = 2.5$, $\Omega = 0.2$ → Fit to SNe. Not too shabby!

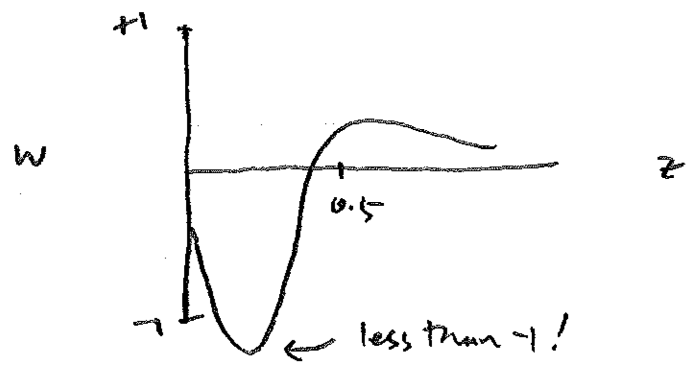
$$d_L = (1+z)^2 R$$

Effective w? Extract from $d_L(z)$

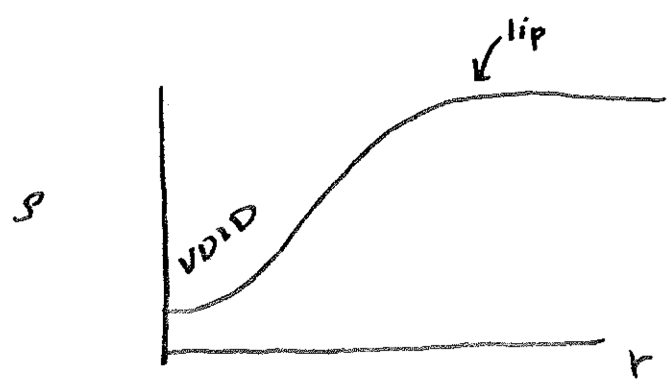
$$w_{eff} = -1 + \tfrac{1}{3}(1+z)\frac{d\ln f}{dz}$$

$$f = \left[\left(\frac{H}{H_0}\right)^2 - \Omega_m (1+z)^3\right] / (1 - \underset{\Omega}{\Omega_m})$$

$$\frac{H}{H_0} = \left[\frac{d}{dz}\left(\frac{H_0 d_L}{1+z}\right)\right]^{-1}$$



fast evolving dark energy



at constant time, this model describes a low density void!

Asymptotes to $\Omega = 1$ RW at large radius.

$H(r=0) \neq H(r \to \infty)$

A more methodical approach – require LTB exactly reproduce d_L and ρ of ΛCDM, along past light cone.

Step 0: "^" means on past light cone

1: $\hat{d}_L(z) = (1+z)\int_0^z \frac{dz'}{H_\Lambda(z')}$, $H_\Lambda(z) = H_0\sqrt{\Omega(1+z)^3 + (1-\Omega)}$

so that $\hat{R}(\hat{r}(z), \hat{t}(z)) = \hat{R} = \frac{1}{1+z}\int_0^z \frac{dz'}{H_\Lambda(z')}$ ✓

2: MATCH $\hat{\rho}(z)\, dV_{LTB} = \rho_{m,\Lambda}\, dV_\Lambda$

rhs $ds^2 = -dt^2 + a^2 d\vec{x}^2$

$$dV_\Lambda = a^3 r_\Lambda^2\, dr_\Lambda\, d\Omega$$

$$\rho_{m,\Lambda} = \frac{3}{8\pi G} H_0^2 \Omega (1+z)^3$$

since $dr_\Lambda = \frac{-dt}{a} = \frac{dz}{a_0 H_\Lambda(z)}$

$$r_\Lambda = \frac{1}{a_0}\int_0^z \frac{dz'}{H_\Lambda(z')}$$

$$\rho_{m,\Lambda}\, dV_\Lambda = \frac{3}{8\pi G} H_0^2 \Omega (1+z)^3 \left(\frac{a}{a_0}\right)^3 \frac{dz}{H_\Lambda(z)}\left[\int_0^z \frac{dz}{H_\Lambda(z)}\right]^2 d\Omega$$

$$= \frac{3\Omega}{8\pi G}\,\frac{H_0^2}{H_\Lambda}\left[\int_0^z \frac{dz'}{H_\Lambda(z')}\right]^2 dz\, d\Omega$$

Next, on the lhs

$$\hat{\rho}\, dV_{LTB} = \hat{\rho}(z) \frac{\hat{R}'}{\sqrt{1+\beta}}\, dr\, \hat{R}^2 d\Omega$$

use this fact to recompute

$$\frac{\partial \hat{R}}{\partial r} = \sqrt{1+\beta} \quad \text{on the light cone, whereby}$$

$$= \hat{\rho}(z)\, \hat{R}^2\, \cancel{d\Omega}\, dr = \frac{3\Omega}{8\pi G} \frac{H_0^2}{H_\Lambda} \left[\int_0^z \frac{dz'}{H_\Lambda(z')} \right]^2 dz\, \cancel{d\Omega}$$

$$\frac{dr}{dz} = \frac{3\Omega}{8\pi G} \frac{H_0^2}{\hat{\rho}(z)\, \hat{R}^2} \frac{1}{H_\Lambda} \left[\int_0^z \frac{dz'}{H_\Lambda(z')} \right]^2$$

3: use $\hat{\rho}(z) = \frac{3}{8\pi G} \Omega H_0^2 (1+z)^3$ above, whereby

$$\frac{dz}{dr} = (1+z)\, H_\Lambda(z) \quad \checkmark$$

4: $\alpha' = \frac{d\alpha}{dr} = 8\pi G \rho R^2 R'$

on light cone $\frac{d\alpha}{dr} = 8\pi G \hat{\rho}\, \hat{R}^2 \hat{R}' = 8\pi G \hat{\rho}\, \hat{R}^2 \sqrt{1+\hat{\beta}}$

[$\hat{\beta} = \beta$ where to get?]

Use $\frac{d\hat{R}}{dr} = \frac{d\hat{R}}{dz}\,\frac{dz}{dr}$

$$= \frac{\partial \hat{R}}{\partial r} + \frac{\partial \hat{R}}{\partial \hat{t}}\,\frac{d\hat{t}}{dr} = \sqrt{1+\beta} + \hat{R}\,(-1)$$

since $\frac{d\hat{t}}{dr} = -\frac{\hat{R}'}{\sqrt{1+\beta}}$ originally

$= -1$ upon revising $\hat{R}'$

$$= \sqrt{1+\beta} - \hat{R}$$

but $\hat{R} = \sqrt{\beta + \frac{\alpha}{\hat{R}}}$

combine to yield

$$\sqrt{1+\beta} = \frac{1}{2}\left[\left(\frac{d\hat{R}}{dr}\right)^2 + 1 - \frac{\alpha}{\hat{R}} \right] \Big/ \left(\frac{d\hat{R}}{dr}\right)$$

At last $$\frac{d\alpha}{dr} = 4\pi G\hat{\rho}\,\hat{R}^2 \left[\left(\frac{d\hat{R}}{dr}\right)^2 + 1 - \frac{\alpha}{\hat{R}} \right] \Big/ \left(\frac{d\hat{R}}{dr}\right)$$

6: Since we already know $\frac{d\alpha}{dr} = 8\pi G\hat{\rho}\,\hat{R}^2\sqrt{1+\beta}$

$$\beta(r) = \left(\frac{d\alpha}{dr} \Big/ 8\pi G\hat{\rho}\,\hat{R}^2\right)^2 - 1$$

Gives everything I need to build Λ-like model

Sol'n is NOT a VOID! SURPRISE!

See Kolb + Lamb arXiv 0911.3852