1. (Eddington luminosity.) Rapidly rotating neutron stars arise from matter from a companion star falling on to the neutron star. Light emitted by a star exerts pressure on the matter around it. The radiation pressure on infalling electrons is proportional to the star's luminosity (energy emitted per unit time). The electrons are blown away from the star if the force on an electron due to radiation pressure is greater than the gravitational force on an electron+proton pair.

   (a) Why?

   (b) Equate the two forces to find the Eddington limit on L.

2. Assume that the luminosity of a neutron star that accretes matter at rate $\dot{M}$ is $\frac{1}{10}$(kinetic energy of infalling matter); and suppose the matter hits the star at about the Kepler velocity $\Omega_K = \sqrt{M/R^3}$ of a particle in circular at the equator orbit.

   (a) Use the Eddington limit to find the maximum rate $\dot{M}$ at which matter can accrete onto a neutron star of mass $1.4M_\odot$ and radius 12 km.

   (b) If the accretion lasts for 10^7 yr, how much mass is accreted?

   (c) Using a rough estimate of the neutron-star's moment of intertia and the freqency for a particle in circular orbit about a nonrotating relativistic star, decide whether the star accretes enough angular momentum to spin it up to Ω_K.

3. (Symplectic product) Consider a particle with a Hamiltonian $H(p,q)$. We'll look at a conserved product of two different perturbations of a solution to Hamilton's equations.

   Let $q(\lambda,t), p(\lambda,t)$ be a family of solutions to Hamilton's equations, and write $\delta q := \left.\frac{d}{d\lambda}q(\lambda,t)\right|_{\lambda=0}$ A solution $\delta p, \delta q$ to the perturbation equations satisfies

$$\delta\dot{q} = \delta\partial_p H = \partial_p \delta H, \qquad \delta\dot{p} = -\delta\partial_q H = -\partial_q \delta H,$$

   where $\delta H = \partial_p H \delta p + \partial_q H \delta q$. Show that if $\hat{\delta}p, \hat{\delta}q$ is a second solution to the perturbed equations, the product

$$\Omega(\hat{\delta}p, \hat{\delta}q; \delta p, \delta q) := \hat{\delta}p\delta q - \delta p\hat{\delta}q \qquad (1)$$

   is conserved: $\frac{d}{dt}\Omega(\hat{\delta}p, \hat{\delta}q; \delta p, \delta q) = 0$.

   Note that for an n-particle system, the same equations are valid if p and q are replaced everywhere by p_i and q^i and repeated indices give implied sums.