Black holes and phase transitions in higher curvature gravity

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Based on joint work with:
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Higher curvature corrections and quantum gravity

Classical gravity seems well-described by the Einstein-Hilbert action.

Quantum corrections generically involve higher curvature corrections:

- Wilsonian approaches.
- $\alpha'$ corrections in string theory.
- Higher dimensional scenarios.
- Relevant when studying generic strongly coupled CFTs under the light of the gauge/gravity correspondence (e.g., 4d CFTs with $a \neq c$).

They are typically argued to be plagued of ghosts.

Lovelock gravities are the most general second order theories free of ghosts when expanding about flat space. 

Lovelock (1971)
Lovelock theory

The action is compactly expressed in terms of differential forms

\[ I = \sum_{k=0}^{K} \frac{c_k}{d - 2k} \left( \int_{\mathcal{M}} I_k - \int_{\partial \mathcal{M}} Q_k \right) \]

where \( K \leq \left[ \frac{d-1}{2} \right] \) and \( c_k \) is a set of couplings with length dimensions \( L^{2(k-1)} \).
Lovelock theory

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where $K \leq \left\lfloor \frac{d-1}{2} \right\rfloor$ and $c_k$ is a set of couplings with length dimensions $L^{2(k-1)}$.

$I_k$ is the extension of the Euler characteristic in $2k$ dimensions

$$I_k = \epsilon_{a_1 \ldots a_d} R^{a_1 a_2} \wedge \cdots \wedge R^{a_{2k-1} a_{2k}} e^{a_{2k+1}} \wedge \cdots \wedge e^{a_d}$$

with $R^{ab} = d\omega^{ab} + \omega^c \wedge \omega^{cb} = \frac{1}{2} R^{ab}_{\mu\nu} \, dx^\mu \wedge dx^\nu$. 
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- \( Q_k \) comes from the GB theorem in manifolds with boundaries \( \text{Myers (1987)} \)

\[ Q_k = k \int_0^1 d\xi \, \epsilon_{a_1 \ldots a_d} \theta^{a_1 a_2} \wedge \tilde{\omega}^{a_3 a_4}(\xi) \wedge \cdots \wedge \tilde{\omega}^{a_{2k-1} a_{2k}}(\xi) \wedge e^{a_{2k+1}} \wedge \cdots \wedge e^{a_d} \]

where \( \theta^{ab} = n^a K^b - n^b K^a \) and \( \tilde{\omega}^{ab}(\xi) \equiv R^{ab} + (\xi^2 - 1) \theta^a_e \wedge \theta^b_e \).
Lovelock theory: lowest order examples

The first two contributions (most general up to $d = 4$) correspond to:

- **The cosmological term:** we set $2\Lambda = -\frac{(d - 1)(d - 2)}{L^2}$ with $c_0 = \frac{1}{L^2}$

- **The EH action (with GH term):** we set $16\pi(d - 3)!G_N = 1$ with $c_1 = 1$
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  \[ c_1 = 1 \]

For \(d \geq 5\), we have the Lanczos-Gauss-Bonnet (LGB) term \((c_2 = \lambda L^2)\),

\[
\mathcal{I}_2 \sim d^d x \sqrt{-g} \left( R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} \right) \\
\mathcal{Q}_2 \sim \sqrt{-h}(KR + \ldots)
\]
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$$I_2 \sim d^d x \sqrt{-g} \left(R^2 - 4R_{\mu \nu} R^{\mu \nu} + R_{\mu \nu \rho \sigma} R^{\mu \nu \rho \sigma}\right)$$

while for $d \geq 7$, the cubic Lovelock Lagrangian ($c_3 = \mu L^4$),

$$I_3 \sim d^d x \sqrt{-g} \left(R^3 + 3R R^{\mu \nu \alpha \beta} R_{\alpha \beta \mu \nu} - 12R R^{\mu \nu} R_{\mu \nu} + 24R^{\mu \nu \alpha \beta} R_{\alpha \mu} R_{\beta \nu} + 16R^{\mu \nu} R_{\nu \alpha} R_{\mu}^{\alpha} + 24R^{\mu \nu \alpha \beta} R_{\alpha \beta \nu \rho} R_{\mu \rho} + 8R^{\mu \nu} R^{\alpha \beta} R_{\nu \sigma} R_{\mu \beta}^{\rho \sigma} + 2R_{\alpha \beta \rho \sigma} R^{\mu \nu \alpha \beta} R_{\mu \nu}^{\rho \sigma}\right)$$
Varying the action with respect to the connection,

\[ \epsilon_{ab_3a_d} \sum_{k=1}^{K} \frac{k c_k}{d-2k} \left( R^{a_3a_4} \wedge \cdots \wedge R^{a_{2k-1}a_{2k}} \wedge e^{a_{2k+1}} \wedge \cdots \wedge e^{a_{d-1}} \right) \wedge T^{a_d} = 0 \]

we can safely impose \( T^a = 0 \) as in standard Einstein’s gravity.
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we can safely impose \( T^a = 0 \) as in standard Einstein's gravity.

The equations of motion, when varying with respect to the vierbein,

\[ \epsilon_{aa_1 \cdots a_{d-1}} \mathcal{F}^{a_1 a_2}_{(1)} \wedge \cdots \wedge \mathcal{F}^{a_2 K-1 a_{2K}}_{(K)} \wedge e^{a_{2K+1}} \wedge \cdots \wedge e^{a_{d-1}} = 0 \]

admit \( K \) constant curvature vacua,

\[ \mathcal{F}^{ab}_{(i)} := R^{ab} - \Lambda \wedge_i e^a \wedge e^b = 0 \]
AdS/dS/flat vacua

Varying the action with respect to the connection,

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\epsilon_{a_1a_2a_3\cdots a_d} \sum_{k=1}^{K} \frac{k}{d-2k} c_k \left( R^{a_3a_4} \wedge \cdots \wedge R^{a_{2k-1}a_{2k}} \wedge e^{a_{2k+1}} \wedge \cdots \wedge e^{a_{d-1}} \right) \wedge T^{a_d} = 0
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\epsilon_{a_1a_2\cdots a_{d-1}} F^{a_1a_2} \wedge \cdots \wedge F^{a_{2K-1}a_{2K}} \wedge e^{a_{2K+1}} \wedge \cdots \wedge e^{a_{d-1}} = 0
$$

admit $K$ constant curvature vacua,

$$
F^{ab}_{(i)} := R^{ab} - \Lambda_i e^a \wedge e^b = 0
$$

The cosmological constants being the roots of the polynomial $\Upsilon[\Lambda]$: 

$$
\Upsilon[\Lambda] := \sum_{k=0}^{K} c_k \Lambda^k = c_K \prod_{i=1}^{K} (\Lambda - \Lambda_i) = 0
$$

Degeneracies arise when 

$$
\Delta := \prod_{i<j} (\Lambda_i - \Lambda_j)^2 = 0
$$
Lovelock black holes

The black hole solution can be obtained via the ansatz

\[ ds^2 = -f(r) \, dt^2 + \frac{dr^2}{f(r)} + \frac{r^2}{L^2} \, d\Sigma_{\sigma,d-2}^2 \]

where \( d\Sigma_{\sigma,d-2} \) is the metric of a maximally symmetric space.

The equations of motion can be nicely rewritten as

\[
\frac{d}{d \log r} \left[ \frac{d}{d \log r} + (d - 1) \right] \left( \sum_{k=0}^{K} c_k \, g^k \right) = 0
\]

where \( g(r) = \frac{\sigma - f(r)}{r^2} \), and easily solved as

\[
\Upsilon[g] = \sum_{k=0}^{K} c_k \, g^k = V_{d-2} \frac{M}{r^{d-1}}
\]

The black hole solution is implicitly given by this polynomial equation.
Lovelock black holes

Each branch, $g_i(r)$, corresponds to a monotonous part of the polynomial,

$$\gamma[g] = \sum_{k=0}^{K} c_k g^k = V_{d-2} M \frac{M}{r^{d-1}}$$

The variation of $r$ translates the curve (y-intercept) rigidly, upwards,

This leads to $K$ branches, $g_i(r)$, associated with each $\Lambda_i$: $g_i(r \to \infty) = \Lambda_i$
The existence of a black hole horizon requires $g_+ = 0$ for planar black holes (recall $g(r) = \frac{\sigma - f(r)}{r^2}$), and

\[ \Upsilon[g_+] = V_{d-2} \frac{M}{r_+^{d-1}} = V_{d-2} M |g_+|^{(d-1)/2} \]

since $g_+ = \frac{\sigma}{r^2}$

- **Planar** case, only the EH-branch has an event horizon.
- **Non-planar** case, $\sigma = \pm 1$, several branches can have the same mass or temperature.
Features of Lovelock theory

Some of the **new features** seemingly **unnatural** or **pathological**

- **Additional couplings**
  - **new scales**

- **Naked singularities**
  - **mass gap**

- **Branches**
  - **multivaluedness**
Some of the new features seemingly unnatural or pathological

- Additional couplings
- Naked singularities
- Branches

\[ \overset{\text{AdS/CFT}}{\underset{\text{instabilities}}{\longrightarrow}} \]

- Constraints
- Cosmic censor
- Phase transitions

\[ \overset{\text{compact domain}}{\underset{\text{new phases?}}{\longrightarrow}} \]

- EH unambiguous
Bold statement:

Quantum gravity in $\text{AdS}_d$ space is equal to a $\text{CFT}_{d-1}$ living at the boundary

The generating function reads

$$\left\langle \exp \left( \int d\mathbf{x} \eta^{ab}(\mathbf{x}) T_{ab}(\mathbf{x}) \right) \right\rangle_{\text{SYM}} = Z_{\text{QG}}[g_{\mu\nu}] \approx \exp(-\mathcal{I}_G[g_{\mu\nu}])$$

where $g_{\mu\nu} = g_{\mu\nu}(z, \mathbf{x})$ such that $g_{ab}(0, \mathbf{x}) = \eta_{ab}(\mathbf{x})$. 
Holography — the AdS/CFT correspondence

**Bold statement:**

Quantum gravity in AdS$_d$ space is equal to a CFT$_{d-1}$ living at the boundary

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$$

where $g_{\mu\nu} = g_{\mu\nu}(z, \mathbf{x})$ such that $g_{ab}(0, \mathbf{x}) = \eta_{ab}(\mathbf{x})$.

5d EH gravity describes 4d CFTs with $a = c$.

Higher curvature corrections are relevant when studying "more general" strongly coupled CFTs.
When $K = 2$:

$$\gamma[\Lambda] = \frac{1}{L^2} + \Lambda + \lambda L^2 \Lambda^2 = 0$$

$$\Lambda_{\pm} = -\frac{1 \pm \sqrt{1 - 4\lambda}}{2\lambda L^2}$$
Warming up: the LGB case

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Each black hole solution *climbs up* a monotonous part of the polynomial.

In the planar case ($\sigma = 0$), just the EH branch ($\Lambda_-)$ has a horizon ($g = 0$).

The EH-branch has $\gamma'[\Lambda_-] > 0$, a positive effective Newton constant.

Every branch *ends up* at a singularity: either $r = 0$ or $\gamma'[g] = 0$. 
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Black holes and phase transitions

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Graviton potentials: unitarity & causality

EOM for perturbations are two derivative.

VACUUM: Coefficient of the kinetic term:

Unitarity

\[ \gamma'[\Lambda] > 0 \]

Boulware, Deser (1985)
Graviton potentials: unitarity & causality

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**VACUUM:** Coefficient of the kinetic term:

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\[ \Upsilon'[\Lambda] > 0 \]

**BLACK HOLE:** at high momentum, EOM in Schrödinger form:

\[ -\hbar^2 \frac{\partial}{\partial y} \psi_i + c_i^2(y) \psi_i = \frac{\omega^2}{q^2} \psi_i \quad , \quad \hbar \equiv \frac{1}{q} \to 0 \]

for \( c_i \) speed of gravitons on radial slices.

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for \( c_i \) speed of gravitons on radial slices.

**Causality**

\[ c_i^2 < 1 \]

Brigante, Liu, Myers, Shenker, Yaida (2008)
Causality violation, $c_i^2 > 1$

The potentials close to the boundary of AdS

\[ c_2^2 \approx 1 + \frac{1}{L^2 \Lambda} \left( \frac{r_+}{r} \right)^{d-1} \left[ 1 + \frac{2(d - 1)}{(d - 3)(d - 4)} \frac{\Lambda \gamma''[\Lambda]}{\gamma'[\Lambda]} \right] \]

\[ c_1^2 \approx 1 + \frac{1}{L^2 \Lambda} \left( \frac{r_+}{r} \right)^{d-1} \left[ 1 - \frac{d - 1}{d - 3} \frac{\Lambda \gamma''[\Lambda]}{\gamma'[\Lambda]} \right] \]

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\]

Causality imposes

\[
- \frac{d-2}{d-4} \leq - \frac{2(d-1)(d-2)}{(d-3)(d-4)} \frac{\Lambda \gamma''[\Lambda]}{\gamma'[\Lambda]} \leq d-2
\]

Causality violations may also occur in the interior of geometry.
Consider a CFT\(_{d-1}\). The leading singularity of the 2-point function is fully characterized by the central charge \(C_T\)

\[
\langle T_{ab}(x) \ T_{cd}(0) \rangle \sim \frac{C_T}{2 \ x^{2(d-1)}} (\ldots)
\]

\[
C_T = \frac{d}{d-2} \frac{\Gamma[d]}{\pi^{\frac{d-1}{2}}} \frac{\Gamma[\frac{d-1}{2}]}{(-\Lambda)^{d/2}}
\]

Osborn, Petkou (1994)

A good parametrization of 3-point functions

Hofman, Maldacena (2008)

This is the expectation value for the total energy flux per unit angle measured in a state created by a local gauge invariant operator

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Osborn, Petkou (1994)

\[ C_T = \frac{d}{d-2} \frac{\Gamma[d]}{\pi^{d-1/2}} \Gamma \left[ \frac{d-1}{2} \right] \frac{\Upsilon'[\Lambda]}{(-\Lambda)^{d/2}} \]

Camanho, Edelstein, Paulos (2010)

The dual theory of a given AdS-branch is unitary,

\[ C_T > 0 \quad \iff \quad \Upsilon'[\Lambda] > 0 \]
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C_T = \frac{d}{d-2} \frac{\Gamma[d]}{\pi^{\frac{d-1}{2}}} \frac{\Gamma \left[ \frac{d-1}{2} \right]}{\left( -\Lambda \right)^{\frac{d}{2}}} \ \Upsilon'[\Lambda]
\]

The dual theory of a given AdS-branch is unitary,

\[
C_T > 0 \quad \iff \quad \Upsilon'[\Lambda] > 0
\]

A good parametrization of 3-point functions

\[
\langle \mathcal{E}(n) \rangle_O = \frac{\langle 0 | O^+ \mathcal{E}(n) O | 0 \rangle}{\langle 0 | O^+ O | 0 \rangle}, \quad \mathcal{E}(n) = \lim_{r \to \infty} r^{d-2} \int_{-\infty}^{\infty} dt \ n^i \ T^0_i(t, r \ n)
\]
Consider a CFT$_{d-1}$. The leading singularity of the 2-point function is fully characterized by the central charge $C_T$

$$\langle T_{ab}(x) T_{cd}(0) \rangle \sim \frac{C_T}{2x^{2(d-1)}} \ldots$$

$$C_T = \frac{d}{d-2} \frac{\Gamma[d]}{\pi^{d/2}} \frac{\Gamma \left[ \frac{d-1}{2} \right]}{\Gamma \left[ \frac{d-1}{2} \right]} \frac{\gamma'[\Lambda]}{(-\Lambda)^{d/2}}$$

The dual theory of a given AdS-branch is unitary,

$$C_T > 0 \iff \gamma'[\Lambda] > 0$$

A good parametrization of 3-point functions

$$\langle \mathcal{E}(n) \rangle_{\mathcal{O}} = \frac{\langle 0 | \mathcal{O}^\dagger \mathcal{E}(n) \mathcal{O} | 0 \rangle}{\langle 0 | \mathcal{O}^\dagger \mathcal{O} | 0 \rangle}, \quad \mathcal{E}(n) = \lim_{r \to \infty} \frac{d-2}{r^{d-2}} \int_{-\infty}^{\infty} dt \ n^i T^0_{i}(t, r n)$$

This is the expectation value for the total energy flux per unit angle measured in a state created by a local gauge invariant operator $\mathcal{O}$
For $O = \epsilon_{ij} T_{ij}$ determined by 2 parameters ($t_2$ and $t_4$) in any CFT.

$$\langle \mathcal{E}(n) \rangle_{\epsilon_{ij} T_{ij}} = \frac{E}{\omega_{d-3}} \left[ 1 + t_2 \left( \frac{n_i \epsilon^*_{ij} n_j}{\epsilon^*_{ij} \epsilon_{ij}} - \frac{1}{d-2} \right) + t_4 \left( \frac{|\epsilon_{ij} n_i n_j|^2}{\epsilon^*_{ij} \epsilon_{ij}} - \frac{2}{d(d-2)} \right) \right]$$

since $\epsilon_{ij}$ is a symmetric and traceless polarization tensor.
For \( \mathcal{O} = \epsilon_{ij} T_{ij} \) determined by 2 parameters \((t_2 \text{ and } t_4)\) in any CFT.

\[
\langle \mathcal{E}(n) \rangle \epsilon_{ij} T_{ij} = \frac{E}{\omega_{d-3}} \left[ 1 + t_2 \left( \frac{n_i \epsilon_{ij}^* \epsilon_{lj} n_j}{\epsilon_{ij}^* \epsilon_{ij}} - \frac{1}{d-2} \right) + t_4 \left( \frac{|\epsilon_{ij} n_i n_j|^2}{\epsilon_{ij}^* \epsilon_{ij}} - \frac{2}{d(d-2)} \right) \right]
\]

since \( \epsilon_{ij} \) is a symmetric and traceless polarization tensor.

Demanding positivity of the different components gives

\[
1 - \frac{1}{d-2} t_2 - \frac{2}{d(d-2)} t_4 \geq 0, \\
1 - \frac{1}{d-2} t_2 - \frac{2}{d(d-2)} t_4 + \frac{1}{2} t_2 \geq 0, \\
1 - \frac{1}{d-2} t_2 - \frac{2}{d(d-2)} t_4 + \frac{d-3}{d-2} (t_2 + t_4) \geq 0.
\]
For $\mathcal{O} = \epsilon_{ij} T_{ij}$ determined by 2 parameters ($t_2$ and $t_4$) in any CFT.

$$\langle \mathcal{E}(n) \rangle_{\epsilon_{ij} T_{ij}} = \frac{E}{\omega_{d-3}} \left[ 1 + t_2 \left( \frac{n_i \epsilon^*_{ij} \epsilon_{lj} n_j}{\epsilon^*_{ij} \epsilon_{ij}} - \frac{1}{d-2} \right) + t_4 \left( \frac{|\epsilon_{ij} n_i n_j|^2}{\epsilon^*_{ij} \epsilon_{ij}} - \frac{2}{d(d-2)} \right) \right]$$

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$$1 - \frac{1}{d-2} t_2 - \frac{2}{d(d-2)} t_4 + \frac{1}{2} t_2 \geq 0,$$
$$1 - \frac{1}{d-2} t_2 - \frac{2}{d(d-2)} t_4 + \frac{d-3}{d-2} (t_2 + t_4) \geq 0.$$

$t_2$ and $t_4$ may be calculated holographically,

$$t_2 = -\frac{2(d-1)(d-2)}{(d-3)(d-4)} \frac{\Lambda^\prime\prime}{\Lambda^{\prime\prime}[\Lambda]} ; \quad t_4 = 0.$$
For $\mathcal{O} = \epsilon_{ij} T_{ij}$ determined by 2 parameters ($t_2$ and $t_4$) in any CFT.

$$\langle \mathcal{E}(n) \rangle \epsilon_{ij} T_{ij} = \frac{E}{\omega_{d-3}} \left[ 1 + t_2 \left( \frac{n_i \epsilon_{ij}^* \epsilon_{lj} n_j}{\epsilon_{ij} \epsilon_{ij}} - \frac{1}{d-2} \right) + t_4 \left( \frac{|\epsilon_{ij} n_i n_j|^2}{\epsilon_{ij}^* \epsilon_{ij}} - \frac{2}{d(d-2)} \right) \right]$$

since $\epsilon_{ij}$ is a symmetric and traceless polarization tensor.

Demanding positivity of the different components gives

$$-\frac{d-2}{d-4} \leq t_2 \leq d - 2$$

$t_2$ and $t_4$ may be calculated holographically,

$$t_2 = -\frac{2(d-1)(d-2)}{(d-3)(d-4)} \frac{\Lambda''''[\Lambda]}{\gamma'[\Lambda]} ; \quad t_4 = 0$$
Restrictions in the Lovelock couplings

\[ \Delta = 0 \]

\[ \mu = \lambda^2 \]

Excluded region
Restrictions in the Lovelock couplings

\[
\begin{align*}
\Delta &= 0 \\
\mu &= \lambda^2 \\
D &= 0 \\
D &= 0
\end{align*}
\]

Excluded region

Causal region

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Restrictions in the Lovelock couplings
Restrictions in the Lovelock couplings

\[ D = 0 \]

Excluded region

Causal region

Stable region

Causal and stable region
Lovelock terms lead to a violation of the KSS bound

\[ \frac{\eta}{s} = \frac{1}{4\pi} \left( 1 - 2 \frac{d - 1}{d - 3} \lambda \right) \frac{\hbar}{k_B} \]
Lovelock terms lead to a violation of the KSS bound

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the ratio being reduced for \( \lambda^{\text{max}} > 0 \)
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the ratio being reduced for \( \lambda^{\max} > 0 \)

- They do contribute to the lower bound of \( \eta/s \)!
The existence of a black hole horizon requires

\[ \gamma[g^+] = \frac{\kappa}{r^d} = \kappa \left( \sqrt{\frac{g^+}{\sigma}} \right)^{d-1} \]

since

\[ g^+ = \frac{\sigma}{r^2} \]
Lovelock black holes: the cosmic censor

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since

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The singularity becomes naked (mass gap)

- \( \lambda > 0, \) for \( \kappa \leq \lambda \) in 5D.
- \( \lambda < 0, \) for \( \kappa \leq \kappa^{*} \) in arbitrary dimension.
The singular solutions are in all cases unstable. Stability imposes a more constraining mass gap.

Naked singularities cannot be reached as the final state of the evolution of generic initial conditions, e.g. collapse.
Instabilities and the cosmic censor

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Naked singularities cannot be reached as the final state of the evolution of generic initial conditions, e.g. collapse.

Figure: Collapse of a shell of radiation (thick line) to a black hole (left) and a naked singularity (right). In the latter case, radiation has no obstacle to escape across (or bouncing on) the singularity.
A new type of (branch) phase transitions

For this talk, we consider $\lambda > 0$ in LGB theory.

In the canonical ensemble, we study processes where the system undergoes a phase transition between thermal AdS$_+ (\Lambda_+, \beta_+)$ and a given BH$_- (\Lambda_, \beta_-)$.

How to deal with solutions that differ in the asymptotics?
A new type of (branch) phase transitions

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In the canonical ensemble, we study processes where the system undergoes a phase transition between thermal AdS$_+$ ($\Lambda_+, \beta_+$) and a given BH$_-$ ($\Lambda_-, \beta_-$).

How to deal with solutions that differ in the asymptotics?

Likely mechanism: thermalon mediated transition.

Camanho, Edelstein, Giribet, Gomberoff (2012)

The two phases and the thermalon

**Figure:** Euclidean sections for (a) empty AdS and (b) bubble hosting a black hole.

The thermalon

**Inner region:** black hole with mass $M_-$, corresponding to the EH branch ($\Lambda_-$).

**Outer region:** asymptotes AdS space with $\Lambda_+$ (and total mass $M_+$).
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**Inner region:** black hole with mass $M_-$, corresponding to the EH branch ($\Lambda_-$).

**Outer region:** asymptotes AdS space with $\Lambda_+$ (and total mass $M_+$).

- **Inner periodicity:** demanding regularity at the black hole horizon.
- **Outer periodicity:** fully determined by continuity.

there is a unique free parameter.
The thermalon: periodicity, temperature and bubble dynamics

For bubble configurations, it is convenient to break the action into bulk and surface pieces, \( \mathcal{M} = \mathcal{M}_- \cup \Sigma \cup \mathcal{M}_+ \)

\[
I = \int_{\mathcal{M}_-} \mathcal{L}^- - \int_{\Sigma} Q^- + \int_{\mathcal{M}_+} \mathcal{L}^+ + \int_{\Sigma} Q^+ - \int_{\partial \mathcal{M}} Q^+
\]

\( Varying \) with respect to the induced vierbein at the bubble, \( a(\tau) \), gives the junction conditions (Israel conditions of GR).

\( \tilde{Q}_{ab} = \delta (Q_+ - Q^-) \delta h_{ab} \mid_{\Sigma} = 0 \iff \dot{a} = \dot{a}(a; \mathcal{M}_\pm) \)

We may fix \( \mathcal{M}_\pm \) so that an equilibrium position exists at \( a = a^\star > r_H \). Each of the two (Euclidean) bulk regions is characterized by \( f_\pm \).

The periodicity of the inner solution is fixed by regularity of the black hole horizon, that of the outer solution gets fully determined by gluing conditions,

\[
\sqrt{f} - (a) \beta = \sqrt{f} + (a) \beta + 2
\]

There is a unique free parameter, say, \( \beta^+ \).
The thermalon: periodicity, temperature and bubble dynamics

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**The thermalon: periodicity, temperature and bubble dynamics**

For *bubble configurations*, it is convenient to break the action into bulk and surface pieces, \( \mathcal{M} = \mathcal{M}_- \cup \Sigma \cup \mathcal{M}_+ \)

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Varying with respect to the induced vierbein at the bubble, \( a(\tau) \), gives the junction conditions (Israel conditions of GR).

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\tilde{Q}_{ab} = \frac{\delta(Q^+ - Q^-)}{\delta h^{ab}} \bigg|_{\Sigma} = 0 \iff \dot{a} = \dot{a}(a; M_\pm)
\]

We may fix \( M_\pm \) so that an equilibrium position exists at \( a = a_\star > r_H \). Each of the two (Euclidean) bulk regions is characterized by \( f_\pm \).

The periodicity of the inner solution is fixed by regularity of the black hole horizon, that of the outer solution gets fully determined by gluing conditions,

\[
\sqrt{f_-(a)} \, \beta_- = \sqrt{f_+(a)} \, \beta_+
\]

There is a unique free parameter, say, \( \beta_+ \).
The phase transition

The **canonical ensemble** at temperature $1/\beta$ is defined by the Euclidean path integral over all metrics which asymptote AdS identified with period $\beta$,

$$Z = \int \mathcal{D}g \, e^{-\hat{I}[g]} \quad \hat{I} = \hat{I}_{\text{bubble}} + \hat{I}_{\text{black hole}}$$

Dominant contributions come from the **saddle points**, $\hat{I}_{\text{cl}} \sim -\log Z = \beta F$

The Euclidean action diverges $\Rightarrow$ **background subtraction**; we obtain

$$\hat{I}_{\text{black hole}} = \beta - M - S$$
The phase transition

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The Euclidean action diverges $\Rightarrow$ background subtraction; we obtain

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Remarkably enough, once the junction conditions are imposed,

$$\hat{I}_{\text{bubble}} = \beta_{+} M_{+} - \beta_{-} M_{-} \quad \Rightarrow \quad \hat{I} = \beta_{+} M_{+} - S$$

which is exactly needed to preserve the thermodynamic interpretation; also

$$\beta_{+} dM_{+} = \beta_{-} dM_{-} = dS$$

the first law of thermodynamics holds for both configurations.
Global thermodynamic stability: sign of the free energy

There is a critical temperature, $T_c(\lambda)$, above which $F$ becomes negative triggering the phase transition.
Global thermodynamic stability: sign of the free energy

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**Figure:** [LEFT] Free energy versus temperature in $d = 5$ for $\lambda = 0.04, 0.06, \ldots, 1/4$ (from right to left). [RIGHT] Bubble potential for $\lambda = 0.1$ and $d = 5, 6, 7, 10$.

The bubble may expand reaching the boundary at finite proper time changing asymptotics and charges: $\Lambda_+ \to \Lambda_-$ and $(M_+, T_+) \to (M_-, T_-)$
On the consistency of higher curvature gravities

- Additional couplings
  - new scales

- Naked singularities
  - mass gap

- Branches
  - multivaluedness

- AdS/CFT
  - instabilities

- Constraints
  - compact domain

- Instabilities
  - cosmic censor
  - new phases?

- Phase transitions
  - EH unambiguous
Lovelock theory is a useful playground for AdS/CFT.

A novel mechanism for phase transitions in higher curvature gravity.

Are these different phases of the dual field theory?

It deserves further exploration. Thank you for your attention!