An extension of the inflationary scenario to the Planck regime

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Will summarize the work of many researchers; especially:

Agullo, Barrau, Bojowald, Cailleatau, Campiglia, Corichi, Grain, Kaminski, Lewandowski,

Mielczarek, Mena, Nelson, Olmedo, Pawlowski, Singh, Sloan, Velhinho ...

ICTP/SAIFR Quantum Gravity School, Sao Paulo, September 2013

Organization

The very early universe offers the best arena to probe both, the deep conceptual issues of quantum gravity and observational tests of theories.

Lecture 1

The Big Bang Singularity & Loop Quantum Cosmology

1. Introduction: The Big Bang Singularity
2. Singularity Resolution in LQC: Basic Results
3. Novel features at the Foundation

Lecture 2

Cosmological Perturbations

1. Classical Perturbation Theory
2. Quantum Fields on classical FLRW Space-times
3. Quantum fields on Quantum FLRW geometries

Lecture 3

An Extension of the Inflationary Scenario to the Planck Regime

1. Success and Limitations of Inflation
2. The LQC Strategy
3. Results: Pre-inflationary dynamics and its implications.

Main References for this talk

- For a summary, see:

AA, Agullo & Nelson PRL 109, 251301 (2012);

Viewpoint article providing a brief global perspective: *Physics: Spotlighting Exceptional Research*, 5, 142 (2012) by P. Singh

- More complete references:

AA, Agullo & Nelson, PRD 87, 043507 (2013); CQG 30, 085014 (2013)

AA & Sloan, GRG (2011), PLB (2009); Corichi & Karami, PRD

AA, Corichi & Singh, PRD (2008); AA, Pawlowski, Singh, PRL & PRD (2006).

Other Results Referred to in the Talk:

- Future Observations:

Agullo & Parker PRD & GRG (2011); Agullo & Shandera JCAP (2012); Ganc & Koamtzu PRD (2012).

- A recent detailed Review of Loop Quantum Cosmology

AA & Singh, CQG (2011).

1. Inflation: Successes and Limitations

• The Very Early Universe provides a natural arena to test quantum gravity: Guidance for formulating in detail the theoretical framework *and* confronting theories with observations.

• Standard tools used currently: FLRW solutions to Einstein's equations and quantum field theory of linear perturbations on it, ignoring the back reaction. Checked by self-consistency in the very early universe.

• Challenge to quantum gravity theories: Extend this theory the Planck regime where general relativity breaks down. Do theoretically self consistent extensions exist? Do they pass the current observational tests? Are there new predictions for future observations?

Goal: Probe these issues using Loop Quantum Gravity. Several related but distinct ideas have been proposed. For pedagogical reasons I will present one continuous thread which is systematic in that it begins with the classical theory and proceeds step by step with all inputs spelled out. Focus will be on the *inflationary scenario* although the framework is general.

Inflationary Paradigm

- Major success: Prediction of inhomogeneities in CMB which serve as seeds for structure formation. Observationally relevant wave numbers in the range $\sim (k_o, 2000k_o)$ (radius of the observable CMB surface $\sim \lambda_o$).

- Rather minimal assumptions:

1. Some time in its early history, the universe underwent a phase of accelerated expansion during which the Hubble parameter H was nearly constant.

2. Starting from this phase till the CMB era, the universe is well-described by a FLRW background with linear perturbations. Only matter: inflaton in a suitable potential.

3. At the onset of this 'slow roll inflationary phase' Fourier modes of quantum fields describing perturbations were in the Bunch-Davis vacuum (at least for co-moving wave numbers in the range $\sim (k_o, 2000k_o)$); and,

4. Soon after a mode exited the Hubble radius, its quantum fluctuation can be regarded as a classical perturbation and evolved via linearized Einstein's equations.

- Then QFT on FLRW space-times (and classical GR) implies the existence of tiny inhomogeneities in CMB seen by the 7 year WMAP data. **All large scale structure emerged from vacuum fluctuations!**

Inflationary Paradigm: Incompleteness

Particle Physics Issues:

• Where from the inflaton? A single inflaton or multi-inflatons? Interactions between inflatons? How are particles/fields of the standard model created during ‘reheating’ at the end of inflation? ...

Quantum Gravity Issues:

• Big bang singularity also in the inflationary models (Borde, Guth & Vilenkin). Is it resolved by quantum gravity as has been hoped since the 1970’s? What is the nature of the quantum space-time that replaces Einstein’s continuum in the Planck regime?

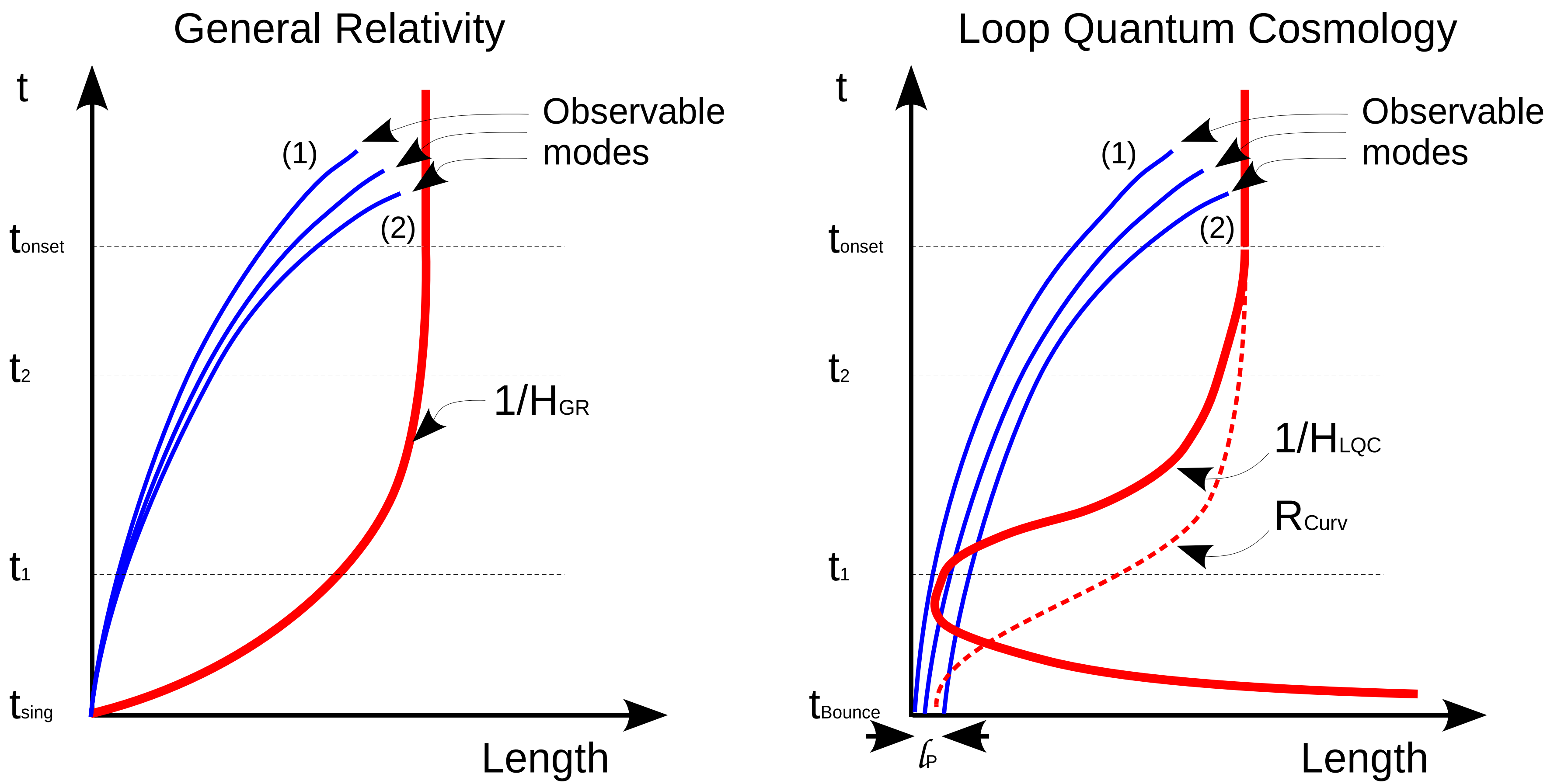
• Does the slow-roll inflation used to explain the WMAP data naturally arise from natural initial conditions ‘at the Beginning’ that replaces the big bang in quantum gravity?

• In classical GR, if we evolve the modes of interest back in time, they become trans-Planckian. Is there a QFT on quantum cosmological space-times needed to adequately handle physics at that stage?

• Can one arrive at the Bunch-Davis vacuum (at the onset of the WMAP slow roll) from more fundamental considerations?

‘Standard’ View & its limitations

Why Planck scale physics could affect the scenario



2. The LQG Strategy

Quantum Gravity Issues:

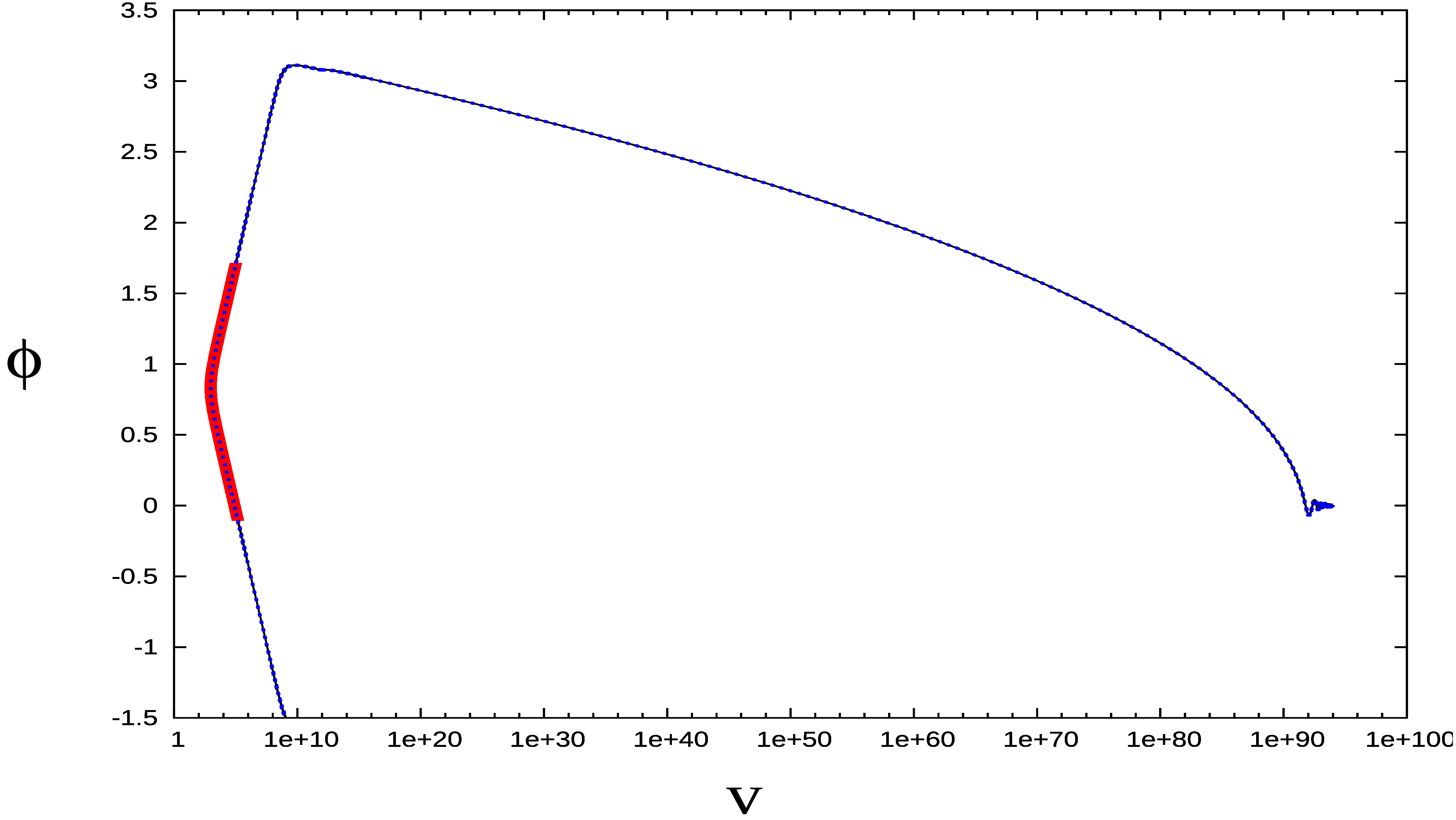
- Big bang singularity also in the inflationary models (Borde, Guth & Vilenkin). Is it resolved by quantum gravity as has been hoped? Nature of the quantum space-time that replaces Einstein's continuum in the Planck regime?

Einstein's Views on the Big Bang Singularity:

> "One may not assume the validity of field equations at very high density of field and matter and one may not conclude that the beginning of the expansion should be a singularity in the mathematical sense." A. Einstein, 1945

- In Loop Quantum Gravity, singularities have been resolved in a large number of cosmological models including the (Flat & Closed) FLRW models, Zero and non-zero Λ, Anisotropic Bianchi models, & Gowdy models that have simplest types of inhomogeneities. Mechanism: Quantum Geometry underlying Loop Quantum Gravity.

Singularity Resolution: $(1/2)m^2\phi^2$ Potential



Expectations values and dispersions of $\hat{V}|_\phi$ for a massive inflaton ϕ with phenomenologically preferred parameters (AA, Pawlowski, Singh). The Big Bang is replaced by a Big Bounce.

General Inhomogeneous Perturbations

• The mainstream strategy in LQG: Don't have a complete quantum gravity theory. Progress has occurred by first truncating the classical theory to the physical problem under consideration and then passing to quantum theory using LQG techniques. Successes: Study of quantum horizons, the graviton propagator, and simple cosmological models.

• For inflation, the sector of physical interest: FLRW background with an inflation ϕ in a suitable potential as matter, together with first order perturbations.

• Our Approach: Use the truncation provided by this cosmological sector. (In numerical simulations, $V(\phi) = (1/2)m^2\phi^2$.) The sector includes inhomogeneities, but as perturbations. Thus, quantum fields representing scalar and tensor perturbations now propagate on a quantum FLRW geometry as discussed in Lecture 2. 'Trans-Planckian' issues are faced squarely. Caveat: Have to check self-consistency of this truncation! Is the back reaction on quantum geometry negligible even in the Planck regime? If so, there we would have a self consistent extension of the inflationary paradigm to the Planck regime.

Cosmological Perturbations: Truncated theory

- Truncated Phase Space $\ni \{(v, \phi;\, \delta h_{ab}(x), \delta\phi(x))$ and their conjugate momenta$\}$

Quantum Theory: Start with $\mathbf{\Psi}(v, \phi;\, \delta h_{ab}(x), \delta\phi(x))$ and proceed to the quantum theory using LQG techniques.

- Test field approximation: $\mathbf{\Psi} = \Psi_o(v, \phi) \otimes \psi(\delta h_{ab}, \phi)$, Ψ_o a physical quantum state in the homogeneous sector. Provides QUANTUM background geometry.
- Linearized constraints $\Rightarrow \psi(\delta h_{ab}, \phi) = \psi(T^{(1)}, T^{(2)}, \mathcal{R};\, \phi)$, where $T^{(1)}, T^{(2)}$ are the tensor modes and $\mathcal{R}$ the scalar mode. In the Planck regime of interest, ϕ serves as the 'internal/relational time'. ψ propagates on the quantum geometry determined by Ψ_o.

- Idea: Choose $\Psi_o(v, \phi)$ to be sharply peaked at an effective LQC solution g^o_{ab}. Such 'coherent states' exist. **First question:** Does the required inflationary phase occur generically in such quantum geometries Ψ_o? To answer this, need to first examine the constraints on the onset of inflation imposed by observations.

Constraints From the 7 year WMAP Data

- The reference co-moving wave number used by WMAP: $k_\star = 8.58\, k_o$

Slow roll parameter $\epsilon = -\dot{H}/H^2 = (3\mathrm{KE})/2\rho$ & $\epsilon_V := 1/16\pi\,\left(V'/V\right)^2$; approximately equal when $\epsilon \ll 1$. (Difference of order ϵ^2).

- Within inflationary models, the power spectrum is given by

$\Delta^2_{\mathcal{R}}(t(k_\star)) = \frac{H^2(t(k_\star))}{\pi\epsilon\,(t(k_\star))\,m^2_{\mathrm{Pl}}}$ and WMAP gives $\Delta^2_{\mathcal{R}}(t(k_\star)) = 2.43 \times 10^{-9}$

and the scalar spectral index is given by $n_s(t(k_\star)) = 1 - \frac{\mathrm{d}\ln\Delta^2_{\mathrm{R}}}{\mathrm{d}\ln \mathrm{k}}\Big|_{k_\star} = 0.968$

with error bars of about $\pm 4.50\%$ for $\Delta_{\mathcal{R}}$ and $\pm 1.25\%$ for n_S.

- For the $(m^2\phi^2/2)$ potential, using GR and these two observed values, one determines inflaton mass and all the parameters at $t = t_{k_\star}$:

$$\phi(t(k_\star)) = \pm 3.15\, m_{\mathrm{Pl}}; \quad \epsilon(t(k_\star)) = 8 \times 10^{-3}\, m^2_{\mathrm{Pl}}; \quad m = 1.21 \times 10^{-6}\, m_{\mathrm{Pl}}\,.$$

(The LQC corrections to these parameters *completely* negligible because $\rho(t_{k_\star}) \approx 7.32 \times 10^{-12} m^4_{\mathrm{Pl}}$.)

- Question: Which effective LQC trajectories, starting from the bounce, pass through this tiny region of phase space within the WMAP error bars?

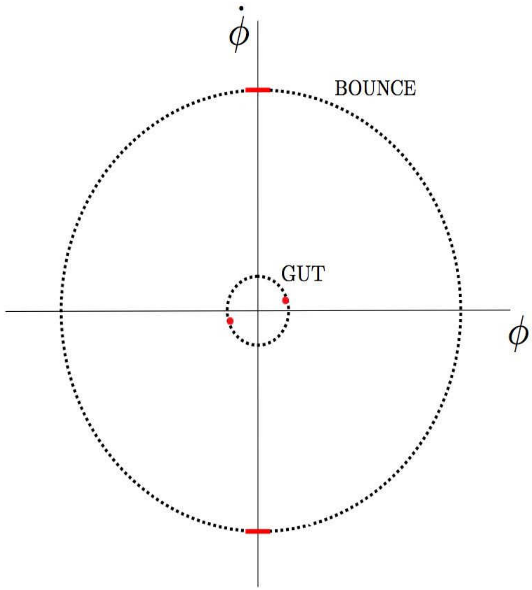
Quantum FLRW Background Geometry Ψ_o

- Let us start with generic data at the bounce **in the effective theory** and evolve. Will the solution enter slow roll at energy scale $\rho \approx 7.32 \times 10^{-12} m_{\rm Pl}^4$ determined from the 7 year WMAP data ? Note: 11 orders of magnitude from the bounce to the onset of the desired slow roll!

- Answer: **YES**. In LQC, $|\phi_{\rm B}| \in (0, 7.47 \times 10^5)$. If $\phi_{\rm B} \geq 0.93$, the data evolves to a solution that encounters the slow roll compatible with the 7 year WMAP data sometime in the future. In this sense, 'almost every' initial data at the bounce evolves to a solution that encounters the desired slow roll sometime in the future. (AA & Sloan, Corichi & Karami)

- Result stronger than the 'attractor' idea because it refers to the specific parameters of the slow roll compatible with WMAP.

- Hence, for the background quantum geometry, we can choose a 'coherent' state Ψ_o sharply peaked at an effective trajectory with $\phi_{\rm B} > 0.93$ and evolve using LQC. WMAP slow roll phase ensured! Choice of $\Psi_o \sim \phi_B$; Free parameter in LQC.



Initial conditions on ψ

• Ψ_o: very special because assumed to be a 'coherent state' at the bounce. But the peak is at a generic point of the FLRW phase space.

• ψ: Hilbert space $\mathcal{H}$ of perturbations on the quantum geometry Ψ_o is spanned by 4th adiabatic order states on the smooth Friedmann metric $\bar{g}_{ab}$. Excellent control.

• Initial conditions: Since Ψ_o (and hence $\bar{g}_{ab}$) is homogeneous and isotropic, demand that: (i) $\psi \in \mathcal{H}$ also invariant under translations and rotations; ii) Ensures that back reaction can be neglected at the bounce; and (iii) Minimizes fundamental uncertainties at the bounce. (For tensor Modes: A quantum version of Penrose's Weyl Curvature Hypothesis.

• Intuitive Physical Meaning: Demanding initial quantum homogeneity and isotropy. Heuristic justification: Because of inflation, the observable universe has size of $\leq 10\ell_{\rm Pl}$ at the bounce. The repulsive force of quantum geometry dilutes all inhomogeneities at this scale. So universe is as homogeneous and isotropic as the uncertainty principle allows it to be!

Recall: Open issues

Let us pause to review the tasks for extending the inflationary paradigm to the Planck regime.

- Big bang singularity also in the inflationary models (Borde, Guth & Vilenkin). Is it resolved by quantum gravity as had been long hoped? What is the nature of the quantum space-time that replaces Einstein's continuum in the Planck regime? $\surd$

- In the systematic evolution from the Planck regime in the more complete theory, does a slow roll phase compatible with the WMAP data arise generically or is an enormous fine tuning needed? $\surd$

- In classical GR, if we evolve the modes of interest back in time, they become trans-Planckian. Is there a QFT on quantum cosmological space-times needed to adequately handle physics at that stage? **Yes!** The basic framework was developed in 2009 (AA, Lewandowski, Kaminski). Can it be further developed to obtain well-defined stress energy operators in this new theory to examine if the back reaction is negligible, i.e. justify the truncation procedure even in the Planck regime?

- Can one arrive at the Bunch-Davis vacuum (at the onset of the WMAP slow roll) from more fundamental considerations? **Yes** for most of the parameter space but **No** on a small window.

QFT on cosmological quantum space-times

First, thanks to the background quantum geometry, trans-Planckian modes pose no problem, provided the test field approximation holds: $\rho_{\rm Pert} \ll \rho_{\rm BG}$ all the way from the bounce to the onset of slow roll.

- Second, surprisingly, truncated dynamics of $\hat{T}^{(1)}, \hat{T}^{(2)}, \hat{\mathcal{R}}$ on the quantum geometry of Ψ_o is mathematically equivalent to that of $\hat{T}^{(1)}, \hat{T}^{(2)}$, $\hat{\mathcal{R}}$ as quantum fields on a smooth space-time with a 'dressed' effective, c-number metric $\bar{g}_{ab}$ (whose coefficients depend on $\hbar$):

$$\bar{g}_{ab}dx^a dx^b = \bar{a}^2(-d\bar{\eta}^2 + d\vec{x}^2)$$

with

$$d\bar{\eta} = \langle \hat{H}_o^{-1/2}\rangle\, [\langle \hat{H}_o^{-1/2} \hat{a}^4 \hat{H}_o^{-1/2}\rangle]^{1/2}\, d\phi; \qquad \bar{a}^4 = (\langle \hat{H}_o^{-1/2}\, \hat{a}^4 \hat{H}_o^{-1/2}\rangle)/\langle \hat{H}_o^{-1}\rangle$$

where H_o is the Hamiltonian governing dynamics of Ψ_o. Analogy with light propagating in a medium.

- Because of this, the mathematical machinery of adiabatic states, regularization and renormalization of the Hamiltonian can be lifted to the QFT on cosmological QSTs under consideration. Result: Full mathematical control on dynamics for computation of the CMB power spectrum, and spectral indices starting from the bounce.

Three Key Questions

1. Does the back-reaction remain negligible as ψ evolves all the way to the onset of the slow roll compatible with WMAP?

If so, our truncation strategy is justified by self-consistency.

2. At the end of the WMAP compatible slow roll, do we recover the observed power spectrum: $\Delta^2_{\mathcal{R}}(k, t_{k^\star}) \approx \frac{H^2(t_{k^\star})}{\pi m^2_{\rm Pl}\epsilon(t_{k^\star})}$?

If so, we would have obtained a quantum gravity completion of the inflationary paradigm.

3. Does $\psi(T^{(1)}_{\bar{k}}, T^{(2)}_{\bar{k}}, \mathcal{R}_{\bar{k}};\, \phi_{\rm B})$ evolve to a state which is indistinguishable from the Bunch Davis vacuum at the onset of slow roll or are there deviations with observable consequences for more refined future observations (e.g. non-Gaussianitities in the bispectrum)?

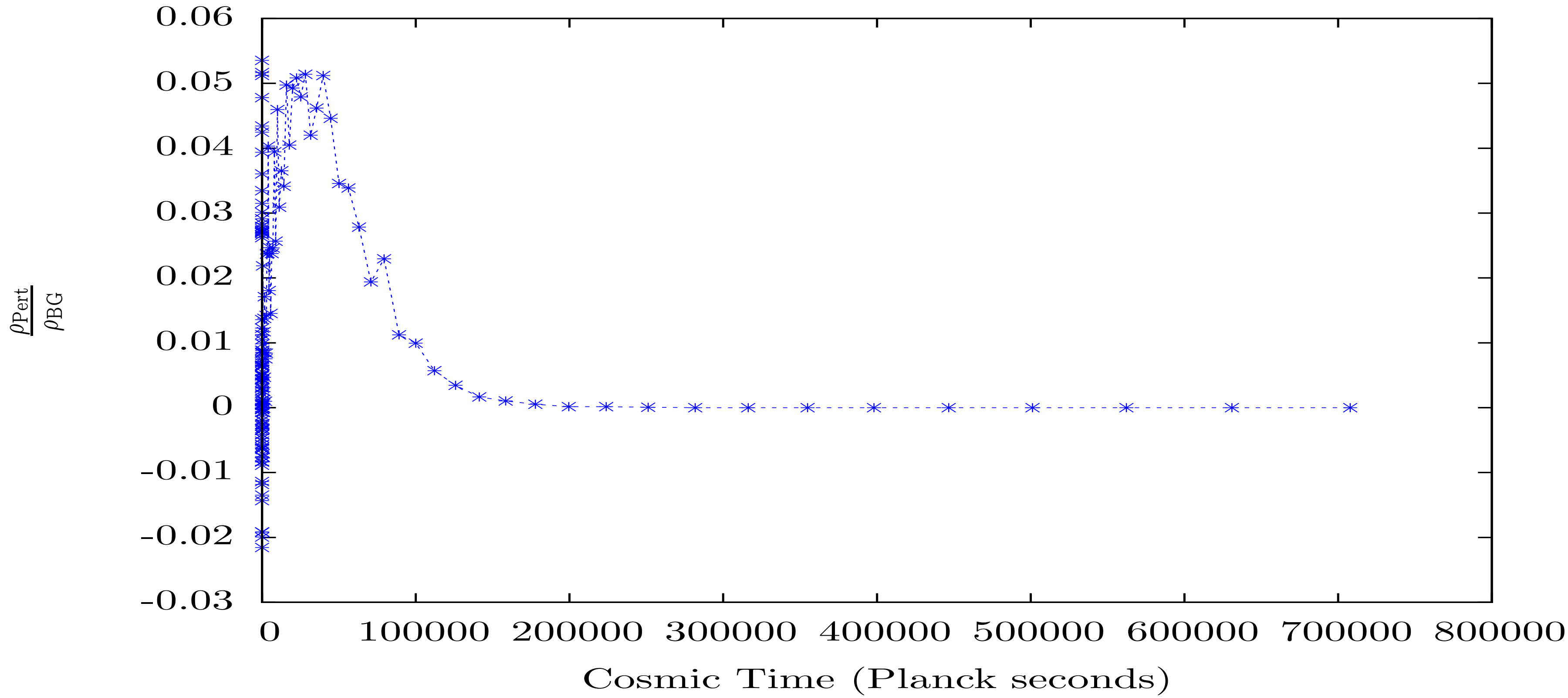
(Agullo & Shandera; Ganc & Komatsu)

If so, we will have observational glimpses into Planck scale physics.

3. Results: Extracting Physics

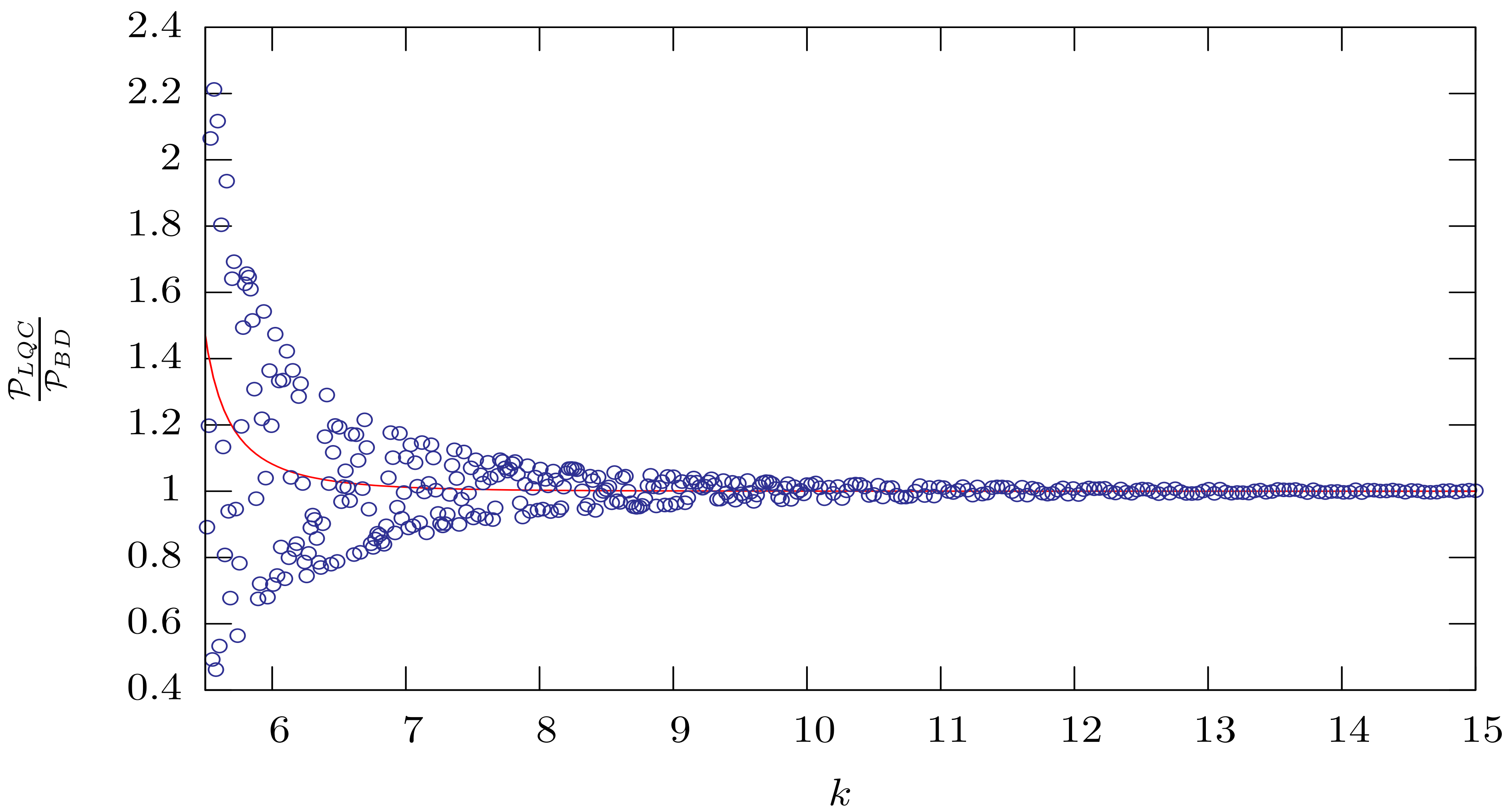
Recall: Because the quantum fields representing perturbations evolve on a background quantum geometry, trans-Planckian modes pose no problem, provided the test field approximation holds: $\rho_{\rm Pert} \ll \rho_{\rm BG}$ all the way from the bounce to the onset of slow roll.

$\rho_{\rm Pert}/\rho_{\rm BG}$ vs time.



Renormalized energy density in ψ is negligible compared to that in Ψ_o all the way from the bounce to the onset of slow roll. Here $\phi_{\rm B} = 1.15 m_{\rm Pl}$.

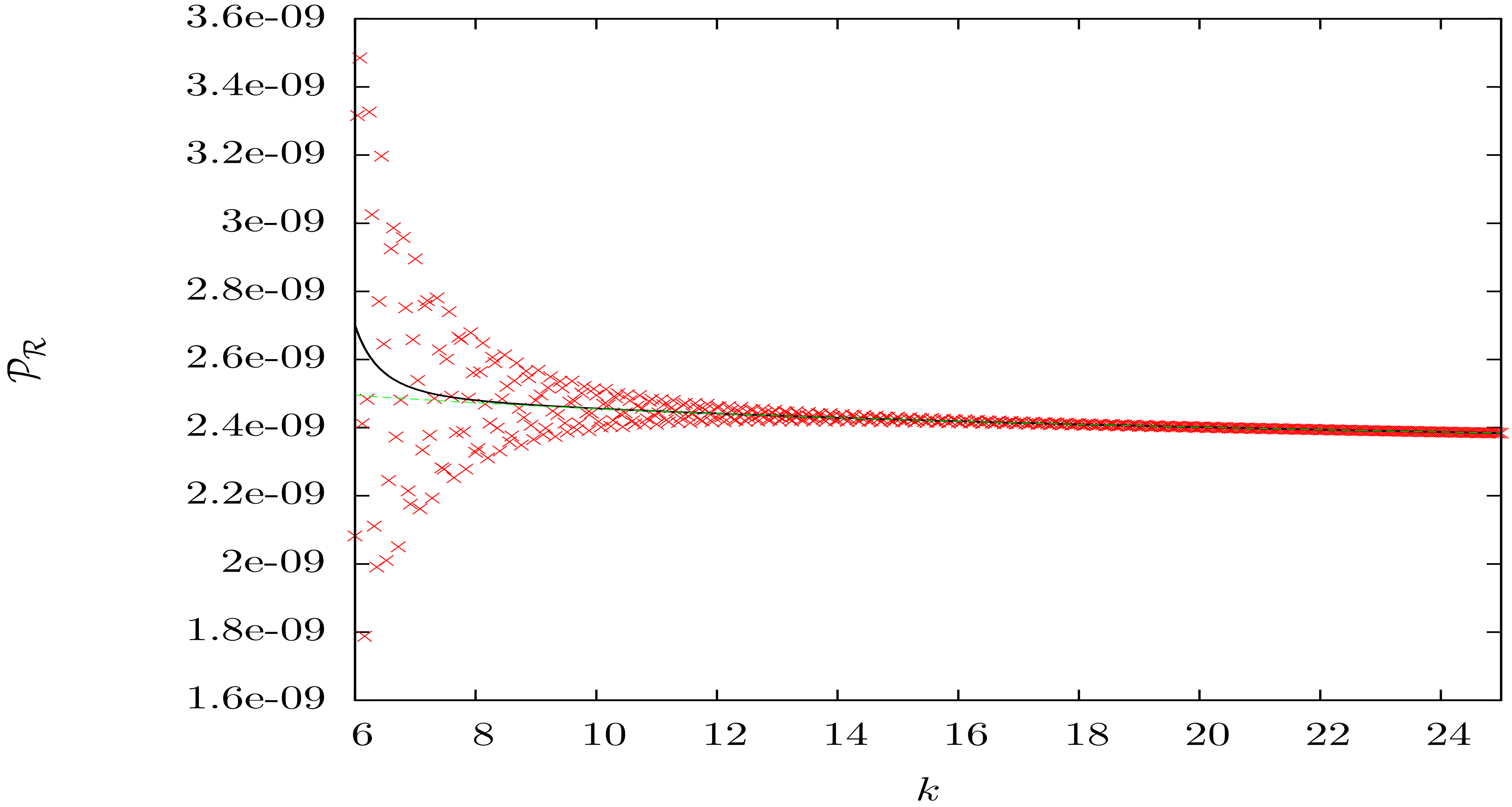
The Scalar Power spectrum: Ratio $(\mathcal{P}_{\rm LQG}/\mathcal{P}_{\rm BD})$



Ratio of the LQC and the standard BD power spectrum for the scalar mode. Blue: Raw data. Red: Binned average. LQC prediction is compatible with observations for $\phi_{\rm B} \geq 1.14 m_{\rm Pl}$.

For $\phi_{\rm B} = 1.2 m_{\rm Pl}$, WMAP $k_{\rm min} = 9 m_{\rm Pl}$. Agreement with standard predictions for $\phi_{\rm B} \geq 1.2 m_{\rm Pl}$. For $\phi_{\rm B} < 1.2 m_{\rm Pl}$: Deviations for future observations.

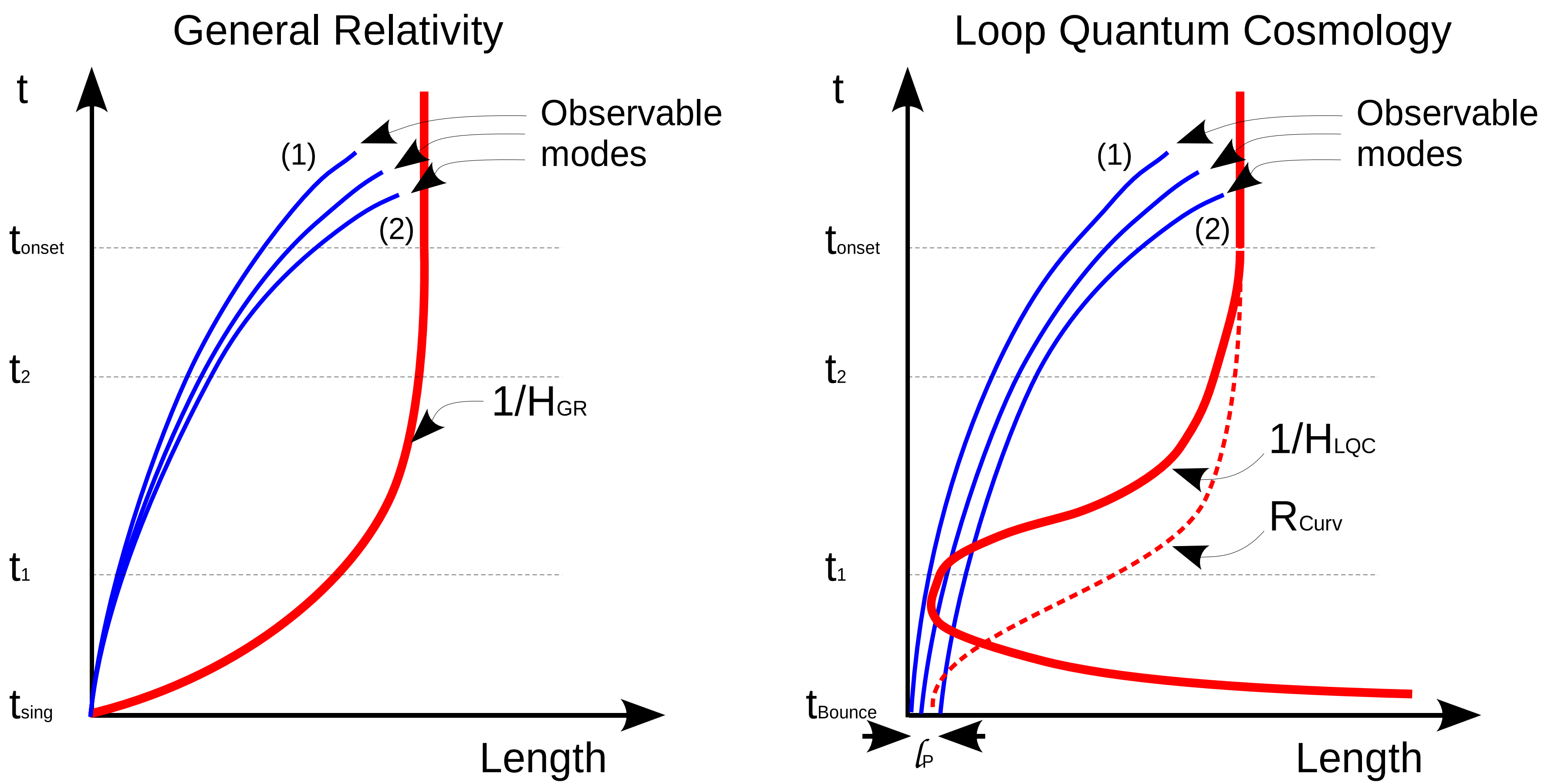
The LQC Tensor Power spectrum



Predicted power spectra for the tensor mode. Black: Binned average. Red: Raw data points.

Understanding the Power Spectrum

Only modes with $\lambda > R_{\rm curv}$, the curvature radius, in the pre-inflationary era are excited and populated at the onset of inflation. Can occur in a narrow window for $\phi_{\rm B} \leq 1.2 m_{\rm Pl}$.



4. Summary: Framework

• The early universe provides an ideal setting to test quantum gravity ideas. Can one obtain a quantum gravity extension of cosmological scenarios to the Planck regime? We focused on inflationary paradigm because it has been extremely successful with structure formation.

The standard theory can be extended using LQG:

• *Background geometry:* Singularity Resolution and precise quantum geometry for the Planck regime. $\surd$

• *Perturbations:* Since they propagate on quantum geometry, using QFT on cosmological quantum geometries (AA, Lewandowski, Kaminski), trans-Planckian issues can be handled systematically provided the test field approximation holds. Analyzed in detail using the renormalized stress-energy of $\hat{T}^{(1)}, \hat{T}^{(2)}, \hat{\mathcal{R}}$ on the quantum geometry of Ψ_o. Detailed numerics show that the approximation does hold in most of the parameter space. (Agullo, AA, Nelson). $\surd$

Detailed calculations in the inflationary paradigm but framework is much more general.

Summary: Implications for Inflation

- Extension: For most of the parameter (ϕ_B) space, modes of observational interest are essentially in the Bunch Davies vacuum at the onset of the WMAP slow roll $\Rightarrow$ Predictions of the standard inflationary scenario for the power spectra, spectral indices & ratio of tensor to scalar modes are recovered starting from Planck era. (Agullo, AA, Nelson) $\surd$

- New Effects: There is a small window in the parameter space, for which some observable modes have excitations over the Bunch-Davies vacuum at the onset of inflation. These give rise certain deviations from standard inflation for future observations (e.g., $r \neq -8n_t$; 3-point functions that will be observed in SDSS;...) A window to probe the Planck era around the LQC bounce. (Agullo, AA, Nelson, Shandera, Ganc, Komatsu) $\surd$

- Note: LQG does not imply that inflation must have occurred because it does not address particle physics issues. The analysis simply assumes that there is an inflaton with a suitable potential. But it does show concretely that many of the standard criticisms of inflation can be addressed in LQG by facing the Planck regime squarely.

The framework is general; can be applied to other viable scenarios.

Main References for this talk

- For a summary, see:

AA, Agullo & Nelson PRL 109, 251301 (2012);

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