Holographic current algebras & BMS4
(ICTP-SAIFR)
w. C. Troessaert

1) Motivation
2) Global current algebras
3) Holographic "
4) Results in 3d & 4d

1) • $\boxed{\text{asymptotic symmetries}}$ in grav. th
$\longrightarrow$ global sym. of dual th.

• generalize grav holo

AdS $\longrightarrow$ flat $\mathscr{I}^+$

• $\boxed{\text{3d Einstein}}$: asy. sym, c.c., solutions,
CS $\to$ WZW $\to$ Liouville

• $\boxed{4d}$ ∞ dim ext. $\boxed{iso(3,1) \subset bms_4^{loc} = 2\times\text{Virasoro} \oplus ST}$
$\quad\quad$ c Kof of S^2

• $l_m, \bar{l}_m$: poles on S^2
Problem: charges L_m, $\bar{L}_m$ $\nearrow \infty$ $\quad \{L_m, L_n\} = (m-n)L_{m+n} \dots$

• Strominger: $\boxed{\text{local Ward identities}}$

$$\boxed{\partial^x_a \langle j^a_{\xi_1}(x)\, j^b_{\xi_2}(y)\, X(z)\rangle = i\,\delta(x-y)\langle j^b_{[\xi_1,\xi_2]}(y)\, X(z)\rangle + i\,\delta(x-z)\langle j^a_{\xi_2}(y)\,\delta_{\xi_1} X(z)\rangle}$$

$\boxed{\text{classical version: current algebra}}$

$\to$ solves the problem

"just what the doctor ordered to deal consistently with bms$_4^{ue}$".

2) Noether current: $\boxed{\delta_{Q_1} : d\, j_{Q_2} = Q_2^i \frac{\delta L}{\delta \phi^i} d^n x}$

$$d\left(\delta_{Q_1} j_{Q_2} - j_{[Q_1,Q_2]} + T_{Q_1}[Q_2, \tfrac{\delta L}{\delta\phi}]\right) = 0$$

$$\Rightarrow \delta_{Q_1} j_{Q_2} = j_{[Q_1,Q_2]} + \underset{\phi}{K_{Q_1,Q_2}} + \underset{0}{\overset{}{\underset{\approx}{T}}} + d(\;)$$

(i) $H^{n-1}(d)$ global current algebra

highly constrained

(ii) $\delta_{Q_1} K_{Q_2,Q_3} - \frac{1}{2} K_{[Q_1,Q_2],Q_3} + \text{cyclic}(1,2,3) \approx 0$

3) Holography: understand gauge transf.

$$\delta_f \phi^i = R^i_\alpha f^\alpha + R^{i\mu}_\alpha \partial_\mu f^\alpha + \dots$$

Noether current $S_f = \left(R^{i\mu}_\alpha f^\alpha \frac{\delta L}{\delta\phi^i} + \dots\right) d^{n-1}x_\mu$

$(n-2)$ form $\quad k_f[\delta\phi] = \frac{1}{2} \delta\phi^i \frac{\partial}{\partial \partial_\nu \phi^i} \frac{\partial}{\partial dx^\nu} S_f + \dots$

$d k_f = 0$ if $\frac{\delta L}{\delta\phi^i} = 0$, $\delta \frac{\delta L}{\delta\phi^i} = 0$, $R^i_\alpha(f^\alpha) \approx 0$

GR: $[k_f]$ 1↔1 with Kof of background

ADM type charges

$$\delta_{f_1} k_{f_2} \sim k_{[f_1,f_2]} + K_{f_1,f_2}$$

- exact case ✓ $K_{f_1,f_2} = 0$
- asymptotic case $x^\mu = (u, r, y^A)$

$r = ct \to \infty$

$$k_f = k_f^{[\mu\nu]} (d^{n-2}x)_{\mu\nu}$$

- current of lower dim. theory

$x^a = (u, y^A)$

- integrability $\quad k_f^{[ur]} = \delta J_f^u \,, \quad k_f^{[Ar]} = \delta J_f^A$

4) $\boxed{3d}$ gauge fixed $\quad ds^2 = e^{2\beta}\frac{V}{r}du^2 - 2e^{2\beta}du\,dr + r^2(d\phi - U du)^2$

$$\beta = o(1) \quad \frac{V}{r} = -\frac{r^2}{\ell^2} + O(1) \,, \quad U = O(r^{-2})$$

residual symmetries: $\quad \ell \neq 0 \quad \xi = Y^+(x^+)\partial_+ + Y^-(x^-)\partial_-$

$x^\pm = \frac{u}{\ell} \pm \phi \qquad$ conformal alg.

$\ell = 0 \quad \xi = Y(\phi)\partial_\phi + (T(\phi) + uY')\partial_u$

bms_3 alg.

general sol:

$$ds^2 = \left(-\frac{r^2}{\ell^2} + M\right)du^2 - 2du\,dr + N du\,d\phi + r^2 d\phi^2$$

$\ell \neq 0 : \quad M = 2\left(\tilde{J}_{++}(x^+) + \tilde{J}_{--}(x^-)\right)$

$N = 2\ell\left(\tilde{J}_{++} - \tilde{J}_{--}\right)$

$\ell = 0 \quad M = \Theta(\phi) \,, \quad N = \tilde{J}(\phi) + u\Theta'$

current algebra: $\quad -\delta_{\xi_2} J^a_{\xi_1} \approx J^a_{[\xi_1,\xi_2]} + K^a_{\xi_1,\xi_2}$

$\partial_a J^a_\xi \approx 0$

$\ell \neq 0 \quad J^\pm_\xi = \frac{1}{4\pi G} Y^\mp \tilde{J}_{\mp\mp}$

$K^\pm_{\xi_1,\xi_2} = \frac{1}{16\pi G}\left[Y_1^{\mp\prime} Y_2^{\mp\prime\prime} - (1 \leftrightarrow 2)\right]$

$\ell = 0 \quad J^u_\xi = \frac{1}{16\pi G}\left[T\Theta + Y\tilde{J}\right] \,, \quad J^\phi = 0$

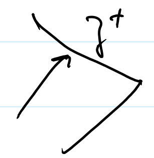
$$K^u_{\xi_1,\xi_2} = \frac{1}{16\pi G}\left[Y_1{}' T_2{}'' + T_1{}' Y_2{}'' - (1 \leftrightarrow 2)\right]$$

not more info than in : $\{L^\pm_m, L^\pm_n\} = (m-n) L^\pm_{m+n} + \frac{c^\pm}{12} m(m^2-1)\delta^0_{m+n}$

$c^\pm = \frac{3l}{2G}$ <u>BH</u>

because spatial comp follow
from current conservation + Fourier analysis
on the circle.

4d) •



$$d\bar{s}^2 = 0\cdot du^2 + d\theta^2 + \sin^2\theta\, d\phi^2 \qquad S^2 \times \mathbb{R} = \mathcal{J}^+$$
$$= 2P^{-2}\, d\zeta\, d\bar{\zeta} \qquad \zeta = \cot\frac{\theta}{2} e^{i\phi}$$

• residual symm: $\xi = Y^A \partial_A + f \partial_u$

Y^A CKV of $d\bar{s}^2$, $f = T(y) + \frac{1}{2} u\, \bar{D}_A Y^A$

a) bms_4^{glob} $T \to$ spherical harm. , Y^A : $l_m = -\zeta^{m+1}\partial_\zeta$

supertranslations $\bar{l}_m$

$m = -1, 0, 1$

Lorentz trsf

b) bms_4^{loc} Laurent series

$T_{m,n} = P_s^{-1} \zeta^m \bar{\zeta}^m$, $l_m, \bar{l}_m$ $\forall m \in \mathbb{Z}$

solut. $\sigma^0(u,\zeta,\bar{\zeta})$, Ψ^0_2, Ψ^0_1, + c.c.

$\dot{\sigma}^0$ news function

↪ Δ non integrable, J^a non conserved current algebra

$$-\delta_{\xi_2} J^a_{\xi_1} + \theta^a_{\xi_2}(-\delta_{\xi_1}\chi) \approx J^a_{[\xi_1,\xi_2]} + K^a_{\xi_1,\xi_2}$$

$$\theta^u_{\xi}(-\delta\chi) = \frac{1}{8\pi G p^2}\left(f \dot{\bar{\tau}}^0 \delta \tau^0 + c.c.\right)$$

$$K^u_{\xi_1,\xi_2} = \frac{1}{8\pi G p^2}\left(\frac{1}{2}\bar{\tau}^0 f_1 \tau^3 Y_2 + \ldots - (1 \leftrightarrow 2) + c.c.\right)$$

$$-\delta_{\xi_3} K^a_{\xi_1,\xi_2} + K^a_{[\xi_1,\xi_2],\xi_3} + \text{cyclic}(1,2,3) = \partial_b N^{[ab]}_{\xi_1,\xi_2,\xi_3}$$

- ultimate aim: exploit for Kerr solution in Carody type formula
- local formulation in terms of current algebras removes maths. inconsistencies
- asymptotic solutions → allow poles as well.