# The Very Early Universe
## Lecture 1.

I. In the $k=0$, FLRW model, one introduces co-moving coordinates $x^i$, corresponding orthonormal triads $\mathring{e}^a{}_i$ and co-triads $\mathring{\omega}_a{}^i$, and a cubical cell $\mathcal{C}$ with $\int_{\mathcal{C}} d^3x = V_0$. Then homogeneous isotropic connection and physical triads are expressed as $A_a^i = c\, V_0^{-1/3}\, \mathring{\omega}_a{}^i$ and $E^a{}_i = p\, V_0^{-2/3}\, \mathring{e}^a{}_i$.
or, $c = \frac{1}{3} V_0^{-2/3} \int_{\mathcal{C}} A_a^i(x)\, \mathring{e}^a{}_i(x)\, d^3x \qquad p = \frac{1}{3} V_0^{-1/3} \int_{\mathcal{C}} E^a{}_i\, \mathring{\omega}_a{}^i\, d^3x$.

1. Using the fundamental Poisson brackets $\{A_a^i(\vec{x}), E_j^b(\vec{x})\} = (k\gamma)\, \delta_a^b\, \delta_j^i\, \delta^3(x,y)$ of LQG, show that we have $\{c, p\} = \frac{k\gamma}{3}$. (Here $k = 8\pi G$)
2. Show that the holonomy of $A_a^i$ along an edge $e_3$ parallel to $x_3$-axis and of length $\mu V_0^{1/3}$ defined by $h_{(e_3,\mu)}(A) = \mathbb{P}\exp \int_{e_3} A_a^i \tau_i\, dl^a$ $\equiv \exp \int_{e_3} c V_0^{-1/3}\, \tau_3\, dx_3$ reduces to $h_{(e_3,\mu)}(A) = \cos\frac{\mu c}{2} + 2\sin\frac{\mu c}{2}\, \tau_3$.
where $\tau_i$ satisfy the normalization $\tau_i \tau_j = \frac{1}{2}\epsilon_{ijk}\tau_k$.

II. Quantum states for FLRW LQC are given by $\Psi(c) = \sum_j \alpha_j\, e^{i\mu_j c}$ where $\mu_j \in \mathbb{R}$ and $\alpha_j \in \mathbb{C}$. The scalar product is given by
$(\Psi_1, \Psi_2) = \lim_{L\to\infty} \frac{1}{2L} \int_{-L}^{L} dc\; \bar{\Psi}_1 \Psi_2$. Show that

1. "Plane waves" $e^{\frac{i\mu c}{2}} =: N_\mu(c)$ form an orthonormal basis with $(N_{\mu_1}, N_{\mu_2}) = \delta_{\mu_1,\mu_2}$ (Note the Kronecker $\delta_{\mu_1,\mu_2}$ **NOT** Dirac $\delta(\mu_1, \mu_2)$)
2. Show that the operators
$\hat{p}\,\Psi(c) := -i\left(\frac{8\pi\gamma \ell_P^2}{3}\right)\frac{d\Psi}{dc}, \qquad \hat{N}_{(\sigma)}\,\Psi(c) := \left(\exp\frac{i\sigma c}{2}\right)\Psi(c)$
are, respectively, self adjoint and unitary, and satisfy the commutation relations expected from the Poisson bracket $\{c, p\} = \frac{k\gamma}{3}$. (Here, $\ell_P^2 = G\hbar$)
3. Show that the matrix elements $(N_\mu(c),\; \hat{N}_{(\sigma)} N_\mu(c))$ fail to be continuous in $\sigma$ (Hint: suffices to show $\lim_{\sigma\to 0} (N_\mu, \hat{N}_\sigma N_\mu) \neq (N_\mu\, \hat{N}_{\sigma=0} N_\mu)$.) This is why operator $\hat{c}$ does not exist on the LQC Hilbert space.
4. Show that none of the LQC states $\Psi(c)$ belongs to the Schrödinger Hilbert space where $\int \bar{\Psi}\Psi\, dc < \infty$.