Lecture # 3.

I. slow roll inflation with quadratic potential $V(\phi) = \frac{1}{2} m^2 \phi^2$

observations provide the power spectrum $P_R(k) = \frac{H^2(t_{k^*})}{\pi \epsilon(t_{k^*}) m_{pl}^2} = 2.43 \times 10^{-9}$ and spectral index $n_s(t_{k^*}) = 0.968$ at the time t_{k^*} when the reference mode k^* exits the Hubble horizon. (All quantities in Planck units). For the quadratic potential, $4\epsilon = (1-n_s)$.

1. show that the Hubble parameter $H(t_{k^*}) = 7.83 \times 10^{-6}\, m_{pl}$ and (the Hubble radius) $R_H(t_{k^*}) = 1.28 \times 10^{5}\, \ell_{pl}$. Using the Friedmann equation $H^2 = \frac{8\pi G}{3} \rho$, find $\rho(t_{k^*})$. How likely is it that quantum gravity effects would be manifest during inflation?

2. show that the Friedmann equation and the equation $\ddot\phi + 3H\dot\phi + V_{,\phi} = 0$ imply the Raychoudhuri equation $(3\ddot a/a) + 4\pi G(\rho + 3p) = 0$ (where $\rho = \frac{1}{2}\dot\phi^2 + V$ and $p = \frac{1}{2}\dot\phi^2 - V$).

3. Using these equations and observational data, show that $\phi(t_{k^*}) = \pm 3.15\, m_{pl}$, $\dot\phi(t_{k^*}) = \mp 1.98 \times 10^{-7} m_{pl}^2$, $m = 1.21 \times 10^{-6} m_{pl}$.

Thus, initial conditions at t_{k^*} and the inflaton mass are determined by observations.

II. In LQC, the scalar curvature at the bounce is universal, $R_B = 62$. Using problem 1 in HW#2, show that this gives a new scale for propagation of tensor modes: $k_{LQC} = \left(\frac{62}{6}\right)^{1/2} \approx 3.21$, or $\lambda_{LQC} \approx 0.997$. Argue that modes with $\lambda > \lambda_{LQC}$ at the bounce will be excited during the pre-inflationary dynamics in LQC and would therefore not be in the BD vacuum at the onset of inflation.