MHD turbulence in the solar corona and solar wind

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Waves, turbulence, low-frequency fluctuations

Consider the 3D MHD equations,

$$\mathbf{u} = \mathbf{u}(x, y, z, t) = \text{plasma velocity}$$

$$\mathbf{B} = \mathbf{B}(x, y, z, t) = \text{magnetic field}$$

$$\rho(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u}) = -\nabla p + \mathbf{J} \times \mathbf{B} + \rho \nu \nabla^2 \mathbf{u} + \mathbf{f}_u(t)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B} + \mathbf{f}_b(t)$$

$$\mathbf{J} = \nabla \times \mathbf{B} = \text{current density}, \quad \nabla \cdot \mathbf{B} = 0 \quad , \quad \nabla \cdot \mathbf{u} = 0$$

And we will assume also a background magnetic field $\mathbf{B_0}$,so $\mathbf{B} = \mathbf{B_0} + \mathbf{b}$.

We numerically (DNS) solve the MHD equations using a pseudospectral code. The fields are evolved in k-space, and we add forcing terms $\mathbf{f}_k^u(t)$ and $\mathbf{f}_k^b(t)$ to achieve a steady state.
The forcing terms are narrow in k-space ($1 \leq k \leq 2$) and include a memory part and a random part. A component of the forcing is of the form:

$$\alpha_{i+1} = m\alpha_i + \sqrt{1-m^2}r_{i+1}$$

with $0 \leq m \leq 1$ the memory parameter and r_i a (uniform) random number.

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It can be seen that

$$< \alpha_n \alpha_{n+l} > \rightarrow m^l, \qquad \text{when} \quad n \rightarrow \infty$$

By constructing a (discrete) time series with $t_n = n\Delta t$,

$$< \alpha(t)\alpha(t+\tau_f) > \rightarrow e^{-t/\tau_f}$$

with $\tau_f = l\Delta t$, $m = 1 - \Delta t/\tau_f$.
So the forcing can be made to have a fixed chosen correlation time τ_f.

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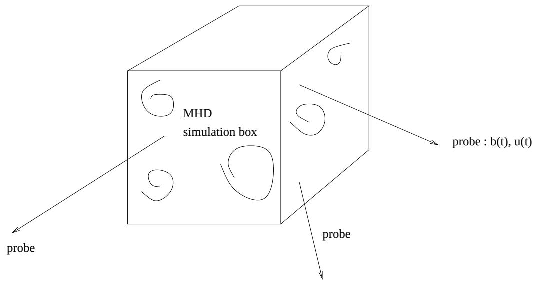
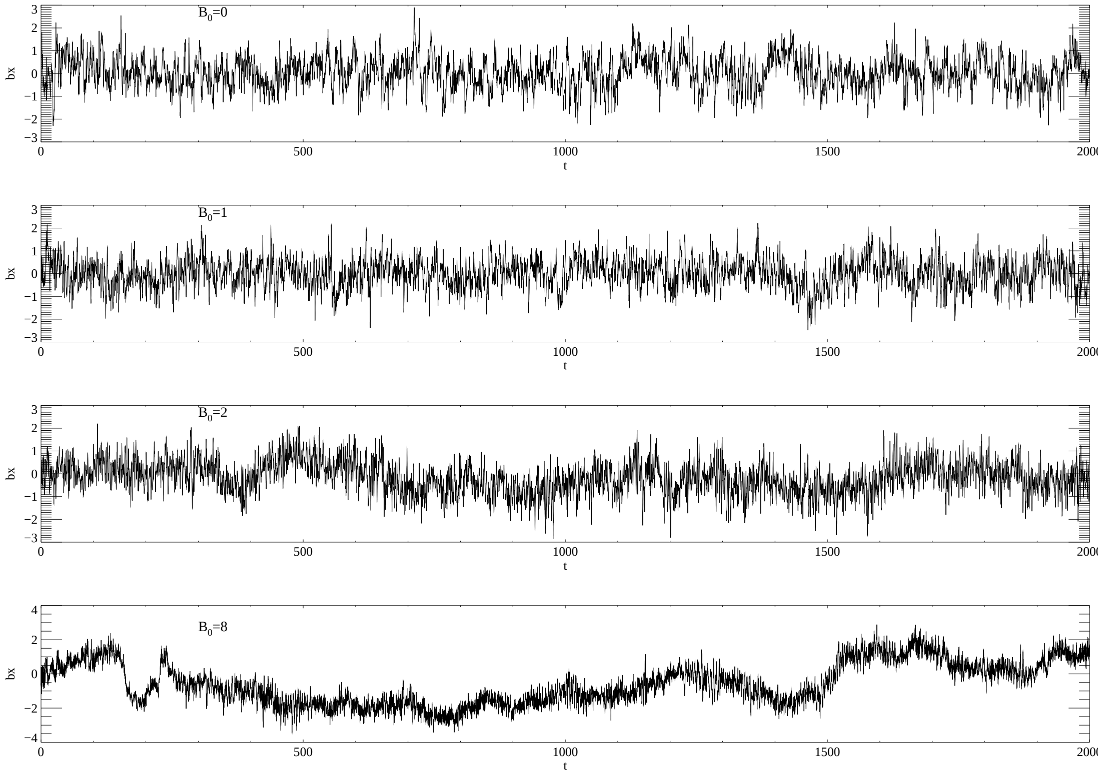
We use units of length $L_0 = 1/2\pi$ size of the box, that is $L_0 = 1$, and $u_0 =< b^2 >^{1/2} (t = 0) =< u^2 >^{1/2} (t = 0) = 1$, and $t_0 = L_0/u_0 = 1$. We chose $\tau_f = 1$.

We consider **probes** inside the box, where we can measure magnetic and velocity fluctuations, $\mathbf{b}(x, y, z, t)$, $\mathbf{u}(x, y, z, t)$ as a function of time.

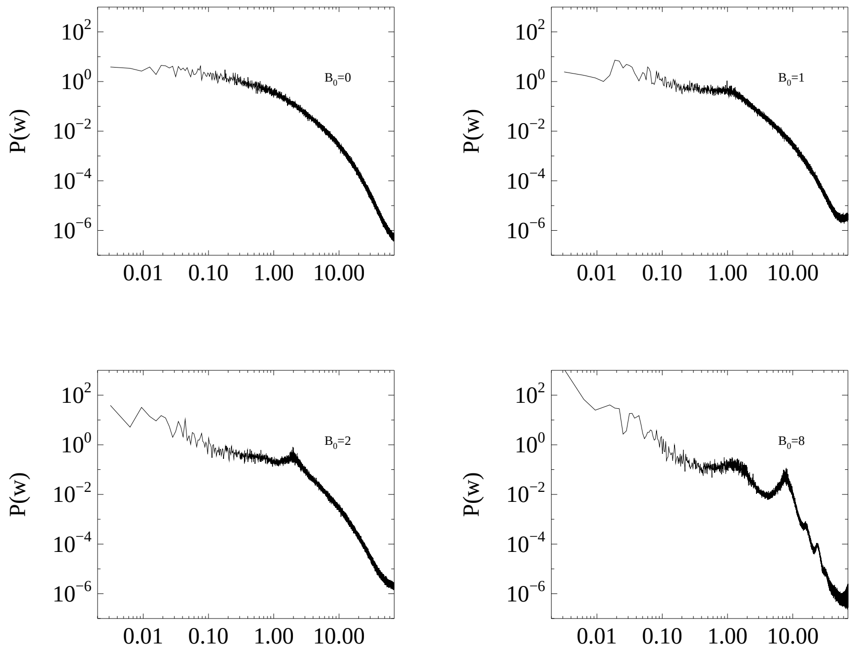


Dmitruk & Matthaeus, Phys. Plasmas 16, 1, 2009.

Time series

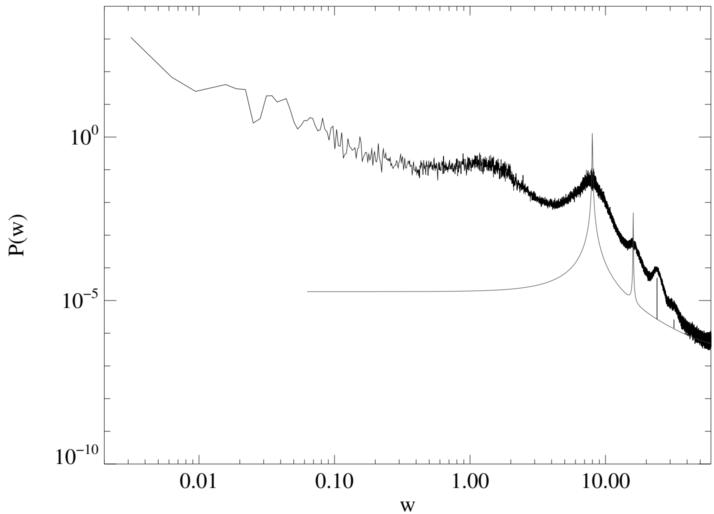


Frequency spectra

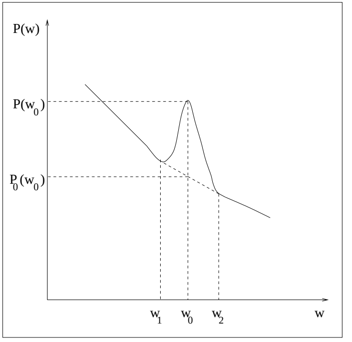


Recall Alfven waves, satisfy,

$$w = \mathbf{k} \cdot \mathbf{B_0} = k_{\parallel} B_0$$



Here $B_0 = 8$ and we see the peaks at multiples of B_0. Also plotted with light line the spectrum in the case of linear ideal MHD (i.e., only waves).



We define the Signal to Noise Ratio,

$$\mathrm{SNR} = \log_{10}\left[\frac{\mathrm{P}(\mathrm{w}_0)}{\mathrm{P}_0(\mathrm{w}_0)}\right].$$

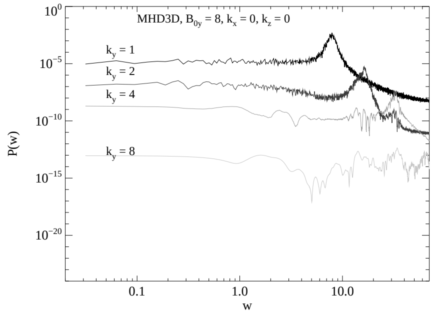
and the Wave Power Ratio,

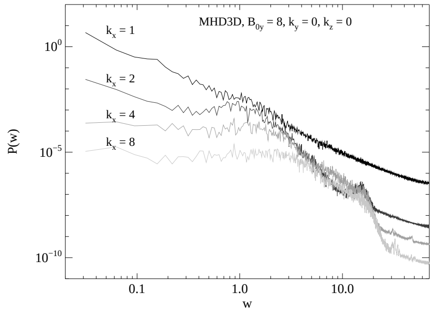
$$\mathrm{WPR} = \frac{\int_{\mathrm{w}_1}^{\mathrm{w}_2}[\mathrm{P}(\mathrm{w}) - \mathrm{P}_0(\mathrm{w})]\mathrm{dw}}{\int_{\mathrm{w}>0}\mathrm{P}(\mathrm{w})\mathrm{dw}}$$

SNR = 0, 0.3, 0.6, 1.5, 3.1 for B_0=0, 1, 2, 8, 16
WPR = 0, 0.1, 0.13, 0.03, 0.02 for B_0=0, 1, 2, 8, 16

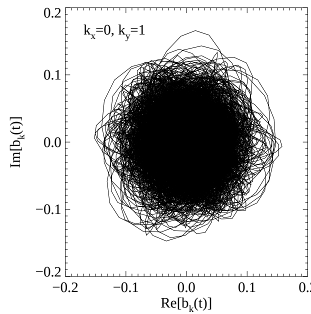
Although waves can be clearly distinguished (large SNR ratio), most of the power (WPR ratio) it is on eddies (turbulent fluctuations).

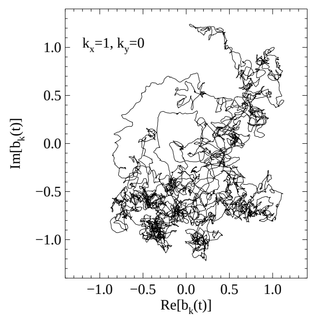
We can also look at the frequency spectrum of individual modes in k-space,





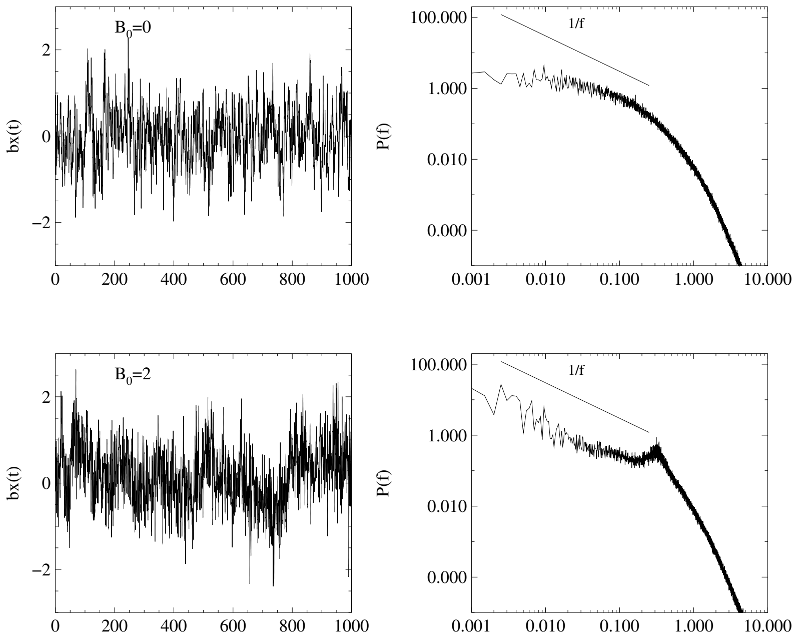
And the real and imaginary parts of a mode b_k, which for the case of a wave should be a circle



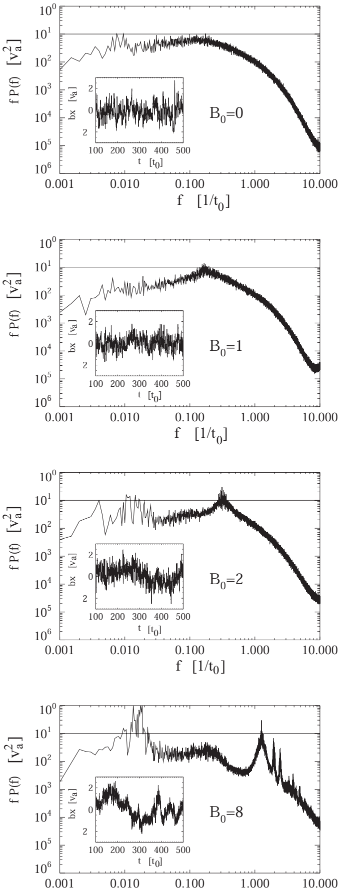


Low-frequency fluctuations, $1/f$ noise

Look at low frequency fluctuations (Dmitruk & Matthaeus, Phys. Rev. E 76, 036305, 2007)

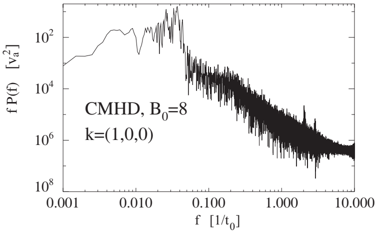


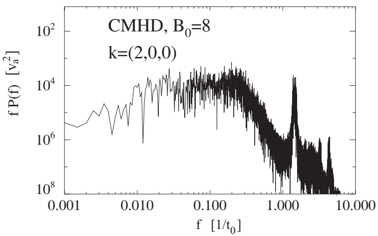
see solar wind observations in Matthaeus & Goldstein, Phys. Rev. Lett. 57, 495, 1986

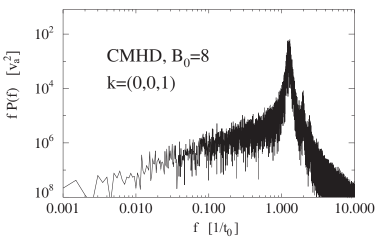


f P(f) [v_a^2]
f [1/t_0]
0.001
0.010
0.100
1.000
10.000
bx [v_a]
t [t_0]
100
200
300
400
500
B_0=0
B_0=1
B_0=2
B_0=8

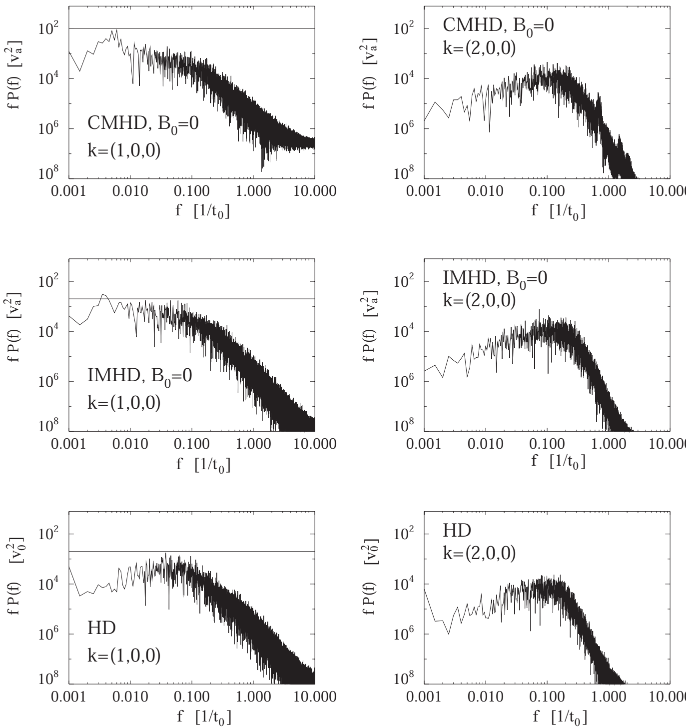
Frequency spectra for different modes $\mathbf{b}(\mathbf{k})$



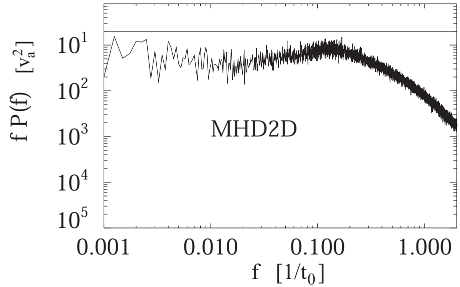


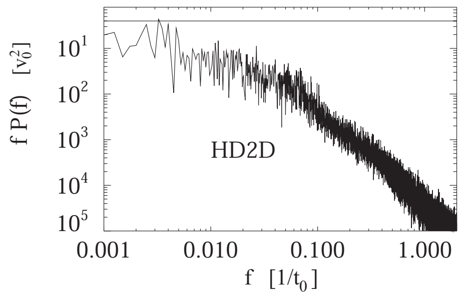


Consider hydrodynamics (3D HD) → no $1/f$!!

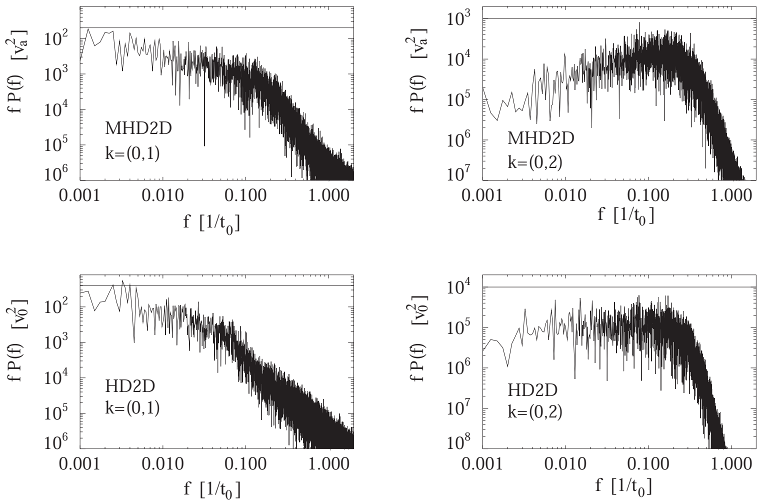


Other systems with low frequency fluctuations: MHD2D, HD2D



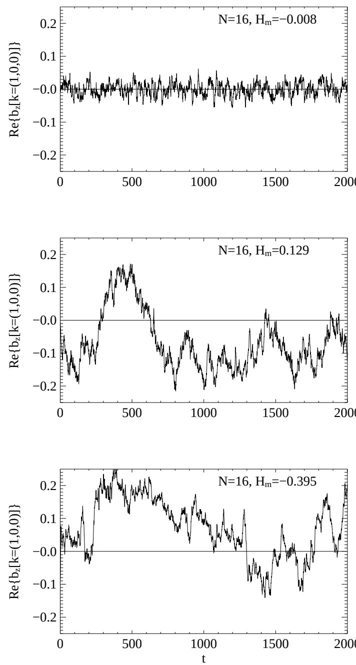


Frequency spectra of individual modes

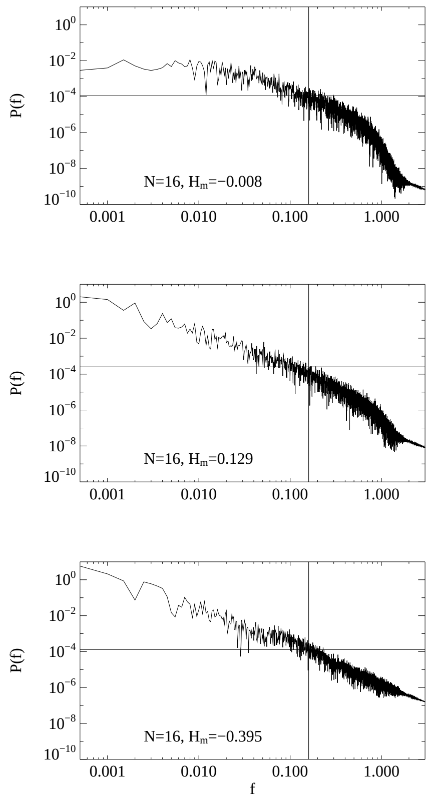


Ideal MHD ($\eta = \nu = 0$) with non-zero magnetic helicity, $H_m < \mathbf{a} \cdot \mathbf{b} >$, where $\nabla \times \mathbf{a} = \mathbf{b}$ is the potential vector. No DC field (i.e. $B_0 = 0$).

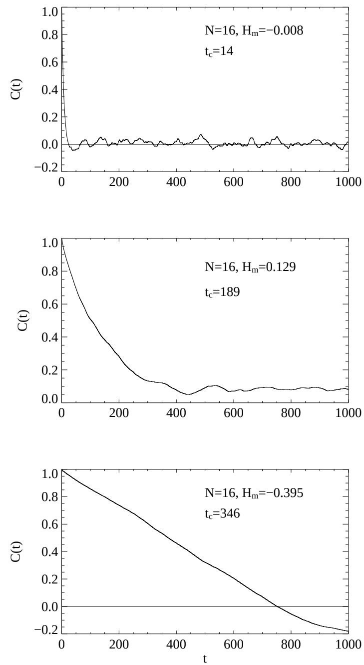
Time behavior of the lowest k mode



Frequency spectra

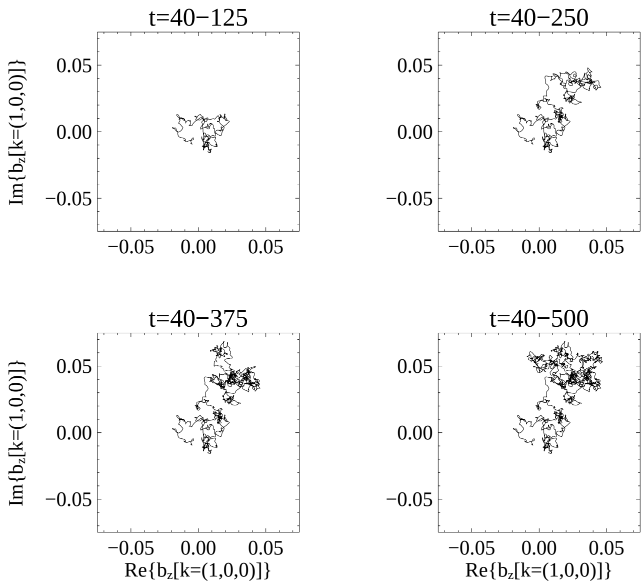


Correlation function $< b(t_0)b(t_0+t) >$

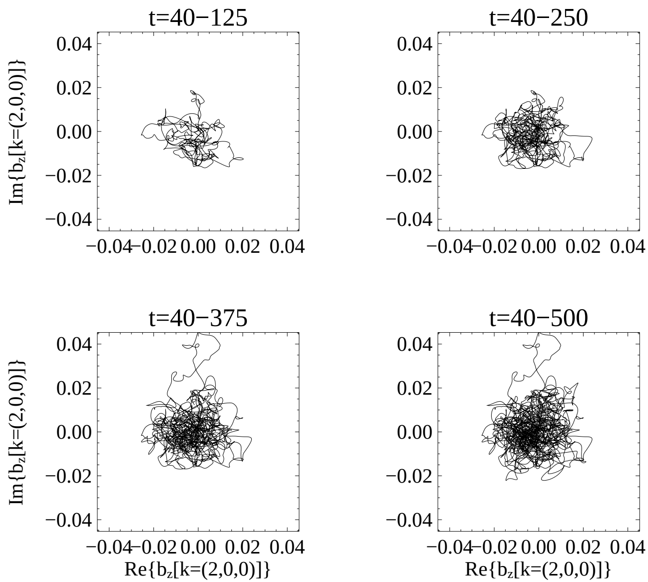


Lack of single correlation time $\rightarrow \ 1/f$!!

Phase space behavior of modes in complex plane, $\mathbf{k} = (1, 0, 0)$



$$\mathbf{k} = (2, 0, 0)$$



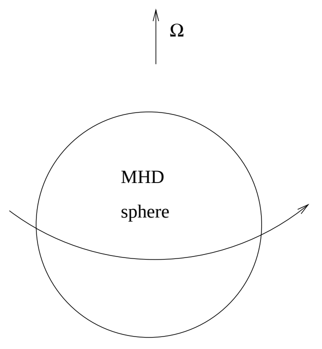
t=40−125
t=40−250
t=40−375
t=40−500
0.04
0.02
0.00
−0.02
−0.04
−0.04 −0.02 0.00 0.02 0.04
Im{b_z[k=(2,0,0)]}
Re{b_z[k=(2,0,0)]}

We also consider MHD and HD inside a sphere, with or without rotation Ω,

$$\frac{\partial \mathbf{u}}{\partial t} + \omega \times \mathbf{u} + 2\mathbf{\Omega} \times \mathbf{u} = -\frac{1}{\rho}\nabla \mathcal{P} + \mathbf{j} \times \mathbf{B}$$

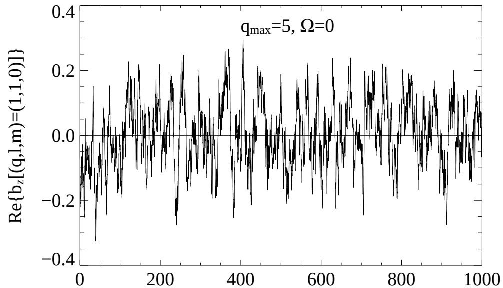
$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B})$$

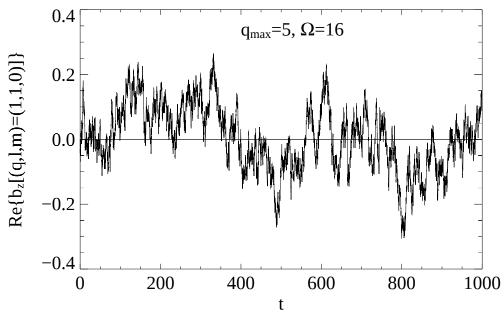
with null boundary conditions, $\mathbf{B} \cdot \hat{n} = 0 \mathbf{u} \cdot \hat{n}$ at the radius $R = 1$ of the sphere.



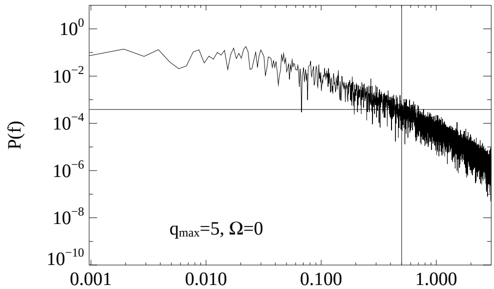
We use a Galerkin spectral code, with Chandrasekar-Kendall functions in the sphere (Dmitruk, Mininni, Pouquet, Servidio, Matthaeus, Phys. Rev. E 2011)

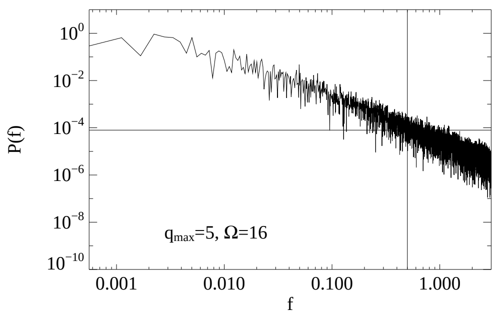
Ideal MHD in the sphere, with rotation



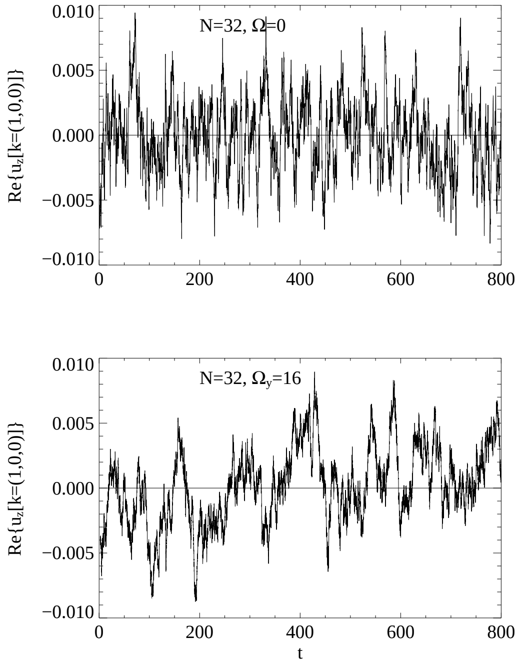


Frequency spectra

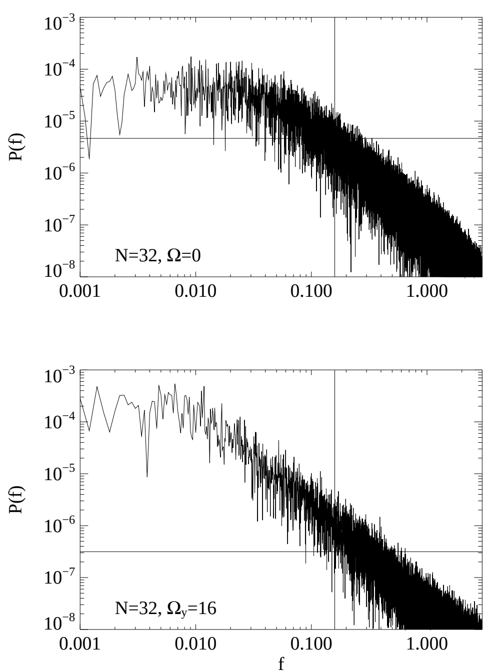




Ideal HD in the sphere, with rotation



Frequency spectra



Recall that in our defined units, $t_0 = L_0/u_0 = 1$, and the eddy turnover time (non-linear time) is

$$\tau_k = l_k/u_k = (ku_k)^{-1}$$

So, for $k = 1$, we get $\tau_k = t_0 = 1$. Following Kolmogorov scaling, $u_k \sim k^{-1/3}$ we can get

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Non-local interactions between large scale modes and a thermal bath of small scale fluctuations give long time fluctuations, i.e., low-frequency fluctuations.

$$\frac{\partial b(k)}{\partial t} = -ik \sum_{k=p+q} u(q)b(p)$$

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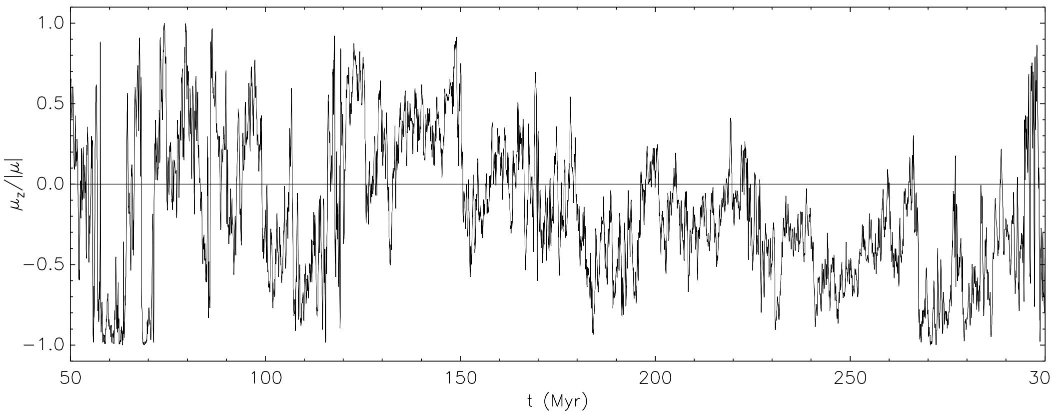
A nice example: geomagnetic field reversals

We solve ideal MHD equations inside a rotating sphere, and consider the dynamics of the magnetic dipole moment

$$\mu = \frac{1}{2}\int \mathbf{r}\times\mathbf{j}\,dV$$



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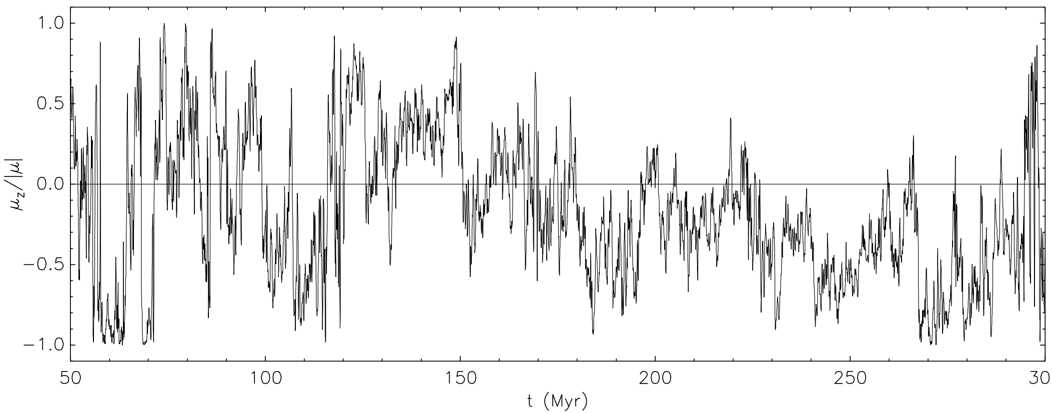
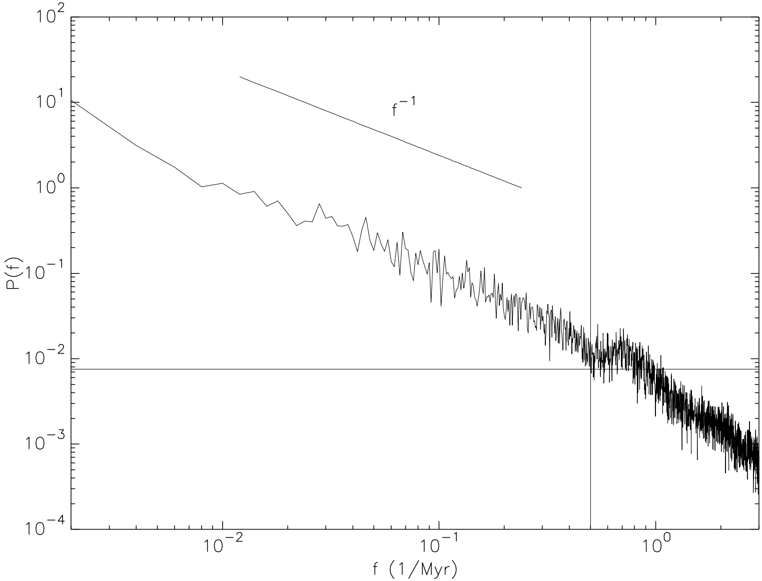


We found **reversals** !! These are long-time fluctuations...

Magnetic dipole $1/f$ frequency spectrum



Waiting time between reversals is compatible with geological observations (Cande-Kent 95).