Cosmology

Map of the infant Universe
Age = 380,000 years

Scale shows temperature. Not much structure at the time, just tiny differences in the temperature of ten parts per million.

Produced by the Planck satellite

Current Age = 13,800,000,000 years
Introduction

Questions in Cosmology

• What is the Universe made off?
• How did it start?
• What physical laws govern the dynamics?
• What objects/structures form as the Universe evolves?
• How and when did they form?
• What will happen in the future?

These question connect to fundamental physics, astronomy and astrophysics. Many have broad appeal.
The Expansion of the Universe

More distant objects are moving away from us faster, with a velocity proportional to their distance.

This is what we call the expansion of the universe.
The History of our Universe
Evolution of the density

\[ 1 + z_{eq} = \frac{1}{a_{eq}} \approx 3600 \]

\[ \Omega_m = 0.3 ; \, \Omega_{\nu} = 0.7 \]
13.7 BILLION YEARS AGO
(Universe 380,000 years old)
Thermal History

\[ 1 + \frac{z_{eq}}{a_{eq}} \approx 3600 \]
\[ \Omega_m = 0.3 \; ; \; \Omega_v = 0.7 \]
The changing Universe

Conditions in the Universe changed dramatically with time.

The history of the Universe can be used to probe physics in many different regimes including ones that have not been probed in the laboratory.

Sometimes we can do this by directly looking at different regions with our telescopes. Sometimes we can do this by analyzing fossils. Sometimes we cannot do it.
Fossils

Penzias & Wilson
The Spectrum of the CMB

\[ n_\gamma \approx 422 \, \text{cm}^{-3} \]
\[ \Omega_\gamma \approx 5 \times 10^{-5} \]
\[ T_\gamma \approx 2.7 \, \text{cm}^{-3} \]

There is also a background of neutrinos with \( T_\nu \approx 2 \, \text{K} \) and \( n_\nu \approx 115 \, \text{cm}^{-3} \). They are detected indirectly through their effect in the expansion history.
BBN: Nuclear physics applied in an expanding universe

The Formation of the light elements: Hydrogen, Helium, Deuterium, Lithium

Check in the local universe by looking at spectra

Start with protons and neutrons

Fossil from the first minutes.
Direct imaging

Scale shows temperature. Not much structure at the time, just tiny differences in the temperature of ten parts per million.

Age = 380,000 years

Current Age = 13,800,000,000 years
Age = 380,000 years

Scale shows the temperature of the universe. No real structure, just tiny differences in the temperature of ten parts per million.

Current Age = 13,800,000,000 years
Anisotropies and gravitational instability

$Z = 28.62$
Large Scale Structure of the Universe

Map of a region of the Universe around us. Each point shows the location of a galaxy.

Sloan Digital Sky Survey
Anisotropies and gravitational instability

If we know the composition we know the relevant dynamical equations but we still need to initial conditions. Initial conditions are not forgotten.

Predictions are statistical in nature.

We typically compute the amplitude of fluctuations as a function of scale or power spectrum.

Fluctuations are fossils from before the big bang itself.
The Cosmic Microwave Background as seen by Planck and WMAP

Scale shows the temperature of the universe. No real structure, just tiny differences in the temperature of ten parts per million.

Current Age = 13,800,000,000 years
Constraints on the basic six-parameter CMB temperature angular power spectrum. The error bars include cosmic variance, whose magnitude

Parameter | Best fit | 68% limits | Best fit | 68% limits
--- | --- | --- | --- | ---
µ | 0.022068 | ±0.04139 | 0.02205 | ±0.04119
K | 8344 | ±215 | 8342 | ±215
z | 12029 | ±160 | 12029 | ±160
H_0 | 3175 | ±37 | 3175 | ±37
CMB | 6825 | ±0 | 6825 | ±0
s | 0 | ±0 | 0 | ±0

Angular scale

Angular scale in degrees

Temperature fluctuation δT [µK]

Angular scale

Multipole moment l

Angular scale

D_l [µK^2]

Multipole moment, ℓ
Temperature differences on different angular scales
Observational Results

### Constraints on the basic six-parameter CDM model using Planck data. The top section contains constraints on the six primary parameters included directly in the estimation process, and the bottom section contains constraints on derived parameters.

**Cosmic Variance**

- Multipole moment, $\ell$
- Angular scale
- $D_\ell [\mu K^2]$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Best fit</th>
<th>68% limits</th>
<th>Planck</th>
<th>68% limits</th>
</tr>
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<tbody>
<tr>
<td>$\Omega_{b}h^2$</td>
<td>0.022068</td>
<td>0.02207 ± 0.00033</td>
<td>0.02203</td>
<td>0.02205 ± 0.00028</td>
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<td>$\Omega_{c}h^2$</td>
<td>0.12029</td>
<td>0.1196 ± 0.0031</td>
<td>0.12038</td>
<td>0.1199 ± 0.0027</td>
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<tr>
<td>$100\theta_{MC}$</td>
<td>1.04122</td>
<td>1.04132 ± 0.00068</td>
<td>1.04119</td>
<td>1.04131 ± 0.00063</td>
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<tr>
<td>$\tau$</td>
<td>0.0925</td>
<td>0.097 ± 0.038</td>
<td>0.0925</td>
<td>0.089 ± 0.014</td>
</tr>
<tr>
<td>$n_s$</td>
<td>0.9624</td>
<td>0.9616 ± 0.0094</td>
<td>0.9619</td>
<td>0.9603 ± 0.0073</td>
</tr>
<tr>
<td>$\ln(10^{10}A_s)$</td>
<td>3.098</td>
<td>3.103 ± 0.072</td>
<td>3.0980</td>
<td>3.089 ± 0.024</td>
</tr>
<tr>
<td>$\Omega_{\Lambda}$</td>
<td>0.6825</td>
<td>0.686 ± 0.020</td>
<td>0.6817</td>
<td>0.685 ± 0.016</td>
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<td>$\Omega_{m}$</td>
<td>0.3175</td>
<td>0.314 ± 0.020</td>
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<td>$\sigma_8$</td>
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<td>$z_{re}$</td>
<td>11.35</td>
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<td>$10^5A_s$</td>
<td>2.215</td>
<td>2.23 ± 0.16</td>
<td>2.215</td>
<td>2.196 ± 0.060</td>
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<td>$\Omega_{b}h^2$</td>
<td>0.14300</td>
<td>0.1423 ± 0.0029</td>
<td>0.14305</td>
<td>0.1426 ± 0.0025</td>
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<td>Age/Gyr</td>
<td>13.819</td>
<td>13.813 ± 0.058</td>
<td>13.8242</td>
<td>13.817 ± 0.048</td>
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<td>$z_{re}$</td>
<td>1090.43</td>
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<tr>
<td>$z_{eq}$</td>
<td>3402</td>
<td>3386 ± 69</td>
<td>3403</td>
<td>3391 ± 60</td>
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**Derived parameters**

- Constraints on the best fit model are indicated by the green shaded area.
- The low-$\ell$ CMB temperature angular power spectrum. The error bars include cosmic variance, whose magnitude is indicated by the green shaded area around the best fit model.
FRW Background

\[ ds^2 = a^2(\tau)[-d\tau^2 + dx^2] \]

\[ \mathcal{H}^2 = \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G a^2}{3} \rho \]

\[ \dot{\rho} = 3\mathcal{H}(\rho + p) \]

\[ \Omega = \frac{8\pi G}{3H^2} \rho \equiv \frac{\rho}{\rho_{\text{crit}}} \]

\[ (1 + z) = \frac{a_{\text{obs}}}{a_{\text{em}}} \]
Take to test particles separated by 10 million light years and trace their position back in time.

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<th>Distance</th>
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<td>14 Billion years</td>
<td>10 Million light years</td>
</tr>
<tr>
<td>1/3 second</td>
<td>1/2 light day</td>
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This happens because of gravity is an attractive force so the expansion of the universe has been decelerating. This implies that as you trace the atom back in time it was both closer but also moving away from us faster.
Figure 8: Conformal diagram of Big Bang cosmology. The CMB at last-scattering (recombination) consists of $10^5$ causally disconnected regions!

$$\tau_{rec} \sim 300 \text{ Mpc}$$

$$ds^2 = a^2(\tau)[-d\tau^2 + dx^2]$$

$$a(\tau) \propto \tau \quad \text{(Radiation era)}$$

$$a(\tau) \propto \tau^2 \quad \text{(Matter era)}$$
Clusters & groups

Galaxies

LSS & CMB

Non-linear scale (z=0)

Horizon Scale

$\frac{1}{H\alpha} = \frac{1}{H}$
Anisotropies in the CMB Temperature

Outline

- Basic equations
- Solution under some simplifying assumption
- Basic parameter dependences

References:

Bashinky & Seljak astro-ph/0310198
Bashinky astro-ph/0405157
Recombination

$T = 0.3 \text{ eV} \ll m_e c^2$

$\tau [h^{-1}\text{Mpc}]$

$\chi_e = n_e/(n_p + n_H)$

Hydrogen is ionized  Hydrogen is neutral

Thomson Scattering
\[ \tau_c = \frac{1}{an_e \sigma_T} \]
Perturbations

Relativistic perturbation theory

\[ ds^2 = a^2(\tau)[-(1 + 2\Phi)d\tau^2 + (1 - 2\Psi)dx^2] \]

Conformal-Newtonian

\[ T_{\mu\nu} = 0 \]

\[
\begin{align*}
T_{a0}^0 &= -(\rho_a + \delta\rho_a) \\
T_{ai}^0 &= (\rho_a + p_a)v_{ia} \\
T_{aj}^i &= \delta_j^i(p_a + \delta p_a) + (\rho_a + p_a)\Pi_{aj}^i \\
v_{ia} &= -\nabla_i u_a \\
\Pi_{aj}^i &= \frac{3}{2}(\nabla^i \nabla_j - \frac{1}{3}\delta_j^i \nabla^2)\pi_a
\end{align*}
\]

\[
\begin{align*}
\delta\rho_a &= (\rho_a + p_a)\delta_a \\
\delta p_a &= c_a^2\delta\rho_a \\
\dot{\delta}_a &= \nabla^2 u_a + 3\dot{\Psi} \\
\dot{u}_a &= c_a^2\delta_a - \mathcal{H}(1 - 3c_a^2)u_a + \nabla^2\pi_a + \dot{\Phi}
\end{align*}
\]
Perturbations

\[ ds^2 = a^2(\tau)[- (1 + 2\Phi)d\tau^2 + (1 - 2\Psi)dx^2] \]

\[ \dot{\delta}_a = \nabla^2 u_a + 3\dot{\Psi} \]

\[ d_a = \delta_a - 3\Psi \]

\[ \dot{d}_a = \nabla^2 u_a \]

\[ \dot{u}_a = c_a^2 d_a - \mathcal{H}(1 - 3c_a^2)u_a + \nabla^2 \pi_a + \Phi + 3c_a^2\Psi \]

\[ \ddot{d}_a + \mathcal{H}(1 - 3c_a^2)\dot{d}_a - c_a^2 \nabla^2 d_a - \nabla^4 \pi_a = \nabla^2(\Phi + 3c_a^2\Psi) \]

\[ \nabla^2 \Psi - 3\gamma \Psi = \gamma(d + 3\mathcal{H}u) \]

\[ \Psi - \Phi = 3\gamma \pi \]

\[ \gamma = 4\pi Ga^2(\rho + p) \]

\[ d = \sum_a x_a d_a \quad u = \sum_a x_a u_a \quad \pi = \sum_a x_a \pi_a \]

\[ x_a = (\rho_a + p_a)/(\rho + p) \]
**Figure Description**

- **Axes:**
  - **X-axis:** Temperature (T) in Kelvin (K) and Kelvin (K).
  - **Y-axis:** Density (n_e/(n_P+n_H)) and Hubble (comoving Mpc).

- **Key Points:**
  - **Recombination:** Indicated by a sharp drop in density at high temperatures.
  - **Decoupling:** Marked by a vertical line indicating the transition to free-streaming.
  - **Mean Free Path:** Illustrated by a linear increase in Hubble as temperature decreases.

- **Graphical Elements:**
  - Graph showing the evolution of density with temperature and Hubble scale.
Components

- Photon-Baryon “fluid”
- Neutrinos
- Cold Dark matter

What determines the temperature of recombination?

\[ \frac{\rho_b}{\rho_\gamma} \propto \Omega_b h^2 = \omega_b \]

Dependence is weak.

Conformal time depends on total amount of radiation and matter.

\[ \tau \propto 1/H \]
\[ H^2 \propto (\rho_\gamma + \rho_\nu + \rho_b + \rho_c) \]
Linear Theory: go to Fourier Space
Note relation between k and angle
Radiation era (Initial Conditions)

\[ \mathcal{H} = \frac{1}{\tau} \quad \gamma = \frac{1}{\tau^2} \quad c^2_{\gamma} = \frac{1}{3} \]

\[ \ddot{d}_\gamma + \frac{1}{3}k^2d_\gamma = -2k^2\Phi \]

\[ -k^2\tau^2\Phi - 6\Phi = 2d_\gamma - 6\frac{d_\gamma}{k^2\tau} \]

\[ d_\gamma'' + \frac{12}{6x + x^3}d_\gamma' + \left[ \frac{1}{3} - \frac{4}{6 + x^2} \right]d_\gamma = 0 \]

\[ x = k\tau \quad \dot{x} = \frac{d}{dx} \]

\[ x \ll 1 \rightarrow d_\gamma \propto 1/x \quad ; \quad \text{constant} \]
Radiation era (Initial Conditions)

\[ d_\gamma = d_{\gamma,ini} \left( \frac{2 \sin \varphi}{\varphi} - \cos \varphi \right) \]

\[ v_\gamma = d_{\gamma,ini} c_\gamma \left( \frac{2 - \varphi^2}{\varphi^2} \sin \varphi - 2 \varphi \cos \varphi \right) \]

\[ \Phi = d_{\gamma,ini} \left( \frac{2 (\varphi \cos \varphi - \sin \varphi)}{\varphi^3} \right) \]

\[ \varphi = c_\gamma x \]

\[ c_\gamma^2 = \frac{1}{3} \]

\[ d_\gamma = d_{\gamma,ini} \left( 1 + \frac{\varphi^2}{6} + \cdots \right) \]

\[ v_\gamma = -d_{\gamma,ini} c_\gamma \frac{\varphi}{3} + \cdots \]

\[ \Phi = d_{\gamma,ini} \left( -\frac{2}{9} + \frac{\varphi^2}{45} + \cdots \right) \]

Only relevant parameter initial amplitude.

Gravitational potential decays

Caution: Quantities outside the horizon are particularly gauge dependent.
Two fluid model

Radiation-matter universe only. Neglect neutrinos (for perturbations).

\[ a(\tau) = \bar{\tau} + \frac{\bar{\tau}^2}{4} \]
\[ \bar{\tau} = 2(\sqrt{2} - 1)\frac{\tau}{\tau_{eq}} \equiv \frac{\tau}{\tau_e} \]

\[ c_c^2 = 0 \quad \pi_c = 0 \]
\[ \ddot{d}_c + \mathcal{H}\dot{d}_c = -k^2\Phi \]

\[ \Phi = -\frac{\gamma}{k^2 + 3\gamma}(d_\gamma - 3\mathcal{H}\frac{\dot{d}}{k^2}) \]

\[ c_{\gamma b}^2 = \frac{1}{3(1 + R_b)} \]
\[ R_b = \frac{3\rho_b}{4\rho_\gamma} \quad \pi_{\gamma b} = 0 \]
\[ \ddot{d}_b + \frac{\mathcal{H}R_b}{(1 + R_b)}\dot{d}_b + c_{\gamma b}^2k^2d_{\gamma b} = -k^2\frac{2 + R_b}{(1 + R_b)}\Phi \]
Two fluid model

$k \tau_e = 20$

$k \tau_e = 2$

$k \tau_e = 0.2$
Two fluid model: Damping

\[ k\tau_e = 20 \]

\[ \ddot{d}_{\gamma b} + \frac{\mathcal{H} R_b}{(1 + R_b)} \dot{d}_{\gamma b} + 2\tau_d k^2 \dot{d}_{\gamma b} + c_{\gamma b}^2 k^2 d_{\gamma b} = -k^2 \frac{2 + R_b}{(1 + R_b)} \Phi \]

\[ \tau_d = \frac{1}{a^2 n_e \sigma_T} \left[ \frac{1}{6} - \frac{7}{45 (1 + R_b)} + \frac{1}{6 (1 + R_b)^2} \right] \]

damping \( \propto \exp[-\Gamma t] \rightarrow \exp(-k^2 \tau_d \tau) \)
\[ \ddot{\gamma} + \frac{\mathcal{H} R_b}{(1 + R_b) \dot{\gamma}} + 2\tau_d k^2 \dot{\gamma} + c_{\gamma b}^2 k^2 \dot{\gamma} = -k^2 \frac{2 + R_b}{(1 + R_b)} \Phi \]

\[ \Theta_{0, eff}^\gamma \equiv \frac{1}{3} \dot{\gamma} + \Phi + \Psi \approx A \frac{e^{-k^2 x_S^2}}{(1 + R_b)^{1/4}} \cos(kS + \delta \varphi) - R_b \Phi \]

\[ k u_{\gamma} = -\frac{\dot{\gamma}}{k} \approx A \sqrt{3} \frac{e^{-k^2 x_S^2}}{(1 + R_b)^{3/4}} \sin(kS + \delta \varphi) \]

\[ S(\tau) = \int_0^\tau c_s d\tau' \quad x_S^2(\tau) = \int_0^\tau \tau_d d\tau' \quad A \approx -\frac{d_{\gamma, ini}(1 + \Delta \gamma)}{3} \]

\[ \Phi = \Psi = -\frac{1}{5} d_{\gamma, ini} \quad \Theta_{0, eff}^\gamma \approx -\frac{1}{15} d_{\gamma, ini} \approx \frac{\Phi}{3} \]
Simple formula

\[
ds^2 = a^2(\tau)[-(1 + 2\Phi)d\tau^2 + (1 - 2\Psi)dx^2]
\]

\[
T_{\text{obs}} = T_{\text{rec}} \frac{P^\mu u_\mu|_{\text{obs}}}{P^\mu u_\mu|_{\text{rec}}}
\]

\[
\frac{dP_\mu}{d\lambda} = \frac{1}{2} \partial_\mu g_{\alpha\beta} P^\alpha P^\beta
\]

\[
\frac{1}{P_0} \frac{dP_0}{d\lambda} = \dot{\Phi} + \dot{\Psi}
\]

\[
\frac{P^\mu u_\mu|_{\text{obs}}}{P^\mu u_\mu|_{\text{rec}}} = \frac{a_{\text{rec}}}{a_{\text{obs}}} [1 + \Phi_{\text{rec}} - \Phi_{\text{obs}} - \hat{n} \cdot \vec{v}_{\text{rec}} + \hat{n} \cdot \vec{v}_{\text{obs}} + \int d\tau (\dot{\Phi} + \dot{\Psi})]
\]
Recombination

$T \ [^\circ K]$

$\chi_e$

$\tau \ [h^{-1} \text{Mpc}]$
Projection

\[
\frac{\delta T}{T}(\hat{n}) \equiv \Theta(\hat{n}) = \sum a_{lm}Y_{lm}(\hat{n}) \quad \text{ISW}
\]

\[
\Theta(\hat{n}) = \int d\tau \dot{g}\left[\frac{d_{\gamma}}{3} + \Phi + \Psi - \hat{n} \cdot \vec{v}_\gamma\right] + g(\dot{\Phi} + \dot{\Psi})
\]

\[
\Theta(\hat{n}) = \int \frac{d^3k}{(2\pi)^3} \zeta(\vec{k}) \int d\tau \dot{g}[\Theta_0^{eff} + u_{\gamma} \frac{\partial}{\partial \tau_0} + g(\dot{\Phi} + \dot{\Psi})]e^{i(\tau_0 - \tau)\vec{k} \cdot \hat{n}}
\]
Two fluid model

\[ \ddot{\gamma} + \frac{\mathcal{H} R_b}{(1 + R_b)} \dot{\gamma} + 2 \tau_d k^2 \dot{\gamma} + c^2 \gamma k^2 \dot{\gamma} = -k^2 \frac{2 + R_b}{(1 + R_b)} \Phi \]

\[ \Theta^0_{\text{eff}} = \frac{1}{3} \dot{\gamma} + \Phi + \Psi \approx A \frac{e^{-k^2 x^2_S}}{(1 + R_b)^{1/4}} \cos(k S + \delta \varphi) - R_b \Phi \]

\[ ku_\gamma = -\frac{\dot{\gamma}}{k} \approx A \sqrt{3} \frac{e^{-k^2 x^2_S}}{(1 + R_b)^{3/4}} \sin(k S + \delta \varphi) \]

\[ S(\tau) = \int_0^\tau c_s d\tau' \quad x^2_S(\tau) = \int_0^\tau \tau_d d\tau' \quad A \approx -\frac{d_{\gamma,\text{ini}}(1 + \Delta \gamma)}{3} \]

\[ \Phi = \Psi = -\frac{1}{5} d_{\gamma,\text{ini}} \quad \Theta^0_{\text{eff}} \approx -\frac{1}{15} d_{\gamma,\text{ini}} \approx \frac{\Phi}{3} \]
Two fluid model

Beware:
The observed spectrum will have additional damping due to the finite width of the last scattering surface.
Planck Spectra

We now know what the state of the plasma was at recombination. We still need to connect it with what we observe.

What is the relation of the peaks in the previous transparency and the power spectrum?
Projection

\[ \langle \zeta(k_1)\zeta(k_2) \rangle = (2\pi)^3 \delta^D(k_1 - k_2) P_\zeta(k) \]

\[ C_l = \frac{2}{\pi} \int k^2 dk P_\zeta(k) \int d\tau [\dot{g} (\Theta_0^{eff} + u_\gamma \frac{\partial}{\partial \tau_0}) + g(\dot{\Phi} + \dot{\Psi})] j_l(k(\tau_0 - \tau)) \]

\[ C_l \approx \frac{2}{\pi} \int k^2 dk P_\zeta(k) |\Theta_0^{eff}|^2 j_l(kD)^2 + |k u_\gamma|^2 j'_l(kD)^2 \]

\[ \int \frac{dx}{x} j_l(x)^2 = \frac{1}{2l(l + 1)} \]

\[ \int \frac{dx}{x} j'_l(x)^2 = \frac{1}{6(l - 1)(l + 2)} \]

\[ \int \frac{dx}{x} j_l(x)j'_l(x) = \frac{\pi}{(2l - 1)(2l - 1)(2l + 3)} \]
Projection Kernels

Structure on scales

\[ k \sim \frac{1}{\tau_{rec}} \]

Structure on scales

\[ k \sim \frac{1}{(\tau_0 - \tau_{rec})} \]
### Angular scale

**Cosmic Variance**

![Image of angular scale plot](image)

### Observational Results

**Recombination**

**Late times**

**Initial Conditions**

**Derived parameters**

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Parameter dependencies

http://space.mit.edu/home/tegmark/cmb/movies.html
What is Cosmic Variance?

\[ P[a_{lm}] \]

\[ C_l[\text{cosmological parameters}] \]

(2l+1) if had all sky

\[ \Theta(\hat{n}) = \int \frac{d^3k}{(2\pi)^3} \zeta(\vec{k}) \int d\tau [g(\Theta_0^{eff} + u, \frac{\partial}{\partial \tau_0}) + g(\Phi + \dot{\Psi})] e^{i(\tau_0 - \tau)\vec{k} \cdot \hat{n}} \]

\[ \langle \zeta(k_1)\zeta(k_2) \rangle = (2\pi)^3 \delta^{D}(k_1 - k_2)P_{\zeta}(k) \]

Statistical nature of predictions is also an issue when dealing with anomalies
Planck constrains on LCDM

Fig. 2. Comparison of the base ΛCDM model parameters for Planck+lensing only (colour-coded samples), and the 68% and 95% constraint contours adding WMAP low-ℓ polarization (WP; red contours), compared to WMAP-9 (Bennett et al. 2012; grey contours).
Geometric Degeneracy

**Fig. 25.** The Planck+WP+highL data combination (samples; colour-coded by the value of $H_0$) partially breaks the geometric degeneracy between $\Omega_m$ and $\Omega_\Lambda$ due to the effect of lensing in the temperature power spectrum. These limits are significantly improved by the inclusion of the Planck lensing reconstruction (black contours). Combining also with BAO (right; solid blue contours) tightly constrains the geometry to be nearly flat.
Additional observable from recombination: polarization

Secondary effects: Lensing, ISW, SZ effect, kSZ, polarization from the EOR
Planck Collaboration: Planck 2013 results. XVIII. Gravitational lensing-infrared background correlation

Fig. 4. Temperature maps of size 1 deg$^2$ at 545 and 857 GHz stacked on the 20,000 brightest peaks (left column), troughs (centre column) and random map locations (right column). The stacked (averaged) temperature maps is in K. The arrows indicate the lensing deflection angle deduced from the gradient of the band-pass filtered lensing potential map stacked on the same peaks. The longest arrow corresponds to a deflection of 6.3$^0$, which is only a fraction of the total deflection angle because of our filtering. This stacking allows us to visualize in real space the lensing of the CMB by the galaxies that generate the CIB. A small and expected set (10) was corrected by hand when displaying the deflection field. We have verified in simulations that this is due to noise in the stacked lensing potential map that shifts the peak. As expected, we see that the temperature maxima of the CIB, which contain a larger than average number of galaxies, deflect light inward, i.e., they correspond to gravitational potential wells, while temperature minima trace regions with fewer galaxies and deflect light outward, i.e., they correspond to gravitational potential hills.

5. Statistical and systematic error budget

The first pass of our pipeline suggests a strong correlation of the CIB with the CMB lensing potential. We now turn to investigate the strength and the origin of this signal. We will first discuss the different contributions to the statistical error budget in Sect. 5.1, and then possible systematic effects in Sect. 5.2. Although the most straightforward interpretation of the signal is that it arises from dusty star-forming galaxies tracing the large-scale mass distribution, in Sect. 5.3 we consider other potential astrophysical origins for the observed correlation.

5.1. Statistical error budget

In this section we discuss any noise contribution that does not lead to a bias in our measurement. The prescription adopted throughout this paper is to obtain the error estimates from the naive Gaussian analytical error bars calculated using the measured auto-spectra of the CIB and lensing potential. We find that these errors are approximately equal to 1.2 times the naive scatter within a `-bin, and we will sometimes use this prescription where appropriate for convenience (as will be stated in the text). This is justified in Appendix A where we consider six different methods of quantifying the statistical errors using both simulations and data. The Gaussian analytical errors, $\hat{C}_{TT}$, are calculated using the naive prescription

$$f_{\text{sky}}(2^\prime + 1)^{-\frac{1}{2}} = \hat{C}_{TT} = \hat{C}_{TT} + C_{TT}^\prime,$$

where as before $f_{\text{sky}}$ is the fraction of the sky that is unmasked, $^\prime = 126$ for our 15 linear bins between $^\prime = 100$ and $^\prime = 2000$, $\hat{C}_{TT}$ and $\hat{C}_{TT}$ are the spectra measured using the data, and $C_{TT}$ is the model cross spectrum. This last term provides a negligible...
\[ \delta \beta = -2 \delta \chi \nabla_{\perp} \Psi, \]

\[ \delta \theta_\chi = \frac{f_K(\chi_* - \chi) \delta \beta}{f_K(\chi_*)} = -\frac{f_K(\chi_* - \chi)}{f_K(\chi_*)} 2 \delta \chi \nabla_{\perp} \Psi \]

\[ f_K(\chi) = \begin{cases} K^{-1/2} \sin(K^{1/2} \chi) & \text{for } K > 0, \text{ closed}, \\ \chi & \text{for } K = 0, \text{ flat}, \\ |K|^{-1/2} \sinh(|K|^{1/2} \chi) & \text{for } K < 0, \text{ open}. \end{cases} \]

\[ \alpha = -2 \int_0^{\chi_*} d\chi \frac{f_K(\chi_* - \chi)}{f_K(\chi_*)} \nabla_{\perp} \Psi(\chi \hat{n}; \eta_0 - \chi). \]
The remaining integral is generally small, and the lensed spectrum only deviates from scale invariance at the largest scales. This is important because the acoustic oscillations and small scale damping give a well defined non-scale-invariant structure to the power spectrum. Asymptotic result of Eq. (4.16) (dashed). Bottom: the fractional change in the unlensed spectrum (dotted), and the lensed spectrum (solid) compared to the large scale acoustic oscillations of the temperature power spectrum only deviates from scale invariance at the largest scales, and the unlensed spectrum (dotted), and the lensed spectrum (solid) compared to the large scale acoustic oscillations of the temperature power spectrum. The calculation was first done in Ref. (90) for the temperature we did in Section 4.2, so we shall not elaborate here. The calculation is rather similar to the one for the temperature we did in Section 4.2, so we shall not elaborate here. The calculation is rather similar to the one for the temperature we did in Section 4.2, so we shall not elaborate here.

5.3.2 Lensed polarization correlation functions

To do this, we want to describe the polarization in the physical basis, about the similar parts of the CDM model.

We shall work from the spin-2 polarization field \( P_{αβ} \) defined by the polarization at \( x \) making an angle \( φ \) with the reference direction, this amounts to rotating the basis by an angle \( r \)

\[
φ \rightarrow φ + r
\]


\[
\frac{l(l+1)C_ℓ^α}{2πK^2} \approx \frac{(1 + 0.15l^{-1/2})}{l^{-1/2}}
\]

If the structures are scale invariant, the change in the polarization in the deflection angle that we have used in the previous section is not expected to be very accurate on small scales, so for a non-scale-invariant structure to the power spectrum, Fig. 6. Top: the lensed power spectrum (solid) and the unlensed spectrum (dotted), and the lensed spectrum (solid) compared to the large scale acoustic oscillations of the temperature power spectrum.
**Geometric Degeneracy**

![WMAP 7 yr](image)

**Fig. 25.** The Planck+WP+highL data combination (samples; colour-coded by the value of $H_0$) partially breaks the geometric degeneracy between $\Omega_m$ and $\Omega_\Lambda$ due to the effect of lensing in the temperature power spectrum. These limits are significantly improved by the inclusion of the Planck lensing reconstruction (black contours). Combining also with BAO (right; solid blue contours) tightly constrains the geometry to be nearly flat.
\(\langle \Theta(l) \Theta^*(1 - L) \rangle_{\Theta} = \delta(L) C_l^{\Theta} - \int \frac{d^2l'}{2\pi} \left[ l' \cdot (1 - l') \psi(1 - l') \langle \Theta(l') \Theta^*(1 - L) \rangle ight. \\
+ \left. l' \cdot (1 - L - l') \psi^*(1 - L - l') \langle \Theta(l) \Theta^*(l') \rangle \right] + \mathcal{O}(\psi^2) \\
= \delta(L) C_l^{\Theta} + \frac{1}{2\pi} \left[ (L - 1) \cdot L C_l^{\Theta} + l \cdot L C_l^{\Theta} \right] \psi(L) + \mathcal{O}(\psi^2). \quad (7.1)\)

\(\hat{\psi}(L) \equiv N(L) \int \frac{d^2l}{2\pi} \hat{\Theta}(l) \hat{\Theta}^*(1 - L) g(l, L),\)

**Fig. 1.** An exaggerated example of the lensing effect on a \(10^6 \times 10^6\) field. Top: (left-to-right) unlensed temperature field, unlensed E-polarization field, spherically symmetric deflection field \(d(\mathbf{n})\). Bottom: (left-to-right) lensed temperature field, lensed E-polarization field, lensed B-polarization field. The scale for the polarization and temperature fields differ by a factor of 10.
$A_L^{\phi} = 0.99 \pm 0.05 \quad (68\%; \text{Planck+lensing+WP+highL}),$
Fig. 15. Two views of the geometric degeneracy in curved $\Lambda$CDM models which is partially broken by lensing. *Left*: the degeneracy in the $\Omega_m$-$\Omega_\Lambda$ plane, with samples from Planck+WP+highL colour coded by the value of $H_0$. The contours delimit the 68% and 95% confidence regions, showing the further improvement from including the lensing likelihood. *Right*: the degeneracy in the $\Omega_K$-$H_0$ plane, with samples colour coded by $\Omega_\Lambda$. Spatially-flat models lie along the grey dashed lines.
CIB-Lensing

545 GHz

Fig. 4. Temperature maps of size 1 deg$^2$ at 545 and 857 GHz stacked on the 20,000 brightest peaks (left column), troughs (centre column) and random map locations (right column). The stacked (averaged) temperature maps is in K. The arrows indicate the lensing deflection angle deduced from the gradient of the band-pass filtered lensing potential map stacked on the same peaks. The longest arrow corresponds to a deflection of 6.3", which is only a fraction of the total deflection angle because of our filtering. This stacking allows us to visualize in real space the lensing of the CMB by the galaxies that generate the CIB. A small and expected offset ($\approx 1'$) was corrected by hand when displaying the deflection field.
The Sunyaev Zeldovich effect

A map of the hot gas in the Universe

Planck catalog of clusters of galaxies

'Bridge' of hot gas connecting two galaxy clusters
The Polarization of the Cosmic Microwave Background

DASI
Overview

- Physical origin of polarization
- The information encoded by polarization
- Summary
The Anisotropies are polarized

Kovac et al.
astro-ph/0209478
The polarization and temperature patterns are correlated.
How is polarization generated?

Thomson Scattering

\[ I_1 \propto \sigma_{Th} \cos^2 \theta \]

\[ I_2 \propto \sigma_{Th} \]
Recombination

$T = 0.3 \text{ eV} \ll m_e c^2$

Thomson Scattering

$\frac{x_e}{n_e/(n_p + n_H)}$

$\tau [h^{-1} \text{Mpc}]$

Hydrogen is ionized  Hydrogen is neutral
The quadrupole created by density perturbations

\[ v(x_0 + \lambda_T \hat{n}) \quad d \sim \lambda_T \]

Doppler shift

\[ \hat{n} \cdot [v(x_0 + \lambda_T \hat{n}) - v(x_0)] \sim \lambda_T \hat{n}_i \hat{n}_j v_{i,j} \]

Polus \sim k \lambda_T v \sim k \lambda_T c_s \delta_{\gamma}
Peaks are out of phase

Polarization peaks at smaller scales

\[ \dot{\delta}_\gamma = -\frac{4}{3} k v_\gamma + 4 \phi \]
When were perturbations created?

Sharp acoustic peaks are difficult to create without inflation

Causal Seeds

Pen, Seljak & Turok (1997)

Negative peak imply fluctuations come from outside horizon

Hu & White (1996)
Spergel & MZ (1997)
Scatterings

+

Anisotropies


Density Pert.

Gravity Waves

Polarization
The quadrupole created by GW

\[ \frac{1}{\nu} \frac{d\nu}{d\tau} \propto \frac{1}{2} (1 - \cos^2 \theta) e^{\pm i2\phi} \dot{h}_t(\tau) \]

\[ P \propto \delta \tau_D \dot{h}_t(\tau_D) \sim k \delta \tau_D \ h_t(\tau_D) \]
Anisotropies created by GW

\[ h_t(k\tau) \propto \frac{3j_1(k\tau)}{k\tau} \]

ISW like effect, produced after recombination

Polarization is produced at recombination
Density pert. & Gravity Waves

\begin{align*}
Q > 0 & \quad U = 0 \\
E < 0 \\
Q < 0 & \quad U = 0 \\
E > 0
\end{align*}

Gravity Waves

\begin{align*}
Q = 0 & \quad U < 0 \\
B < 0 \\
Q = 0 & \quad U > 0 \\
B > 0
\end{align*}
After Recombination

Tight Coupling

Free Streaming

Observer Today
1. The height of the bump is given by the optical depth
2. The quadrupole is produced by the free streaming of the monopole at recombination, $k \sim 2/\left(\tau_{reio} - \tau_{rec}\right)$ produces the biggest quadrupole
3. Angular scale $l \sim (\tau_o - \tau_{reio})/(\tau_{reio} - \tau_{rec})$
Gravitational Lensing

Lensing induced B modes

Q > 0  U = 0
E < 0

Q = 0  U < 0
B < 0

Q = 0  U > 0
B > 0

[\{l(l+1)c/(2\pi)^2\}]^{1/2}

Temperature
Density modes
Polarization
Gravity Waves

B lensing
Structure formation
$N_{200} \sim 3 \times 10^7$

Navarro et al 2006
The Core vs. Cusp problem

Weinberg et al. 1306.0913

Fig. 1. The cusp-core problem. (Left) An optical image of the galaxy F568-3 (small inset, from the Sloan Digital Sky Survey) is superposed on the dark matter distribution from the “Via Lactea” cosmological simulation of a Milky Way-mass cold dark matter halo (Diemand et al. 2007). In the simulation image, intensity encodes the square of the dark matter density, which is proportional to annihilation rate and highlights low mass substructure. (Right) The measured rotation curve of F568-3 (points) compared to model fits assuming a cored dark matter halo (blue solid curve) or a cuspy dark matter halo with an NFW profile (red dashed curve, concentration $c = 9.2$, $V_{200} = 110\, \text{km\,s}^{-1}$). The dotted green curve shows the contribution of baryons (stars+gas) to the rotation curve, which is included in both model fits. An NFW halo profile overpredicts the rotation speed in the inner few kpc. Note that the rotation curve is measured over roughly the scale of the 40 kpc inset in the left panel.
The Satellite problem
The Satellite problem

Weinberg et al. 1306.0913

Fig. 2. The missing satellite and "too big to fail" problems. (Left) Projected dark matter distribution (600 kpc on a side) of a simulated, $10^{12} M_\odot$ CDM halo (Garrison-Kimmel, Boylan-Kolchin, & Bullock, in preparation). As in Figure 1, the numerous small subhalos far exceed the number of known Milky Way satellites. Circles mark the nine most massive subhalos. (Right) Spatial distribution of the "classical" satellites of the Milky Way. The central densities of the subhalos in the left panel are too high to host the dwarf satellites in the right panel, predicting stellar velocity dispersions higher than observed. The diameter of the outer sphere in the right panel is 300 kpc; relative to the simulation prediction (and to the Andromeda galaxy) the Milky Way’s satellite system is unusually centrally concentrated (Yniguez et al. 2013).
Fig. 1.— The nine rising curves show the largest virialized mass scale as a function of time for different values of $Q$. Structures with $M \lesssim M_{eq}$ (horizontal line) are seen to all virialize about a factor $Q^{-3/2}$ after the end of the radiation-dominated epoch (shaded, left), whereas for later times, the virialized mass scale asymptotes to about $Q^{3/2}$ times the horizon mass (shaded, upper left). Cooling is inefficient in the
The composite X-ray/optical image (556 kpc on a side) of the galaxy cluster Abell 1689 at redshift $z = 0.18$. The purple haze shows X-ray emission of the $T \sim 10^8$ K gas, obtained by the Chandra X-ray Observatory. Images of galaxies in the optical band, colored in yellow, are from observations performed with the Hubble Space Telescope. The long arcs in the optical image are caused by the gravitational lensing of background galaxies by matter in the galaxy cluster, the largest system of such arcs ever found (Credit: X-ray: NASA/CXC/MIT; Optical: NASA/STScI).

(b) The galaxy cluster SPT-CL J2106-5844 at $z = 1.133$, the most massive cluster known at $z > 1$ discovered via its Sunyaev-Zel’dovich (SZ) signal ($M_{200} \approx 1.3 \times 10^{15} M_\odot$). The color image shows the Magellan/LDSS3 optical and Spitzer/IRAC mid-IR measurements (corresponding to the blue-green-red color channels). The frame subtends $4.8 \times 4.8$ arcmin, which corresponds to $2.4 \times 2.4$ Mpc at the redshift of the cluster. The solid yellow contours correspond to the South Pole Telescope SZ significance values, as labeled, where dashed yellow contours are used for the negative significance values. Adapted from Foley et al. 2011.

Images of Abell 1835 ($z = 0.25$) at (a) X-ray, (b) optical, and (c) millimeter wavelengths, exemplifying the regular multiwavelength morphology of a massive, dynamically relaxed cluster. All three images are centered on the X-ray peak position and have the same spatial scale, 5.2 arcmin or $\sim1.2$ Mpc on a side (extending out to $\sim r_{200}$; Mantz et al. 2010a). Figure credits: (a) X-ray: Chandra X-ray Observatory/A. Mantz; (b) optical: Canada-France-Hawaii Telescope/A. von der Linden et al.; (c) millimeter: Sunyaev Zel’dovich Array/D. Marrone.
Counting clusters

Fig. 1. The distribution on the sky of the Planck SZ cluster sub-sample used in this paper, with the 35% mask overlaid.

Planck cluster counts: The $\sigma_8$ problem

constraints from Planck primary CMB

constraints from Planck SZ clusters

Fig. 2. Constraints on the mass variance $\sigma_8$ as a function of the matter density parameter $\Omega_m$, from Planck primary CMB and Planck SZ clusters.
Counting galaxies

Fig. 1 Role of feedback in modifying the galaxy luminosity function
The motivation comes from the gravitational instability of cold gas-rich disks, which provides the scaling, although the normalization depends on feedback physics. For the global law, in terms of SFR and gas mass per unit area, SN regulation provides the observed efficiency of about 2% which fits essentially all local star–forming galaxies. One finds from simple momentum conservation that \[ \text{SFE} = \frac{v_{\text{cool}} \times m_{\text{SN}}}{\text{E}_{\text{initial}}}. \]

Here, \( v_{\text{cool}} \) is the SN-driven swept-up shell velocity at which approximate momentum conservation sets in and \( m_{\text{SN}} \) is the mass formed in stars per SNII, in this case for a Chabrier (2003) initial mass function (IMF). This is a crude estimator of the efficiency of SN momentum input into the interstellar medium, but it reproduces the observed global normalization of the star formation law.

The fit applies not only globally but to star formation complexes in individual galaxies such as M51 and also to starburst galaxies. The star formation law is known as the Schmidt-Kennicutt law (Kennicutt et al., 2007), and its application reveals that molecular gas is the controlling gas ingredient. In the outer parts of galaxies, where the molecular fraction is reduced due to the ambient UV radiation field and lower surface density, the SFR per unit gas mass also declines (Bigiel et al., 2011).

For disk instabilities to result in cloud formation, followed by cloud agglomeration and consequent star formation, one also needs to maintain a cold disk by accretion of cold gas. There is ample evidence of a supply of cold gas, for example in the M33 group. Other spiral galaxies show extensive reservoirs of HI in their outer regions, for example NGC 6946 (Boomsma et al., 2008) and UGC 2082 (Heald et al.,

Fig. 2 Schmidt-Kennicutt laws on nearby (including Local Group galaxies as shaded regions) and distant galaxies, as well as Milky Way Giant Molecular Clouds (Krumholz et al., 2012). The solid line is similar to equation (1).

Fig. 8 Black hole mass versus spheroid velocity dispersion (luminosity-weighted within one effective radius), from McConnell et al. (2011).
Fig. 5.— The reconstructed matter power spectrum: the stars show the power spectrum from combining ACT and WMAP data (top panel). The solid and dashed lines show the nonlinear and linear power spectra respectively from the best-fit CDM model with spectral index of $n_s = 0.96$ computed using CAMB and HALOFIT (Smith et al. 2003). The data points between $0.02 < k < 0.19$ Mpc$^{-1}$ show the SDSS DR7 LRG sample, and have been deconvolved from their window functions, with a bias factor of 1.18 applied to the data. This has been rescaled from the Reid et al. (2010) value of 1.3, as we are explicitly using the Hubble constant measurement from Riess et al. (2011) to make a change of units from $h Mpc$ to Mpc. The constraints from CMB lensing (Das et al. 2011), from cluster measurements from ACT (Sehgal et al. 2011), CCCP (Vikhlinin et al. 2009) and BCG halos (Tinker et al. 2011), and the power spectrum constraints from measurements of the Lyman–$\alpha$ forest (McDonald et al. 2006) are indicated. The CCCP and BCG masses are converted to solar mass units by multiplying them by the best-fit value of the Hubble constant, $h = 0.738$ from Riess et al. (2011). The bottom panel shows the same data plotted on axes where we relate the power spectrum to a mass variance, $\Delta M^2/M$, and illustrates how the range in wavenumber $k$ (measured in Mpc$^{-1}$) corresponds to range in mass scale of over 10 orders of magnitude. Note that large masses correspond to large scales and hence small values of $k$. This highlights the consistency of power spectrum measurements by an array of cosmological probes over a large range of scales.
Neutrinos

![Graphs showing CMB lensing power spectrum and shear correlation function ξ+ from Planck and SPT, compared to predictions from CFHTLenS.](image)

**FIG. 1:** The CMB lensing power spectrum (top) data points from *Planck* (green squares) and SPT (blue squares) and the shear correlation function ξ+ from CFHTLenS (bottom), compared to predictions for parameters from samples of the *Planck* CMB+WP+BAO MCMC chains with non-linear corrections [18, 19]. In both cases, the data is systematically lower than theory, although the significance is somewhat lower than the eye would suggest in the case of CFHTLenS due to correlations between data points.

![Probability distributions for total neutrino mass Σmν.](image)

**FIG. 2:** Marginalized likelihoods for Σmν. The datasets are colour coded in the legend, but the solid line is for (I), the dashed line is for (II) and the dotted line is for (III). It is clear that inclusion of lensing leads to a preference for Σmν > 0 which is compatible with that coming from the SZ cluster counts and that there is a strong preference (~ 4σ) in the case of dataset (III).