# Effective Field Theories

## Lecture 1

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May 2013 / Brazil

# Outline

- Introduction
- Reasons for using an EFT
- Dimensional Analysis and Power Counting
- Examples
- Loops
- Decoupling
- Field Redefintions and the Equations of Motion
- One-loop matching in a HQET example
- SCET

# Basic Idea

You can make quantitative predictions of observable phenomena without knowing everything.

The computations have some small (non-zero) error.

Can improve on the accuracy by adding a finite number of additional parameters, in a systematic way.

Key concept is locality — as a result one can factorize quantities into some short distance parameters (coefficients in the Lagrangian), and long distance operator matrix elements.

# Examples

Chemistry and atomic physics depend on the interactions of atoms.

The interaction Hamiltonian contains non-relativistic electrons and nuclei interacting via a Coulomb potential, plus electromagnetic radiation.

The only property of the nucleus we need is the electric charge $Z$.

The quark structure of the proton, weak interactions, GUTs, etc. are irrelevant.

A more accurate calculation includes recoil corrections and needs $m_p$.

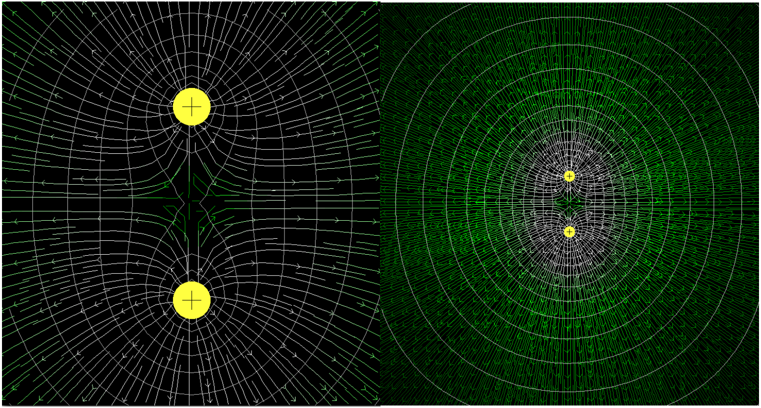
The Hyperfine interaction needs $\mu_p$

Charge radius, ...

Weak interactions, ...

If one is interested in atomic parity violation, weak interactions are the leading contribution, and cannot be treated as a small correction.

# Multipole Expansion



The field far away looks just like a point charge.

$$V(r) = \sum c_{lm} Y_{lm}(\Omega) \frac{1}{r} \left(\frac{a}{r}\right)^l$$

At the classical level, expand in $a/r$.

$c_{lm}$ expected to be of order unity, once $a^l$ has been factored out.

Need more multipoles for a better description of the field.

Effective theory is a local quantum field theory with a finite number of low energy parameters.

There is a systematic expansion in a small parameter like $a/r$ for the multipole expansion. [called power counting]

Keep as many terms as you need to reach the desired accuracy.

It is a quantum theory — one can compute radiative corrections (loops), renormalize the theory, etc. just as for QED or QCD.

All the non-trivial effects are due to quantum corrections. Otherwise, just series expand.

EFT is the low-energy limit of a “full theory”

It is not a Lagrangian with form-factors $e \to e\, F(q^2/M^2)$

These are non-local, contain an infinite amount of information, and lead to a violation of power counting.

It is not just a series expansion of amplitudes in the full theory

$$F(q^2/M^2) \to F(0) + F'(0)\frac{q^2}{M^2} + \ldots$$

though it looks like this at tree-level.

The EFT is an interacting quantum theory in its own right.

One can compute using it without ever referring to the full theory from which it came.

The EFT has a different divergence structure from the full theory. The renormalization procedure is part of the definition of a field theory, not some irrelevant detail.

If you are given a full theory, can compute the EFT Lagrangian — matching.

# Examples of EFT

In some cases, one can compute the EFT from a more fundamental theory (typically, if it is weakly coupled).

- The Fermi theory of weak interactions is an expansion in $p/M_W$, and can be computed from the $SU(2) \times U(1)$ electroweak theory in powers of $1/M_W$, $\alpha_s(M_W)$, $\alpha(M_W)$ and $\sin^2\theta$.
- The heavy quark Lagrangian (HQET) can be computed in powers of $\alpha_s(m_Q)$ and $1/m_Q$ from QCD.
- NRQCD/NRQED
- SCET

# Examples

Chiral perturbation theory: Describes the low energy interactions of mesons and baryons.

The full theory is QCD, but the relation between the two theories (and the degrees of freedom) is non-perturbative.

$\chi$PT has parameters that are fit to experiment. Has been enormously useful.

Standard Model — don't know the more fundamental theory, and we all hope there is one.

Can use EFT ideas to parameterize new physics in terms of a few operators in studying, for example, precision electroweak measurements.

# Dependence on high energy

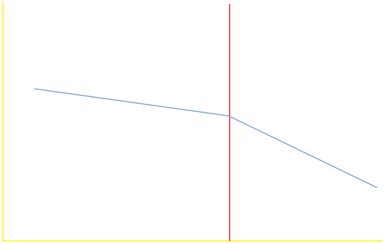
High energy dynamics irrelevant:

H energy levels do not depend on $m_t$ — but this depends on what is held fixed as $m_t$ is varied.

Usually, one takes low energy parameters such as $m_p$, $m_e$, $\alpha$ from low energy experiments, and then uses them in the Schrödinger equation.

But instead, hold high energy parameters such as $\alpha(\mu)$ and $\alpha_s(\mu)$ fixed at $\mu \gg m_t$.

$$m_t \frac{\mathrm{d}}{\mathrm{d}m_t}\left(\frac{1}{\alpha}\right) = -\frac{1}{3\pi}$$



The proton mass also depends on the top quark mass,

$$m_p \propto m_t^{2/27}$$

There are constraints from the symmetry of the high energy theory:

For example, the chiral lagrangian preserves $C$, $P$ and $CP$ because QCD does.

More interesting case: Non-relativistic quantum mechanics satisfies the spin-statistics theorem because of causality in QED.

# Reasons for using EFT

- Every theory is an effective theory: Can compute in the standard model, even if there are new interactions at (not much) higher energies.
- Greatly simplifies the calculation by only including the relevant interactions: Gives an explicit power counting estimate for the interactions.
- Deal with only one scale at a time: For example the $B$ meson decay rate depends on $M_W$, $m_b$ and $\Lambda_{\rm QCD}$, and one can get horribly complicated functions of the ratios of these scales. In an EFT, deal with only one scale at a time, so there are no functions, only constants.

- Makes symmetries manifest: QCD has spontaneously broken chiral symmetry, which is manifest in the chiral Lagrangian, and heavy quark spin-flavor symmetry which is manifest in HQET. These symmetries are only true for certain limits of QCD, and so are hidden in the QCD Lagrangian.
- Sum logs: Use renormalization group improved perturbation theory. The running of constants is not small, e.g. $\alpha_s(M_Z) \sim 0.118$ and $\alpha_s(m_b) \sim 0.22$. Fixed order perturbation theory breaks down. Sum logs of the ratios of scales (such as $M_W/m_b$).

- Efficient way to characterize new physics: Can include the effects of new physics in terms of higher dimension operators. All the information about the dynamics is encoded in the coefficients. [This also shows it is difficult to discover new physics using low-energy measurements.]
- Include non-perturbative effects: Can include $\Lambda_{\rm QCD}/m$ corrections in a systematic way through matrix elements of higher dimension operators. The perturbative corrections and power corrections are tied together. [Renormalons]

# Dimensional Analysis

Effective Lagrangian (neglect topological terms)

$$L = \sum c_i O_i = \sum L_D$$

is a sum of local, gauge and Lorentz invariant operators.

The functional integral is

$$\int \mathcal{D}\phi \; e^{iS}$$

so $S$ is dimensionless.

Kinetic terms:

$$S = \int \mathrm{d}^d x \, \bar{\psi} \, i \not{D} \, \psi, \qquad S = \int \mathrm{d}^d x \, \frac{1}{2} \partial_\mu \phi \, \partial^\mu \phi$$

so

$$0 = -d + 2 \, [\psi] + 1, \qquad 0 = -d + 2 \, [\phi] + 2$$

Dimensions given by

$$[\phi] = (d-2)/2, \qquad [\psi] = (d-1)/2, \qquad [D] = 1, \qquad [g A_\mu] = 1$$

Field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + \ldots$ so $A_\mu$ has the same dimension as a scalar field.

$$[g] = 1 - (d-2)/2 = (4-d)/2$$

In $d = 4$,

$$[\phi] = 1, \qquad [\psi] = 3/2, \qquad [A_\mu] = 1, \qquad [D] = 1, \qquad [g] = 0$$

Only Lorentz invariant renormalizable interactions (with $D \leq 4$) are

$$\begin{aligned} D = 0: &\quad 1 \\ D = 1: &\quad \phi \\ D = 2: &\quad \phi^2 \\ D = 3: &\quad \phi^3, \bar\psi\psi \\ D = 4: &\quad \phi\bar\psi\psi, \phi^4 \end{aligned}$$

and kinetic terms which include gauge interactions.

Renormalizable interactions have coefficients with mass dimension $\geq 0$.

In $d = 2$,

$$[\phi] = 0, \qquad [\psi] = 1/2, \qquad [A_\mu] = 0, \qquad [D] = 1, \qquad [g] = 1$$

so an arbitrary potential $V(\phi)$ is renormalizable. Also $(\bar\psi\psi)^2$ is renormalizable.

In $d = 6$,

$$[\phi] = 2, \qquad [\psi] = 5/2, \qquad [A_\mu] = 2, \qquad [D] = 1, \qquad [g] = -1$$

Only allowed interaction is $\phi^3$.

# What Fields to use for EFT?

Not always obvious: Low energy QCD described in terms of meson fields.

NRQCD/NRQED and SCET: Naive guess does not work. Need multiple gluon fields.

Effective Lagrangian:

$$L_D = \frac{O_D}{M^{D-d}}$$

so in $d = 4$,

$$L_{\text{eft}} = L_{D \le 4} + \frac{O_5}{M} + \frac{O_6}{M^2} + \dots$$

An infinite number of terms (and parameters)

# Power Counting

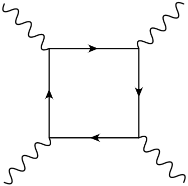
If one works at some typical momentum scale $p$, and neglects terms of dimension $D$ and higher, then the error in the amplitudes is of order

$$\left(\frac{p}{M}\right)^{D-4}$$

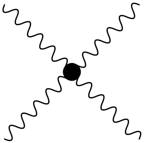
A non-renormalizable theory is just as good as a renormalizable theory for computations, provided one is satisfied with a finite accuracy.

Usual renormalizable case given by taking $M \to \infty$.

# Photon-Photon Scattering



(a)



(b)

$$L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\alpha^2}{m_e^4} \left[ c_1 \left( F_{\mu\nu} F^{\mu\nu} \right)^2 + c_2 \left( F_{\mu\nu} \tilde{F}^{\mu\nu} \right)^2 \right].$$

(Terms with only three field strengths are forbidden by charge conjugation symmetry.)

$e^4$ from vertices, and $1/16\pi^2$ from the loop.

An explicit computation gives

$$c_1 = \frac{1}{90}, \qquad c_2 = \frac{7}{90}.$$

Scattering amplitude

$$A \sim \frac{\alpha^2 \omega^4}{m_e^4}$$

and

$$\sigma \sim \left( \frac{\alpha^2 \omega^4}{m_e^4} \right)^2 \frac{1}{\omega^2} \frac{1}{16\pi} \sim \frac{\alpha^4 \omega^6}{16\pi m_e^8} \times \frac{15568}{22275}$$

$$A \propto \frac{1}{m_e^4}$$

determined by the operator dimension.

# Proton Decay

The lowest dimension operator in the standard model which violates baryon number is dimension 6. Natural explanation of baryon number conservation.

$$L \sim \frac{qql}{M_G^2}$$

This gives the proton decay rate $p \to e^+ \pi^0$ as

$$\Gamma \sim \frac{m_p^5}{16\pi M_G^4}$$

or

$$\tau \sim \left( \frac{M_G}{10^{15}\,\text{GeV}} \right)^4 \times 10^{30} \text{ years}$$

# Neutrino Masses

The lowest dimension operator in the standard model which gives a neutrino mass is dimension five,

$$\mathcal{L} \sim \frac{(HL)^2}{M_S}$$

This gives a Majorana neutrino mass of ($v \sim 246\ \text{GeV}$)

$$m_\nu \sim \frac{v^2}{M_S}$$

or a seesaw scale of $6 \times 10^{15}$ GeV for $m_\nu \sim 10^{-2}$ eV.

Absolute scale of masses not known. Only $\Delta m^2$ measured.

# Rayleigh Scattering

Scattering of light from atoms

$$L = \psi^\dagger \left( i\partial_t - \frac{p^2}{2M} \right) \psi + a_0^3 \, \psi^\dagger \psi \left( c_1 E^2 + c_2 B^2 \right)$$

$$A \sim c_i \, a_0^3 \, \omega^2$$

$$\sigma \propto a_0^6 \, \omega^4 .$$

Scattering goes as the fourth power of the frequency, so blue light is scattered about 16 times mores strongly than red.

$a_0^3$ dimensional analysis.