Effective Field Theories

Lecture 4

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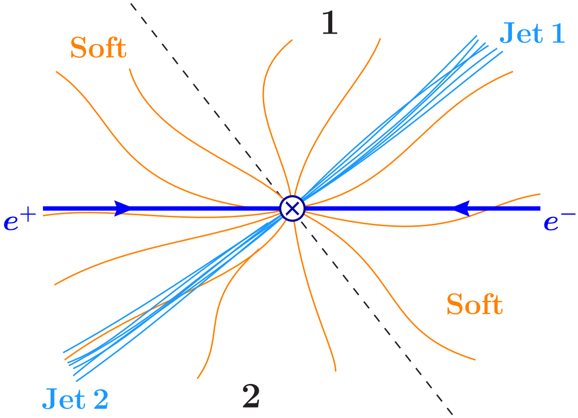
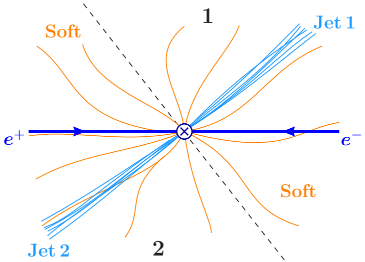
SCET

Soft-Collinear Effective Theory — describes energetic particles.

$$e^+e^- \to q\bar{q} \to \text{jet} + \text{jet}$$

Two-Jet events





$$E_{\rm CM}^2 = Q^2 \gg M_{\rm jet}^2 \gg \Lambda_{\rm QCD}^2$$

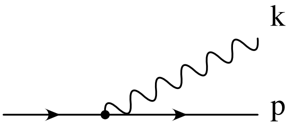
Narrow energetic jets.

Power counting:

$$M_{\rm jet}^2 = Q^2\lambda^2 \qquad \Lambda_{\rm QCD}^2 = Q^2\lambda^4 = \frac{M^4}{Q^2} \qquad \lambda = \sqrt{\frac{\Lambda_{\rm QCD}}{Q}}$$

Theory is called $\text{SCET}_{\rm I}$.

Infrared Singularities in Radiation



The intermediate propagator is

$$\frac{1}{(p+k)^2 - m^2} = \frac{1}{2p\cdot k + k^2} = \frac{1}{2E_p\,\omega_k - 2\,|\mathbf{p}|\,|\mathbf{k}|\cos\theta}$$

For massless particles, $E_p = |\mathbf{p}|$ and $\omega_k = |\mathbf{k}|$

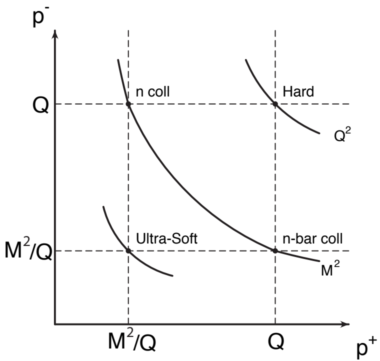
$$2E\omega\,(1-\cos\theta)$$

singularities as $\omega \to 0$ (soft) and $\theta \to 0$ (collinear).

SCET degrees of freedom (modes)

$$p^+ = E - p_z, \qquad p^- = E + p_z$$

- Light Cone Coordinates:
- Hard Modes: $p^2 \sim Q^2$
  integrated out
- Collinear modes: $p^2 \sim M^2$
- Ultra-Soft modes: $p^2 \sim M^4/Q^2$



Sudakov Double Logs

In exclusive processes, there are two powers of

$$\mathsf{L} = \ln Q^2/M^2$$

at each order in perturbation theory.

These are the Sudakov double-logarithms. Lead to a large radiative corrections and a rapid breakdown of fixed order perturbation theory.

General perturbative structure of $F_E(Q)$

$$L = \log Q^2/M^2$$

$$\begin{aligned}
F_E(Q) = &\quad 1 && \text{LO} \\
&+ \alpha_s^1 \left(L^2 + L^1 + L^0 \right) && \text{NLO} \\
&+ \alpha_s^2 \left(L^4 + L^3 + L^2 + L^1 + L^0 \right) && \text{N}^2\text{LO} \\
&+ \alpha_s^3 \left(L^6 + L^5 + L^4 + L^3 + L^2 + L^1 + L^0 \right) && \text{N}^3\text{LO}
\end{aligned}$$

The α_s^n term has powers of L up to L^{2n}. $2n + 1$ terms at order n

- The $\alpha_s L^2$, $\alpha_s^2 L^4$, $\alpha_s^3 L^6$ series is called LL_{FO}.
- The $\alpha_s L$, $\alpha_s^2 L^3$, $\alpha_s^3 L^5$ series is called NLL_{FO}.

Structure of series

The series for $\log F_E(Q^2)$ takes a simpler form

$$\begin{aligned}\log F_E &= \alpha_s \left(\mathsf{L}^2 + \mathsf{L} + \mathsf{L}^0\right) \\ &\quad +\alpha_s^2 \left(\mathsf{L}^3 + \mathsf{L}^2 + \mathsf{L} + \mathsf{L}^0\right) \\ &\quad +\alpha_s^3 \left(\mathsf{L}^4 + \ldots + \mathsf{L}^0\right) + \ldots\end{aligned}$$

with the α_s^n term having power of L upto L^{n+1}. $n+2$ terms at order n.

RGE counting: L f_0 is LL, f_1 is NLL, etc.

$$\log F_E = \mathsf{L}\, f_0(\alpha_s \mathsf{L}) + f_1(\alpha_s \mathsf{L}) + \alpha_s\, f_2(\alpha_s \mathsf{L}) + \ldots$$

Resummation

$$A = \begin{pmatrix} 1 & & & & \\ \alpha_s L^2 & \alpha_s L & \alpha_s & & \\ \alpha_s^2 L^4 & \alpha_s^2 L^3 & \alpha_s^2 L^2 & \alpha_s^2 L & \alpha_s^2 \\ \alpha_s^3 L^6 & & \dots & & \\ \vdots & & & & \end{pmatrix}$$

In the leading-log regime $L \sim 1/\alpha_s$, the various terms are of order

$$A = \begin{pmatrix} 1 & & & & \\ \frac{1}{\alpha_s} & 1 & \alpha_s & & \\ \frac{1}{\alpha_s^2} & \frac{1}{\alpha_s} & 1 & \alpha_s & \alpha_s^2 \\ \frac{1}{\alpha_s^3} & & \dots & & \\ \vdots & & & & \end{pmatrix}.$$

Resummation: Exponentiated Form

Exponentiated form:

$$\log A = \begin{pmatrix} \alpha_s L^2 & \alpha_s L & \alpha_s & & \\ \alpha_s^2 L^3 & \alpha_s^2 L^2 & \alpha_s^2 L & \alpha_s^2 & \\ \alpha_s^3 L^4 & \alpha_s^3 L^3 & \alpha_s^3 L^2 & \alpha_s^3 L & \alpha_s^3 \\ \alpha_s^4 L^5 & & \ldots & & \\ \vdots & & & & \end{pmatrix}$$

In the leading-log regime:

$$\log A = \begin{pmatrix} \frac{1}{\alpha_s} & 1 & \alpha_s & & \\ \frac{1}{\alpha_s} & 1 & \alpha_s & \alpha_s^2 & \\ \frac{1}{\alpha_s} & 1 & \alpha_s & \alpha_s^2 & \alpha_s^3 \\ \frac{1}{\alpha_s} & & \ldots & & \\ \vdots & & & & \end{pmatrix}.$$

Resummation: Exponentiated Form

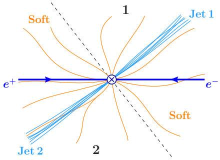
$$
\begin{aligned}
\log A &= \frac{1}{\alpha_s} f_0 + f_1 + \alpha_s f_2 + \ldots \\
&= \frac{1}{\alpha_s}\left[f_0 + \alpha_s f_1 + \alpha_s^2 f_2 + \ldots\right]
\end{aligned}
$$

so that f_1 and f_2 are corrections to $\log A$. However,

$$
\begin{aligned}
A &= \exp\left[\frac{1}{\alpha_s} f_0 + f_1 + \alpha_s f_2 + \ldots\right] \\
&= e^{\frac{1}{\alpha_s} f_0} \times e^{f_1} \times e^{\alpha_s f_2} \times \ldots
\end{aligned}
$$

Must include the LL and NLL series.

Notation



Introduce null vectors

$$n^\mu = (1, \mathbf{n}) \qquad \bar{n}^\mu = (1, -\mathbf{n}) \qquad \mathbf{n} \cdot \mathbf{n} = 1$$

which satisfy

$$n^2 = 0, \qquad \bar{n}^2 = 0, \qquad \bar{n} \cdot n = 2.$$

If there are many directions, use

$$n_i^\mu = (1, \mathbf{n}_i) \qquad \bar{n}_i^\mu = (1, -\mathbf{n}_i)$$

In the back-to-back case, $\bar{n}_1 = n_2$, $\bar{n}_2 = n_1$.

Light-Cone Coordinates

$$p^+ = n \cdot p \qquad p^- = \bar{n} \cdot p, \qquad n \cdot p_\perp = \bar{n} \cdot p_\perp = 0$$

$\perp$ only has two components.

$$\begin{aligned} p^\mu &= \frac{1}{2}(\bar{n} \cdot p)\, n^\mu + \frac{1}{2}(n \cdot p)\, \bar{n}^\mu + p_\perp^\mu \\ &= \frac{1}{2} p^- n^\mu + \frac{1}{2} p^+ \bar{n}^\mu + p_\perp^\mu \end{aligned}$$

$$a \cdot b = \frac{1}{2} a^+ b^- + \frac{1}{2} a^- b^+ + a_\perp \cdot b_\perp$$

If $\mathbf{n} = \hat{\mathbf{z}}$

$$p^+ = E - p_z \qquad p^- = E + p_z$$

$$p^2 = p^+ p^- + p_\perp^2 = m^2$$

For an energetic particle $E \gg m$ moving in the $+z$ direction,

$$p^+ = E - p_z \approx \frac{m^2}{2E} \qquad\qquad p^- = E + p_z \approx 2E$$

Pick **n** to be near the direction of the particle:

SCET power counting $\lambda \ll 1$

$$n - \text{collinear}: \qquad \left(p^+ \sim \lambda^2 Q, p^- \sim 1 Q, p_\perp \sim \lambda Q\right)$$

$$\bar{n} - \text{collinear}: \qquad \left(p^+ \sim 1 Q, p^- \sim \lambda^2 Q, p_\perp \sim \lambda Q\right)$$

n and $\bar{n}$ collinear quarks and gluons.

Can also have ultrasoft gluons:

$$\left(p^+ \sim \lambda^2 Q, p^- \sim \lambda^2 Q, p_\perp \sim \lambda^2 Q\right)$$

$$n - \text{collinear}: \qquad p_n = \left(p^+ \sim \lambda^2 Q, p^- \sim 1 Q, p_\perp \sim \lambda Q\right)$$

$$\bar{n} - \text{collinear}: \qquad p_{\bar{n}} = \left(p^+ \sim 1 Q, p^- \sim \lambda^2 Q, p_\perp \sim \lambda Q\right)$$

$$\text{usoft}: \qquad p_{us} = \left(p^+ \sim \lambda^2 Q, p^- \sim \lambda^2 Q, p_\perp \sim \lambda^2 Q\right)$$

$$p_n^2 \sim p_{\bar{n}}^2 \sim Q^2\lambda^2 \qquad\qquad p_{us}^2 \sim Q^2\lambda^4$$

$$p_n + p_n \to p_n \qquad p_{\bar{n}} + p_{\bar{n}} \to p_{\bar{n}} \qquad p_{us} + p_{us} \to p_{us}$$

$$p_n + p_{us} \to p_n \qquad\qquad p_{\bar{n}} + p_{us} \to p_{\bar{n}}$$

$$p_n + p_{\bar{n}} \to (Q, Q, \lambda Q) \qquad\qquad (p_n + p_{\bar{n}})^2 \sim Q^2 \quad \text{hard}$$

γ-matrix convention

$$\boldsymbol{\gamma} = \begin{pmatrix} 0 & \boldsymbol{\sigma} \\ -\boldsymbol{\sigma} & 0 \end{pmatrix}, \qquad \gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Then

$$\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\sigma^{0i} = i\gamma^0\gamma^i = \begin{pmatrix} -i\sigma^i & 0 \\ 0 & i\sigma^i \end{pmatrix}$$

$$\sigma^{ij} = i\gamma^i\gamma^j = \begin{pmatrix} \sigma^k & 0 \\ 0 & \sigma^k \end{pmatrix}$$

$n = (1,0,0,1)$, $\bar{n} = (1,0,0,-1)$

$$\not{n} = \begin{pmatrix} 0 & 1-\sigma^3 \\ 1+\sigma^3 & 0 \end{pmatrix} \qquad \not{\bar{n}} = \begin{pmatrix} 0 & 1+\sigma^3 \\ 1-\sigma^3 & 0 \end{pmatrix}$$

$$P_n = \frac{\not{n}\not{\bar{n}}}{4} = \begin{pmatrix} \frac{1-\sigma^3}{2} & 0 \\ 0 & \frac{1+\sigma^3}{2} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$P_{\bar{n}} = \frac{\not{\bar{n}}\not{n}}{4} = \begin{pmatrix} \frac{1+\sigma^3}{2} & 0 \\ 0 & \frac{1-\sigma^3}{2} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$P_n = \frac{\not{n}\not{\bar{n}}}{4}$$
$$P_{\bar{n}} = \frac{\not{\bar{n}}\not{n}}{4}$$

P_n and $P_{\bar{n}}$ are two orthogonal projection operators.

$$1 = P_n + P_{\bar{n}} \qquad P_n^2 = P_n \qquad P_{\bar{n}}^2 = P_{\bar{n}} \qquad 0 = P_n P_{\bar{n}} = P_{\bar{n}} P_n$$

Also

$$\psi = P_n\,\psi \qquad \overline{\psi} = \overline{\psi}\,P_{\bar{n}}$$

The Dirac equation is

$$\begin{pmatrix} -m & E - \boldsymbol{p}\cdot\boldsymbol{\sigma} \\ E + \boldsymbol{p}\cdot\boldsymbol{\sigma} & -m \end{pmatrix} \begin{pmatrix} \phi \\ \chi \end{pmatrix} = 0$$

Helicity $h = \pm 1$:

$$\boldsymbol{p}\cdot\boldsymbol{\sigma}\,\psi = h\,|\mathbf{p}|\,\psi$$

For $E > 0, \mathbf{p} = p\,\hat{\mathbf{z}}, h = 1$:

$$\sqrt{2E}\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} e^{-iE(t-z)}$$

$E > 0, \mathbf{p} = p\,\hat{\mathbf{z}}, h = -1$:

$$\sqrt{2E}\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} e^{-iE(t-z)}$$

For antiparticles, $E < 0$.
For an antiparticle moving in the $+z$ direction, $\mathbf{p} = -p\hat{\mathbf{z}}$
For an antiparticle with helicity h, $\boldsymbol{p} \cdot \boldsymbol{\sigma}\, \psi = h\, |\mathbf{p}|\, \psi$
For $E < 0, \mathbf{p} = -p\,\hat{\mathbf{z}}, h = 1$:

$$\sqrt{2E} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} e^{iE(t-z)}$$

$E < 0, \mathbf{p} = -p\,\hat{\mathbf{z}}, h = -1$:

$$\sqrt{2E} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} e^{iE(t-z)}$$

particle with $h = +1$ and antiparticle with $h = -1$ ($\gamma_5 = 1$):

$$\sqrt{2E} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} e^{\mp iE(t-z)}$$

particle with $h = -1$ and antiparticle with $h = 1$ ($\gamma_5 = -1$):

$$\sqrt{2E} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} e^{\mp iE(t-z)}$$

Which one decided by sign of E in $e^{\mp iEt}$.

$$1 = \frac{\not{n}\not{\bar{n}}}{4} + \frac{\not{\bar{n}}\not{n}}{4}$$
$$\psi = \xi_n + \Xi_n$$

Ξ_n is the small component, and can be integrated out.

$$\xi_n = \frac{\not{n}\not{\bar{n}}}{4}\,\xi_n = P_n\,\xi_n$$

is the SCET spinor.

$$\not{n}\,\xi_n = \not{n}P_n\,\xi_n = \not{n}\frac{\not{n}\not{\bar{n}}}{4}\,\xi_n = 0$$

In HQET,

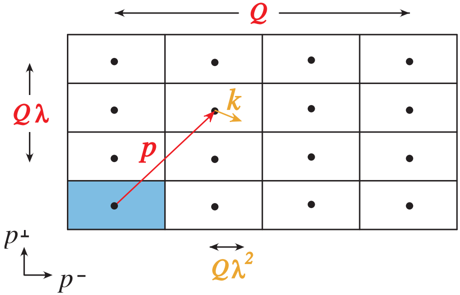
$$P^\mu = m_Q v^\mu + k^\mu$$

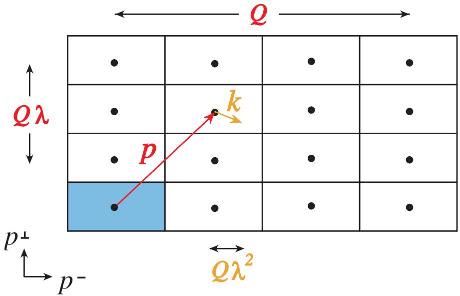
In SCET,

$$P^\mu = (0, p^-, p_\perp) + k^\mu$$

$$p^- \sim 1 \qquad p_\perp \sim \lambda \qquad k^\mu \sim \lambda^2$$

p is called the label momentum. Note that $p \neq 0$ (zero-bin)





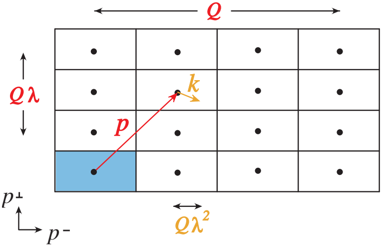
$$\int \mathrm{d}P \to \sum_p \int \mathrm{d}k$$

In dimensional regularization, can integrate over $k \in (-\infty, \infty)$

At a vertex:

$$\delta(P_1 - P_2) \to \delta_{p_1,p_2}\, \delta(k_1 - k_2)$$

$$\psi(x) = \int \frac{d^4P}{(2\pi)^4}\delta(P^2)\theta(P^0)\left[u(p,h)a(p,h)e^{-iP\cdot x} + v(p,h)b^\dagger(p,h)e^{iP\cdot x}\right]$$
$$= \psi^{(+)}(x) + \psi^{(-)}(x)$$



$$\psi^{(+)}(x) = \sum_{p\neq 0} e^{-ip\cdot x}\,\xi^{(+)}_{n,p}(x)$$
$$\psi^{(-)}(x) = \sum_{p\neq 0} e^{ip\cdot x}\,\xi^{(-)}_{n,p}(x)$$

$$\psi^{(+)}(x) = \sum_p e^{-ip\cdot x}\xi^{(+)}_{n,p}(x)$$
$$\psi^{(-)}(x) = \sum_p e^{ip\cdot x}\xi^{(-)}_{n,p}(x)$$

where the labels on the fields are $n, p = (0, p^-, p_\perp)$
$\bar{n}\cdot p = p^- = E + p \approx 2E > 0$.

$\xi^{(\pm)}_{n,p}(x)$ is the field for n-collinear particles: P^μ close to p^μ (within λ^2).

Use spinors that satisfy

$$\not{n}\,\xi^{(\pm)}_{n,p} = 0$$

The small corrections to the spinors are expanded out.

Define (note the minus sign)

$$\xi_{n,p}(x) = \xi^{(+)}_{n,p}(x) + \xi^{(-)}_{n,-p}(x)$$

So that

$\bar{n} \cdot p > 0$: destroy particles
$\bar{n} \cdot p < 0$: create antiparticles

Define label momentum operator $\mathcal{P}$,

$$\mathcal{P}^\mu \, \xi_{n,p}(x) = p^\mu \xi_{n,p}$$

so that the total momentum is $\mathcal{P} + i\partial$.

$$\xi_n(x) = e^{-ip\cdot x}\, \xi_{n,p}(x)$$

and

$$i\partial^\mu\, \xi_n(x) = e^{-ip\cdot x}\, (\mathcal{P} + i\partial)^\mu\, \xi_{n,p}(x)$$

The $e^{-ip\cdot x}$ factors at an interaction vertex cancel by label momentum conservation.

Can apply the same procedure to gluons,

$$A^\mu_{n,q}(x), \qquad \left[A^\mu_{n,q}(x)\right]^* = A^\mu_{n,-q}(x),$$

$\bar{n}\cdot q > 0$ destroy gluons, and $\bar{n}\cdot q < 0$ create gluons.

QCD propagator:

$$\frac{\not{p}}{p^2} = \frac{\frac{1}{2}\not{n}\,\bar{n}\cdot p + \frac{1}{2}\not{\bar{n}}\,n\cdot p + \not{p}_\perp}{p^+p^- + (p_\perp)^2}$$

$$n - \text{collinear} : \left(p^+ \sim \lambda^2 Q,\ p^- \sim 1\,Q,\ p_\perp \sim \lambda Q\right)$$

SCET propagator:

$$\frac{\not{p}}{p^2} \to \frac{\frac{1}{2}\not{n}\,\bar{n}\cdot p}{p^+p^- + (p_\perp)^2}$$
$$\to \frac{\frac{1}{2}\not{n}\,\bar{n}\cdot p}{(p^+ + k^+)\,p^- + (p_\perp)^2}$$

using $p \to p + k$, $k \sim \lambda^2 Q^2$.

$$\mathcal{L} = \overline{\psi}\left(i\not{D}\right)\psi$$

$$\psi = e^{-ip\cdot x}\left[\xi_{n,p}(x) + \Xi_{n,p}(x)\right]$$

p is the label momentum, k is the Fourier transform of x.

$$\mathcal{L} = \left[\overline{\xi}_{n,p}(x)\ P_{\bar{n}} + \overline{\Xi}_{n,p}(x)\ P_n\right]\left(\not{p} + i\not{D}\right)\left[P_n\ \xi_{n,p}(x) + P_{\bar{n}}\ \Xi_{n,p}(x)\right]$$

$$P_{\bar{n}}\ \not{p}\ P_n = \frac{\bar{\not{n}}\not{n}}{4}\left[\frac{1}{2}\not{n}\bar{n}\cdot p + \frac{1}{2}\bar{\not{n}}n\cdot p + \not{p}_\perp\right]\frac{\not{n}\bar{\not{n}}}{4}$$

$$\not{n}\not{n} = 0 \qquad \not{n}\not{p}_\perp\not{n} = 0 \qquad \not{n}\bar{\not{n}}\not{n} = 2(n\cdot\bar{n})\not{n} = 4\not{n}$$

$$P_{\bar{n}}\, \not{p}\, P_n = \frac{\not{\bar{n}}}{4}\,[4\not{n}]\,\frac{\not{\bar{n}}}{4}\, \left(\frac{1}{2} n\cdot p\right) = \not{\bar{n}}\, P_n \left(\frac{1}{2} n\cdot p\right)$$

$$P_n\, \not{p}\, P_{\bar{n}} = \not{n}\, P_{\bar{n}} \left(\frac{1}{2} \bar{n}\cdot p\right)$$

$$\begin{aligned} P_n\, \not{p}\, P_n &= \frac{\not{n}\not{\bar{n}}}{4}\left[\frac{1}{2}\not{n}\bar{n}\cdot p + \frac{1}{2}\not{\bar{n}} n\cdot p + \not{p}_\perp\right]\frac{\not{n}\not{\bar{n}}}{4} \\ &= \frac{\not{n}\not{\bar{n}}}{4}\left[\not{p}_\perp\right]\frac{\not{n}\not{\bar{n}}}{4} = \not{p}_\perp P_n \end{aligned}$$

$$\mathcal{L} = \bar{\xi}_{n,p}(x) \frac{\bar{\not{n}}}{2} (in \cdot D) \, \xi_{n,p}(x) + \bar{\Xi}_{n,p}(x) \frac{\not{n}}{2} (\bar{n} \cdot p + i\bar{n} \cdot D) \, \Xi_{n,p}(x)$$
$$+ \bar{\Xi}_{n,p}(x) \left(\not{p}_\perp + i\not{D}_\perp \right) \xi_{n,p}(x) + \bar{\xi}_{n,p}(x) \left(\not{p}_\perp + i\not{D}_\perp \right) \Xi_{n,p}(x)$$

The Ξ field has a kinetic term of order Q, and can be integrated out:

$$\mathcal{L} = \bar{\xi}_{n,p}(x) \left[(in \cdot D) + \left(\not{p}_\perp + i\not{D}_\perp \right) \frac{1}{\bar{n} \cdot p + i\bar{n} \cdot D} \left(\not{p}_\perp + i\not{D}_\perp \right) \right] \frac{\bar{\not{n}}}{2} \, \xi_{n,p}(x)$$

A similar term for $\bar{n}$-collinear quarks with $n \leftrightarrow \bar{n}$

The gluon field A_μ is split into three pieces

$$A^\mu \to A^\mu_{us} + A^\mu_n + A^\mu_{\bar n}$$

The scaling is

$$\begin{aligned} A^\mu_n &: \left(\lambda^2, 1, \lambda\right) Q \\ A^\mu_{\bar n} &: \left(1, \lambda^2, \lambda\right) Q \\ A^\mu_n &: \left(\lambda^2, \lambda^2, \lambda^2\right) Q \end{aligned}$$

$$A^\mu_n(x) = e^{-iq\cdot x} A_{n,q}(x)$$

$\bar{n}$-collinear gluons do not couple to n-collinear quarks, since the resultant quark is off-shell by Q^2.
If we use the label operator $\mathcal{P}$, and define

$$iD_c^\mu = \mathcal{P}^\mu - gT^a A_c^\mu$$

$$\mathcal{L} = \bar{\xi}_{n,p}(x) \left\{ [in \cdot (D_{us} + D_n)] + \left(i\not{D}_{n,\perp} \right) \frac{1}{i\bar{n} \cdot D_n} \left(i\not{D}_{\perp,n} \right) \right\} \frac{\not{\bar{n}}}{2}\, \xi_{n,p}(x)$$

Drop the D_{us} in the $\perp$ terms and denominator.

Infinite number of collinear gluon interactions from the $1/(i\bar{n} \cdot D_n)$ interaction.