São Paulo International Schools on Theoretical Physics
Nonlinear Optics and Nanophotonics

Lecturers for Week 1

Prof. Cid B. de Araújo, Recife, Brazil
Prof. Sergey K. Turitsyn, Birmingham, UK
Dr. Miguel C. Soriano, Palma de Mallorca, Spain
Prof. Marcel Clerc, Santiago, Chile

Prof. Yuri S. Kivshar, Canberra, Australia
Lecture 1: Introduction to Nonlinear Spectroscopy

Lecture 2: Basics of Random Lasers
Nonlinear technologies in fibre optics

Prof. Sergey Turitsyn, UK

Lecture 1 – Introduction to nonlinear fibre optics
Lecture 2 – Raman technologies in optical communications
Lecture 3 – Nonlinear effects in optical communications
Dynamics and applications of delay-coupled semiconductor lasers

Dr. Miguel C. Soriano, Spain

From basic properties to applications:
- Introduction to semiconductor lasers with delayed optical feedback
- Introduction to synchronization of networks of delay-coupled semiconductor lasers
- Applications of chaotic semiconductor lasers

Spatiotemporal chaotic localized states in optics

Prof. Marcel Clerc, Chile

http://www.dfi.uchile.cl/marcel/
Guided Optics, Solitons, and Metamaterials

Prof. Yuri Kivshar, Australia

- Linear and nonlinear guided-wave optics
- Introduction to solitons; optical solitons
- Metamaterials: history and promises

http://wwwrsphyssse.anu.edu.au/nonlinear
Linear and Nonlinear Guided-Wave Photonics

- Optical waveguides
- Waveguide dispersion
- Pulse propagation in waveguides
- Optical nonlinearities
- Self-phase modulation
- Phase matching and harmonic generation
- Plasmonics
- Photonic crystals
From electronics to photonics

- **Electronic components**
  Speed of processors is saturated due to high heat dissipation, frequency dependent attenuation, crosstalk, impedance matching, etc.

- **Photonic integration**
  Light carrier frequency is 100,000 times higher, therefore a potential for faster transfer of information

- **Photonic interconnects**
  already demonstrate advantages of photonics for passive transfer of information
The photonic chip

Processing of the information all-optically

Need to scale down the dimensions

Waveguides: photonic wires

The incident and reflected wave create a pattern that does not change with z – wg mode
Photonic elements

Splitter
What happens to the light in a waveguide

- **Waveguide propagation losses**
  Light can be dissipated or scattered as it propagates

- **Dispersion**
  Different colours travel with different speed in the waveguide

- **Nonlinearities at high powers**
  At high power, the light can change the refractive index of the material that changes the propagation of light.
Waveguide loss: mechanisms

- Intrinsic/material
- Scattering due to inhomogeneities:
  - Rayleigh scattering: $\alpha_R \sim \lambda^{-4}$;
  - Side wall roughness
- Waveguide bending
Waveguide loss: description

\[ P(z) = P_0 \exp(-\alpha z) \]

\( \alpha \) [cm\(^{-1}\)] — attenuation constant

\[ \alpha_{dB} = -10 \log_{10} \left( \frac{P(z)}{P_0} \right) \]

3 dB loss = 50% attenuation

Often propagation loss is measured in dB/cm

\[ \alpha_{dB/cm} = -\frac{10}{L} \log_{10} \left( \frac{P(L)}{P_0} \right) = 4.343\alpha \]

Typical loss for waveguides 0.2 dB/cm for fibres 0.2 dB/km
Dispersion - Mechanisms

- Material (chromatic)
- Waveguide
- Polarisation
- Modal
Material dispersion

- Related to the characteristic resonance frequencies at which the medium absorbs the electromagnetic radiation through oscillations of bound electrons.

Sellmeier equation (far from resonances)

\[ n^2(\lambda) = 1 + \sum_{j=1}^{m} \frac{B_j \lambda^2}{\lambda^2 - \lambda_j^2}, \]

where \( \lambda_j \) are the resonance wavelengths and \( B_j \) are the strength of \( j \)th resonance.

- For short pulses (finite bandwidth): different spectral components will travel with different speed \( c/n(\lambda) \) giving rise to Group Velocity Dispersion (GVD).
Group velocity dispersion

Accounted by the dispersion of the propagation constant:

\[ \beta(\omega) = n(\omega) \frac{\omega}{c} = \beta_0 + \beta_1 (\omega - \omega_0) + \frac{1}{2} \beta_2 (\omega - \omega_0)^2 + \cdots, \]

\[ \beta_m = \left( \frac{d^m \beta}{d \omega^m} \right)_{\omega = \omega_0} \quad (m = 0, 1, 2, \ldots). \]

\[ \beta_1 = \frac{1}{v_g} = \frac{n_g}{c} = \frac{1}{c} \left( n + \omega \frac{dn}{d\omega} \right), \]

\( v_g \) is the group velocity, \( n_g \) is the group index

GVD is quantified by the dispersion parameter

\[ D = \frac{d\beta_1}{d\lambda} \approx \frac{\lambda}{c} \frac{d^2 n}{d\lambda^2} \]

measured in \([\text{ps/(km nm)}]\)

D>0 – anomalous dispersion; D<0 – normal dispersion
At different wavelengths the mode has a different shape. This geometrical consideration leads to shift in the dispersion curves. The effect is more pronounced in high index and narrow waveguides, e.g. photonic nanowires.
Usual waveguides are strongly birefringent, therefore the propagation constants for x and y polarisation will be different.

The two polarisations will travel with different speed inside the waveguide

\[ \Delta T = \left| \frac{L}{v_{gx}} - \frac{L}{v_{gy}} \right| = L |\beta_{1x} - \beta_{1y}| \]

Time delay between two pulses of orthogonal polarisation
A pulse is a superposition (interference) of monochromatic waves:

\[ A(z, t) = \int_{-\infty}^{\infty} A(z, \omega) \exp(i \omega t) d\omega \]

Each of these components will propagate with slightly different speed, but also their phase will evolve differently and the pulse will be modified:

velocity \neq \text{ph. velocity} and duration (profile) will change
Group velocity

- As a result of the dispersion, the pulse (the envelope) will propagate with a speed equal to the group velocity.

\[ v_g \equiv \frac{d\omega}{dk} = \frac{c}{n} \left( 1 - \frac{\lambda}{n} \frac{dn}{d\lambda} \right) \]

- One can define a group index as \( n_g = \frac{c}{v_g} \)

\[ n_g = \frac{c}{v_g} = \left( n - \frac{\lambda}{d\lambda} \right) \]

Possibility for slow, superluminal, or backward light

Index which the pulse will feel
Pulse broadening

The finite bandwidth ($\delta \lambda$) of the source leads to a spread of the group velocities $\delta v_g$:

$$\delta v_g = \frac{dv_g}{d\lambda} \delta \lambda = \frac{c\lambda}{n^2} \left( \frac{d^2 n}{d\lambda^2} - \frac{2}{n} \left( \frac{dn}{d\lambda} \right)^2 \right)$$

Then a short pulse will experience a broadening $\delta t$ after propagation $L$ in the material:

$$\delta t = \frac{L}{v_g} \frac{\delta v_g}{v_g} = LD \delta \lambda$$

where $$D = \frac{\lambda}{c} \left( \frac{d^2 n}{d\lambda^2} \right).$$

Dispersion coefficient
Pulse chirp

Normal dispersion

Anomalous dispersion

Positive chirp

Negative chirp
Short pulse propagation in dispersive media

The propagation of pulses is described by the propagation equation:

\[ i \frac{\partial A}{\partial z} - \frac{\beta_2}{2} \frac{\partial^2 A}{\partial t^2} = 0, \quad \text{where} \quad \beta_2 = -\frac{\lambda^2}{2\pi c} D \]

This is a partial differential equation, usually solved in the frequency domain.

\[ i \frac{\partial \tilde{A}}{\partial z} + \frac{\beta_2}{2} \omega^2 \tilde{A} = 0, \quad \implies \quad \tilde{A}(z, \omega) = \tilde{A}(0, \omega) \exp \left( \frac{\beta_2}{2} \omega^2 z \right) \]

Important parameter:

Dispersion length \( L_D = \frac{T_0^2}{|\beta_2|} \)

The length at which the dispersion is pronounced

\( T_0 \) pulse width
Example: Gaussian pulse

\[ A(0, t) = \exp\left(-\frac{t^2}{2T_0^2}\right) \]

\[ A(z, t) = \frac{T_0}{(T_0 - i\beta_2z)^{1/2}} \exp\left(-\frac{t^2}{2(T_0^2 - i\beta_2z)}\right) \]

A Gaussian pulse maintain its shape with propagation, but its width increases as

\[ T(z) = T_0\left[1 + \left(\frac{z}{L_D}\right)^2\right]^{1/2} \]
How to compensate the spreading due to dispersion?

The dispersion needs to be compensated or close wavepackets will start overlapping.

This is usually done by dispersion compensator devices placed at some distances in the chip, or through proper dispersion management.

Material nonlinearity can balance the dispersion and pulses can propagate with minimum distortion.
What is nonlinearity?

Linear:

\[ \begin{align*}
+ & \quad + \\
\end{align*} \quad = \quad 2 \]
Nonlinearity: interaction

\[ + \quad = \quad ? \quad 0,1,2 \]
Mechanical systems

Large amplitude oscillations of a pendulum

The force is no more linear with the amplitude
Extreme nonlinearities
Optical nonlinearities

1. **Electronic**
The light electric field distorts the clouds displacing the electrons. Due to anharmonic motion of bound electrons (Similar to the nonlin. pendulum). *Fast response (10fs), high power kW - GW*

2. **Molecular orientation**
due to anisotropic shape of the molecules they have different refractive index for different polarisation. The light field can reorient the molecules. *Response 1ps – 10ms, 1kW – 1mW*

3. **Thermal nonlinearities**
due to absorption the material can heat, expand, and change refractive index (thermo-optic effect) *1-100ms, 1mW*
Optical nonlinearities

4. **Photorefractive**
   due to photo-excitation of charges, their separation in the material and electro-optic effect, 1-10s, <1\(\mu\)W

5. **Atomic**
   due to excitation of atomic transitions

6. **Semiconductor**
   due to excitation of carriers in the conduction bands

7. **Metal**
   due to deceleration of the free electrons next to the surface

**Classification:**
Non-resonant and resonant nonlinearities depending on the proximity of resonances
Nonlinear optics

1958-60: Invention of the laser

1964: Townes, Basov and Prokhorov shared the Nobel prize for their fundamental work leading to the construction of lasers

1981: Bloembergen and Schawlow received the Nobel prize for their contribution to the development of laser spectroscopy. One typical application of this is nonlinear optics which means methods of influencing one light beam with another and permanently joining several laser beams
Medium polarisation

- Separation of charges gives rise to a dipole moment (model of bound electron clouds surrounding nucleus)
- Dipole moment per unit volume is called **Polarisation**

This is similar to a mass on a spring

\[ F = -kx \]

When the driving force is too strong, the oscillations become anharmonic.
Optical polarisation

\[ P = \varepsilon_0 \left( \chi^{(1)} \cdot E + \chi^{(2)} : EE + \chi^{(3)} : EEE + \cdots \right) \]

- \( \chi^{(j)} \) \((j=1,2,\ldots)\) is j\(^{th}\) order susceptibility;
- \( \chi^{(j)} \) is a tensor of rank j+1;
- for this series to converge \( \chi^{(1)}E >> \chi^{(2)}E^2 >> \chi^{(3)}E^3 \)
- \( \chi^{(1)} \) is the linear susceptibility (dominant contribution). Its effects are included through the refractive index (real part) and the absorption \( \alpha \) (imaginary part).
Nonlinear refraction

- The refractive index is modified by the presence of optical field:

\[ n(\lambda, I) = n_0(\lambda) + n_2 I \]

where \( n_0(\lambda) \) is the linear refractive index, \( I = (nc\varepsilon_0/2)|E|^2 \) is the optical intensity, \( n_2 = 12\pi^2\chi^{(3)}/n_0c \) \( 3\chi^{(3)}/4\varepsilon_0n_0^2c \) is the nonlinear index coefficient

- This intensity dependence of the refractive index leads to a large number of nonlinear effects with the most widely used:
  - Self-phase modulation
  - Cross phase modulation
Self phase modulation

- SPM – self-induced phased shift experienced by the optical pulse with propagation

\[ \phi = nk_0L = \left(n_0 + n_2I\right)k_0L \]

where

- \( k_0 = \frac{2\pi}{\lambda} \) vacuum wavenumber,
- \( L \) is the propagation length

however \( I = I(t) \) hence \( \phi = \phi(t) \)

\[ \omega(t) = \omega_0 + \delta\omega = \omega_0 - \frac{d\phi}{dt} \]

What does this mean?

Generation of new frequencies
Spectral broadening

Measured spectral broadening of pulses depending on \( \phi_{\text{max}} \)
Optical solitons

What happens to intense pulses in dispersive media?

- Normal Dispersion
- Nonlinearity increases the dispersion

\[ \omega(t) = \omega_0 - \frac{d\phi}{dt} \]

- Anomalous Dispersion
- Nonlinearity counteract the dispersion

- Nonlinearity can fully balance the dispersion: **Optical Soliton**
Non-resonant $\chi^{(3)}$ nonlinearities in optical waveguides

A figure of merit for the efficiency of a nonlinear process: $IL_{eff}$

$$(IL_{eff})_{bulk} = \left( \frac{P}{\pi w_0^2} \right) \frac{\pi w_0^2}{\lambda} = \frac{P}{\lambda}$$

$$= \int_0^L I(z) \exp(-\alpha z) \, dz = \frac{P}{\pi w_0^2 \alpha} [1 - \exp(-\alpha L)].$$

$$F = \frac{(IL_{eff})_{wg}}{(IL_{eff})_{bulk}} = \frac{\lambda}{\pi w_0^2 \alpha}$$

for $\lambda=1.55\mu m$, $w_0=2\mu m$, $\alpha=0.046 cm^{-1}$ (0.2dB/cm) $\rightarrow F \sim 2 \times 10^4$
$\chi^{(2)}$ nonlinearity in noncentrosymmetric media

$P^{(2)} = \chi^{(2)} E E$
Nonlinear frequency conversion

Can use $\chi^{(2)}$ or $\chi^{(3)}$ nonlinear processes. Those arising from $\chi^{(2)}$ are however can be achieved at lower powers.
Frequency mixing

Three wave mixing

\[ \omega_1 \uparrow \quad \omega_1 \downarrow \quad \omega_2 \]

Sum frequency generation

\[ \omega_1 \uparrow \quad \omega_2 \downarrow \quad \omega_2 \]

Difference freq. generation

Four wave mixing

THG

\[ \omega_1 \uparrow \quad \omega_1 \downarrow \quad \omega \]

FWM

\[ \omega_2 \uparrow \quad \omega_1 \downarrow \quad \omega_3 \]

\[ \omega_1 \downarrow \quad \omega_4 \]
$\chi^{(2)}$ parametric processes

- Anisotropic materials: crystals (..................)

$$P_i = \sum_{jk} \chi_{ijk}^{(2)} E_j^{\alpha_a} E_k^{\alpha_b} \quad E^{\alpha_a} = E_0 \sin(\omega_a t), \quad E^{\alpha_b} = E_0 \sin(\omega_b t)$$

$$P_i \propto E_j^{\alpha_a} \sin(\omega_a t) \times E_k^{\alpha_b} \sin(\omega_b t) \quad \Rightarrow \quad \sin[(\omega_a + \omega_b)t] \quad \text{SFG}$$

$$\sin[(\omega_a - \omega_b)t] \quad \text{DFG}$$

- Due to symmetry and when $\chi^{(2)}$ dispersion can be neglected, it is better to use the tensor $d_{ijk} = \frac{1}{2} \chi_{ijk}^{(2)}$

- In lossless medium, the order of multiplication of the fields is not significant, therefore $d_{ijk}=d_{ikj}$. (only 18 independent parameters)
Second harmonic generation

Input light beam

Nonlinear crystal

output light beams

1. Energy conservation
   \[ \omega_1 + \omega_1 = \omega_2 \]

2. Momentum conservation
   Phase matching
   \[ k_1 + k_1 = k_2 \]
   \[ n_1 = n_2 \]
Phase matching: SHG

At all z positions, energy is transferred into the SH wave. For a maximum efficiency, we require that all the newly generated components interfere constructively at the exit face. (the SH has a well defined phase relationship with respect to fundamental)

The efficiency of SHG is given by:

\[ SH \propto L^2 \frac{\sin^2(\Delta k L/2)}{(\Delta k L/2)^2} \]

\[ \Delta k = k_2 - 2k_1 \]

Coherence length: SH is out-of-phase

\[ L_c = \frac{\pi}{\Delta k} \]
In most crystals, due to dispersion of phase velocity, the phase matching cannot be fulfilled. Therefore, efficient SHG cannot be realised with long crystals.

**Methods for achieving phase matching:**
- dielectric waveguide phase-matching (*difficult*)
- non-collinear phase-matching
- birefringent phase-matching
- quasi phase-matching
Phase matching

1. **Waveguide phase matching:**
   \[ n^{SH\,\text{eff}} = n^{FF\,\text{eff}}; \]
   usually \( n^{SH\,\text{eff}} > n^{FF\,\text{eff}} \) due to waveguide dispersion (see slide 16/1)

   - **Bulk crystal**
     - \( k_1 \rightarrow k_1 \)
     - \( k_2 \rightarrow \Delta k \)
   - **Crystalline waveguide**
     - \( k_1 \rightarrow k_1 \)
     - \( k_2 \rightarrow \Delta k \)

   Need to take care of the overlap of the modes of the FF and SH.

2. **Non-collinear phase matching:** (not suitable in waveguide geometry)

   - **Collinear**
     - \( k_1 \rightarrow k_1 \)
     - \( k_2 \rightarrow \Delta k \)
   - **Non-collinear**
     - \( k_1 \rightarrow k_2 \)
4. Quasi-phase matching

The ferroelectric domains are inverted at each $L_c$. Thus the phase relation between the pump and the second harmonic can be maintained.
Quasi-phase matching: advantages

- Use any material
  smallest size $\Lambda=4\mu m$

- Multiple order phase-matching

- Noncritical phase-matching
  propagation along the crystalline axes

- Complex geometries
  chirped or quasi-periodic poling for multi-wavelength or broadband conversion
Four wave mixing (FWM)

- In isotropic materials, the lower nonlinear term is the cubic $\chi^{(3)}$.
- It also exist in crystalline materials.

NL Polarization:

$$P_{NL} = \varepsilon_0 \chi^{(3)} : EEE$$
FWM: Description

- Four waves $\omega_1$, $\omega_2$, $\omega_3$, $\omega_4$, linearly polarised along $x$

$$E = \frac{1}{2} \hat{x} \sum_{j=1}^{4} E_j \exp[i(k_j z - \omega_j t)] + \text{c.c.}$$

where $k_j = n_j \omega_j / c$ is the wavevector

$$P_{NL} = \frac{1}{2} \hat{x} \sum_{j=1}^{4} P_j \exp[i(k_j z - \omega_j t)] + \text{c.c.}$$

$$P_4 = \frac{3\varepsilon_0}{4} \chi^{(3)}_{xxxx} |E_4|^2 E_4 + 2(|E_1|^2 + |E_2|^2 + |E_3|^2)E_4$$

$$+ 2E_1E_2E_3 \exp(i\theta_+) + 2E_1E_2E_3^* \exp(i\theta_-) + \cdots$$

$$\theta_+ = (k_1 + k_2 + k_3 - k_4)z - (\omega_1 + \omega_2 + \omega_3 - \omega_4)t,$$

$$\theta_- = (k_1 + k_2 - k_3 - k_4)z - (\omega_1 + \omega_2 - \omega_3 - \omega_4)t.$$
FWM- Phase matching

- Linear PM: \( \Delta k = k_3 + k_4 - k_1 - k_2 \)
- However, due to the influence of SPM and CPM, Net phase mismatched: \( \kappa = \Delta k + \gamma(P_1 + P_2) \)
  \[ \gamma_j = n'_2 \omega_j / (cA_{\text{eff}}) \approx \gamma \]
  
- Phase matching depends on power.
- For the degenerate FWM: \( \kappa = \Delta k + 2\gamma P_0 \)
- Coherence length:
  \[ L_{\text{coh}} = \frac{2\pi}{|\kappa|} \]
FWM applications

Supercontinuum generation: Due to the combined processes of cascaded FWM, SRS, soliton formation, SPM, CPM, and dispersion
Plasmonics
Introduction to plasmonics

Boundary conditions TM (p) wave

\[ H_{y1} = H_{y2} \]
\[ \varepsilon_1 E_{z1} = \varepsilon_2 E_{z2} \]

\( z > 0: H = A_1 e^{i\beta x} e^{-k_2 z} \)
\( z < 0: H = A_2 e^{i\beta x} e^{k_1 z} \)

\[ k_2 = -\frac{\varepsilon_2}{\varepsilon_1} \]

TM equation

\[ \frac{\partial^2 H_y}{\partial z^2} + (k_0^2 \varepsilon - \beta^2) H_y = 0 \]

\( \text{Im}(\beta) \) defines the propagation;
\( k_1 \) and \( k_2 \) define the penetration

Dispersion relation for TM waves

\[ \beta = \frac{\omega}{c} \sqrt{\frac{\varepsilon_1(\omega)\varepsilon_2}{\varepsilon_1(\omega) + \varepsilon_2}} \]
Dispersion relation of SPP

Surface plasmon frequency

\[ \omega_{sp} = \frac{\omega_p}{\sqrt{1 + \varepsilon_2}} \]

1. Large wavevector, short \( \lambda \): Optical frequencies, X-ray wavelengths. Sub-wavelength resolution!

2. The maximum propagation and maximum confinement lie on opposite ends of Dispersion Curve

Example:

air-silver interface

\[ \lambda_0 = 450 \text{nm} \quad L \approx 16 \mu \text{m and } z \approx 180 \text{ nm.} \]
\[ \lambda_0 = 1.5 \mu \text{m} \quad L \approx 1080 \mu \text{m and } z \approx 2.6 \mu \text{m.} \]
SPP waveguides

- SPPs at either surface couple giving symmetric and anti-symmetric modes
  - Symmetric mode pushes light out of metal: lower loss
  - Anti-symmetric mode puts light in and close to metal, higher loss
- Metal strips: Attenuation falls super-fast with \( t \), so does confinement
Plasmonic waveguides

- To counteract the losses while keeping strong confinement (100nm), new designs are explored:
  - V-grooves
  - Slot-waveguides
Periodic photonic structures and photonic crystals
Braggs vs. Resonant Reflection

W.H. Bragg          W.L. Bragg
born in 1890 in Adelaide

(Nobel Prize in Physics 1915)

WILLIAM LAWRENCE BRAGG

The diffraction of X-rays by crystals

Nobel Lecture, September 6, 1922*

1a) Normal Plane NaCl Crystal.
Photonic Crystals

Braggs: 1915
Nobel prise -
X-ray diffraction

PRL 58 (1987):
Sajeev John;
Eli Yablonovitch

Manipulation of light in direction of periodicity: dispersion, diffraction, emission
Bragg grating in photonics

Bragg condition: \( \lambda_B = 2n \Lambda/m \), where \( n = (n_1 + n_2)/2 \)

The reflections from the periodic layers result in a formation of a photonic bandgap.
Bragg grating in a waveguide written in glass by direct laser writing MQ University (2008)
Waveguide arrays

Bragg condition

\[ \lambda_B = \Lambda \sin \alpha_m / m \]

- Period \(\approx 5\mu m\)
- \(\Delta n \approx 0.5\)

In PCFs the Bragg reflections are realised for small angles and light propagates along z axis freely. The reflection is negligible.

A defect, where waves with certain propagation constant can propagate, but they are reflected by the surrounded by two Bragg reflectors.
Linear waveguide arrays

Dispersion relation

\[ k_z = 2c \cos(k_x D) \]

D: distance between waveguides

First Brillouin zone

\[ i \frac{dE_n}{dz} + c(E_{n+1} + E_{n-1}) = 0 \]

anomalous diffraction

normal diffraction
Waveguide Array Diffraction

Assuming a discrete Floquet-Bloch function:

\[ a_n = \exp\left[i(k_z z + nk_x D)\right] \]

\[ k_z = 2c \cdot \cos(k_x D) \]

- Relative phase difference between adjacent waveguides determines discrete diffraction
- Dispersion relation is periodic

\[ k_x D = 0 \]  
normal diffraction

\[ k_x D = \frac{\pi}{2} \]  
zero diffraction

\[ k_x D = \pi \]  
anomalous diffraction
Fibres and crystals

Larger contrast is achieved in photonic crystal fibres (PCF) or photonic crystals (PC).
Phrame-by-Phrame Photonics