Axion-Higgs Unification

Michele Redi

1208.6013 with A. Strumia

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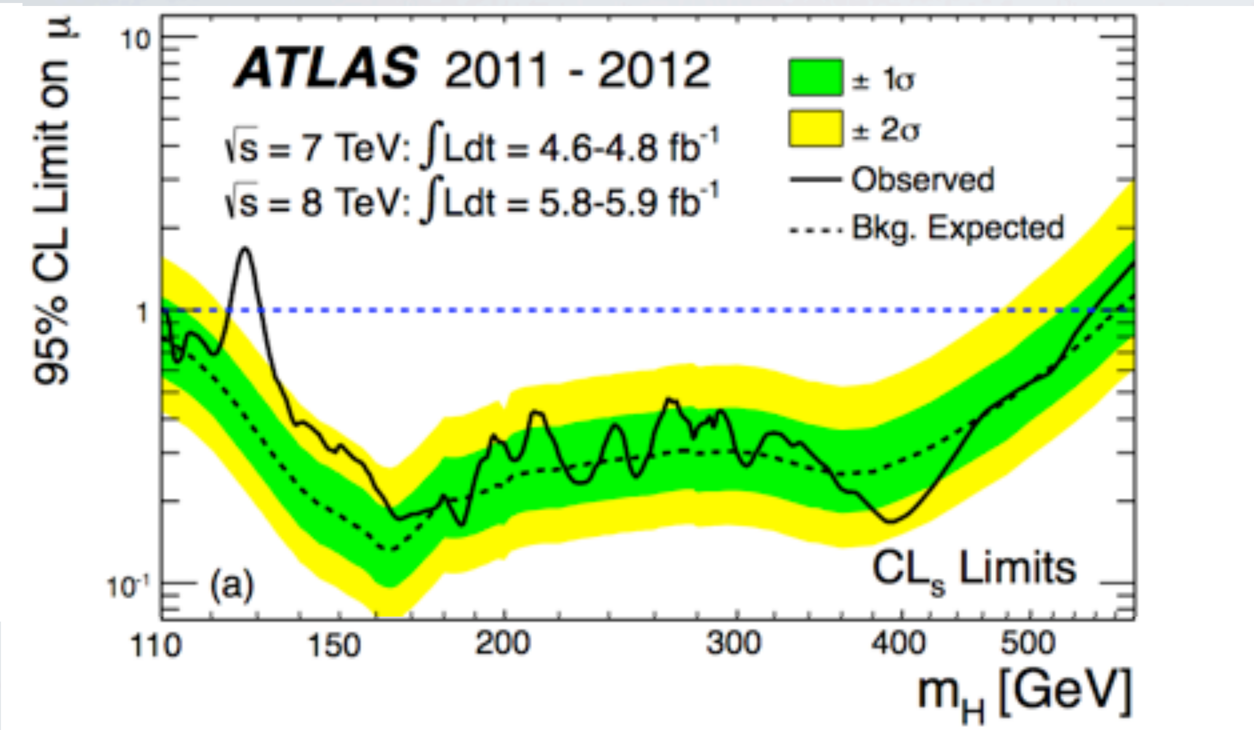
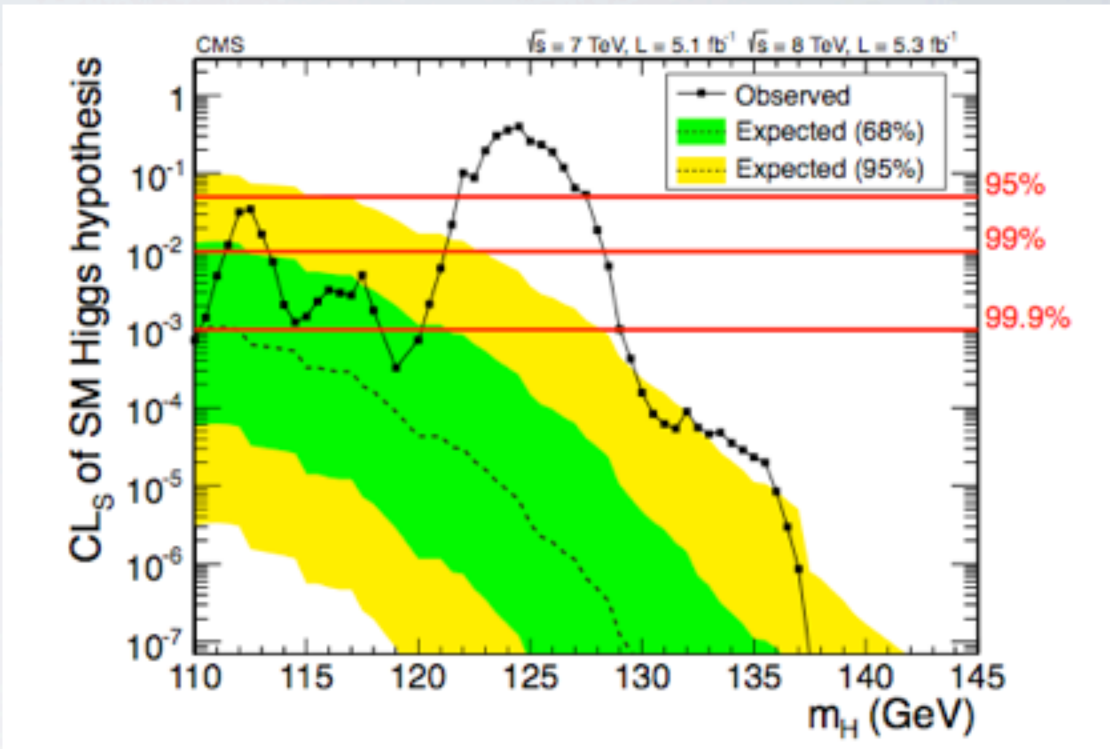




$$m_h \approx 125\,\mathrm{GeV}$$

What is the scale of new physics?

FLAVOR HAS FOUND NOTHING

$$\Lambda > 10^5\,\mathrm{TeV}$$

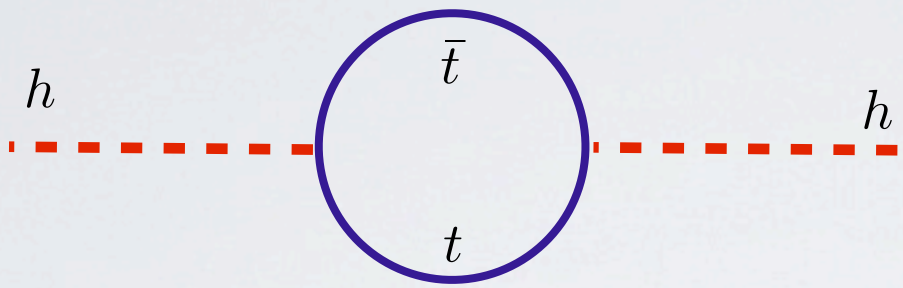
LEP HAS FOUND NOTHING

$$\Lambda > 5 - 10\,\mathrm{TeV}$$

LHC HAS FOUND THE HIGGS + NOTHING

$$\Lambda > \text{few} \times \text{TeV}$$

Hierarchy Problem:



$$\delta m_h^2 = -\frac{3\lambda_t^2}{8\pi^2}\Lambda_t^2$$

$$\Lambda < 1\,\mathrm{TeV}$$

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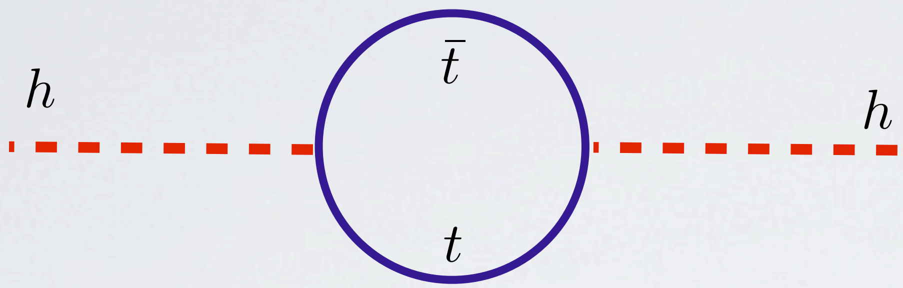

$$\delta m_h^2 = -\frac{3\lambda_t^2}{8\pi^2}\Lambda_t^2$$

$$\Lambda < 1\,\mathrm{TeV}$$

Suspicion:
Maybe naturalness was not a good guide

$$\Lambda \gg \mathrm{TeV?}$$

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- Explains why we have not seen anything

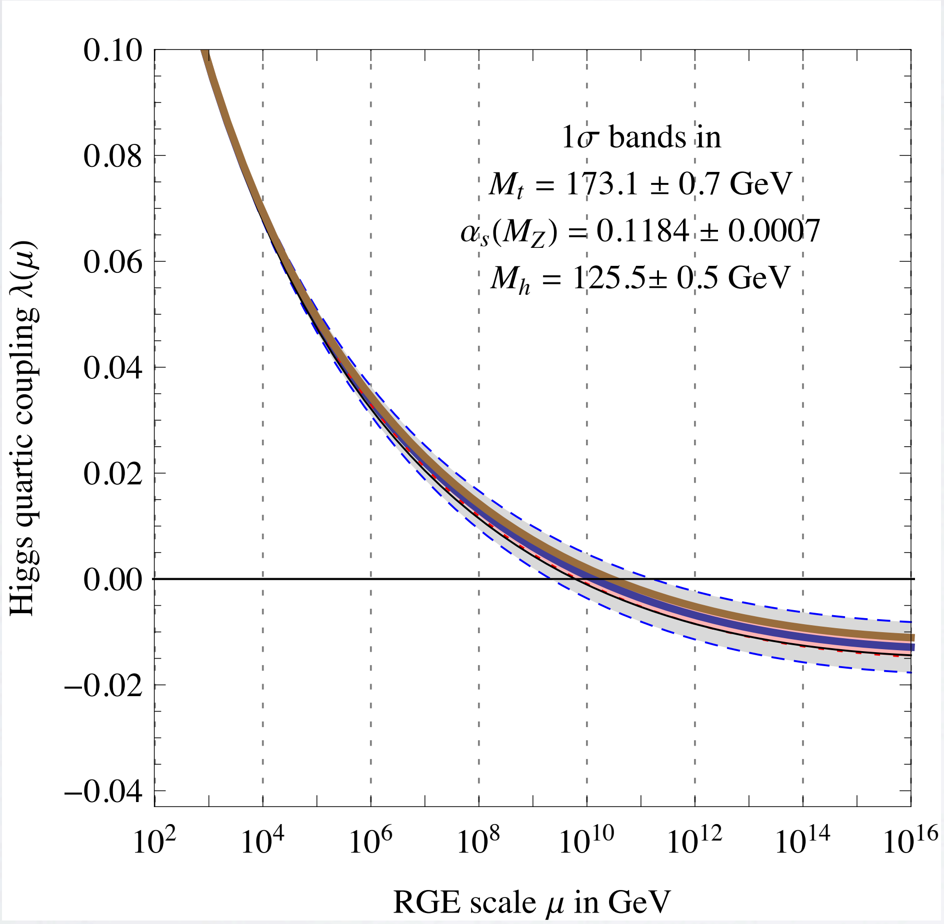
$$\Lambda \gg \text{TeV}?$$

- Explains why we have not seen anything

- Higgs VEV could be tuned anthxxxx (as c.c.)

HINTS ?

- **Running:**



$$V(h) = m^2h^2/2 + \lambda h^4/4$$

De Grassi et al. '12

Quartic almost zero at high scale for 125 GeV Higgs

- Strong CP problem:

$$\frac{\theta}{32\pi^2}\int d^4x\,\epsilon^{\mu\nu\rho\sigma}\,Tr[G_{\mu\nu}G_{\rho\sigma}] \qquad\qquad \theta < 10^{-10}$$

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Axions are most elegant solution

$$\theta \to \frac{a(x)}{f}$$

Axions are Goldstone bosons of a symmetry anomalous under QCD

$$m_a \sim \frac{m_\pi f_\pi}{f}$$

$$f > 10^9 \, \text{GeV} \longrightarrow m_a < 10^{-3} \, \text{eV}$$

Axions can be dark matter

$$\frac{\rho_a}{\rho_{\text{DM}}} \approx \theta_i^2 \left(\frac{f}{2 - 3 \times 10^{11} \, \text{GeV}} \right) \xrightarrow{\theta_i \sim 1} f \approx 10^{11} \, \text{GeV}$$

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- **Neutrino masses**

$$\frac{1}{\Lambda}(LH)^2 \qquad m_\nu \propto \frac{v^2}{\Lambda}$$

- **Unification**

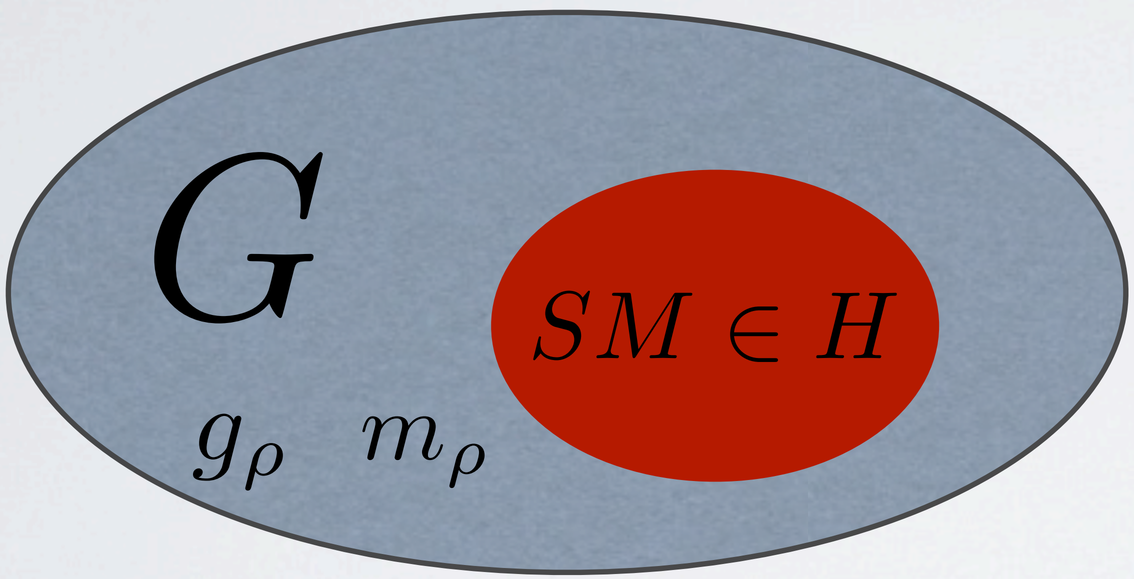
$$\Lambda < M_{GUT}$$

Georgi, Kaplan '80s

GOLDSTONE HIGGS

The Higgs could be a remnant of strong dynamics.

Most compelling Higgs as Nambu-Goldstone boson:



$$GB = \frac{G}{H}$$

$$\delta m_h^2 \sim N_c \frac{y_t^2}{8\pi^2} m_\rho^2$$

Ex:

$$\frac{SO(5)}{SU(2)_L \otimes SU(2)_R} \xrightarrow{f = \frac{m_\rho}{g_\rho}} GB = (2,2)$$

Agashe , Contino, Pomarol, '04

Deviation from SM: $\mathcal{O}\left(\frac{v^2}{f^2}\right)$

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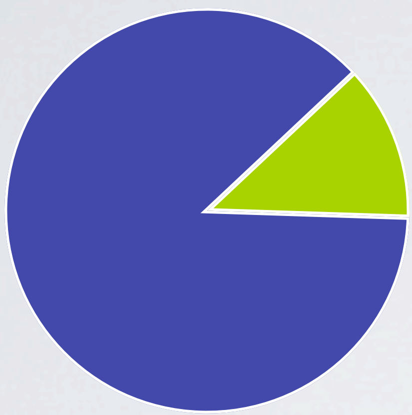
Higgs is an angle,



$0 < h < 2\pi f$ $\longrightarrow$ $\text{TUNING} \propto \frac{f^2}{v^2}$

Small Tuning $f < TeV$

Natural spectrum:

$m_\rho \sim 3\,\text{TeV}$

$f = 0.5 - 1\,\text{TeV}$

$m_h = 125\,\text{GeV}$

$m_W = 80\,\text{GeV}$

0

Partial compositeness:

D. B. Kaplan '92
Contino-Pomarol, '04

Strong sector:
Higgs + (top)
m_ρ g_ρ

Elementary:
SM Fermions
+ Gauge Fields

Partial compositeness:

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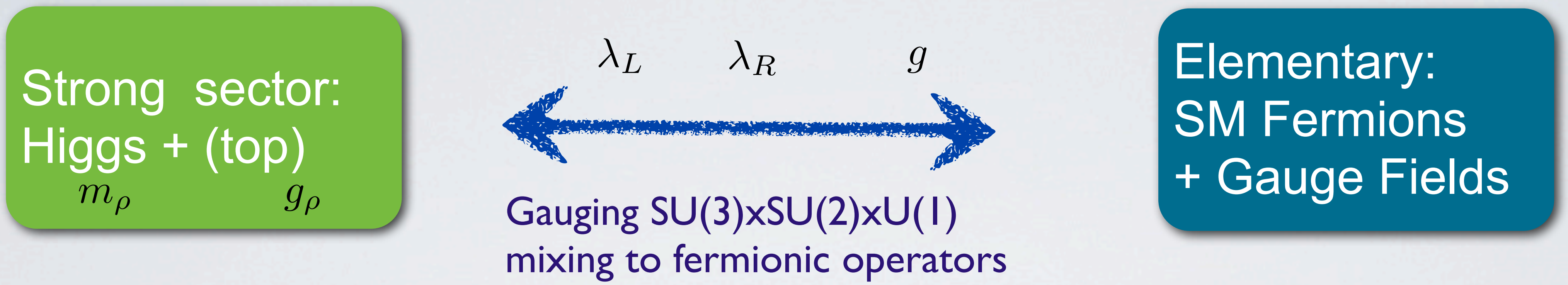

They talk through linear couplings:

$$\mathcal{L}_{gauge} = g\, A_\mu J^\mu$$

$$\mathcal{L}_{mixing} = \lambda_L \bar{f}_L O_R + \lambda_R \bar{f}_R O_R \quad \xrightarrow{\epsilon \sim \frac{\lambda}{Y}} \quad y_{SM} = \epsilon_L \cdot Y \cdot \epsilon_R$$

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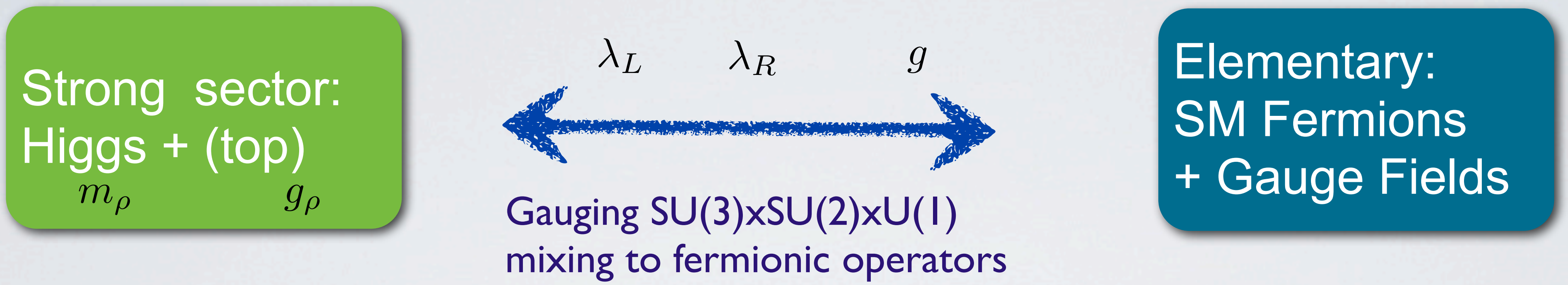

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\+ phenomenology greatly ameliorated

\- big dynamical assumptions

Many challenges:

- flavor

$$m_\rho > 20\,\mathrm{TeV}$$

- precision tests

$$m_\rho > 3\,\mathrm{TeV}$$

- direct exclusion

$$m_f > 0.7\,\mathrm{TeV} \qquad m_\rho > 2\,\mathrm{TeV}$$

- smart model building

$$f \geq 10^{11}\,\mathrm{GeV}$$

- 125 GeV Higgs can be explained

- Axions + Higgs can be unified

HIGGS MASS

Higgs potential is generated by the couplings that break the global symmetry. Minimally gauge and Yukawa couplings.

$$V(h) = \sum_i a_i \sin^{2i}\left(\frac{h}{f}\right)$$

Electro-weak scale:

$v \ll f$ ⟶ a_i must be tuned

Higgs mass is then "predicted".

Coleman-Weinberg effective potential:

$$V(h)_{gauge} = \frac{9}{2}\int \frac{d^4p}{(2\pi)^4} \ln\left[1 + \frac{1}{4}\frac{\Pi_1(p^2)}{\Pi_0(p^2)}\sin^2\frac{h}{f}\right]$$

$$\langle J^a_\mu(q) J^a_\mu(-q)\rangle = (\eta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2})\,\Pi^a(q^2)$$

$$\Pi_0 = \Pi^a$$

$$\langle J^{\hat a}_\mu(q) J^{\hat a}_\mu(-q)\rangle = (\eta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2})\,\Pi^{\hat a}(q^2)$$

$$\Pi_1 = 2(\Pi^{\hat a} - \Pi^a)$$

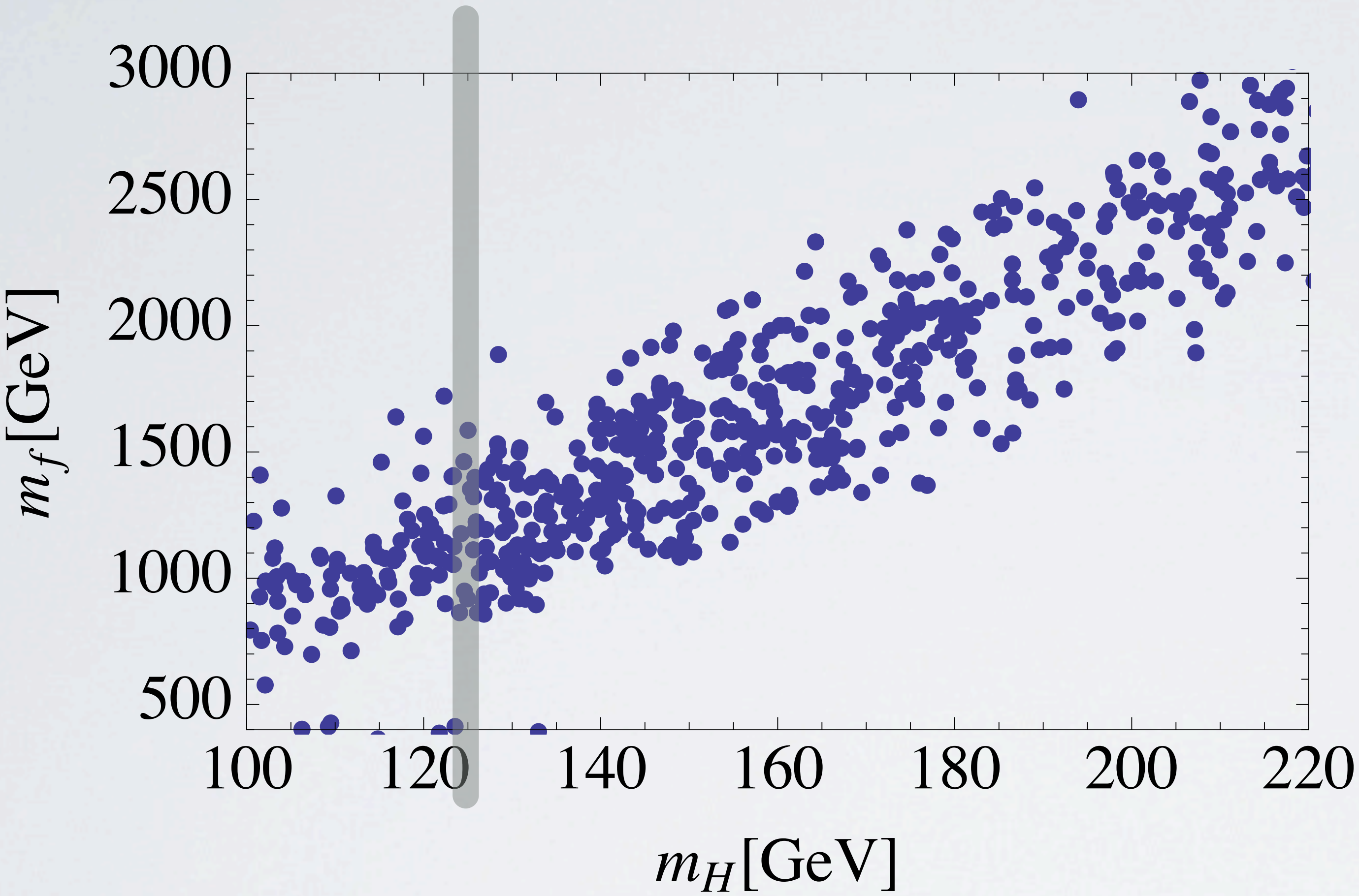
Potential is finite with a single SO(5) multiplet:

$$V(h)_{gauge} \approx \frac{9}{4}\frac{g^2}{(4\pi)^2}\frac{m_\rho^4}{g_\rho^2}\log\left[\frac{m_a^2}{m_\rho^2}\right]\sin^2\frac{h}{f}$$

Divergences effectively cut-off by resonances

CHM5:

De Curtis, MR, '11
MR, Tesi, '12



$$f = 800\,\mathrm{GeV}$$

$$m_h \sim \sqrt{\frac{N_c}{2}\frac{y_t}{\pi}\frac{m_f}{f}}\,v$$

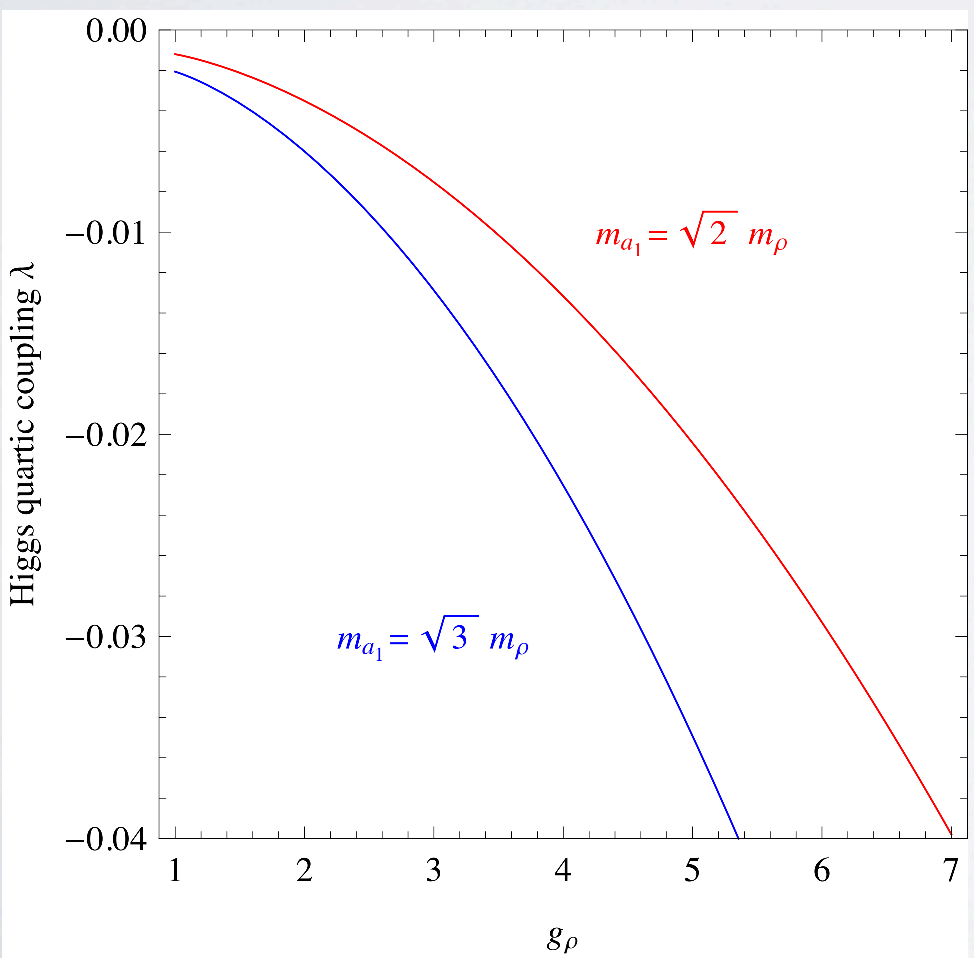
Partners around ~ 1 TeV

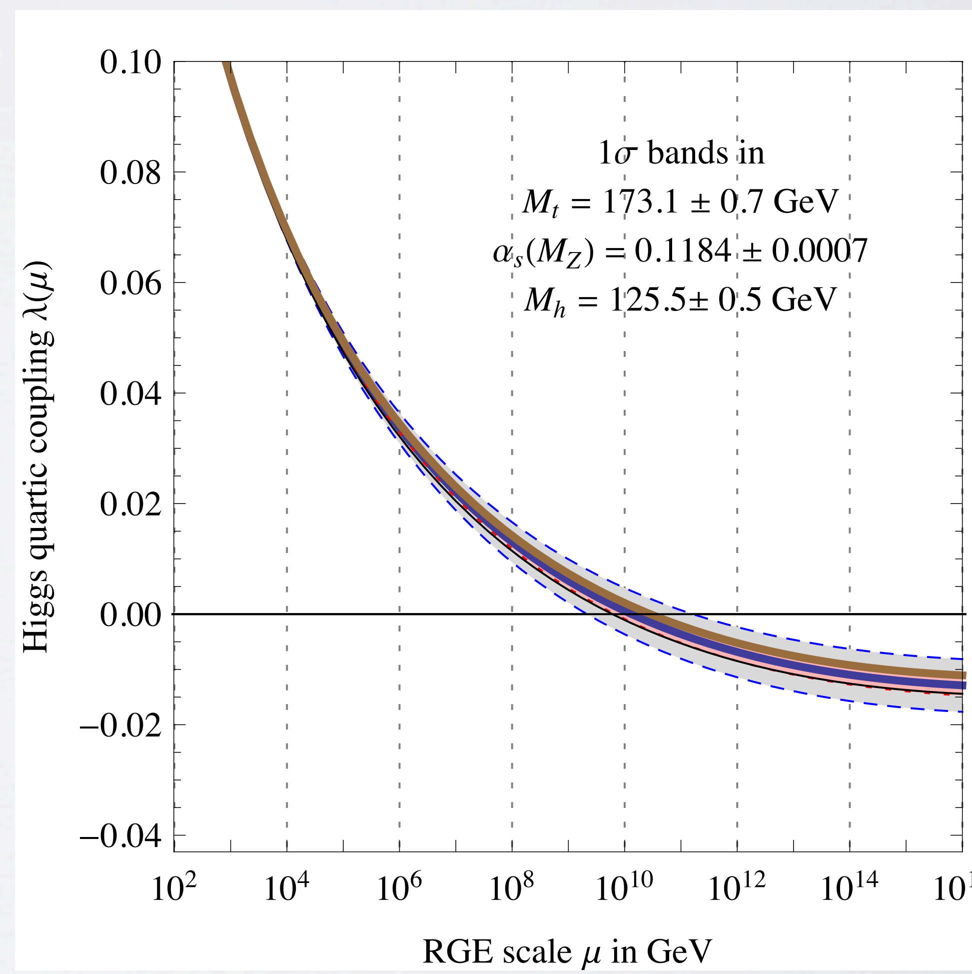
Quartic boundary condition:

$$V(h, m_\rho)_{gauge} = \frac{9}{2}\int_{m_\rho}^{\infty}\frac{d^4p}{(2\pi)^4}\ln\left[1+\frac{1}{4}\frac{\Pi_1(p^2)}{\Pi_0(p^2)}\sin^2\frac{h}{f}\right]$$

$$= \int_{m_\rho}^{\infty}\frac{d^4p}{(2\pi)^4}\left[\frac{9}{8}\frac{\Pi_1(p^2)}{\Pi_0(p^2)}\sin^2\frac{h}{f} - \frac{9}{64}\frac{\Pi_1(p^2)^2}{\Pi_0(p^2)^2}\sin^4\frac{h}{f} + \dots\right]$$

$$\lambda(m_\rho) = -3\, a_1\, g_{SM}^2 \frac{g_\rho^2}{(4\pi)^2} + O\left(\frac{g_{SM}^4}{(4\pi)^2}\right)$$





Quartic contributions:

$$\cancel{\frac{g_\rho^4}{(4\pi)^2}} \qquad \frac{g_{SM}^2\, g_\rho^2}{(4\pi)^2} \qquad \frac{g_{SM}^4}{(4\pi)^2}$$

Quartic contributions:

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- **Leading order cancellation:**

$$\lambda(\Lambda) \sim g_{\rm SM}^2 \frac{g_\rho^2}{(4\pi)^2} \sim \text{few}\, 10^{-2}$$

125 GeV Higgs requires weak coupling (large n)

- **Subleading order cancellation:**

$$\lambda(\Lambda) \sim \frac{g_{\rm SM}^4}{(4\pi)^2} \sim 10^{-3}$$

AXION-HIGGS

Basic idea:
Axion and Higgs originate from the same dynamics.
f is fixed by dark matter and the electro-weak scale is tuned.

$$\frac{G}{H} \xrightarrow{f \approx 10^{11}\ \mathrm{GeV}} \mathrm{Higgs} + \mathrm{singlet}$$

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- QCD anomaly from new fermions (KSVZ)

- QCD anomaly from SM fermions (DFSZ)

HIGGS + KSVZ AXION

Kim-Shifman-Vainstein-Zakharov:

Add new colored fermions + complex scalar

$$\Psi_Q \to e^{i\alpha_Q \gamma_5} \Psi_Q, \qquad \sigma \to e^{-2i\alpha_Q} \sigma$$

$$L = L_{\rm SM} + \bar{\Psi}_Q \partial \Psi_Q + |\partial_\mu \sigma|^2 + (\lambda \, \sigma \, \bar{\Psi}_Q \Psi_Q + {\rm h.c.}) - V(\sigma)$$

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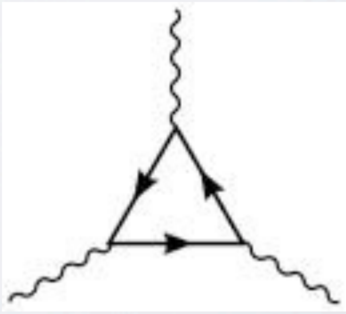
Spontaneous PQ symmetry breaking

$$f \approx \langle \sigma \rangle \qquad\qquad a = \sqrt{2}\, {\rm Im}[\sigma]$$

PQ symmetry anomalous under QCD



$$\frac{G}{H} = \frac{SU(6)_L \times SU(6)_R}{SU(6)_{L+R}}$$

Under $SU(5)_{SM}$

$$\mathbf{35} = \mathbf{24} \oplus \mathbf{5} \oplus \bar{\mathbf{5}} \oplus \mathbf{1}$$

One Higgs doublet.
Two massless singlets are axion candidates.

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One Higgs doublet.
Two massless singlets are axion candidates.

Under SM 33 charged scalars acquire mass.

$$m \approx \frac{g_{SM}}{4\pi}\Lambda$$

UV realization: SU(n) gauge theory with 6 flavors

Fermions	$\mathrm{U}(1)_Y$	$SU(2)_L$	$SU(3)_c$	$SU(n)_{\mathrm{TC}}$
D	$\frac{1}{3}$	1	$\bar{3}$	n
L	$-\frac{1}{2}$	2	1	n
N	0	1	1	n
$\bar{D}$	$-\frac{1}{3}$	1	3	$\bar{n}$
$\bar{L}$	$\frac{1}{2}$	$\bar{2}$	1	$\bar{n}$
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$$(\bar{5}+1, n) \oplus (5+1, \bar{n})$$

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$$(\bar{5}+1, n) \oplus (5+1, \bar{n})$$

$$\langle D\bar{D}\rangle = \langle L\bar{L}\rangle = \langle N\bar{N}\rangle \approx \Lambda^3 \qquad H \sim (L\bar{N}) - (\bar{L}N)^*$$

Singlets:

$$D\bar{D} \qquad L\bar{L} \qquad N\bar{N}$$

$$D\bar{D} + L\bar{L} + N\bar{N} \xrightarrow{U(1)\times SU(n)^2_{TC} \text{ anomaly}} \frac{g_{TC}}{4\pi}\Lambda$$

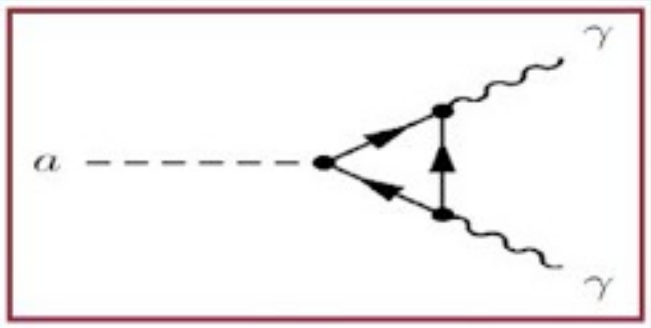
Axions couple to photon and gluons through anomalies

$$\frac{a\,E}{32\pi^2 f}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}$$

$$\frac{aN}{32\pi^2\,f}\epsilon^{\mu\nu\rho\sigma}Tr[G_{\mu\nu}G_{\rho\sigma}]$$

$$E = \sum Q_{PQ}Q_{em}^2$$

$$N = \sum Q_{PQ}T_{SU(3)}^2$$



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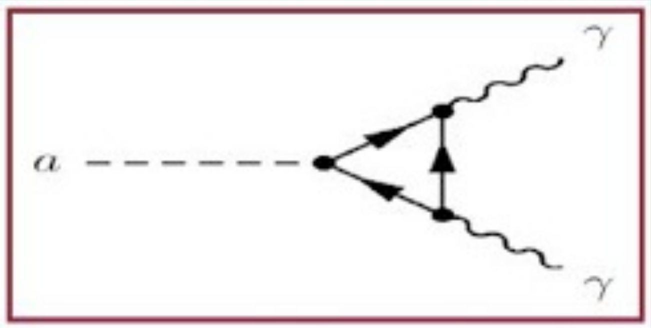
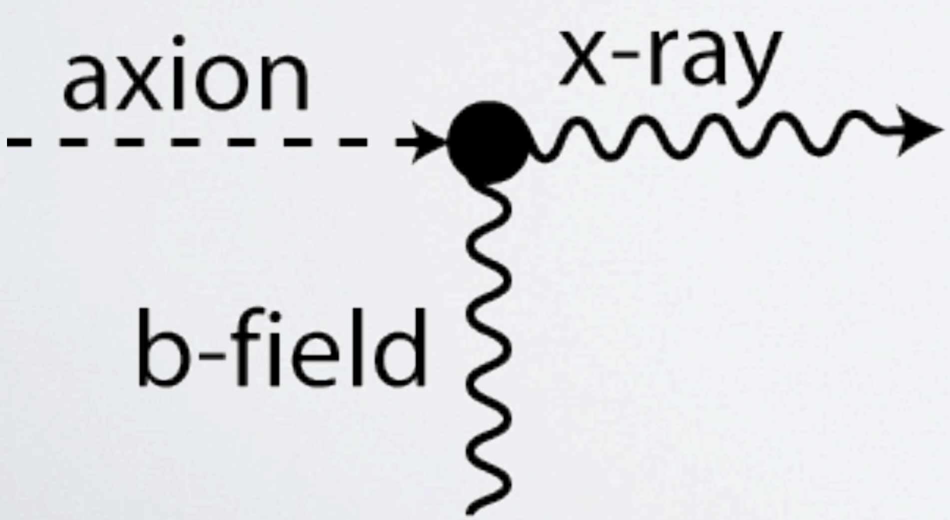
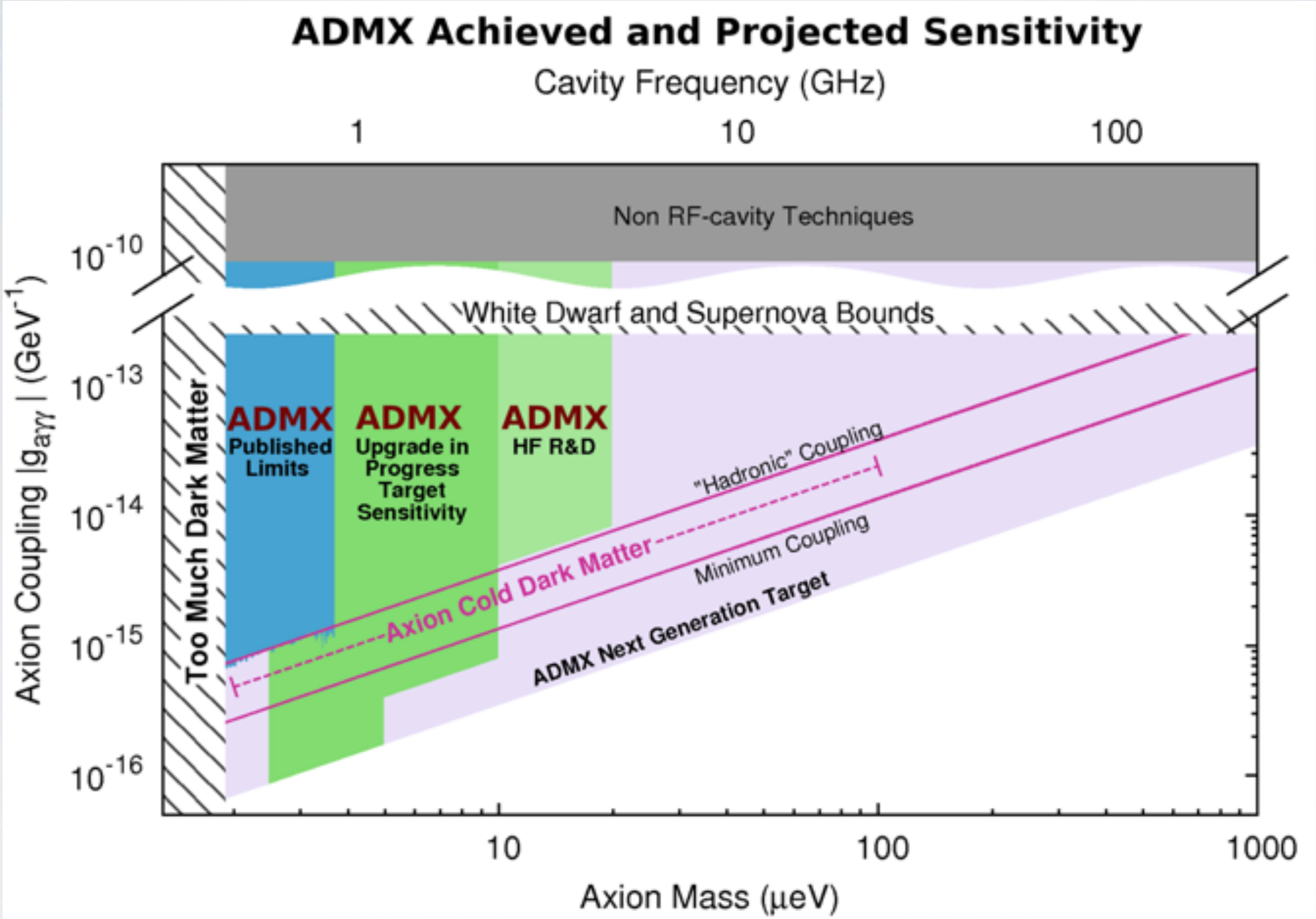


Experiments measure conversion of axion to photons



$$g_{a\gamma\gamma} = \frac{2(E/N - 1.92)}{10^{16}\,\mathrm{GeV}}\frac{m_a}{\mu\mathrm{eV}}$$

$$m_a \sim \frac{N f_\pi m_\pi}{2f}\frac{\sqrt{m_u m_d}}{m_u + m_d}$$



$$\frac{E}{N} < 1.92 + 3.5\sqrt{\frac{0.3\,\mathrm{GeV/cm^3}}{\rho_{DM}}} \qquad (m_a = 1.9 - 3.55 \times 10^{-6}\,\mathrm{ev})$$

Flavor:

$$\frac{1}{\Lambda_1^2}(qu)(L\bar{N}) \qquad \frac{1}{\Lambda_2^2}(\bar{q}\bar{u})(\bar{L}N)$$

Flavor: $\frac{1}{\Lambda_1^2}(qu)(L\bar{N})$ $\frac{1}{\Lambda_2^2}(\bar{q}\bar{u})(\bar{L}N)$

a) If UV interactions respect singlets symmetry

$$\frac{4D-3L-6N}{\sqrt{102}}, \quad \frac{L-2N}{\sqrt{3}} \qquad \frac{E}{N}=-\frac{5}{6}$$

b) If all Yukawas allowed

$$\frac{D-3L+3N}{\sqrt{30}} \qquad \frac{E}{N}=-\frac{16}{3}$$

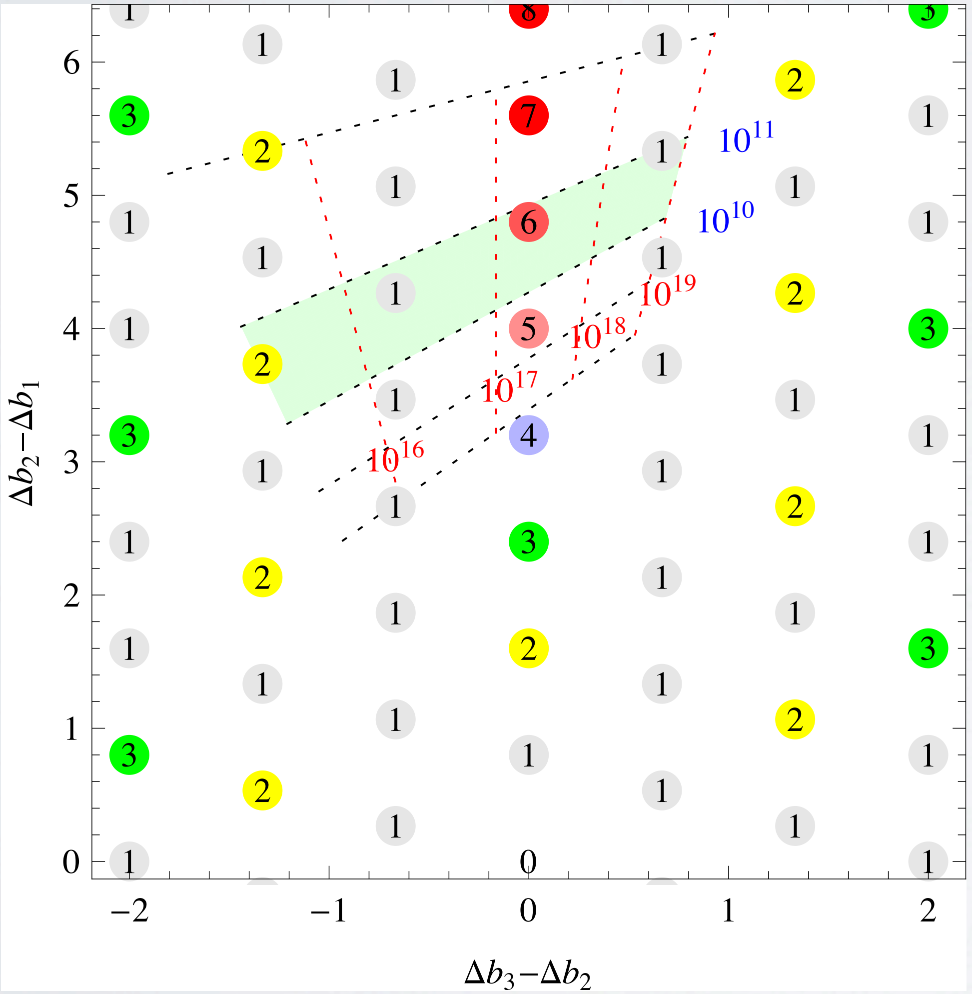
c) SU(5) is gauged

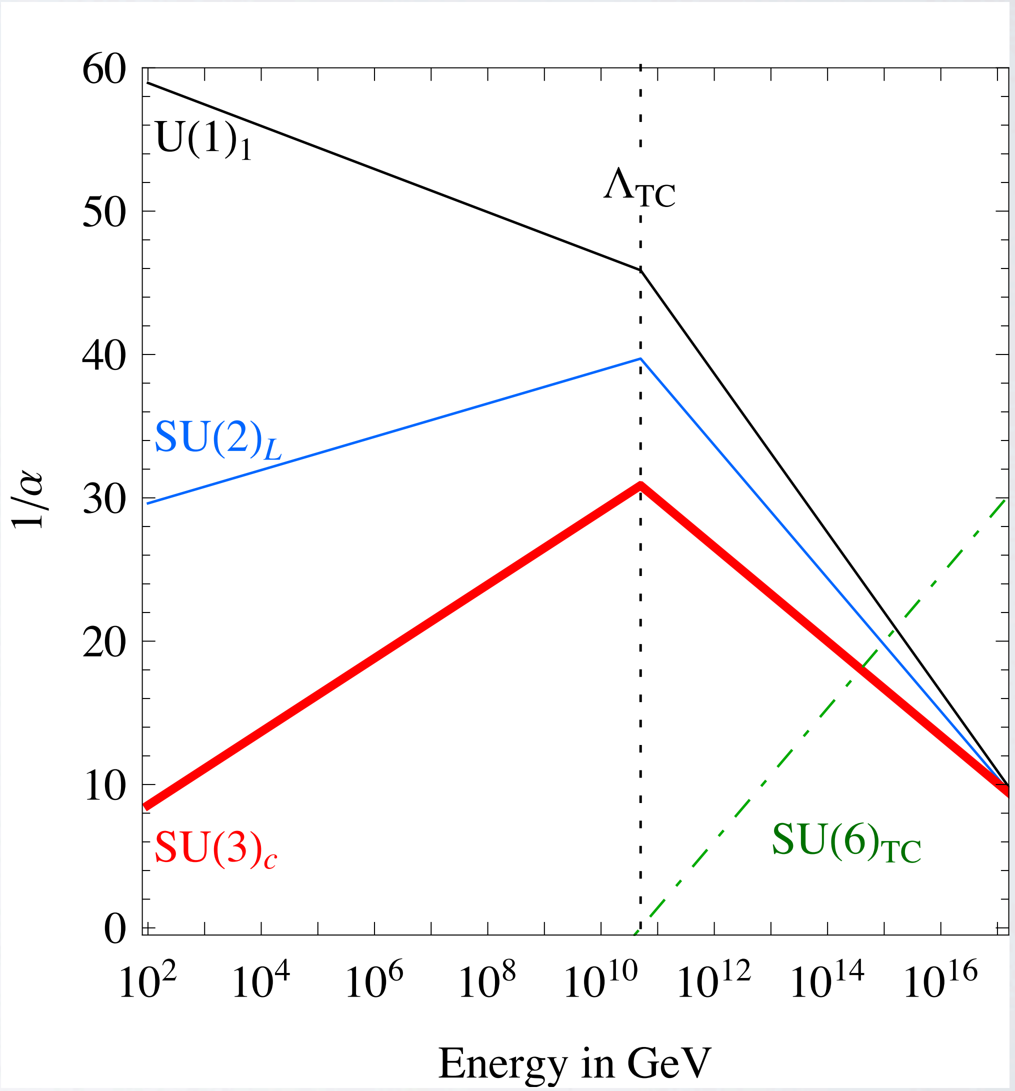
$$\frac{D+L-5N}{\sqrt{30}} \qquad \frac{E}{N}=\frac{8}{3}$$

Incomplete SU(5) multiplets can improve unification ("unificaxion")

Giudice, Rattazzi, Strumia '12

Ex: D, L, Q, U, N





HIGGS + DFSZ AXION

Dine-Fischler-Srednicki-Zhitnitsky:
Two Higgs doublets and complex singlet

$$\sigma \to e^{4i\alpha}\sigma, \qquad q_{L,R} \to e^{i\alpha} q_{L,R} \qquad H_u \to e^{-2i\alpha} H_u, \qquad H_d \to e^{-2i\alpha} H_d$$

$$f = \sqrt{v_u^2 + v_d^2 + |\sigma|^2}$$

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Ex:

$$\frac{G}{H} = \frac{SU(6)}{SO(6)} \qquad\qquad SO(6) \supset SO(4) \otimes \mathrm{U}(1)_{\mathrm{PQ}}$$

$$\mathbf{20}' = (\mathbf{2},\mathbf{2})_{\pm\mathbf{2}} \oplus (\mathbf{1},\mathbf{1})_{\pm\mathbf{4}} \oplus (\mathbf{1},\mathbf{1})_{\mathbf{0}} \oplus (\mathbf{3},\mathbf{3})_{\mathbf{0}}$$

UV realization: SO(n) gauge theory with 6 flavors

Fermions	$U(1)_Y$	$SU(2)_L$	$SU(3)_c$	$SO(n)_{TC}$	$U(1)_{PQ}$
L	$-\frac{1}{2}$	2	1	n	0
$\bar{L}$	$\frac{1}{2}$	$\bar{2}$	1	n	0
N	0	1	1	n	2
$\bar{N}$	0	1	1	n	-2

$$H_1 \sim LN$$

$$\langle L\bar{L}\rangle = \langle N\bar{N}\rangle = \Lambda^3$$

$$H_2 \sim \bar{L}\bar{N}$$

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$$H_1 \sim LN$$

$$\langle L\bar{L}\rangle = \langle N\bar{N}\rangle = \Lambda^3$$

$$H_2 \sim \bar{L}\bar{N}$$

Yukawas must respect PQ

$$\frac{1}{\Lambda_t^2}(q_L t_R^c)^\dagger (L\,N) + \frac{1}{\Lambda_b^2}(q_L b_R^c)^\dagger (\bar{L}\,\bar{N}) + \text{h.c.}$$

Anomalies:

$$E_{TC} = 0$$

$$\frac{E}{N} = \frac{8}{3}$$

PARTIAL COMPOSITENESS

$$\frac{G}{H} = \frac{SO(6)}{SO(5)} \simeq \frac{SU(4)}{\mathrm{Sp}(4)}$$

Gripaios, Pomarol, Riva, Serra '09
Redi, Tesi '12
Galloway et. al. '10

5 GBs:

$$5 = (2, 2) + 1$$

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Gauging of SM gauge symmetry preserves

$$SU(2)_L \times U(1)_Y \times U(1)_{PQ}$$

Under $U(1)_{PQ}$ singlet shifts.

Sp(n) theories with 4 flavors

Fermions	$U(1)_Y$	$SU(2)_L$	$SU(3)_c$	$Sp(n)_{TC}$	$U(1)_{PQ}$
D	0	2	1	n	+1
S	$+\frac{1}{2}$	1	1	n	-1
$\bar{S}$	$-\frac{1}{2}$	1	1	n	-1

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Fermions	$\mathrm{U}(1)_Y$	$SU(2)_L$	$SU(3)_\mathrm{c}$	$\mathrm{Sp}(n)_\mathrm{TC}$	$\mathrm{U}(1)_\mathrm{PQ}$
D	0	2	1	n	$+1$
S	$+\frac{1}{2}$	1	1	n	-1
$\bar{S}$	$-\frac{1}{2}$	1	1	n	-1

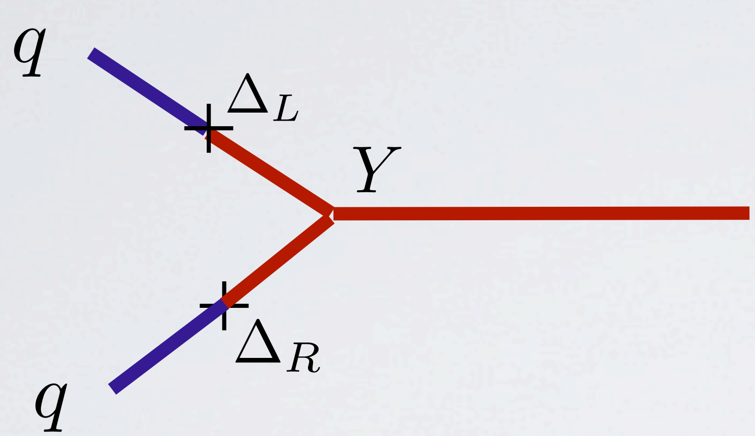
Difficult to generate QCD anomaly

$(qu)(DS)$ $\qquad$ $(qu)(DS)(S\bar{S})$

We can be build models with partial compositeness

$$m\psi\Psi + M\Psi\Psi + g_\mathrm{TC}\Psi\Psi H$$

Partial compositeness



$$y_{SM} = \epsilon_L \cdot Y \cdot \epsilon_R$$

$$\psi_{SM} \Psi_{comp} \qquad \Psi_{comp} \in G$$

We can choose

$$10 = (2,2) + (3,1) + (1,3)$$

$$q_L \subset (2,2) \qquad u_R, d_R, e_R \subset (1,3)$$

Mixing induces a PQ charge on the SM fermions.

$$\delta q_L = 0 \qquad \delta u_R = -\frac{1}{\sqrt{2}} u_R \qquad \delta d_R = -\frac{1}{\sqrt{2}} d_R \qquad \delta e_R = -\frac{1}{\sqrt{2}} e_R$$

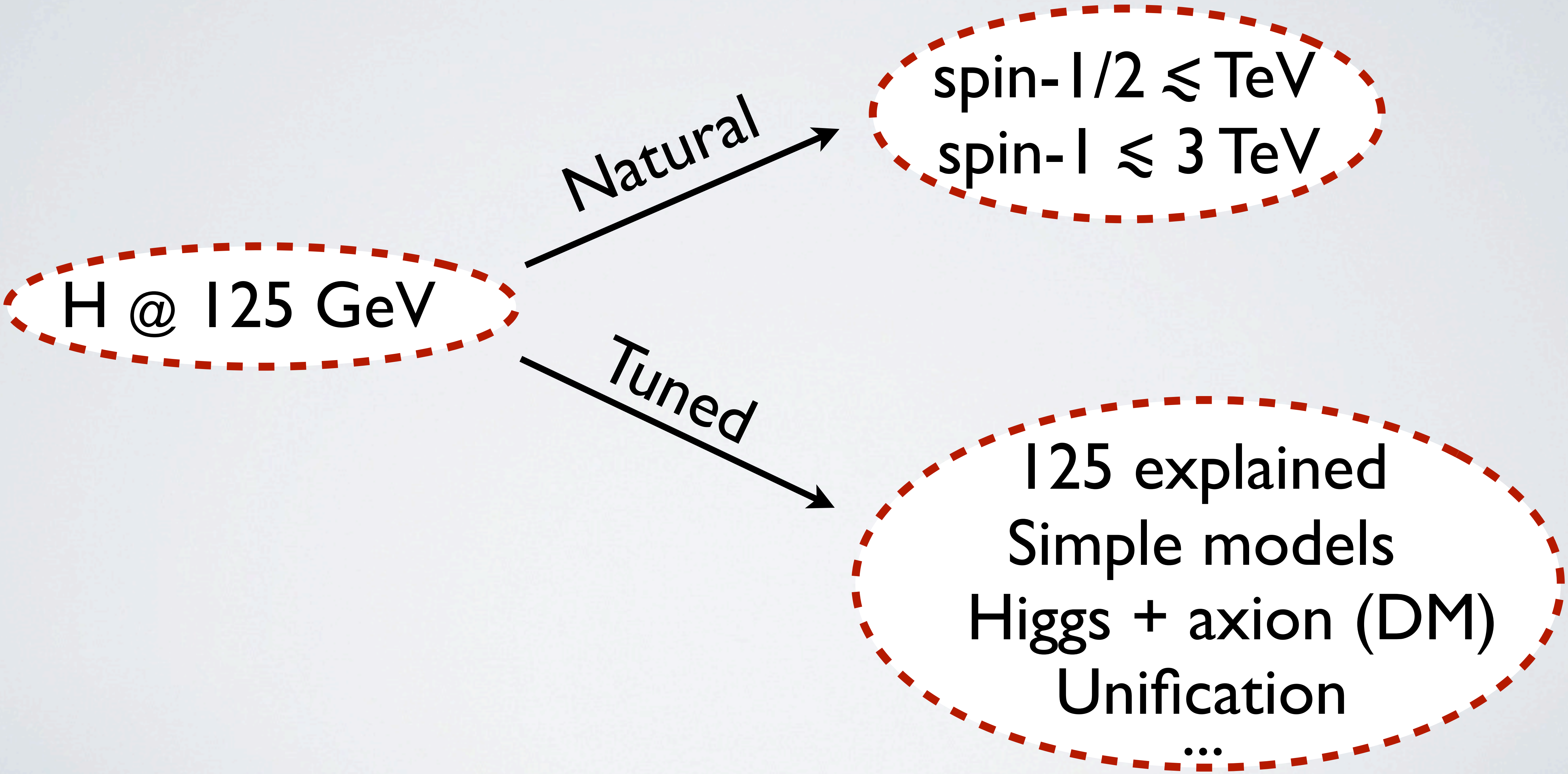
PQ symmetry is anomalous due to uR, dR, eR rotations

$$N = 2N_F$$

$$E = 2\left[\left(\frac{4}{9} + \frac{1}{9}\right) 3 + 1\right] N_F + E_{\rm TC}$$

$$\frac{E}{N} = \frac{8}{3} + \frac{E_{TC}}{6} \qquad\qquad E_{TC} \sim n$$

GOLDSTONE HIGGS



OBRIGADO !

Fermions can couple to 6=(2,2)+2 × 1

$$q_L \to \frac{1}{\sqrt{2}} \begin{pmatrix} b_L \\ -ib_L \\ t_L \\ it_L \\ 0 \\ 0 \end{pmatrix} \qquad t_R \to \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ i\cos\theta\, t_R \\ \sin\theta\, t_R \end{pmatrix}$$

Fermions can couple to 6=(2,2)+2 × 1

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For $\theta = \frac{\pi}{4}$ singlet becomes exact GB

PQ symmetry is anomalous due to tR rotations

$$E = 2\left[\left(\frac{4}{9} + \frac{1}{9}\right) 3 + 1\right] N_F + E_{\text{TC}}$$

$$\frac{E}{N} = \frac{8}{3} + \frac{E_{\text{TC}}}{6} \qquad E_{TC} \sim n$$

Neutrino masses can be generated by see-saw mechanism

$$\frac{1}{\Lambda_\nu^2}(l\nu_R^c)^\dagger(LN) + m(\nu_R^c)^2 + h.c \longrightarrow \lambda\, l H_1 \nu_R^c + m(\nu_R^c)^2 + h.c.$$

$$m_\nu \sim \frac{\lambda^2 v^2}{m}$$

If no right-handed neutrinos

$$\frac{1}{\Lambda_\nu^5}(l\bar{L})^2 N^2 \longrightarrow \frac{1}{\Lambda_\nu^3}(lH_u)^2\sigma^2 + \ldots$$

Leptogenesis?