GLUON PROPAGATOR WITH DYNAMICAL QUARKS

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Based on:
Outline of the talk

- Motivation

- Main dynamical features of gluon propagator

- Inclusion of quark loops

- The effect of unquenching

- Comparison with lattice
Motivation

- In recent years, fruitful synergy between lattice and SDEs.

- Most SDE studies focus on Green's functions of pure Yang-Mills


- Majority of lattice simulations works in the quenched limit (no dynamical quarks)


- Must make the transition to real-world QCD
- **New unquenched lattice data** for gluon and ghost propagators


- **New SDE-based algorithm** for estimating the quark-loop effects on the gluon propagator

  A.C. A., D. Binosi and J. Papavassliou,
Lattice results

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These lattice results suggest that in the presence of dynamical quarks:

1. Gluon propagator continues to saturate in the deep IR.
2. Overall suppression in the IR and intermediate regions.
3. Interpreting the saturation as a result of the gluon mass generation, i.e.

\[ \Delta^{-1}(0) = m^2(0) \]

Inclusion of quarks \(\rightarrow\) Heavier gluon mass

We want to understand these features using the SDE
The gluon SDE (Landau gauge)

\[ \Delta^{-1}(q^2) = q^2 + \sum_{i=1}^{6} a_i \]

Quenched

Unquenched

\[ \Delta_{N_f}^{-1}(q^2) = \Delta^{-1}(q^2) + a_7 \]

\[ \Delta(q^2) = [1 + G(q^2)]\Delta(q^2); \]
IR finiteness means:

\[ \Delta^{-1}(q^2) = q^2 J(q^2) \quad \longrightarrow \quad \Delta_m^{-1}(q^2) = q^2 J_m(q^2) - m^2(q^2), \]

Coupled system of integral equations

\[ J_m(q^2) = 1 + \int_k K_1(k, q, m^2, \Delta), \]
\[ m^2(q^2) = \int_k K_2(k, q, m^2, \Delta), \]

In the limit \( q^2 \to 0 \)

\[ K_2(q^2, m^2, \Delta_m) \neq 0 \]

because of the inclusion of the massless poles.
Gluon mass generation in a nutshell

The gauge invariant generation of a gluon mass proceeds through the implementation of the Schwinger mechanism.

It requires the existence of a very special type of nonperturbative vertices:

1. they make possible that the SDE of the gluon propagator yields $\Delta^{-1} (0) \neq 0$;

2. they guarantee that the WIs and STIs of the theory remain intact - before and after mass generation;

3. they decouple from on-shell amplitudes.

R. Jackiw and K. Johnson, Phys. Rev. D 8, 2386 (1973)
Special vertices

- Contain **massless poles**

- They are completely **longitudinally coupled**

\[ P^{\alpha'\alpha}(q) P^{\mu'\mu}(r) P^{\nu'\nu}(p) \tilde{V}^{\alpha'\mu'\nu'}_{\alpha\mu\nu}(q, r, p) = 0. \]

- **Poles of nonperturbative origin** \( \Rightarrow \) (colored) tightly bound composite states bound states (vanishing mass).

- These bound-state poles act as composite, longitudinally coupled **Nambu-Goldstone bosons**, maintaining gauge invariance (but, not associated with the spontaneous breaking of any continuous symmetry).
Gauge invariance requires the simultaneous replacement

\[ \Delta^{-1}(q^2) = q^2 J(q^2) \quad \rightarrow \quad \Delta^{-1}_m(q^2) = q^2 J_m(q^2) - m^2(q^2), \]

\[ \tilde{\Gamma} \quad \rightarrow \quad \tilde{\Gamma}' = \tilde{\Gamma}_m + \tilde{V}, \]

The new vertex is given by

\[ \tilde{\Gamma}'_{\alpha\mu\nu}(q, r, p) = \left[ \tilde{\Gamma}_m(q, r, p) + \tilde{V}(q, r, p) \right]_{\alpha\mu\nu} \]

The gauge invariance requires that

\[ q^\alpha \tilde{V}_{\alpha\mu\nu}(q, r, p) = m^2(r^2) P_{\mu\nu}(r) - m^2(p^2) P_{\mu\nu}(p), \]

An explicit example:

Schwinger mechanism "Turned off"

\[ q^\alpha \tilde{\Gamma}_{\alpha\mu\nu}(q, r, p) = p^2 J(p^2) P_{\mu\nu}(p) - r^2 J(r^2) P_{\mu\nu}(r), \]

Schwinger mechanism "Turned on"

\[ q^\alpha \tilde{\Gamma}'_{\alpha\mu\nu}(q, r, p) = q^\alpha \left[ \tilde{\Gamma}_m(q, r, p) + \tilde{V}(q, r, p) \right]_{\alpha\mu\nu} = \left[ p^2 J_m(p^2) - m^2(p^2) \right] P_{\mu\nu}(p) - \left[ r^2 J_m(r^2) - m^2(r^2) \right] P_{\mu\nu}(r) = \Delta^{-1}_m(p^2) P_{\mu\nu}(p) - \Delta^{-1}_m(r^2) P_{\mu\nu}(r), \]
The complete gluon mass equation

\[ m^2(q^2) = -g^2 C_A D(q^2) \int_{k} m^2(k^2) \Delta^{\mu}_{\rho}(k) \Delta^{\nu\rho}(k + q) K_{\mu\nu}(k, q). \]

where
\[ K_{\mu\nu}(k, q) = [(k + q)^2 - k^2] \left[ 1 - [Y(k + q) + Y(k)] \right] g_{\mu\nu} + [Y(k + q) - Y(k)] (q^2 g_{\mu\nu} - 2q_{\mu}q_{\nu}). \]

Diagrammatically

\[ m^2(q^2) = D(q^2) q^{\mu} \times \left( \begin{array}{c}
\text{Diagram 1} \\
\text{Diagram 2} \\
\text{Diagram 3}
\end{array} \right) \]

The solution depends on a subtle interplay between the shape of the full \( \Delta(q^2) \) and the kernel \( K_{\mu\nu}(k, q) \).
Solution of the mass equation

- Positive definite and monotonically decreasing gluon mass

- Solution normalized to coincide with lattice value $\Delta^{-1}(0) \approx 0.14 \text{ GeV}^{-2}$, namely $m = 375$ MeV.
We will assume that the main bulk of the quark contribution comes from the diagram $a_7$ (fully dressed quark loop), i.e.

\[ \tilde{\Delta}^{-1}_{N_f}(q^2) = \tilde{\Delta}^{-1}(q^2) + a_7 + \text{“subleading corrections”} \]

\[ a_7 = X(q^2) = \]
Subleading contributions

- There will be a nonlinear propagation of the changes induced due to $X(q)$, which will also affect the original subset of purely Yang-Mills graphs ($a_1 - a_6$) → **Internal gluon propagator and the three-gluon vertex gets modified.**

- We assume that the inclusion of two light quark flavors ($m = 300$ MeV) may be considered as a "perturbation" to the quenched case.

- Our **operating assumption** is that these effects may be **relatively small** compared to those originating from graph $a_7$ (quark loop)
The quark loop is transverse

Moreover, we have that

- No direct influence on the value of $\Delta(0)$;
- However modifies it indirectly, due to the change in the overall shape of $\Delta(q^2)$ throughout the entire range of momenta.

The quark-gluon vertex

- In the PT-BFM scheme, the contribution to the gluon self-energy due to the quark loop has a special ingredient: the fully-dressed quark-gluon vertex.

- The PT-BFM quark-gluon vertex satisfies the Ward identity:

  \[ p_3^\mu \hat{\Gamma}_\mu (p_1, p_2, p_3) = S^{-1}(-p_1) - S^{-1}(p_2) \]

- Instead of the conventional Slavnov-Taylor identity:

  \[ p_3^\mu \Gamma_\mu (p_1, p_2, p_3) = F(p_3)[S^{-1}(-p_1)H(p_1, p_2, p_3) - H(p_2, p_1, p_3)S^{-1}(p_2)] \]
Ansatz for the longitudinal part

• The most general tensorial structure for the longitudinal part is

\[ \Gamma_\mu(p_1, p_2, p_3) = L_1 \gamma_\mu + L_2 (p_1 - p_2)(p_1 - p_2)_\mu + L_3 (p_1 - p_2)_\mu + L_4 \sigma_{\mu\nu}(p_1 - p_2)^\nu \]

• Using the WI we find the form factors

\[
L_1 = \frac{A(p_1) + A(p_2)}{2}; \quad L_2 = \frac{A(p_1) - A(p_2)}{2\left(p_1^2 - p_2^2\right)}; \quad L_3 = -\frac{B(p_1) - B(p_2)}{p_1^2 - p_2^2}; \quad L_4 = 0.
\]

where the functions \( A(p) \) and \( B(p) \)

\[
S^{-1}(k) = -i \left[ A(k) k + B(k) \right] = -i A(k) \left[ k - M(k) \right]
\]

and the dynamical mass is defined as the ratio

\[
M(k) = B(k)/A(k)
\]

• The resulting vertex is known as Ball Chiu (BC) vertex

The unquenching formula

$\Delta_{Nf}(q^2)$ may be expressed as a deviation from $\Delta(q^2)$:

$$\Delta_{Nf}(q^2) = \frac{\Delta(q^2)}{1 + X(q^2)F^2(q^2) + \lambda^2(q^2)}$$

where

$$\lambda^2(q^2) = m^2_{Nf}(q^2) - m^2(q^2),$$

measures the difference induced to the gluon mass due to the inclusion of quarks.

SDE-lattice synergy – Full system

\[ \Delta_{N_f}^{-1}(q^2) = \left[1 + G(q^2)\right]^{-2} \overline{X}(q^2) + \frac{\lambda(q^2)}{\text{lattice}} + m_{N_f}^2(q^2) - m^2(q^2) + \Delta^{-1}(q^2) \]

\[ \overline{X}(q^2) P_{\mu\nu}(q) = \]

\[ m_{N_f}^2(q^2) = \int_k \mathcal{K}_2(k, q, \Delta_{N_f}) \]

\[ (\rightarrow \rightarrow)^{-1} = m_0 + (\rightarrow \rightarrow)^{-1} + \rightarrow \rightarrow \rightarrow \rightarrow \]

\[ 1 + G(q^2) \approx F^{-1}(q^2) \]
**Ingredients**

\[
\Delta_q(q^2) = \frac{\Delta(q^2)}{1 + \left\{ \frac{\lambda}{1 + G(q^2)} \right\}^2 \Delta(q^2)}
\]

Use the relation

\[
F^{-1}(q^2) = 1 + G(q^2) + L(q^2)
\]

\[
L(q^2) \ll G(q^2)
\]

\[
1 + G(q^2) \approx F^{-1}(q^2)
\]


Gluon propagator and ghost dressing function renormalized at $\mu=4.3$ GeV
Calculating the quark loop - $X(q^2)$

$$S^{-1}(k) = -i \left[ A(k) k^2 - B(k) \right] = -i A(k) [k^2 - \mathcal{M}(k)]$$

**Quark wave function**

- Inverse of the quark wave function
  - $m_u = 1.2$ MeV
  - $m_d = 95.0$ MeV
  - $m_s = 1.51$ GeV

**Quark Masses**

- **Dynamical quark mass**
  - $m_u = 1.2$ MeV
  - $m_d = 95.0$ MeV
  - $m_s = 1.51$ GeV

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**N_f | $\alpha_s (4.3 \text{ GeV})$**

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<tr>
<td>2+1+1</td>
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The case where $\lambda(q^2) = 0 \rightarrow$ gluon mass equation turned off

Since $X(0)=0 \rightarrow$ saturation point remains intact.

Supression in the intermediate region is clear.
Full treatment: gluon mass equation turned on

- Convergence process for $\lambda(q^2) \neq 0$
Comparison with the lattice

Quenched lattice propagator
The effect of two quarks

Unquenched lattice propagator

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The effect of two quarks

Unquenched SDE result

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The effect of adding heavier quarks

Unquenched SDE result

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The dynamical gluon mass

\[ N_f \] \[ m(0) \]

\[ \begin{array}{|c|c|}
\hline
N_f & m(0) \\
\hline
0 & 375 \text{ MeV} \\
2 & 413 \text{ MeV} \\
2+1+1 & 425 \text{ MeV} \\
\hline
\end{array} \]

Unquenched effects on the ghost propagators

- The ghost SDE is given by

\[
(iD^{-1}(q^2) = q^2 + ig^2 C_A \int_k \Gamma^\mu \Delta_{\mu\nu}(k) \Gamma^\nu(k, q) D(q + k).}
\]

Increases

Suppresses

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Conclusions

- **New SDE-based** method for estimating quark effects

- **Unquenched** gluon propagator computed as a deviation from the quenched one.

- **Quark loops suppress** intermediate and infrared region.

- **Gluon mass increases.**

- **Good agreement with lattice.**

- **Apply this method to TC models**, or QCD-like theories with fermions in higher representation.
Additional Slides
Simplifying the kernel

- Perturbative one-loop expression for $Y(k^2)$

$$Y_R(k^2) = -\frac{1}{(4\pi)^2} \frac{5}{4} \log \frac{k^2}{\bar{\mu}^2}.$$  

- Rescaling

$$Y_R(k^2) \to C Y_R(k^2)$$

- $C$ arbitrary parameter, models additional corrections.
Contribution at zero momentum

• In the limit \( q^2 \to 0 \)

\[
\hat{X}(0) = -\frac{2g^2}{d-1} \int \frac{1}{A^2(k^2 - \mathcal{M}^2)^2} \left\{ A \left[ (2 - d)k^2 + d\mathcal{M}^2 \right] + 2A'k^2(k^2 + \mathcal{M}^2) - 4k^2B'\mathcal{M} \right\}.
\]

by virtue of the identity

\[
\int_{k} k^2 f'(k) + \frac{d}{2} \int_{k} f(k) = 0,
\]


setting

\[
f(k) = \left[ A(k)(k^2 - \mathcal{M}^2(k)) \right]^{-1}
\]

\[
\hat{X}(0) = 0
\]

• No \textbf{direct} influence on the value of \( \Delta(0) \);
• However modifies it \textbf{indirectly}, due to the change in the overall shape of \( \Delta(q^2) \) throughout the entire range of momenta.