Craig Roberts

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Bound-state Problem in Continuum QCD
Discover the meaning of confinement
Determine its connection with DCSB (dynamical chiral symmetry breaking)
Expose and explain their signals in observables
... so experiment and theory together can map the nonperturbative behaviour of the strong interaction

In my view, it is unlikely that two phenomena, so critical in the Standard Model and tied to the dynamical generation of a single mass-scale, can have different origins and fates.
Exploit opportunities provided by new data on hadron elastic and transition form factors

- Chart infrared evolution of QCD’s coupling and dressed-masses
- Reveal correlations that are key to baryon structure
- Expose facts & fallacies in modern descriptions of hadron structure

Precision experimental study of (far) valence region, and theoretical computation of distribution functions and distribution amplitudes

- Computation is critical
- Without it, no amount of data will reveal anything about the theory underlying the phenomena of strong interaction physics
Mesons

Craig Roberts: Bound state problem in continuum QCD (87p)
Dressed-quark propagator

- Starting point for the continuum bound-state problem.
- Two crucial elements, derived from the contraction
  \[ D_{\mu\nu}(k-q) \Gamma_{\nu}(q,k) \]
  - Dressed gluon propagator
  - Dressed quark-gluon vertex
- Qin-Chang interaction is a representation of the gluon propagator that is consistent with all available information on QCD’s gauge sector.
  - Maris-Tandy interaction fails that test in the far infrared.
- Dressed-quark-gluon vertex
  - That is where the action is!

Interaction model for the gap equation
Mesons in Quantum Field Theory

- Mass and “Wave Function” are obtained from a Bethe-Salpeter equation
  - Generalisation of the Lippmann-Schwinger equation
- General structure is complicated, any truncation must preserve the crucial symmetries: Poincaré-covariance, vector and axial-vector Ward-Green-Takahashi identities
- The pion ... Nature’s strong-interaction messenger ... is a critical example

Takahashi Green-Ward Identities
This class of identities have been known for more than 60 years.

They have been used for 19 years in order to construct a symmetry-preserving kernel for the Bethe-Salpeter equation.

For the last 5 years we’ve known how to construct a symmetry-preserving kernel given an arbitrary quark-gluon vertex.

Axial-Vector vertex
Satisfies an inhomogeneous Bethe-Salpeter equation

Kernels of these equations are completely different
But they must be intimately related
What is $\Gamma_\mu$?


**Nonperturbative fermion boson vertex function in gauge theories** - He, Han-Xin hep-th/0202013


Gauge principle is fundamental in formulating the Standard Model. 

Fermion-gauge-boson couplings are the inescapable consequence and the primary determining factor for observable phenomena. 

Vertices describing such couplings are simple in perturbation theory; yet existence of strong-interaction bound-states guarantees that many phenomena within the SM are nonperturbative. 

Unified treatment and solution of the familiar longitudinal Ward-Green-Takahashi identity and its less well known transverse counterparts.

Novel consequences for the dressed-fermion-gauge-boson vertex

Practical corollaries of transverse Ward–Green–Takahashi identities

Si-xue Qin\textsuperscript{a, b}, Lei Chang\textsuperscript{c}, Yu-xin Liu\textsuperscript{a}, Craig D. Roberts\textsuperscript{d, e}, Sebastian M. Schmidt\textsuperscript{f}

- Longitudinal WGT identity expresses properties of the divergence of the vertex

- Transverse identities relate to its curl (as Faraday’s law of induction involves an electric field)

- The last two terms in each identity arise in computing the momentum space expression of a nonlocal axial-vector/vector vertex, whose definition involves a gauge-field-dependent line integral

- But ... practical progress can be made without knowing their precise forms
Practical corollaries of transverse Ward–Green–Takahashi identities

Si-xue Qin, Lei Chang, Yu-xin Liu, Craig D. Roberts, Sebastian M. Schmidt

- Using symmetries alone, it is readily established that DCSB demands that dressed fermions possess anomalous chromo- and electro-magnetic moments, which are large on the domain within which DCSB is effective.
- This is the “final” word.
- Evidence had slowly been accumulating since 1985

Simple vertex in perturbation theory $\gamma_\mu$

→ 12 distinct terms when strong interactions are turned on

Amongst them, one with the unique structure; i.e., an anomalous magnetic moment term

Follows, algebraically, that gauge theories coupled to fermions with a dynamically generated mass MUST possess an anomalous (chromo/electro)-magnetic moment, whose magnitude is driven by the strength of DCSB
Lattice-QCD
- \( m = 115 \text{ MeV} \)

Nonperturbative result is *two orders-of-magnitude* larger than the perturbative computation

- *This level of magnification is typical of DCSB*
- *cf.*

Quark mass function:
- \( M(p^2=0) = 400 \text{ MeV} \)
- \( M(p^2=10 \text{ GeV}^2) = 4 \text{ MeV} \)

Sao Paulo: II Workshop on Perspectives in Nonperturbative QCD, 12-13 May 14
Modern $\Gamma_{\mu}(q,k)$

Tracing masses of ground-state light-quark mesons

Phys. Rev. C 85, 052201(R) – Published 7 May 2012

Lei Chang and Craig D. Roberts

- Describes the best-informed vertex available today
  - Contains all the Ball-Chiu terms
    They are unique as the kinematic-singularity-free solution of the longitudinal vector WGT identity
  - And two of the terms critical for expressing the AMMs

\[ \Gamma_{\mu}(p_1,p_2) = \Gamma_{\mu}^{acm4}(p_1,p_2) + \Gamma_{\mu}^{acm5}(p_1,p_2), \quad (8) \]

with \( k = p_1 - p_2 \), \( T_{\mu\nu} = \delta_{\mu\nu} - k_\mu k_\nu/k^2 \), \( a_{\mu}^T := T_{\mu\nu}a_{\nu} \)

\[ \Gamma_{\mu}^{acm4} = [\ell^T_{\mu} \gamma \cdot k + i\gamma^T_{\mu} \sigma_{\nu\rho} \ell_{\nu} k_{\rho}] \tau_4(p_1,p_2), \quad (9) \]

\[ \Gamma_{\mu}^{acm5} = \sigma_{\mu\nu} k_\nu \tau_5(p_1,p_2), \quad (10) \]

\[ \tau_4 = \frac{2\tau_5(p_1,p_2)}{\mathcal{M}(p_1^2,p_2^2)}, \quad (11) \]

- The chromo AMM is crucial to explaining the splitting between parity partners, such as $a_1$-$\rho$ mass splitting, and connecting it with DCSB
Pion’s Bethe-Salpeter amplitude

Solution of the Bethe-Salpeter equation

\[ \Gamma_{\pi^j}(k; P) = \tau_{\pi^j} \gamma_5 \left[ iE_{\pi}(k; P) + \gamma \cdot P F_{\pi}(k; P) \right. \\
\left. + \gamma \cdot k_k \cdot P G_{\pi}(k; P) + \sigma_{\mu\nu} k_\mu P_\nu H_{\pi}(k; P) \right] \]

Dressed-quark propagator

\[ S(p) = \frac{1}{i\gamma \cdot p A(p^2) + B(p^2)} \]

Axial-vector Ward-Takahashi identity entails

\[ \int \pi E_{\pi}(k; P = 0) = B(k^2) \]

Pseudovector components necessarily nonzero. Cannot be ignored!

Owing to DCSB & Exact in Chiral QCD

Miracle: two body problem solved, almost completely, once solution of one body problem is known.
The quark level Goldberger-Treiman relation shows that DCSB has a very deep and far reaching impact on physics within the strong interaction sector of the Standard Model; viz.,

Goldstone's theorem is fundamentally an expression of equivalence between the one-body problem and the two-body problem in the pseudoscalar channel.

This emphasises that Goldstone's theorem has a pointwise expression in QCD.

Hence, pion properties are an almost direct measure of the dressed-quark mass function.

Thus, enigmatically, the properties of the massless pion are the cleanest expression of the mechanism that is responsible for almost all the visible mass in the universe.
Dichotomy of the pion Mass Formula for $0^-$ Mesons

- Mass-squared of the pseudoscalar hadron
- Sum of the current-quark masses of the constituents;
  e.g., pion = $m_u^\zeta + m_d^\zeta$, where “$\zeta$” is the renormalisation point
Dichotomy of the pion Mass Formula for $0^-$ Mesons

\[ f_{H_5} m_{H_5}^2 = \rho_{H_5} \mathcal{M}_{H_5} \]

Pseudovector projection of the Bethe-Salpeter wave function onto the origin in configuration space

- Namely, the pseudoscalar meson’s leptonic decay constant, which is the strong interaction contribution to the strength of the meson’s weak interaction

Maris, Roberts and Tandy
Dichotomy of the pion Mass Formula for $0^-$ Mesons

\[ \int H_5 \, m_{H_5}^2 = \rho_{H_5} \mathcal{M}_{H_5} \]

- Pseudoscalar projection of the Bethe-Salpeter wave function onto the origin in configuration space
  - Namely, a pseudoscalar analogue of the meson’s leptonic decay constant
Consider the case of light quarks; namely, $m_q \approx 0$

- If chiral symmetry is dynamically broken, then
  
  * $f_{H_5} \rightarrow f_{H_5}^0 \neq 0$
  * $\rho_{H_5} \rightarrow - \langle q \overline{q} \rangle / f_{H_5}^0 \neq 0$

  both of which are independent of $m_q$

Hence, one arrives at the corollary

$$m_{H_5}^2 = 2m_q \frac{-\langle \overline{q}q \rangle}{f_{H_5}^0}$$

The so-called “vacuum quark condensate.” It’s actually contained within hadrons.

Gell-Mann, Oakes, Renner relation 1968

$m_{\pi}^2 \propto m$
Radial excitations of Pseudoscalar meson

- Hadron spectrum contains 3 pseudoscalars \( I^G(J^P)L = 1^-(0-)S \) masses below 2GeV: \( \pi(140) \); \( \pi(1300) \); and \( \pi(1800) \)

  the pion

- Constituent-Quark Model suggests that these states are the 1\(^{\text{st}}\) three members of an \( n^1S_0 \) trajectory; i.e., ground state plus radial excitations

- But \( \pi(1800) \) is narrow (\( \Gamma = 207 \pm 13 \)); i.e., surprisingly long-lived & decay pattern conflicts with usual quark-model expectations.

  - \( S_{Q-bar Q} = 1 \bigoplus L_{\text{Glue}} = 1 \Rightarrow J = 0 \)

  & \( L_{\text{Glue}} = 1 \Rightarrow 3S_1 \bigoplus 3S_1 \) (Q-bar Q) decays are suppressed

  - Perhaps therefore it’s a hybrid?

Radial excitations & Hybrids & Exotics

\[ \Rightarrow \text{wave-functions with support at long-range} \]

\[ \Rightarrow \text{sensitive to confinement interaction} \]

Understanding confinement “remains one of The greatest intellectual challenges in physics”

Craig Roberts: Bound state problem in continuum QCD (87p)

Sao Paulo: II Workshop on Perspectives in Nonperturbative QCD, 12-13 May 14

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exotic mesons: quantum numbers not possible for quantum mechanical quark-antiquark systems

hybrid mesons: normal quantum numbers but non-quark-model decay pattern

BOTH suspected of having “constituent gluon” content
Radial excitations of Pseudoscalar meson

- Valid for ALL Pseudoscalar mesons
  - When chiral symmetry is dynamically broken, then
    - \( \rho_{H5} \) is finite and nonzero in the chiral limit, \( M_{H5} \to 0 \)
  - A “radial” excitation of the \( \pi \)-meson, is not the ground state, so
    \[
    m^2_{\pi \text{ excited state}} \neq 0 > m^2_{\pi \text{ ground state}} = 0 \quad \text{(in chiral limit, } M_{H5} \to 0)\]

- Putting this things together, it follows that

\[
f_{H5} = 0
\]

for ALL pseudoscalar mesons, except \( \pi (140) \), in the chiral limit

Flip side: if no DCSB, then all pseudoscalar mesons decouple from the weak interaction!

Dynamical Chiral Symmetry Breaking
- Goldstone’s Theorem – impacts upon every pseudoscalar meson
This is fascinating because in quantum mechanics, decay constants of a radial excitation are suppressed by factor of roughly $\frac{1}{3}$

- Radial wave functions possess a zero
- Hence, integral of $"r \, R_n=2(r)^2"$ is quantitatively reduced compared to that of $"r \, R_n=1(r)^2"$

However, only a symmetry can ensure that something vanishes completely.
The suppression of $f_{\pi 1}$ is a useful benchmark that can be used to tune and validate lattice QCD techniques that try to determine the properties of excited state mesons.

- When we first heard about [this result] our first reaction was a combination of “that is remarkable” and “unbelievable”.
- CLEO: $\tau \to \pi(1300) + \nu_{\tau}$
  \[ \Rightarrow f_{\pi 1} < 8.4\text{MeV} \]

_Diehl & Hiller_

_hep-ph/0105194_

- Lattice-QCD check:
  $16^3 \times 32$-lattice, $a \sim 0.1$ fm,
  two-flavour, unquenched
  \[ \Rightarrow f_{\pi 1}/f_{\pi} = 0.078 \ (93) \]

- Full ALPHA formulation is required to see suppression, because PCAC relation is at the heart of the conditions imposed for improvement (determining coefficients of irrelevant operators)

_Craig Roberts: Bound state problem in continuum QCD (87p)_
Some New Challenges

- Heavy-light systems using the DCSB-improved kernel
  - Rainbow-ladder kernel cannot work because cancellations required for its accuracy don’t happen.

- Computation of spectrum of hybrid and exotic mesons

  **exotic mesons**: quantum numbers not possible for quantum mechanical quark-antiquark systems
  **hybrid mesons**: normal quantum numbers but non-quark-model decay pattern

  *BOTH* suspected of having “constituent gluon” content

  - It appears that this will need to be treated as a three-body problem (dressed-q+dressed-qbar+dressed-g) because best attempts so far using Bethe-Salpeter equation have failed.

- Equally pressing, some might say more so, is the three-quark problem; viz., baryons in **QCD**.
  - Understanding that problem will help in making predictions for exotics
Unification of Meson & Baryon Properties

- Correlate the properties of meson and baryon ground- and excited-states within a *single, symmetry-preserving framework*
  - Symmetry-preserving means:
    - Poincaré-covariant & satisfy relevant Ward-Takahashi identities
- Constituent-quark model has hitherto been the most widely applied spectroscopic tool; whilst its weaknesses are emphasized by critics and acknowledged by proponents, it is of continuing value because there is nothing better that is yet providing a bigger picture.
- Nevertheless,
  - no connection with quantum field theory & therefore not with QCD
  - not symmetry-preserving & therefore cannot veraciously connect meson and baryon properties
DSEs & Baryons

- **Dynamical chiral symmetry breaking (DCSB)**
  - has enormous impact on meson properties.
    - *Must be included in description and prediction of baryon properties.*

- **DCSB** is essentially a quantum field theoretical effect.
  
  In quantum field theory
  - Meson appears as pole in four-point quark-antiquark Green function
    - → Bethe-Salpeter Equation
  - Nucleon appears as a pole in a six-point quark Green function
    - → Faddeev Equation.

- Poincaré covariant Faddeev equation sums all possible exchanges and interactions that can take place between three dressed-quarks

- Tractable equation is based on the observation that an interaction which describes colour-singlet mesons also generates *nonpointlike* quark-quark (diquark) correlations in the colour-antitriplet channel

\[
\text{SU}_c(3): \quad 3 \otimes 3 = \bar{3} \oplus 6
\]
Linear, Homogeneous Matrix equation

- Yields wave function (Poincaré Covariant Faddeev Amplitude) that describes quark-diquark relative motion within the nucleon

- Scalar and Axial-Vector Diquarks . . .
  - Both have “correct” parity and “right” masses
  - In Nucleon’s Rest Frame Amplitude has $s$–, $p$– & $d$–wave correlations

S-wave contributes only 37% to normalisation of wave function
Why should a pole approximation produce reliable results?

Quark-quark scattering matrix
- A pole approximation is used to arrive at the Faddeev-equation
Consider the rainbow-gap and ladder-Bethe-Salpeter equations

\[
S(p)^{-1} = i \gamma \cdot p + m + \int \frac{d^4 q}{(2\pi)^4} g^2 D_{\mu \nu}(p - q) \frac{\lambda^a}{2} \gamma_\mu S(q) \frac{\lambda^a}{2} \gamma_\nu(q, p),
\]

\[
\Gamma(k; P) = -\int \frac{d^4 q}{(2\pi)^4} g^2 D_{\mu \nu}(p - q) \frac{\lambda^a}{2} \gamma_\mu S(q + P) \Gamma(q; P) S(q) \frac{\lambda^a}{2} \gamma_\nu.
\]

- In this symmetry-preserving truncation, colour-antitriplet quark-quark correlations (diquarks) are described by a very similar homogeneous Bethe-Salpeter equation

\[
\Gamma_{qq}(k; P) C^\dagger = -\left( \frac{1}{2} \right) \int \frac{d^4 q}{(2\pi)^4} g^2 D_{\mu \nu}(p - q) \frac{\lambda^a}{2} \gamma_\mu S(q + P) \Gamma_{qq}(q; P) C^\dagger S(q) \frac{\lambda^a}{2} \gamma_\nu.
\]

- Only difference is factor of $\frac{1}{2}$

- Hence, an interaction that describes mesons also generates diquark correlations in the colour-antitriplet channel
SU(2) isospin symmetry of hadrons might emerge from mixing half-integer spin particles with their antiparticles.

Remarks

- Diquark correlations are *not* inserted by hand
  Such correlations are a dynamical consequence of strong-coupling in QCD
- The same mechanism that produces an almost massless pion from two dynamically-massive quarks;
  i.e., DCSB, forces a strong correlation between two quarks in colour-antitriplet channels within a baryon
  – an indirect consequence of Pauli-Gürsey symmetry
- Diquark correlations are electromagnetically active and non-pointlike
  – Typically, $r_{0^+} \sim r_\pi$ & $r_{1^+} \sim r_\rho$ (actually 10% larger)
  – They have soft form factors
Baryon Spectrum

Proton
Anti-proton
Neutron
Lambda
π^+
K^0
π^0
J/ψ
One method by which to validate QCD is computation of its hadron spectrum and subsequent comparison with modern experiment. Indeed, this is an integral part of the international effort in nuclear physics.

For example, the $N^*$ programme and the search for hybrid and exotic mesons together address the questions:
- which hadron states and resonances are produced by QCD?
- how are they constituted?

This intense effort in hadron spectroscopy is a motivation to extend the research just described and treat ground- and excited-state hadrons with $s$-quark content. (New experiments planned in Japan)

Key elements in a successful spectrum computation are:
- symmetries and the pattern by which they are broken;
- the mass-scale associated with confinement and DCSB;
- and full knowledge of the physical content of bound-state kernels. All this is provided by the DSE approach.
Contact-Interaction Kernel

- Vector-vector contact interaction

\[ g^2 D_{\mu \nu}(p - q) = \delta_{\mu \nu} \frac{4\pi \alpha_{\text{IR}}}{m_G^2} \]

- \( m_G = 800\text{MeV} \) is a gluon mass-scale
  - dynamically generated in QCD

- Gap equation:

- DCSB: \( M \neq 0 \) is possible so long as \( \alpha_{\text{IR}} > \alpha_{\text{IR}}^{\text{critical}} = 0.4\pi \)

- Observables require \( \alpha_{\text{IR}} = 0.93\pi \)
Symmetry-preserving treatment of vector×vector contact interaction is useful tool for the study of phenomena characterised by probe momenta less-than the dressed-quark mass, \( M \).

Because: \textit{For experimental observables determined by probe momenta }\( Q^2<M^2 \), contact interaction results are not realistically distinguishable from those produced by the most sophisticated renormalisation-group-improved kernels.

Symmetry-preserving regularisation of the contact interaction serves as a useful surrogate, opening domains which analyses using interactions that more closely resemble those of QCD are as yet unable to enter.

They’re critical in attempts to use data as tool for charting nature of the quark-quark interaction at long-range; i.e., identifying signals of the running of couplings and masses in QCD.
Symmetry-preserving treatment of vector-vector contact-interaction: series of papers establishes strengths & limitations.

  *Features and flaws of a contact interaction treatment of the kaon*
  Chen Chen, L. Chang, C. D. Roberts, S. M. Schmidt, Shaolong Wan and D. J. Wilson

  *Electric dipole moment of the rho-meson*
  M. Pitschmann, C.-Y. Seng, M. J. Ramsey-Musolf, C. D. Roberts, S. M. Schmidt and D. J. Wilson

  *Spectrum of Hadrons with Strangeness*
  Chen Chen, L. Chang, C.D. Roberts, Shaolong Wan and D.J. Wilson

  *Nucleon and Roper electromagnetic elastic and transition form factors*
  D. J. Wilson, I. C. Cloët, L. Chang and C. D. Roberts

  *π- and ρ-mesons, and their diquark partners, from a contact interaction*
  H.L.L. Roberts, A. Bashir, L.X. Gutiérrez-Guerrero, C.D. Roberts and David J. Wilson

  *Masses of ground and excited-state hadrons*
  H.L.L. Roberts, Lei Chang, Ian C. Cloët and Craig D. Roberts

  *Abelian anomaly and neutral pion production*

  *Pion form factor from a contact interaction*
  L. Xiomara Gutiérrez-Guerrero, A. Bashir, I. C. Cloët & C. D. Roberts
Spectrum of Hadrons with Strangeness

- Solve gap equation for $u$ & $s$-quarks

<table>
<thead>
<tr>
<th>$m_u$</th>
<th>$m_s$</th>
<th>$m_s/m_u$</th>
<th>$M_0$</th>
<th>$M_u$</th>
<th>$M_s$</th>
<th>$M_s/M_u$</th>
<th>$\kappa_0^{1/3}$</th>
<th>$\kappa_{\pi}^{1/3}$</th>
<th>$\kappa_K^{1/3}$</th>
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<tr>
<td>0.007</td>
<td>0.17</td>
<td>24.3</td>
<td>0.36</td>
<td>0.37</td>
<td>0.53</td>
<td>1.43</td>
<td>0.241</td>
<td>0.243</td>
<td>0.246</td>
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</table>

- Input ratio $m_s/m_u = 24$ is consistent with modern estimates
- Output ratio $M_s/M_u = 1.43$ shows dramatic impact of DCSB, even on the $s$-quark: $M_s - m_s = 0.36$ GeV = $M_0$
  
  ... This is typical of all DSE and lattice studies
- $\kappa$ = in-hadron condensate rises slowly with mass of hadron
Spectrum of Mesons with Strangeness

- Solve Bethe-Salpeter equations for mesons and diquarks

Fig. 2 Left panel: Pictorial representation of Table 2. Circles – computed ground-state masses; squares – computed masses of radial excitations; diamonds – empirical ground-state masses in Row 2; and triangles – empirical radial excitation masses in Row 4. Right panel: Circles – computed splitting between the first radial excitation and ground state in each channel; and triangles – empirical splittings, where they are known. The dashed line marks a splitting of 0.1 GeV.
Spectrum of Mesons with Strangeness

- Solve Bethe-Salpeter equations for mesons and diquarks

✓ Computed values for ground-states are greater than the empirical masses, where they are known.
✓ Typical of DCSB-corrected kernels that omit resonant contributions; i.e., do not contain effects that may phenomenologically be associated with a meson cloud.
Spectrum of Diquarks with Strangeness

- Solve Bethe-Salpeter equations for mesons and **diquarks**

![Graphs showing the spectrum of diquarks with strangeness](image)

**Fig. 3** Left panel: Pictorial representation of Table 4. **Diamonds** – ground-state diquark masses in Row 1; **circles** – ground-state meson masses in Row 2; **triangles** – masses of diquark first radial excitations in Row 3; and **squares** – masses of meson radial excitations in Row 4. Right panel: **Diamonds** – for diquarks, computed splittings between first radial excitation and ground state; and **circles** – for mesons, computed splitting between the first radial excitation and ground state in each channel. The **dashed line** marks a splitting of 0.1 GeV.
From apparently simple material, one arrives at a powerful elucidative tool, which provides numerous insights into baryon structure; e.g.,

There is a causal connection between $m_\Delta - m_N$ & $m_{1^+} - m_{0^+}$

Physical splitting grows rapidly with increasing diquark mass difference
Spectrum of Diquarks with Strangeness

- Solve Bethe-Salpeter equations for mesons and diquarks

- Level ordering of diquark correlations is same as that for mesons.
- In all diquark channels, except scalar, mass of diquark’s partner meson is a fair guide to the diquark’s mass:
  - Meson mass bounds the diquark’s mass from below;
  - Splitting always less than 0.13 GeV and decreases with increasing meson mass
- Scalar channel “special” owing to DCSB
Bethe-Salpeter amplitudes are couplings in Faddeev Equation

Magnitudes for diquarks follow precisely the meson pattern

Owing to DCSB, FE couplings in $\frac{1}{2}^-$ channels are 25-times weaker than in $\frac{1}{2}^+$!
Spectrum of Baryons with Strangeness

- Solved all Faddeev equations, obtained masses and eigenvectors of the octet and decuplet baryons.

Fig. 4  **Left panel:** Pictorial representation of octet masses in Table 6. **Circles** — computed masses; and **diamonds** — empirical masses. On the horizontal axis we list a particle name with a subscript that indicates its row in the table; e.g., $N_1$ means nucleon column, row 1. In this way the labels step through ground-state, radial excitation, parity partner, parity partner’s radial excitation. **Right panel:** Analogous plot for the decuplet masses in Table 6.
Comparison with bare masses from ANL-Osaka Coupled Channels Collaboration

Searched for best fit, including \((CC_{\text{BARE}}-\text{DSE})^2\)

ANL-Osaka bare masses provide excellent description of enormous amount of data

<table>
<thead>
<tr>
<th></th>
<th>(P_{11})</th>
<th>(S_{11})</th>
<th>(S_{11})</th>
<th>(P_{33})</th>
<th>(P_{33})</th>
<th>(D_{33})</th>
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<td>1.39</td>
<td>1.84</td>
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<td>(</td>
<td>\text{Rel. Err.}</td>
<td>)</td>
<td>0</td>
<td>11.3%</td>
<td>11.1%</td>
<td>7.9%</td>
</tr>
</tbody>
</table>

\(\text{rms } |\text{Rel. Err.}| = 9.4 \pm 5.7 \%\)

This is remarkable, given that the DSE results have neither been optimised nor tuned in any way.
Structure of Baryons with Strangeness

Baryon structure is flavour-blind

Table 7  Contact interaction Faddeev amplitudes for each of the octet baryons and their low-lying excitations. The superscript in the expression $s^4$ or $a^x$ is a diquark enumeration label associated with Eq. (31), except for [2, 3] and [6, 8], which are the $I = 0$ combinations in Eq. (49).

<table>
<thead>
<tr>
<th></th>
<th>$s^1$</th>
<th>$s^2$</th>
<th>$s^{[2,3]}$</th>
<th>$a^4_1$</th>
<th>$a^4_2$</th>
<th>$a^5_1$</th>
<th>$a^5_2$</th>
<th>$a^6_1$</th>
<th>$a^6_2$</th>
<th>$a_1^{[6,8]}$</th>
<th>$a_2^{[6,8]}$</th>
<th>$a^0_1$</th>
<th>$a^0_2$</th>
<th>$P_{J=0}$</th>
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</thead>
<tbody>
<tr>
<td>$(P = +, n = 0)$</td>
<td></td>
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</tr>
<tr>
<td>$N$</td>
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Baryon structure is flavour-blind

Table 7 Contact interaction Faddeev amplitudes for each of the octet baryons and their low-lying excitations. The superscript in the expression $s^I$ or $a^I$ is a diquark enumeration label associated with Eq. (31), except for [2, 3] and [6, 8], which are the $I = 0$ combinations in Eq. (49).

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- J\(_{qq}\) = 0 content of J=1/2 baryons is almost independent of their flavour structure
- Radial excitation of ground-state octet possess zero scalar diquark content!
- This is a consequence of DCSB
- Ground-state (1/2)\(^+\) possess unnaturally large scalar diquark content
- Orthogonality forces radial excitations to possess (almost) none at all!
Spectrum of Hadrons with Strangeness

- Solved all Faddeev equations, obtained masses and eigenvectors of the octet and decuplet baryons.

Fig. 4  **Left panel:** Pictorial representation of octet masses in Table 6. **Circles** – computed masses; and **diamonds** – empirical masses. On the horizontal axis we list a particle name with a subscript that indicates its row in the table; e.g., $N_1$ means nucleon column, row 1. In this way the labels step through ground-state, radial excitation, parity partner, parity partner’s radial excitation. **Right panel:** Analogous plot for the decuplet masses in Table 6.
Spectrum of Hadrons with Strangeness

- Solved all Faddeev equations, obtained masses and eigenvectors of the octet and decuplet baryons.

- This level ordering has long been a problem in CQM variants with linear or HO confinement potentials.

- Correct ordering owes to DCSB:
  - Positive parity diquarks have Faddeev equation couplings 25-times greater than negative parity diquarks.

- Explains why approaches within which DCSB cannot be realised (CQM variants) or simulations whose parameters suppress DCSB will both have difficulty reproducing experimental ordering.
Charting the Interaction

- Interaction in QCD is not momentum-independent
  - Behaviour for $Q^2 > 2\text{GeV}^2$ is well know; namely, renormalisation-group-improved one-gluon exchange
  - Computable in perturbation theory
- Known = there is a “freezing” of the interaction below a scale of roughly 0.4GeV, which is partly why momentum-independent interaction works
- Unknown
  - Infrared behavior of the interaction, which is responsible for
    - Confinement
    - DCSB
  - How is the transition to $\text{pQCD}$ made and is it possible to define a transition boundary?
DSE Studies
- Phenomenology of gluon

- Wide-ranging study of $\pi$ & $\rho$ properties

- Effective coupling
  - Agrees with pQCD in ultraviolet
  - Saturates in infrared
    - $\alpha(0)/\pi = 8 - 15$
    - $\alpha(m_G^2)/\pi = 2 - 4$

- Running gluon mass
  - Gluon is massless in ultraviolet
    - in agreement with pQCD
  - Massive in infrared
    - $m_G(0) = 0.67 - 0.81$ GeV
    - $m_G(m_G^2) = 0.53 - 0.64$ GeV

Qin et al., *Phys. Rev. C* 84 042202(Rapid Comm.) (2011)
Rainbow-ladder truncation

Craig Roberts: Bound state problem in continuum QCD (87p)
- quark’s mass depends on its momentum.
- Mass function can be calculated and is depicted here.
- Continuum- and Lattice-QCD are in agreement: the vast bulk of the light-quark mass comes from a cloud of gluons, dragged along by the quark as it propagates.

\[ S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)} \]


Mass from nothing!
Electron is a good probe because it is structureless

Structureless fermion, or simply structured fermion, $F_1 = 1$ & $F_2 = 0$, so that $G_E = G_M$ and hence distribution of charge and magnetisation within this fermion are identical

Proton’s electromagnetic current

$$J_\mu(P', P) = ie \bar{u}_p(P') \Lambda_\mu(Q, P) u_p(P),$$

$$= ie \bar{u}_p(P') \left( \gamma_\mu F_1(Q^2) + \frac{1}{2M} \sigma_{\mu\nu} Q_\nu F_2(Q^2) \right) u_p(P).$$

$F_1 = $ Dirac form factor $F_2 = $ Pauli form factor

$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2),$ $G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$

$G_E = $ Sachs Electric form factor
If a nonrelativistic limit exists, this relates to the charge density

$G_M = $ Sachs Magnetic form factor
If a nonrelativistic limit exists, this relates to the magnetisation density
Nucleon form factors

- For the nucleon & Δ-resonance, studies of the Faddeev equation exist that are based on the 1-loop renormalisation-group-improved interaction that was used efficaciously in the study of mesons
  - *Toward unifying the description of meson and baryon properties*
    G. Eichmann, I.C. Cloët, R. Alkofer, A. Krassnigg and C.D. Roberts
  - *Survey of nucleon electromagnetic form factors*
    I.C. Cloët, G. Eichmann, B. El-Bennich, T. Klähn and C.D. Roberts
  - *Nucleon electromagnetic form factors from the Faddeev equation*

- These studies retain the scalar and axial-vector diquark correlations, which we know to be necessary and sufficient for a reliable description

- In order to compute form factors, one needs a photon-nucleon current
Vertex must contain the dressed-quark anomalous magnetic moment

- Composite nucleon must interact with photon via nontrivial current constrained by Ward-Green-Takahashi identities
- DSE $\rightarrow$ BSE $\rightarrow$ Faddeev equation plus current $\rightarrow$ nucleon form factors
- NB. Diquarks are electromagnetically active. Cannot ignore the photon-diquark coupling.

Oettel, Pichowsky, Smekal

Survey of nucleon electromagnetic form factors
I.C. Cloët et al, arXiv:0812.0416 [nucl-th],

Unification of meson and nucleon form factors.

Very good description.

Quark’s momentum-dependent anomalous magnetic moment has observable impact & materially improves agreement in all cases.
Distribution of charge and magnetisation within the proton

- Data before 1999
  - Looks like the structure of the proton is simple
  - Distribution of charge and magnetisation are the same
  - Always like that in particle-physics-like pictures of hadrons

\[ \frac{\mu_p G_E^P(Q^2)}{G_M^P(Q^2)} \]
Distribution of charge and magnetisation within the proton

- The properties of JLab (high luminosity) enabled a new technique to be employed.
- First data released in 1999 and paint a VERY DIFFERENT PICTURE

\[ \frac{\mu_p G_E^P(Q^2)}{G_M^P(Q^2)} \]

Which is correct? How is the difference to be explained?
The Electric and Magnetic Elastic Proton Form Factor Ratio $G_E^p / G_M^p$

One of the fundamental goals of nuclear physics is to understand the structure and behavior of strongly interacting matter in terms of its basic constituents, quarks and gluons. An important step towards this goal is the characterization of the internal structure of the nucleon; the elastic electric and magnetic form factors of the proton and neutron are key ingredients of this characterization. The elastic electromagnetic form factors are directly related to the charge and current distributions inside the nucleon; these form factors are among the most basic observables of the nucleon.

The challenge of understanding the nucleon’s structure and dynamics has occupied a central place in nuclear physics; many experimental and theoretical physicists have spent a considerable amount of effort in the last five decades to understand it. A break-through was made towards this goal in the last decade and a half, when two JLab Hall A experiments extracted the elastic electromagnetic form factor ratio of the proton, $G_{Ep}/G_{Mp}$, from the measured recoil proton polarization components, using the polarization transfer method. A third experiment in Hall C at JLab has pushed the highest Q2 limit to 8.5 GeV2 using the same method.

The form factor ratio data from all three JLab recoil polarization experiments are shown in Figure 1, which display a strikingly different Q2 dependence than the Rosenbluth results. The most important feature of the data from JLab is the decrease of the ratio to Q2= 8.5 GeV2, which indicates that $G_{Ep}$ falls faster with increasing Q2 than $G_{Mp}$ and demonstrates that the spatial extension of charge is larger than that of magnetization. This is the first definitive experimental indication that the Q2 dependence of $G_{Ep}$ and $G_{Mp}$ is different. In the third experiment in Hall C, the ratio is decreasing, however with a strong indication that the linear behavior has softened toward a possible constancy of the ratio at Q2 values beyond the range covered so far.

**References:**

I.C. Cloët, C.D. Roberts, A.W. Thomas, 

- DSE result Dec 08
- DSE result including the anomalous magnetic moment distribution
- Highlights again the critical importance of DCSB in explanation of real-world observables.
DSE studies indicate that the proton has a very rich internal structure.

The JLab data, obtained using the polarisation transfer method, are an accurate indication of the behaviour of this ratio.

The pre-1999 data (Rosenbluth) receive large corrections from so-called 2-photon exchange contributions.

\[ \frac{\mu_p G_E^p(Q^2)}{G_M^p(Q^2)} \]

Proton plus proton-like resonances

Does this ratio pass through zero?
Origin of the zero & its location

- The Pauli form factor is a gauge of the distribution of magnetization within the proton. Ultimately, this magnetization is carried by the dressed quarks and influenced by correlations amongst them, which are expressed in the Faddeev wave function.

- If the dressed quarks are described by a momentum-independent mass function, $M=\text{constant}$, then they behave as Dirac particles with constant Dirac values for their magnetic moments and produce a hard Pauli form factor.
Alternatively, suppose that the dressed quarks possess a momentum-dependent mass function, $M=M(p^2)$, which is large at infrared momenta but vanishes as their momentum increases.

At small momenta they will then behave as constituent-like particles with a large magnetic moment, but their mass and magnetic moment will drop toward zero as the probe momentum grows. (Remember: Massless fermions do not possess a measurable magnetic moment)

Such dressed quarks produce a proton Pauli form factor that is large for $Q^2 \sim 0$ but drops rapidly on the domain of transition between nonperturbative and perturbative QCD, to give a very small result at large $Q^2$. 

Craig Roberts: Bound state problem in continuum QCD (87p)
Origin of the zero & its location

- The precise form of the $Q^2$ dependence will depend on the evolving nature of the angular momentum correlations between the dressed quarks.
- From this perspective, existence, and location if so, of the zero in
  \[ \mu_p G_{Ep}(Q^2)/G_{Mp}(Q^2) \]
  are a fairly direct measure of the location and width of the transition region between the nonperturbative and perturbative domains of QCD as expressed in the momentum dependence of the dressed-quark mass function.
- Hard, $M=\text{constant}$
  \[ \rightarrow \] Soft, $M=M(p^2)$
Origin of the zero & its location

- One can anticipate that a mass function which rapidly becomes partonic—namely, is very soft—will not produce a zero
- We’ve seen that a constant mass function produces a zero at a small value of $Q^2$
- And also seen and know that a mass function which resembles that obtained in the best available DSE studies and via lattice-QCD simulations produces a zero at a location that is consistent with extant data.
- There is opportunity here for very constructive feedback between future experiments and theory.

Visible Impacts of DCSB

- Apparently small changes in $M(p)$ within the domain $1<p(\text{GeV})<3$ have striking effect on the proton’s electric form factor.
- The possible existence and location of the zero is determined by behaviour of $Q^2 F^p_2(Q^2)$.
- Like the pion’s PDA, $Q^2 F^p_2(Q^2)$ measures the rate at which dressed-quarks become parton-like:
  - $F^p_2=0$ for bare quark-partons 
  - Therefore, $G_E^p$ can’t be zero on the bare-parton domain.
Visible Impacts of DCSB

Follows that the

✓ possible existence
✓ and location of a zero in the ratio of proton elastic form factors

\[ \left[ \mu_p G_{Ep}(Q^2)/G_{Mp}(Q^2) \right] \]

are a direct measure of the nature of the quark-quark interaction in the Standard Model.

Leads to Prediction neutron:proton

\[ G_{En}(Q^2) > G_{Ep}(Q^2) \] at \( Q^2 > 4\text{GeV}^2 \)
Flavor Decomposition of the Elastic Nucleon Electromagnetic Form Factors

Abstract

The $u$- and $d$-quark contributions to the elastic nucleon electromagnetic form factors have been determined by using experimental data on $G_E^n$, $G_M^n$, $G_E^p$, and $G_M^p$. Such a flavor separation of the form factors became possible up to negative four-momentum transfer squared $Q^2=3.4$ GeV$^2$ with recent data on $G_E^p$ from Hall A at Jefferson Lab. For $Q^2$ above 1 GeV$^2$, for both the $u$ and the $d$ quark, the ratio of the Pauli and Dirac form factors, $F_2/F_1$, was found to be almost constant in sharp contrast to the behavior of $F_2^u/F_1^u$ for the proton as a whole. Also, again for $Q^2>1$ GeV$^2$, both $F_2^d$ and $F_1^d$ are roughly proportional to $1/Q^4$, whereas the dropoff of $F_2^u$ and $F_1^u$ is more gradual.

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DOI: 10.1103/PhysRevLett.106.252003
Flavor separation of proton form factors

Very different behavior for $u$ & $d$ quarks

Means apparent scaling in proton $F2/F1$ is purely accidental

$Q^4F_2^q/\kappa$

$Q^4F_1^q$

Cates, de Jager, Riordan, Wojtsekhowski, PRL 106 (2011) 252003
Diquark correlations!

- Poincaré covariant Faddeev equation
  - Predicts scalar and axial-vector diquarks
- Proton's singly-represented $d$-quark more likely to be struck in association with $1^+$ diquark than with $0^+$
  - Form factor contributions involving $1^+$ diquark are softer
- Doubly-represented $u$-quark is predominantly linked with harder $0^+$ diquark contributions
- Interference produces zero in Dirac form factor of $d$-quark in proton
  - Location of the zero depends on the relative probability of finding $1^+$ & $0^+$ diquarks in proton
  - Correlated, e.g., with valence $d/u$ ratio at $x=1$
Adding dressed-quark AMM (sensibly modifying the current operator) can quantitatively improve the description of data.

But it’s the presence of diquark correlations that explains the difference between u- and d-quark distributions within the nucleon.

Craig Roberts: Bound state problem in continuum QCD (87p)
Far valence domain

$x \approx 1$
Endpoint of the far valence domain: $x \approx 1$, is especially significant
- All familiar PDFs vanish at $x=1$; but ratios of any two need not
- Under DGLAP evolution, the value of such a ratio is invariant.

Thus, e.g.,
- $\lim_{x \to 1} \frac{d_v(x)}{u_v(x)}$
  is unambiguous, scale invariant, nonperturbative feature of QCD.

∴ keen discriminator between frameworks that claim to explain nucleon structure.

Furthermore, Bjorken-$x=1$ corresponds strictly to the situation in which the invariant mass of the hadronic final state is precisely that of the target; viz., elastic scattering.

∴ Structure functions inferred experimentally on $x \approx 1$
  are determined theoretically by target's elastic form factors.
Neutron Structure

Function at high-x

- Valence-quark distributions at $x=1$
  - Fixed point under DGLAP evolution
  - Strong discriminator between theories

Algebraic formula

$$\left. \frac{d_v(x)}{u_v(x)} \right|_{x \to 1} = \frac{P^{p,d}_1}{P^{p,u}_1} = \frac{2}{3} P^{p,a}_1 + \frac{1}{3} P^{p,m}_1 + \frac{2}{3} P^{p,s}_1 + \frac{1}{3} P^{p,a}_1 + \frac{2}{3} P^{p,m}_1$$

Measures relative strength of axial-vector/scalar diquarks in proton

- $P^{p,s}_1$ = contribution to the proton's charge arising from diagrams with a scalar diquark component in both the initial and final state
- $P^{p,a}_1$ = kindred axial-vector diquark contribution
- $P^{p,m}_1$ = contribution to the proton's charge arising from diagrams with a different diquark component in the initial and final state.
Neutron Structure Function at high-\( x \)

Deep inelastic scattering – the Nobel-prize winning quark-discovery experiments

Reviews:
- S. Brodsky et al.
  NP B441 (1995)
- W. Melnitchouk & A.W. Thomas
  PL B377 (1996) 11
- N. Isgur, PRD 59 (1999)
- R.J. Holt & C.D. Roberts
  RMP (2010)

\[ \frac{F_{2n}}{F_{2p}} \]

\( x > 0.9 \)

\( d/u = 1/2 \)

SU(6) symmetry

\( d/u = 0.28 \)

DSE: "realistic"

pQCD, uncorrelated \( \psi \)

DSE: "contact"

\( d/u = 0.18 \)

0\(^+\) \( \psi\psi \) only, \( d/u = 0 \)

Melnitchouk, Accardi et al.
Phys.Rev. D84 (2011) 117501

Melnitchouk, Arrington et al.

Distribution of neutron’s momentum amongst quarks on the valence-quark domain

Craig Roberts: Bound state problem in continuum QCD (87p)
Neutron Structure
Function at high-x

Deep inelastic scattering – the Nobel prize winning quark-discovery experiments

Reviews:
- S. Brodsky et al.
  NP B441 (1995)
- W. Melnitchouk & A.W. Thomas
  PL B377 (1996) 11
- N. Isgur, PR D19 (1999)
- R.J. Holt & C.D. Roberts
  RMP (2010)

NB.
\[ d/u \mid_{x=1} = 0 \text{ means there are no valence } d\text{-quarks in the proton!} \]

JLab12 can solve this enigma

Distribution of neutron’s momentum amongst quarks on the valence-quark domain

Craig Roberts: Bound state problem in continuum QCD (87p)
Nucleon spin structure at very high $x$

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ABSTRACT

Dyson–Schwinger equation treatments of the strong interaction show that the presence and importance of nonperturbative quark correlations within the nucleon are a natural consequence of dynamical chiral symmetry breaking. Using this foundation, we deduce a collection of simple formulae, expressed in terms of quark appearance and mixing probabilities, from which one may compute ratios of longitudinal-spin-dependent $u$- and $d$-quark parton distribution functions on the domain $x \approx 1$. A comparison with predictions from other approaches plus a consideration of extant and planned experiments shows that the measurement of nucleon longitudinal spin asymmetries on $x \approx 1$ can add considerably to our capacity for discriminating between contemporary pictures of nucleon structure.

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Nucleon spin structure at very high $x$

- Similar formulae for nucleon longitudinal structure functions.
- Plainly, existing data cannot distinguish between modern pictures of nucleon structure.
- Empirical results for nucleon longitudinal spin asymmetries on $x \approx 1$ promise to add greatly to our capacity for discriminating between contemporary pictures of nucleon structure.

NB. pQCD is actually model-dependent: assumes $SU(6)$ spin-flavour wave function for the proton's valence-quarks and the corollary that a hard photon may interact only with a quark that possesses the same helicity as the target.
Tensor Charge: $\sigma_{\mu\nu}$ current

$$\delta q = \int_0^1 dx \left(h_1^q(x) - h_1^\bar{q}(x)\right)$$

- $h_{1T} = \text{distribution of transversely polarized quarks inside a transversely polarised proton}$

- $\delta q = \text{Light-front number-density of quarks with transverse polarisation parallel to that of the proton minus that of quarks with transverse polarisation antiparallel}$
  - Bias in quark polarisation induced by polarisation of parent proton

- Value of tensor charge places constraints on some extensions of the Standard Model \(<\text{PRD85 (2012) 054512}\>\)

- Current knowledge of transversity: SIDIS @HERMES, COMPASS, JLab

- No gluon transversity distribution => transversity is suppressed at low-$x$, hence large-$x$ behavior important => JLab12 a useful tool. Transversity will be measured at JLab12 (Hall-A E12-09-018-SIDIS; CLAS12; and SoLID)
Direction of motion

Tensor Charge: \( \sigma_{\mu\nu} \) current

\[
\delta q = \int_0^1 dx \left( h^q_1(x) - h^\bar{q}_1(x) \right)
\]

1. Jlab 12 Projection
2. Anselmino et al., 1303.3822 [hep-ph]
3. Pitschmann et al. (DSE) (2014) [including axial-vector diquarks but contact interaction]
4. Hecht et al. (DSE), PRC64 (2001) 025204 [only scalar diquarks]
5. Cloët et al., PLB659 (2008) 214
8. Gockeler et al., PLB627 (2005) 113
10. He et al., PRD52 (1995) 2960

Big shift from including axial-vector diquark correlations? d-quark can now be unpaired.

Evolution to 4GeV²
DSEs: A practical, predictive, unifying tool for fundamental physics

- Exact results proved in QCD, amongst them:
  - Quarks are not Dirac particles and gluons are nonperturbatively massive
  - Dynamical chiral symmetry breaking is a fact.
    It’s responsible for 98% of the mass of visible matter in the Universe
  - Goldstone’s theorem is fundamentally an expression of equivalence between the one-body problem and the two-body problem in the pseudoscalar channel
  - Confinement is a dynamical phenomenon
    It cannot in principle be expressed via a potential
  - The list goes on ...

DSEs are a single framework, with IR model-input turned to advantage,
“almost unique in providing an unambiguous path from a defined interaction → Confinement & DCSB → Masses → radii → form factors → distribution functions → etc.”

Craig Roberts: Bound state problem in continuum QCD (87p)

Sao Paulo: II Workshop on Perspectives in Nonperturbative QCD, 12-13 May 14