The role of Asymptotic Freedom for the Pseudocritical Temperature in Magnetized Quark Matter

Ricardo L.S. Farias
Departamento de Física
Universidade Federal de Santa Maria

In Collaboration with: G. Krein (IFT), K.P. Gomes (UFSJ) and M.B. Pinto (UFSC)

II Workshop on Perspectives in Nonperturbative QCD
Outline

- Motivation
- Magnetic Fields and chiral symmetry breaking - magnetic catalysis
- Lattice Results - disagreements between lattice results and model calculations regarding $T_{PC} \times B$
- Including asymptotic freedom in NJL model
- Results and perspectives
Motivation

- Why magnetic fields are interesting for QCD matter?
Motivation

- Signatures of QGP in Heavy-ion collisions (large $T$ and low $\mu$)?

- Compact Stars: quark stars? neutron stars? or hybrid stars (large $\mu$ (400 MeV) and low $T$)?

\textbf{In this regimes LARGE magnetic fields are present!!!}
Cartoon of QCD phase diagram

B effects on:
- position of CP
- confinement trans.
- chiral transition
- stars
- 2SC and CFL
- QGP
- Early Universe

credits: GSI Darmstadt
Motivation

Strong magnetic fields may be produced in non central heavy ion collisions:


Heavy-ion collisions:

Temporarily $B \lesssim 10^{19}$ G

Motivation

- Strong magnetic fields are also present in magnetars: C. Kouveliotou et al., Nature 393, 235 (1998).

magnetars:

at surface $B \lesssim 10^{15}$ G
larger in the interior, $B \sim 10^{18-20}$ G?
E. J. Ferrer et al., PRC 82, 065802 (2010)


Motivation

- We need to understand quark confinement and chiral symmetry breaking.
- Moreover deconfinement and chiral symmetry restoration at finite temperature and/or density - (NEW PHASES)
- In heavy ion colliders: two beams of charged particles in opposite direction.
- Ext. magnetic field: short-time, large magnitude, QCD out of equilibrium?
- ...
- magnetic field as another axis of the QCD phase diagram!
Amplitudes of magnetic fields

earth - 0.6 Gauss

magnet - 100 Gauss

neutron stars (surface of magnetars)

$\sim 10^{15} \ldots 10^{15} \text{G} \Rightarrow eB^{1/2} \sim 1 \text{MeV}$

RHIC/LHC - $eB^{1/2}$

$= 0.1 \ldots 0.5 \text{GeV}$. The strongest magnetic field ever achieved in the lab. ($10^{17} \text{G}$)

early universe - $eB^{1/2} \sim 2 \text{GeV}$
Motivation

- The behavior of QCD under extreme conditions: temperature, density, external magnetic fields
- Problem: QCD is nonperturbative in relevant scales
- Lattice: Signal problem!!!
To make progress

- We use Quantum Field Theory (in medium)
- Experiments - most expensive!!!
- Effective models (just a few degrees of freedom)
- Lattice (limitations...)

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Lattice Results

At vanishing baryon density and magnetic field, lattice QCD simulations predict that there is a \textit{crossover} transition at a pseudo critical temperature $T_{pc}$.

First Lattice Results


- indications that chiral $T_{PC}$ is increasing as a function of $B$.

Results in agreement with effective models: NJL, PNJL, QMM…
However:

- They use the bare quark masses used correspond to a pion mass in the range $m_\pi = 200 - 480$ MeV, i.e. a very heavy pion.

- These results have been confirmed by:

- For light quark masses that correspond to the physical pion mass of $m_\pi = 140$ MeV, their simulations show a $T_{PC}$ which is a decreasing function of the magnetic field $B$.

- The basic mechanism seems to be that the MC at $T = 0$ turns into IMC for $T$ around $T_c$.
- The results suggest that the $T_{PC}$ is a nontrivial function of the quark masses.
Recent Lattice Results X Effect. models

lattice definitions:

\[ \Sigma_f(B, T) = \frac{2m_f}{m^2 \pi f^2} \left[ \langle \bar{\psi}_f \psi_f \rangle - \langle \bar{\psi}_f \psi_f \rangle_0 \right] + 1 \]

\[ \Delta \Sigma_{u,d}(B, T) = \Sigma_{u,d}(B, T) - \Sigma_{u,d}(0, T). \]


Right Panel: CM and IMC
Recent Lattice Results


$\Sigma_u - \Sigma_0$

$T$ (MeV)

$CM(T=0)$ and $IMC(high T)$
SU(2) NJL model

The standard two flavor NJL model is defined by a fermionic Lagrangian density

\[ \mathcal{L}_{\text{NJL}} = \bar{\psi} (i \slashed{\partial} - m) \psi + G \left[ (\bar{\psi} \psi)^2 - (\bar{\psi} \gamma_5 \bar{\tau} \psi)^2 \right] \]

In mean field approximation (MFA):

\[ \mathcal{F}_{\text{NJL}} = \frac{(M - m)^2}{4G} + \frac{i}{2} \text{tr} \int \frac{d^4 p}{(2\pi)^4} \ln[-p^2 + M^2] \]
NJL at finite $T$, $\mu$ and $B$

To study the effect of $B$ in the chiral transition at finite $T$ and $\mu$, a dimensional reduction is induced:

$$p_0 \rightarrow i(\omega_\nu - i\mu)$$

$$p^2 \rightarrow p_z^2 + (2n + 1 - s)|q_f|B$$

$$s = \pm 1$$

$$n = 0, 1, 2...$$

$$\int_{-\infty}^{+\infty} \frac{d^4 p}{(2\pi)^4} \rightarrow i \frac{T|q_f|B}{2\pi} \sum_{\nu=-\infty}^{\infty} \sum_{n=0}^{\infty} \int_{-\infty}^{+\infty} \frac{dp_z}{2\pi}$$

$$\mathcal{F}_{NJL} = \frac{(M - m)^2}{4G} + \mathcal{F}_{NJL}^\text{vac} + \mathcal{F}_{NJL}^\text{mag} + \mathcal{F}_{NJL}^\text{med}$$
NJL at finite $T$, $\mu$ and $B$

\[
\mathcal{F}_{\text{vac}}^{\text{NJL}} = -2N_c N_f \int \frac{d^3p}{(2\pi)^3} (p^2 + M^2)^{1/2}
\]

\[
\mathcal{F}_{\text{vac}}^{\text{NJL}} = \frac{N_c N_f}{8\pi^2} \left\{ M^4 \ln \left[ \frac{(\Lambda + \epsilon_\Lambda)}{M} \right] - \epsilon_\Lambda \Lambda \left[ \Lambda^2 + \epsilon_\Lambda^2 \right] \right\}
\]

\[
\epsilon_\Lambda = \sqrt{\Lambda^2 + M^2}
\]

\[
\mathcal{F}_{\text{mag}}^{\text{NJL}} = -\frac{N_c}{2\pi^2} \sum_{f=u}^{d} (|q_f| B)^2 \left\{ \zeta^{(1,0)}(-1, x_f) - \frac{1}{2} [x_f^2 - x_f] \ln(x_f) + \frac{x_f^2}{4} \right\}
\]

\[
\mathcal{F}_{\text{med}}^{\text{NJL}} = -\frac{N_c}{2\pi} \sum_{f=u}^{d} \sum_{k=0}^{\infty} \alpha_k |q_f| B \int_{-\infty}^{+\infty} \frac{dp_z}{2\pi} \left\{ T \ln[1 + e^{-[E_{p, k(B)}+\mu]/T}] + \mu \to -\mu \right\}
\]
Numerical Implementation: difficulties

- $T = 0$ OK!
- $T \neq 0$ We need to numerically perform the sums over the Landau levels, or find suitable approximations...
- Job becomes easier in the high magnetic field regime: $\sqrt{eB} \gg T$
- But this is not so in the low magnetic field regime, with $\sqrt{eB} \lesssim T$, and $\Omega/T \lesssim 1$ (requires to sum up to very large Landau levels)
- Any numerical computation to become quickly expensively (integrations involved no high $T$)

Here we discuss a more natural alternative for dealing with the cases of low magnetic fields!!!
Euler-Maclaurin formula

\[
\sum_{k=a}^{b} f(k) = \int_{a}^{b} f(x) \, dx + \frac{1}{2} [f(a) + f(b)] + \sum_{i=1}^{n} \frac{b_{2i}}{(2i)!} \left[ f^{(2i-1)}(b) - f^{(2i-1)}(a) \right] \\
+ \int_{a}^{b} \frac{B_{2n+1}(\{x\})}{(2n+1)!} f^{(2n+1)}(x) \, dx
\]

\( b_{i} \) are the Bernoulli numbers, defined by the generating function

\[
\frac{x}{\exp(x) - 1} = \sum_{n=0}^{\infty} b_{n} \frac{x^{n}}{n!}
\]

\( B_{n}(x) \) are the Bernoulli polynomials, with generating function

\[
\frac{z \exp(zx)}{\exp(z) - 1} = \sum_{n=0}^{\infty} B_{n}(x) \frac{z^{n}}{n!}
\]
Reliability of the use of the EM formula

\[ L\tilde{Y} = \sum_{k=0}^{\infty} \int_{-\infty}^{+\infty} \frac{dz}{2\pi} \ln \left[ 1 - e^{-E(z,\Omega,T,B,k)} \right] \]

where

\[ E(z,\Omega,T,B,k) = \sqrt{z^2 + \frac{\Omega^2}{T^2} + (2k+1)\frac{eB}{T^2}} \]

\[ L\tilde{X} = \sum_{k=0}^{\infty} \int_{-\infty}^{+\infty} \frac{dz}{2\pi} \frac{1}{E(z,\Omega,T,B,k)} \frac{1}{e^{E(z,\Omega,T,B,k)} - 1} \]
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NJL at finite $T$, $\mu$ and $B$

$$E_{p,k}(B) = \sqrt{p_{z}^{2} + 2k|q_{f}|B + M^{2}}$$

where $M$ is the effective self consistent quark mass

$$M = m + \frac{N_{c}N_{f}MG}{\pi^{2}} \left\{ \Lambda \sqrt{\Lambda^{2} + M^{2}} - \frac{M^{2}}{2} \ln \left[ \left( \frac{\Lambda + \sqrt{\Lambda^{2} + M^{2}}}{M} \right)^{2} \right] \right\}$$

$$+ \frac{N_{c}MG}{\pi^{2}} \sum_{f=u}^{d} |q_{f}|B \left\{ \ln[\Gamma(x_{f})] - \frac{1}{2} \ln(2\pi) + x_{f} - \frac{1}{2} (2x_{f} - 1) \ln(x_{f}) \right\}$$

$$- \frac{N_{c}MG}{2\pi^{2}} \sum_{f=u}^{d} \sum_{k=0}^{\infty} \alpha_{k} |q_{f}|B \int_{-\infty}^{\infty} \frac{dp_{z}}{E_{p,k}(B)} \left\{ \frac{1}{e^{[E_{p,k}(B)+\mu]/T} + 1} + \frac{1}{e^{[E_{p,k}(B)-\mu]/T} + 1} \right\}$$
Quark Condensate

\[
\langle \bar{\psi}_f \psi_f \rangle = -\frac{N_c M}{2\pi^2} \left\{ \Lambda \sqrt{\Lambda^2 + M^2} - \frac{M^2}{2} \ln \left[ \frac{(\Lambda + \sqrt{\Lambda^2 + M^2})^2}{M^2} \right] \right\}
\]

\[
-\frac{N_c M}{2\pi^2} |q_f| B \left\{ \ln[\Gamma(x_f)] - \frac{1}{2} \ln(2\pi) + x_f - \frac{1}{2} (2x_f - 1) \ln(x_f) \right\}
\]

\[
+ \frac{N_c M}{4\pi^2} \sum_{k=0}^{\infty} \alpha_k |q_f| B \int_{-\infty}^{\infty} \frac{dp_z}{E_{p,k}(B)} \left\{ \frac{1}{e^{[E_{p,k}(B)+\mu]/T}+1} + \frac{1}{e^{[E_{p,k}(B)-\mu]/T}+1} \right\}
\]

Where: \( E_{p,k}(B) = \sqrt{p_z^2 + 2k|q_f|B + M_f^2} \)\\
\( |q_u| = 2e/3, |q_d| = e/3 \)

\( x_f = M_f^2 / (2|q_f|B) \)

\( \alpha_k = 2 - \delta_{0k} \)

We use Gaussian natural units \( 1 \text{ GeV}^2 = 1.44 \times 10^{19} \text{ G} \)
Magnetic Catalysis - NJL $T=0$


V. P. Gusynin, V. A. Miransky, I. A. Shovkovy, PLB 349, 477-483 (1995)

The phenomenological values of quantities such as the pion mass $m_\pi$, the pion decay constant $f_\pi$, and the quark condensate $\langle \bar{\psi}_f \psi_f \rangle$ are used to fix $G$, $\Lambda$, and $m$. Here, we choose the set $\Lambda = 650$ MeV and $G = 5.022$ GeV$^{-2}$ with $m = 5.5$ MeV in order to reproduce $f_\pi = 93$ MeV, $m_\pi = 140$ MeV, and $\langle \bar{\psi}_f \psi_f \rangle^{1/3} = -250$ MeV in the vacuum.
NJL results at finite $T$ and $B$

- Gap equation: $M_f = m_f - 2G \sum_f \langle \overline{\psi}_f \psi_f \rangle$
- where $\langle \overline{\psi}_f \psi_f \rangle$ is the quark condensate of flavor $f$

These evaluations have been considered at more sophisticated levels:
- Polyakov Loop (PNJL, EPNJL)
- Chiral PT (Andresen)
- including strangeness
- beyond MFA (FRG) (Fukushima)
- despite those refinements in model calculations, no qualitative changes in TPC X B.
Exceptions?

1) Bag model calculation (First order PT) - E. S. Fraga and L. F. Palhares, Phys. Rev. D 86, 016008 (2012)


- All predict a decreasing $T_{PC}$ with $B$

- In 1 and 2 it is probably related to their treatment of vacuum fluctuations and related renormalization issues…
Exceptions?


F. Bruckmann, G. Endrodi and T. G. Kovacs, JHEP 1304, 112 (2013) - IMC is the result of the back-reaction of the gluons due to the coupling of the magnetic field to the sea quarks.

Be careful with the approximations!

The chiral phase transition and the role of vacuum fluctuations
Jens O. Andersen, Rashid Khan, Lars T. Kyllingstad (Norwegian U. Sci. Tech.).

e.g., QM model, chiral symmetry breaking takes place in the meson sector, and as a consequence, the vacuum contribution to the free energy from the fermions is sometimes omitted.

In the NJL model, on the other hand, this term is responsible for the chiral symmetry breaking, and so it cannot be neglected.

When making such simplifications, however, it is important to know exactly what one is discarding, in order not to “throw the baby out with the bath water”


It was recently shown that neglecting the fermion vacuum contribution to the QM free energy changes the order of the phase transition!
Phase Diagram of Strong Interactions in an External Magnetic Field
Published in PoS FACESQCD (2010) 020
Recently...

Deconfinement and chiral restoration within the SU(3) Polyakov--Nambu--Jona-Lasinio and entangled Polyakov--Nambu--Jona-Lasinio models in an external magnetic field

Published in Phys.Rev. D89 (2014) 016002
DOI: 10.1103/PhysRevD.89.016002

IMC with critical temperature for the deconfinement phase transition in pure gauge $T_0 = T_0(B)$
Recently...

A search for inverse magnetic catalysis in thermal quark-meson models

E. S. Fraga, B. W. Mintz, J. Schaffner-Bielich

(Submitted on 15 Nov 2013)

We explore the parameter space of the two-flavor thermal quark-meson model and its Polyakov loop-extended version under the influence of a constant external magnetic field \( B \). We investigate the behavior of the pseudo critical temperature for chiral symmetry breaking taking into account the likely dependence of two parameters on the magnetic field: the Yukawa quark-meson coupling and the parameter \( T_0 \) of the Polyakov loop potential. Under the constraints that magnetic catalysis is realized at zero temperature and the chiral transition at \( B=0 \) is a crossover, we find that the quark-meson model leads to thermal magnetic catalysis for the whole allowed parameter space, in contrast to the present picture stemming from lattice QCD.

"If one takes the usual parameter fixing in the vacuum, \( g(0) = 3.3 \), there is no continuous function \( g(B) \) that could lead to inverse magnetic catalysis in the QM model at finite temperature and zero quark chemical potential, unless the chiral transition is of first order"
Effective quark theories as the NJL model can be motivated by -> QCD integrating out gluonic degrees of freedom.

Although some features of confinement can be enforced by means of extending the model with the Polyakov loop...

the running with energy scales of the effective coupling, as e.g. due to asymptotic freedom, is lost.

we have examined the effect of introducing a running coupling $G$ motivated by asymptotic freedom.
Our Purpose

Let us recall the important result by Miransky and Shovkovy (for \( eB \gg \Lambda_{QCD}^2 \))


The leading order running of the QCD coupling constant \( \alpha_s \) is given by

\[
\frac{1}{\alpha_s} \sim b \ln \frac{eB}{\Lambda_{QCD}^2}
\]

with \( b = (11N_c - 2N_f)/12\pi \)

As \([\alpha_s] = [G\Lambda^2]\)

we propose for the NJL coupling, at \( T = 0 \),

the interpolating formula

\[
G(B) = \frac{G_0}{1 + \alpha \ln \left( 1 + \beta \frac{eB}{\Lambda_{QCD}^2} \right)}
\]
Our Purpose

\[ G(B) = \frac{G_0}{1 + \alpha \ln \left( 1 + \beta \frac{eB}{\Lambda_{QCD}^2} \right)} \]

with \( G_0 = 5.022 \text{ GeV}^{-2} \), which is the value of the coupling at \( B=0 \).

\[ \alpha \text{ and } \beta \text{ are fixed to obtain a reasonable description of the lattice average } \left( \frac{\Sigma_u + \Sigma_d}{2} \right) \text{ for } T=0 \]

At high temperatures, \( \alpha_s \) also runs as the inverse of

\[ \ln \left( \frac{T}{\Lambda_{QCD}} \right) \]
Our Purpose

However, the values of T used in the lattice simulations

\[ T \leq \Lambda_{QCD} \]

are not high enough to justify the use of such a running for G.

Moreover, the exact dependence of the coupling with B AND T is not known presently.

\[ G(B) \simeq G_0 \left(1 - \alpha \beta eB / \Lambda_{QCD}^2\right) \]


Our Ansatz:

\[ G(B, T) = G(B) \left(1 - \gamma \frac{|eB|}{\Lambda_{QCD}^2} \frac{T}{\Lambda_{QCD}}\right) \]

\[ \gamma \]

for reasonable lattice

\[ \frac{(\Sigma_u + \Sigma_d)}{2} \] at high T.
Numerical Results - quark(u) condensate

$$\langle \bar{\psi} u \psi \rangle \text{[MeV]}$$

- $eB = 1.0 \text{ GeV}^2$
- $eB = 0.8 \text{ GeV}^2$
- $eB = 0.6 \text{ GeV}^2$
- $eB = 0.4 \text{ GeV}^2$
- $eB = 0.2 \text{ GeV}^2$
- $eB = 0.0$

MC at $T=0$

IMC at high $T$
Numerical Results - Thermal Mass

$M(\text{MeV})$ vs $T(\text{MeV})$

- $eB = 1.0 \text{ GeV}^2$
- $eB = 0.8 \text{ GeV}^2$
- $eB = 0.6 \text{ GeV}^2$
- $eB = 0.4 \text{ GeV}^2$
- $eB = 0.2 \text{ GeV}^2$
- $eB = 0.0$

MC at $T=0$

IMC at high $T$
Numerical Results - condensate average

- MC at $T=0$
- Good results at $T=0$
- IMC at high $T$

Numerical Results - condensate difference

- MC at T=0
- good results at T=0
- IMC at high T

Pseudocritical temperature $T_{PC} \times B$

we consider the physical point with nonzero $m$

at high temperatures $\rightarrow$ crossover

chiral symmetry is partially restored.

One can only establish a pseudocritical temperature $T_{PC}$

we use the location of the peaks for the vacuum normalized quark condensates, where the thermal susceptibilities are:

$$\chi_T = -m_\pi \frac{\partial \sigma}{\partial T} \text{ where } \sigma = \frac{\langle \bar{\psi}_u \psi_u \rangle(B, T) + \langle \bar{\psi}_d \psi_d \rangle(B, T)}{\langle \bar{\psi}_u \psi_u \rangle(B, 0) + \langle \bar{\psi}_d \psi_d \rangle(B, 0)}$$
Numerical Results - normalized thermal susceptibility

\[ \chi_T(T) = \frac{eB}{1.0 \text{ GeV}^2} \]

- \( eB = 1.0 \text{ GeV}^2 \)
- \( eB = 0.8 \text{ GeV}^2 \)
- \( eB = 0.6 \text{ GeV}^2 \)
- \( eB = 0.4 \text{ GeV}^2 \)
- \( eB = 0.2 \text{ GeV}^2 \)
- \( eB = 0.0 \)

\( T [\text{MeV}] \)

\( \chi_T \) ranges from 0.0 to 3.0.

The graph shows how the thermal susceptibility \( \chi_T \) varies with temperature \( T \) for different values of \( eB \).
Numerical Results - $T_{pc} \times B$

Left panel: our results

Possible Questions:

- Similar effect in QMM? in progress...
- If we increase the magnetic field?

![Graph showing the effect of different magnetic fields on the quark-antiquark potential at various temperatures.](image)
Possible Questions:

- MC at T=0
- IMC at high T

Graph showing the variation of M[MeV] with T[MeV] for different values of $eB$.

- $eB = 2.0 \text{ GeV}^2$
- $eB = 1.7 \text{ GeV}^2$
- $eB = 1.5 \text{ GeV}^2$
- $eB = 1.2 \text{ GeV}^2$
- $eB = 1.1 \text{ GeV}^2$
Recently...

Magnetic inverse catalysis in the (2+1)-flavor Nambu--Jona-Lasinio and Polyakov--Nambu--Jona-Lasinio models
$SU(3)$ PNJL + $G(B)$ (fitted with the lattice)
Final Remarks

- Our aim -> understanding discrepancies between effective model predictions and recent lattice results. The behavior of $T_{PC}$ as a function of $B$

- we have examined the effect of introducing a running coupling $G$ motivated by asymptotic freedom.
Final Remarks

- Our assumption of the decrease of G with B and T $\rightarrow$ mimicking asymptotic freedom in QCD

- This represents a concrete realization of the back reaction of the sea quarks and confirms its potential importance on explaining the IMC as stressed in the recent literature.
Final Remarks

- But the running of $G$ with $T$ is crucial to obtain results with IMC (lattice results!)
Perspectives

- Asymptotic freedom in QMM (in progress collaboration with G. Krein and M.B. Pinto)
- B effects on BEC BCS crossover in progress (in progress - collaboration with R.O. Ramos)
- B effects on the Langevin Dynamics
- $SDE + T + B$ ???
This talk was based in:

The Importance of Asymptotic Freedom for the Pseudocritical Temperature in Magnetized Quark Matter

Thank you for your attention!

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