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STRONGLY INTERACTING QUANTUM PARTICLES IN ONE DIMENSION

AN EXACT APPROACH TO LOW-DIMENSIONAL QUANTUM MAGNETISM

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October 8th 2014

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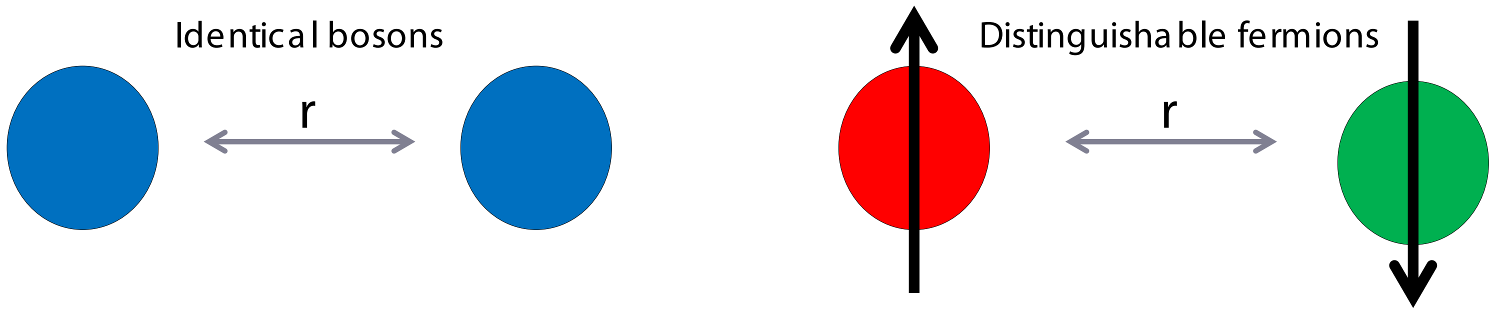
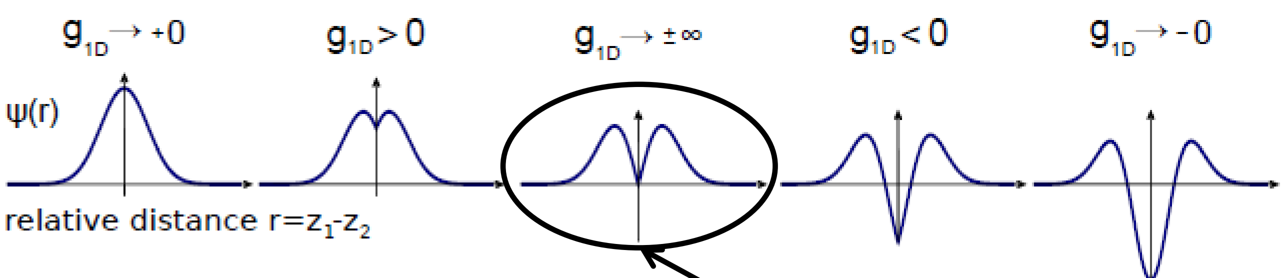
A ONE DIMENSIONAL WORLD



Relative wave function



Interaction

$$g_{1D}\,\delta(r)$$

Source: G. Zürn, thesis

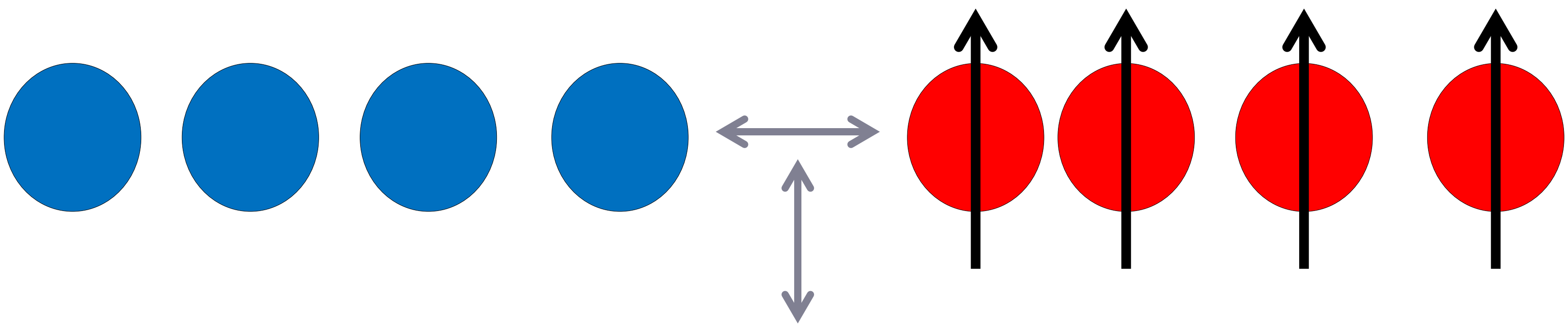
Strong interactions -> Impenetrability!

STRONGLY INTERACTING BOSONS

$|g_{1D}| \rightarrow \infty$ limit

Tonks (1936)-Girardeau (1960) gas
of impenetrable bosons

Mapping identical bosons to spin-polarized fermions. Girardeau (1960).



$$\Psi_B(z_1, \ldots, z_N) = |\Psi_F(z_1, \ldots, z_N)|$$

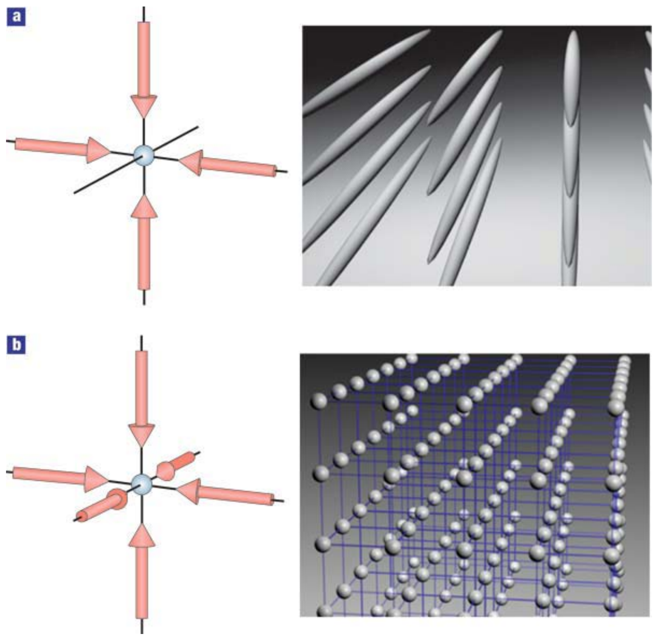
Impenetrable bosons

Antisymmetrized fermions

Lieb-Liniger (1963) used Bethe ansatz to
solve N boson problem for any g>0

EXPERIMENTAL REALIZATION

Optical lattices



I. Bloch, Nature Physics **1**, 23 (2005)

Confinement-induced resonances

Maxim Olshanii
Phys. Rev. Lett. **81**, 938 (1998)

$$g_{1D} = \frac{2\hbar^2 a_{3D}}{m a_\perp^2} \frac{1}{1 - C a_{3D}/a_\perp}$$

Divergent at specific point depending on lattice and 3D Feshbach resonance

EXPERIMENTAL REALIZATION

Tonks–Girardeau gas of ultracold atoms in an optical lattice

Belén Paredes[1], Artur Widera[1,2,3], Valentin Murg[1], Olaf Mandel[1,2,3], Simon Fölling[1,2,3], Ignacio Cirac[1], Gora V. Shlyapnikov[4], Theodor W. Hänsch[1,2] & Immanuel Bloch[1,2,3]

Nature **429**, 277 (2004)

Observation of a One-Dimensional Tonks-Girardeau Gas

Toshiya Kinoshita, Trevor Wenger, David S. Weiss*

Science **305**, 1125 (2004)

Experimentally produced and probed the Tonks-Girardeau gas on the repulsive side g>0

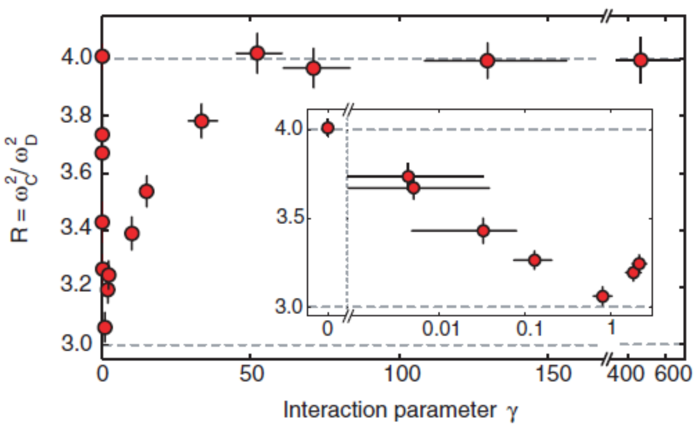
EXPERIMENTAL REALIZATION

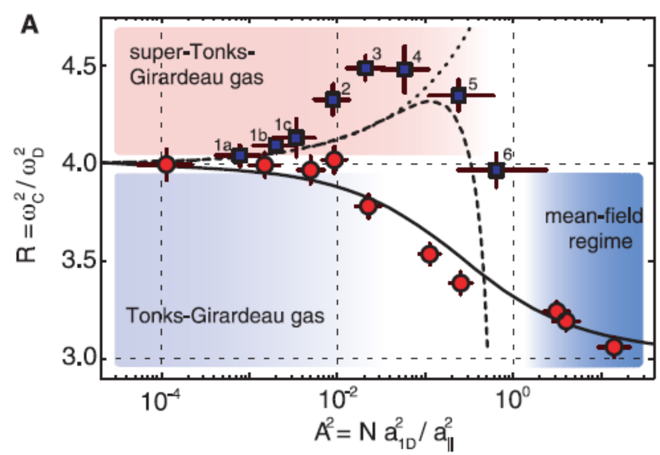
Realization of an Excited, Strongly Correlated Quantum Gas Phase

Elmar Haller,[1] Mattias Gustavsson,[1] Manfred J. Mark,[1] Johann G. Danzl,[1] Russell Hart,[1] Guido Pupillo,[2,3] Hanns-Christoph Nägerl[1]*

Science **325**, 1224 (2009)

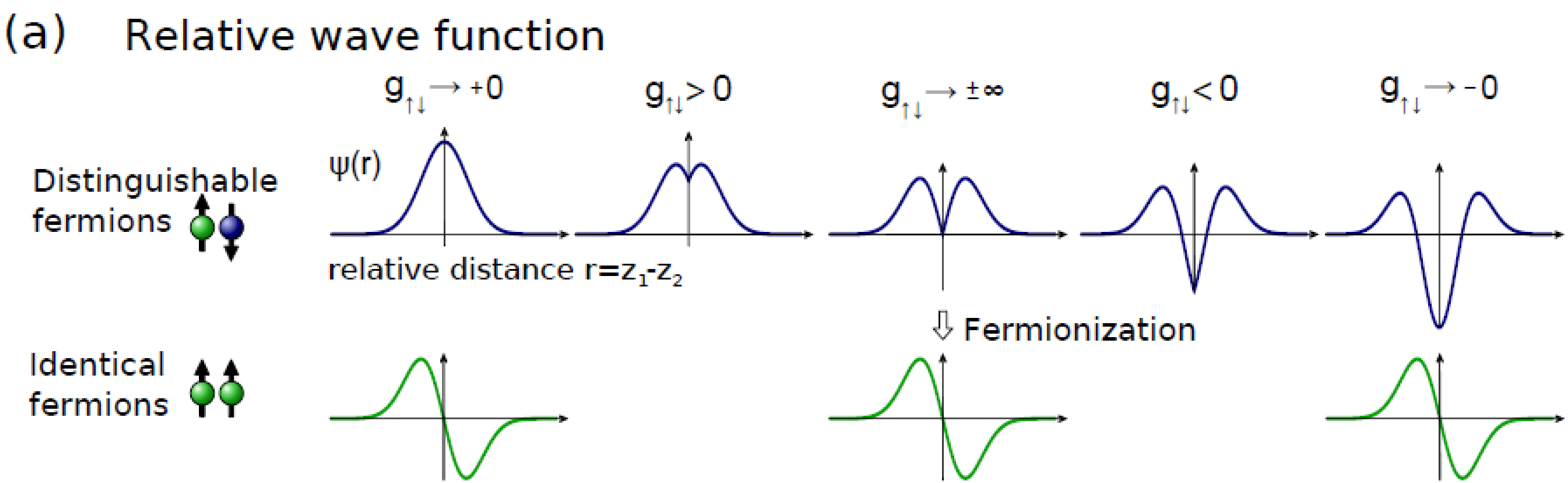
Study of the crossover from g>0 to g<0 in the strongly-interacting regime.





1D FERMIONS – A FRONTIER

Two kinds of relative motion for two-body states!

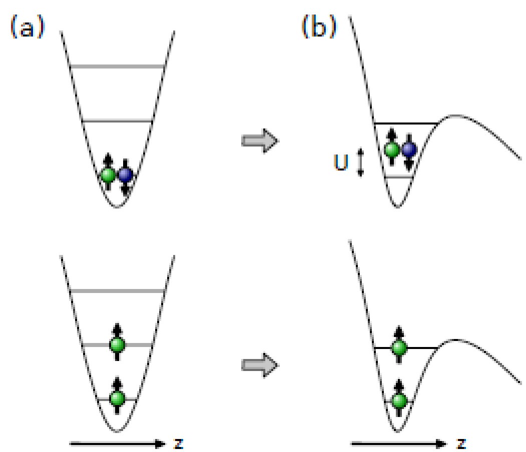


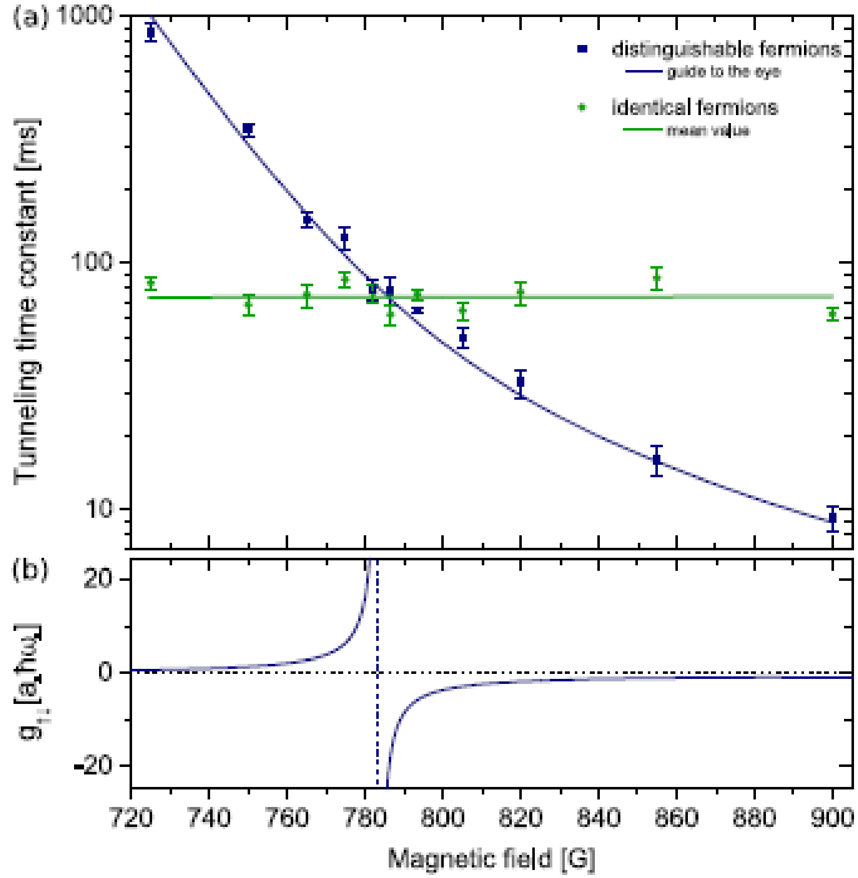
Source: G. Zürn, thesis

Fermionization of two fermions in a 1D harmonic trap:
G. Zürn *et al.*, Phys. Rev. Lett. **108**, 075303 (2012).

EXPERIMENTAL REALIZATION

Two-body tunneling experiments

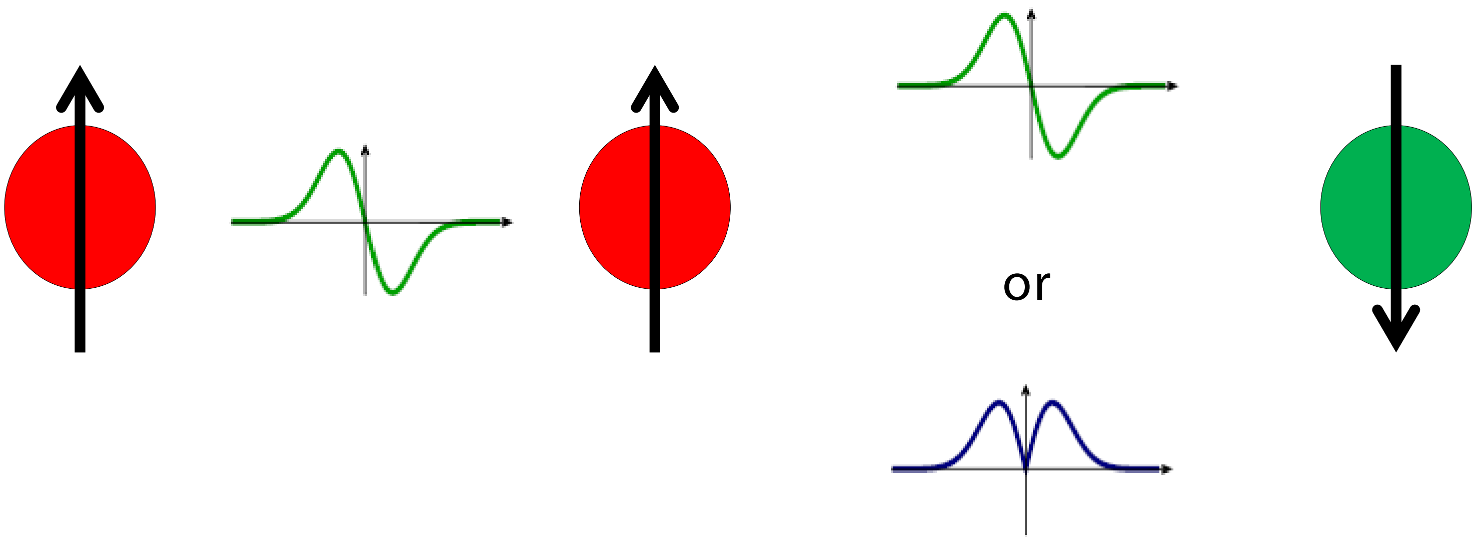




Fermionization of two fermions in a 1D harmonic trap:
G. Zürn *et al.*, Phys. Rev. Lett. **108**, 075303 (2012).

THREE FERMIONS

Relative wave functions. What should we take?



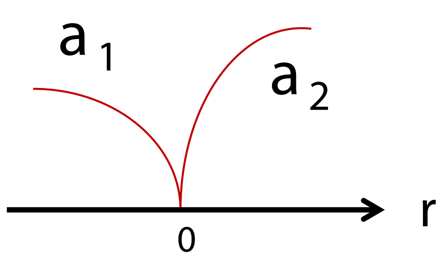
???

Conjecture: Use the symmetric choice for non-identical pairs for any N-body system

M.D. Girardeau, Phys. Rev. A **82**, 011607(R) (2010).

THREE FERMIONS

Let's keep an open mind!



Non-identical relative wave function

Two strict conditions:

1) In limit g_{1D}->∞, relative wave functions have not vanish at zero for identical and non-identical pairs!

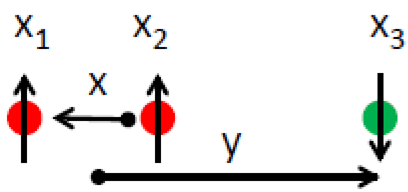
2) Identical fermions must have odd relative wave functions!

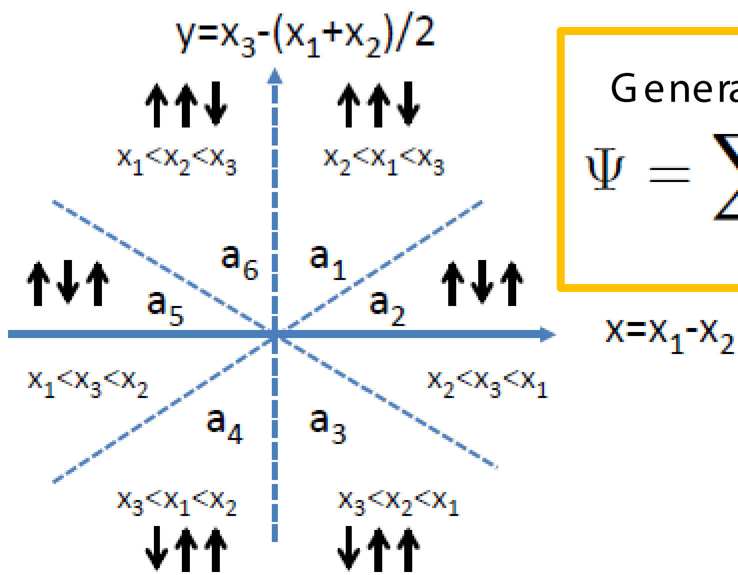
IDEA: Keep a_1 and a_2 as free parameters and do a variation!

A.G. Volosniev *et al.*, arXiv:1306.4610 (2013)
Nature Communications, in press (October 2014).

THREE FERMIONS - SOLUTION

Split space in patches



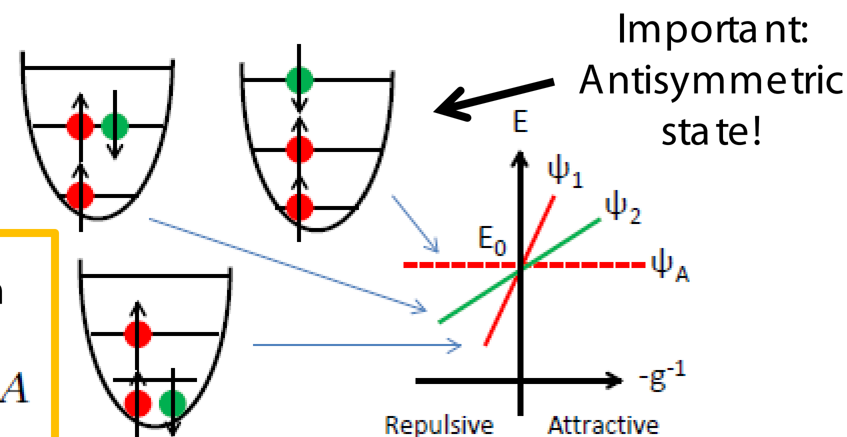


General solution

$$\Psi = \sum_i a_i \Psi_A$$

Pauli and parity reduces problem to a_1, a_2, and a_3.

Spectrum on resonance



Optimize derivative!

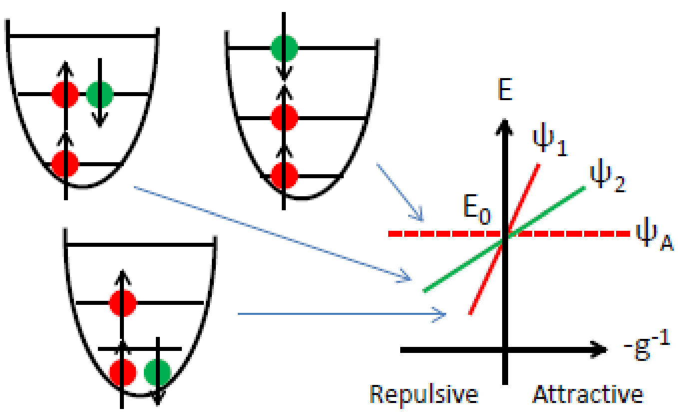
$$K = -\frac{\partial E}{\partial g^{-1}} = g^2 \frac{\sum_{ij} \int \prod_{k=1}^{N} dx_k |\Psi|^2 \delta(x_i - x_j)}{\langle \Psi | \Psi \rangle}$$

THREE FERMIONS - SOLUTION

$$K = \frac{27}{8\sqrt{2\pi}} \frac{(a_1 - a_2)^2 + (a_2 - a_3)^2}{a_1^2 + a_2^2 + a_3^2}$$

Extremizing solutions are:

$a_1 = a_2 = a_3$	Non-interacting state
$a_1 = a_3$ and $a_2 = 0$	Excited state, even parity
$2a_1 = 2a_3 = -a_2$	Ground state, odd parity

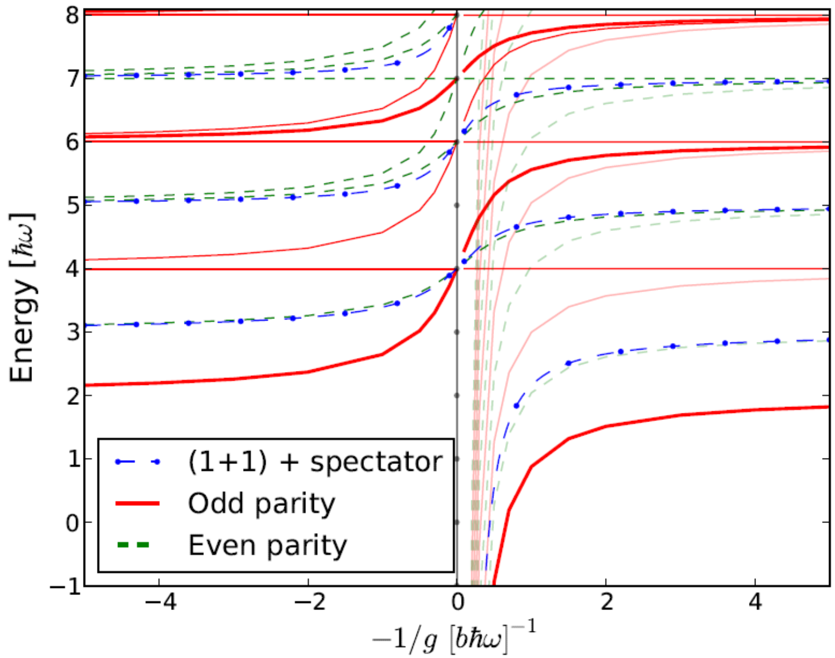


IMPORTANT: Coefficients are generally NOT the same!

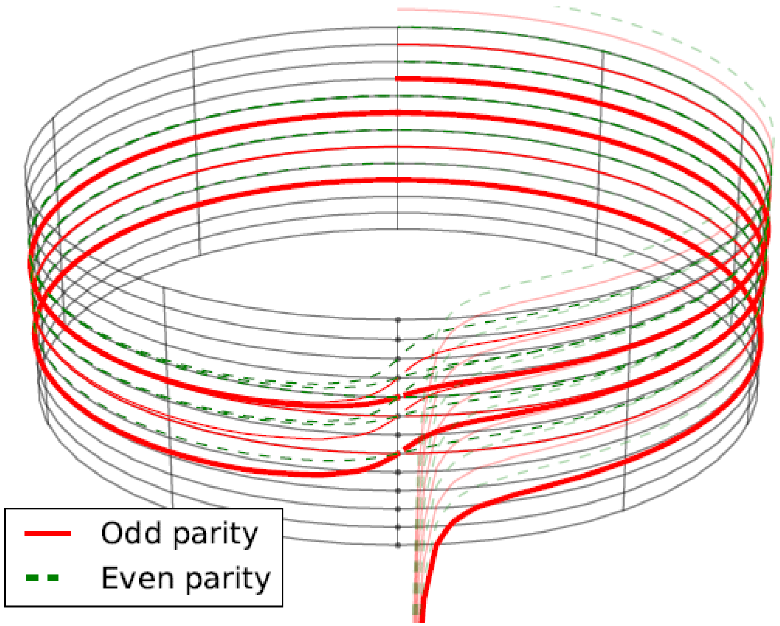
A.G. Volosniev *et al.*, arXiv:1306.4610 (2013)
Nature Communications, in press (October 2014).

HARMONICALLY TRAPPED SYSTEMS

Standard style



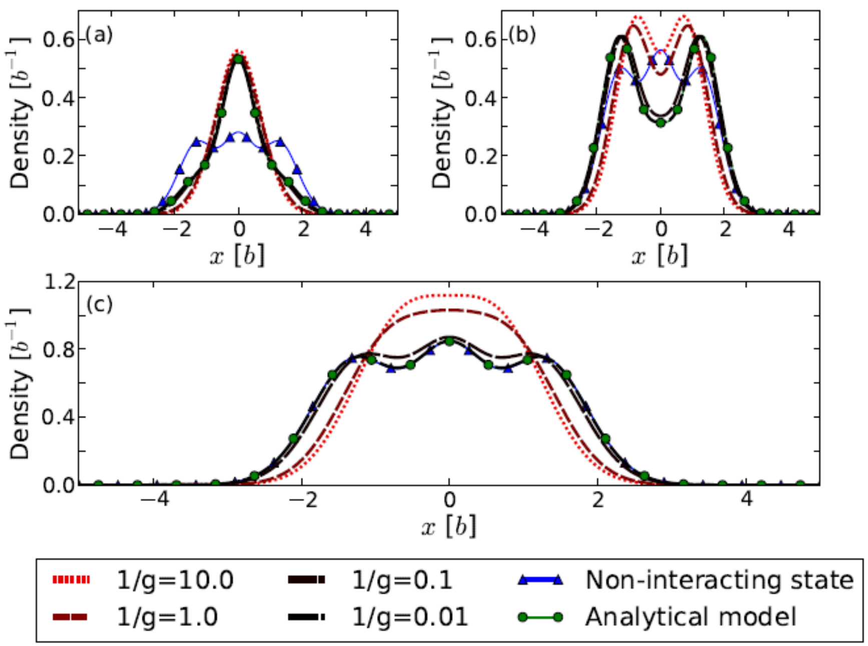
Elegant style



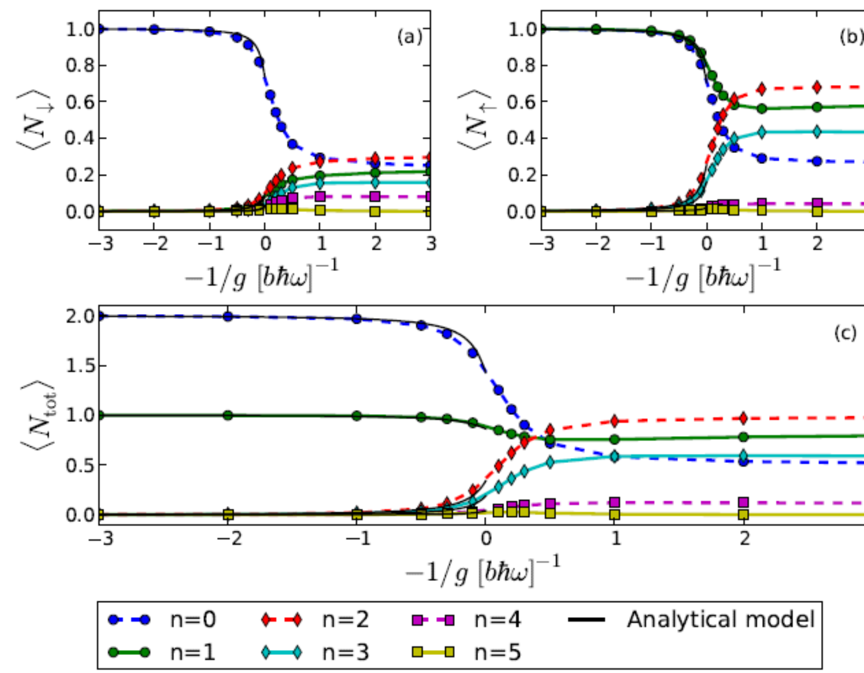
E.J. Lindgren *et al.,* New J. Phys. **16**, 063003 (2014).
S.E. Gharashi and D. Blume, Phys. Rev. Lett. **111**, 045302 (2013).

GROUND STATE PROPERTIES

Trap density



Occupation numbers

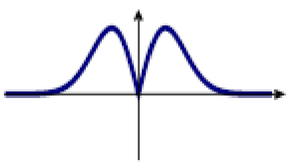


E.J. Lindgren *et al.* New Journal of Physics **16**, 063003 (2014).

FERMIONIZATION OF FERMIONS

It is different from identical bosons and spin-polarized fermions!

The 'democratic' solution or trivial Bose-Fermi mapping uses:



between all non-identical pairs.

In the 2+1 case it is NOT a relevant eigenstate but rather a linear combination!

$$\Psi_{BF} = (8^{1/2}\Psi_{gs} + \Psi_{non})/3$$

BUT can we tell the difference in experiments?

S.E. Gharashi and D. Blume, Phys. Rev. Lett. **111**, 045302 (2013)
A.G. Volosniev *et al.*, arXiv:1306.4610 (2013)

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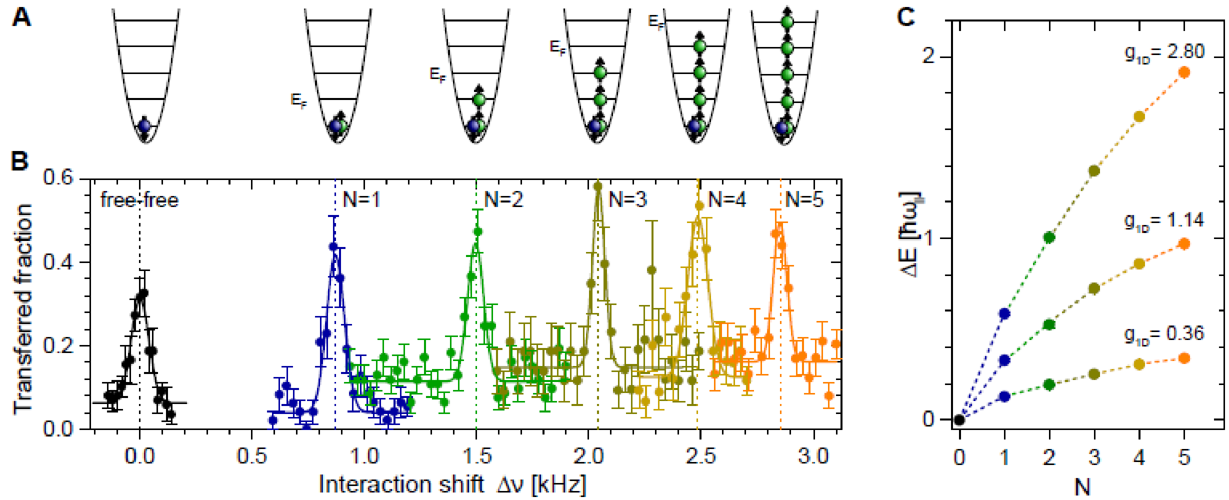
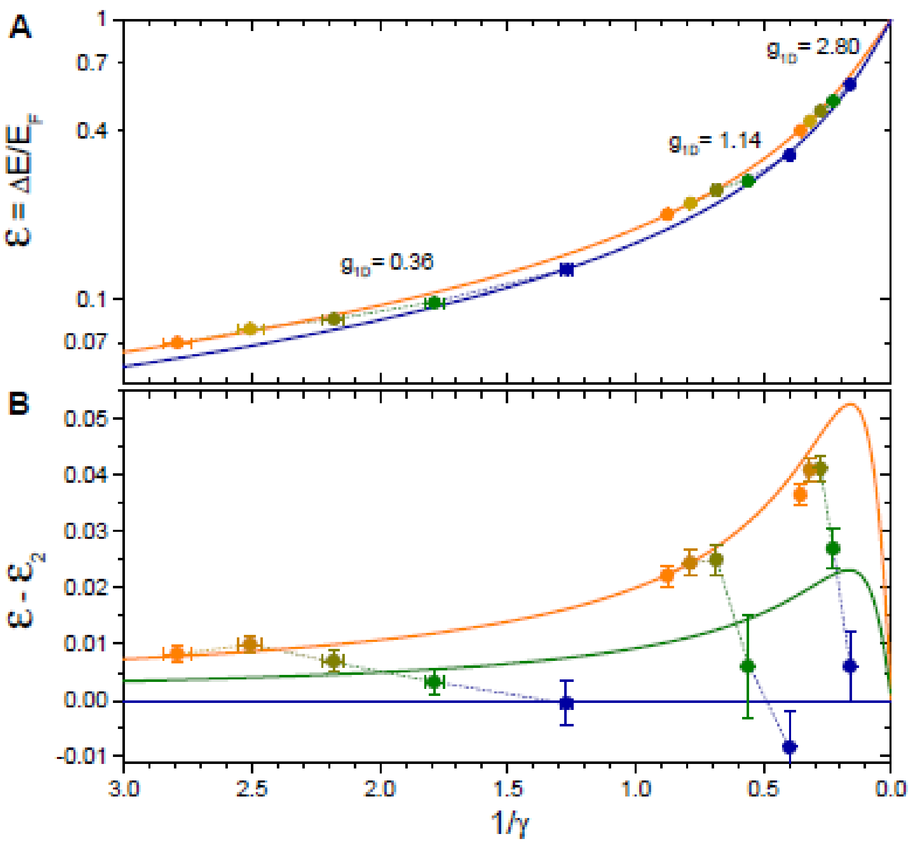
Selim Jochim experiments in Heidelberg.





From Few to Many: Observing the Formation of a Fermi Sea One Atom at a Time

A. N. Wenz *et al.*, Science **342**, 457 (2013)

Green solid line from
S.E. Gharashi, K.M. Daily, and D. Blume, Phys. Rev. A **86**, 042702 (2012).

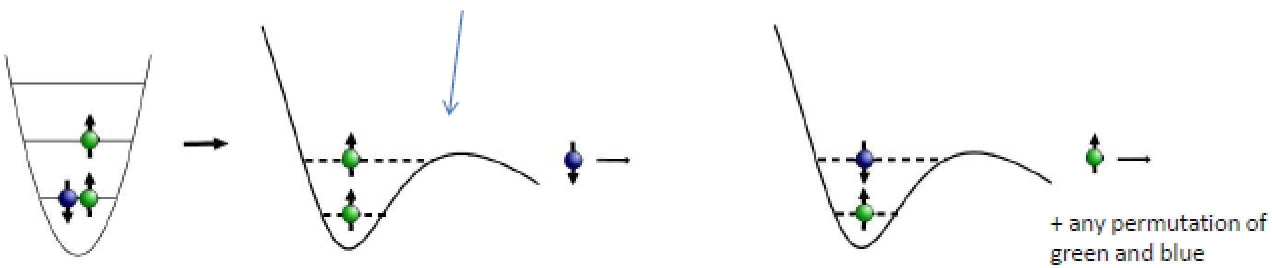
Orange 'many-body' line
J.B. McGuire, J. Math. Phys. **6**, 432 (1965).
G.E. Astrakharchik and I. Brouzos, Phys. Rev. A **88**, 021602 (2013).

EXPERIMENTAL SIGNATURE

Do tunneling experiments!

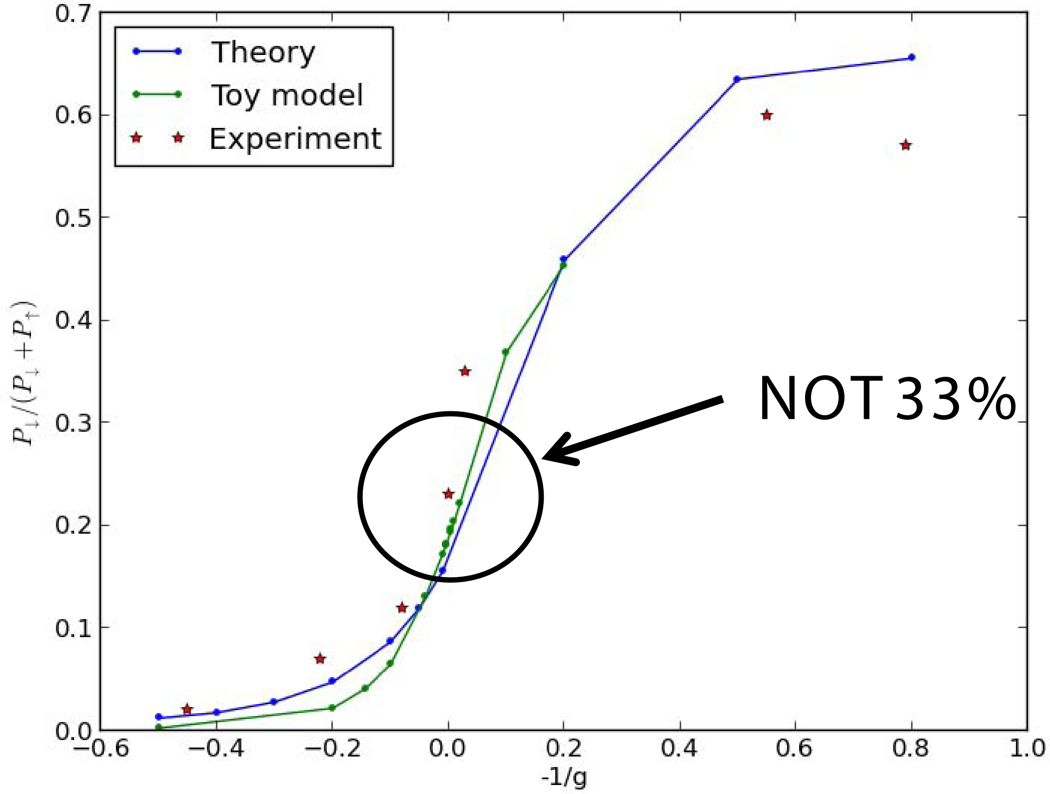
F. Serwane *et al.*, Science **332**, 336 (2011).

G. Zürn *et al.*, Phys. Rev. Lett. **108**, 075303 (2012).



Source: G. Zürn

THEORY VS. EXPERIMENT



Theory by Lindgren and Volosniev.
Data from Jochim group (G. Zürn)

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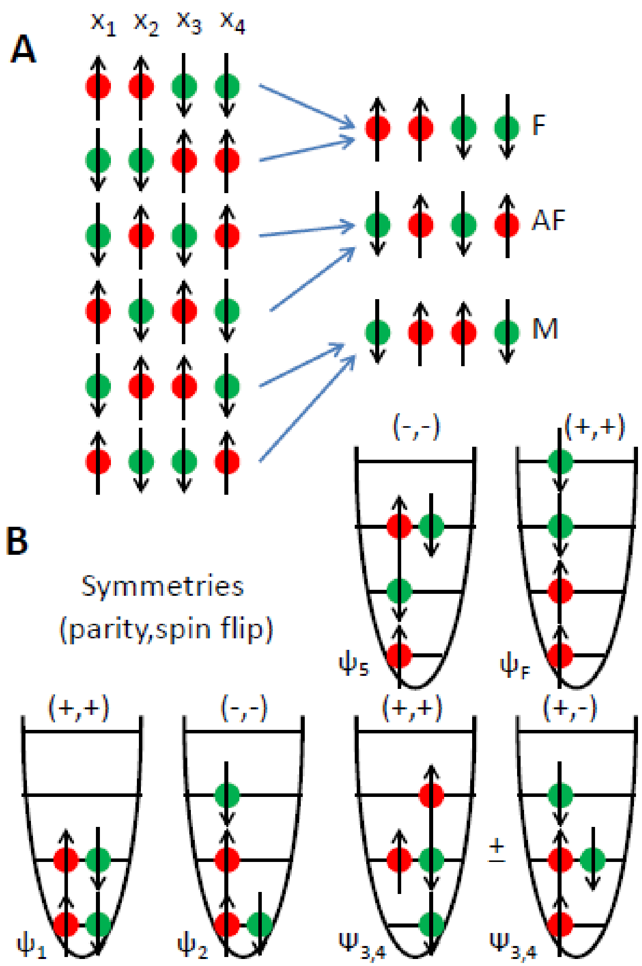
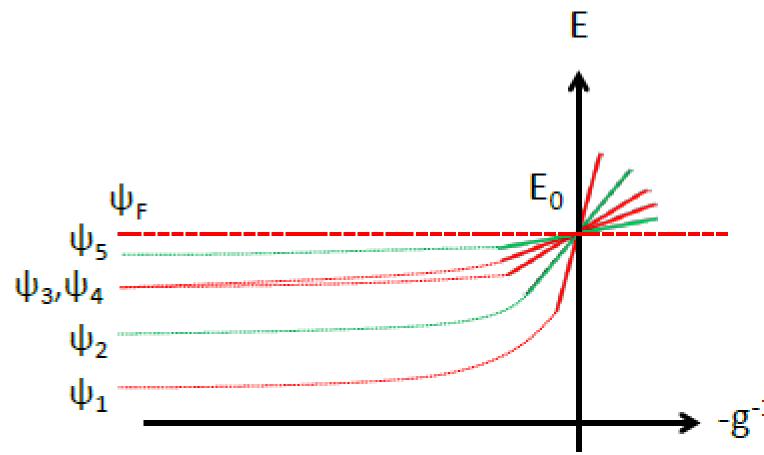
FOUR-BODY SYSTEMS

A.G. Volosniev *et al.*, arXiv:1306.4610 (2013)
Nature Communications, in press (October 2014).





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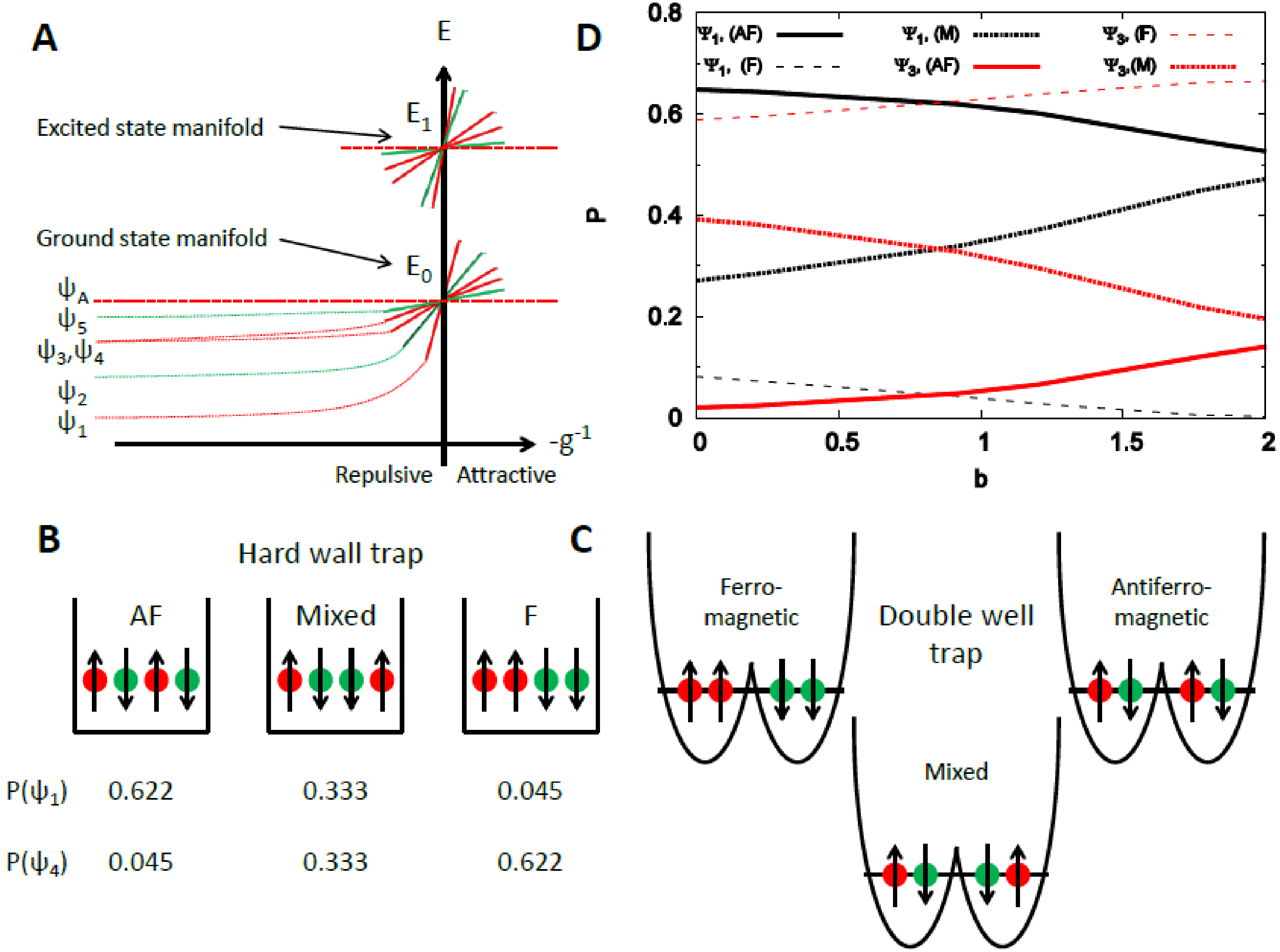
$$V(x) = \frac{1}{2}m\omega^2(|x| - b)^2$$



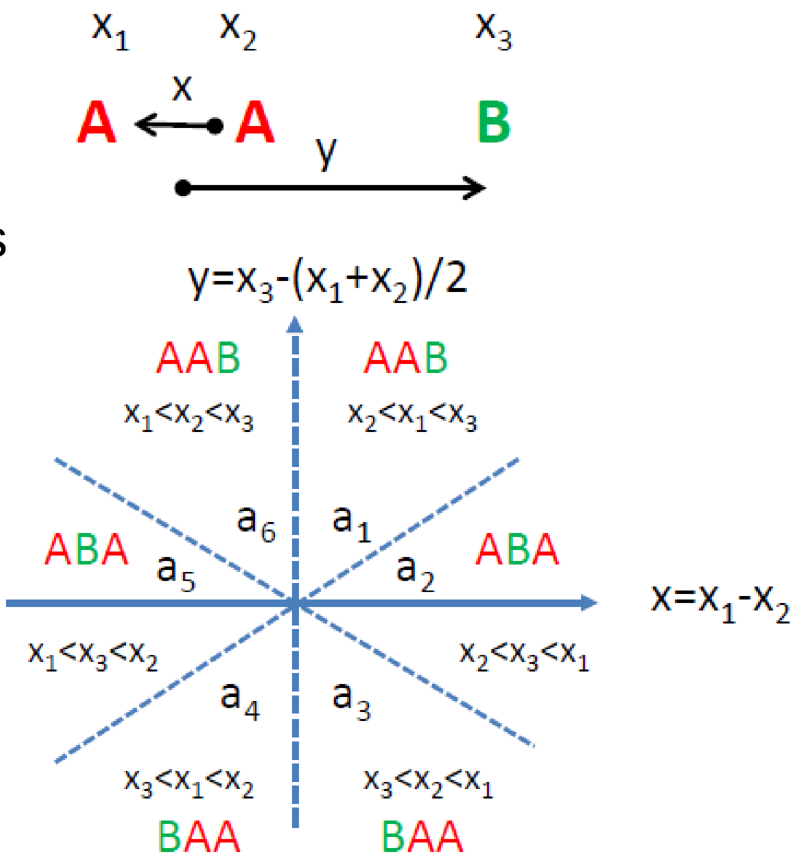
A.G. Volosniev *et al.*, arXiv:1306.4610 (2013)
Nature Communications, in press (October 2014).

THREE TWO-COMPONENT BOSONS

Strong AB interactions

No AA interactions

No BB interactions



N.T. Zinner *et al.*, EPL **107**, 60003 (2014).

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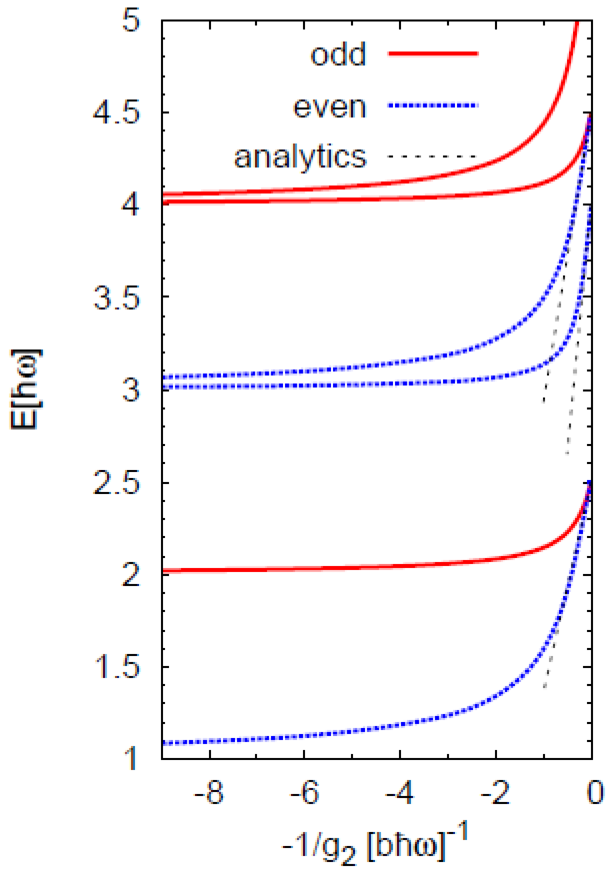
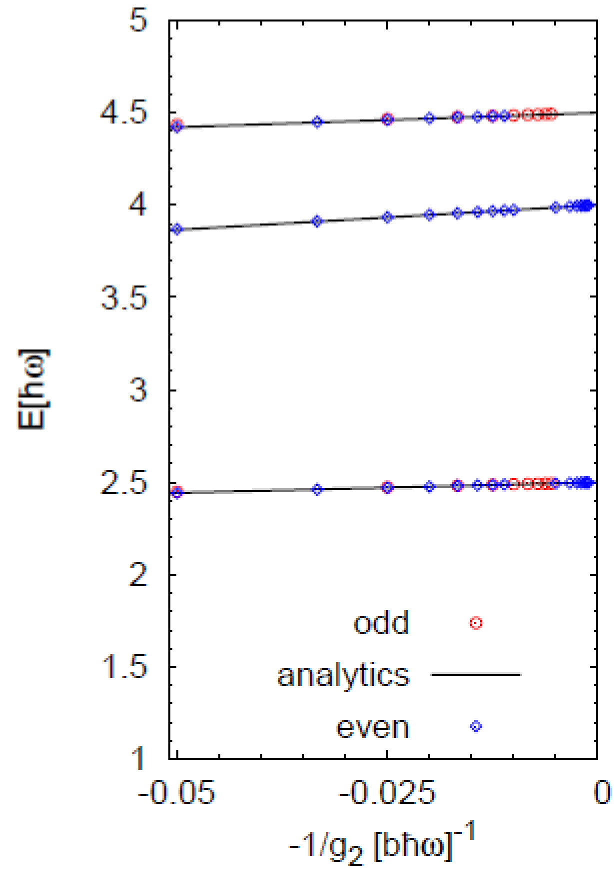
Stochastic variational calculations

x_1 x_2 x_3

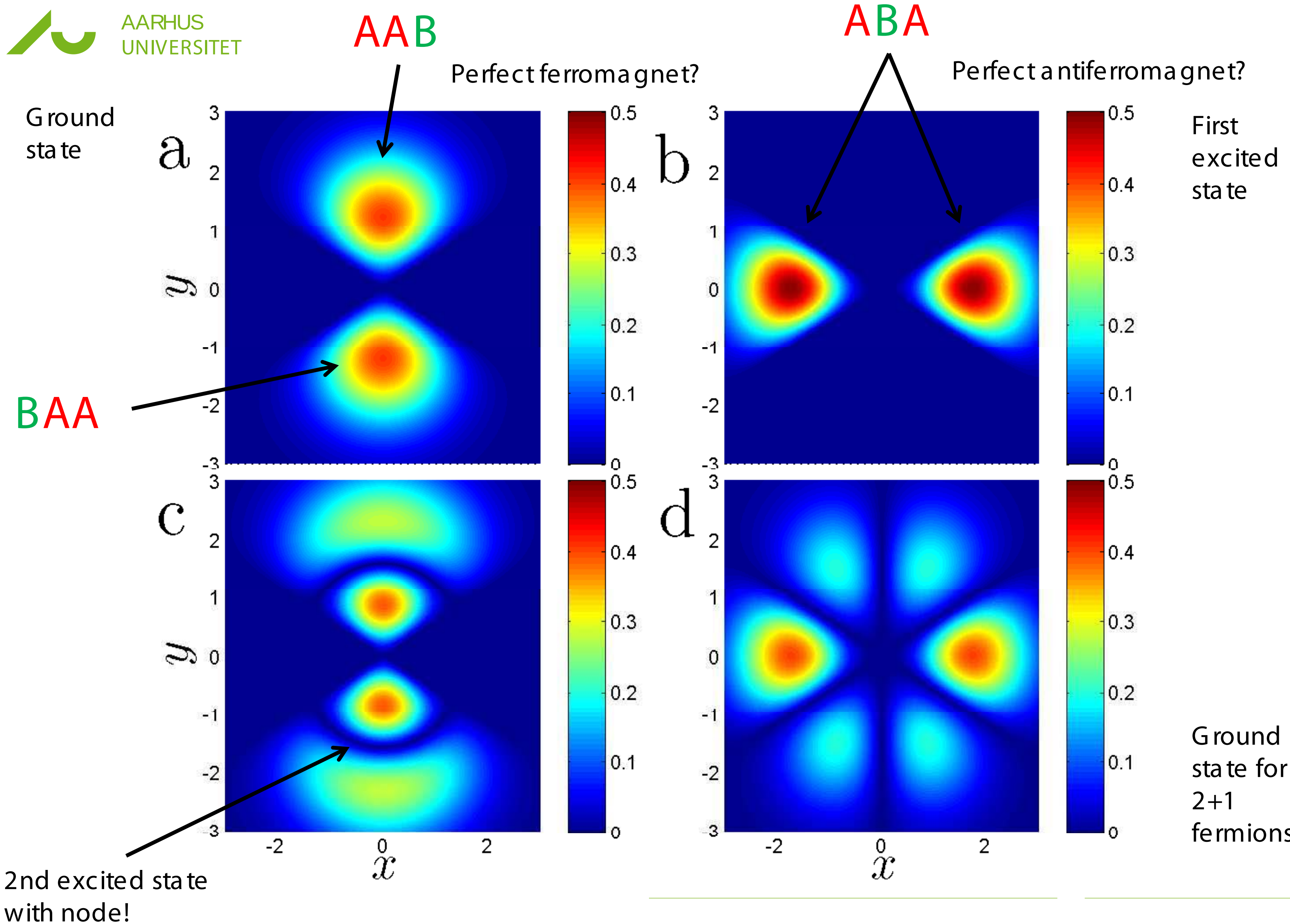
A A B

$$H = \sum_{i=1}^{3} \frac{p_i^2}{2m} + g_2\delta(x_1 - x_3) + g_2\delta(x_2 - x_3)$$





N.T. Zinner *et al.*, EPL **107**, 60003 (2014).

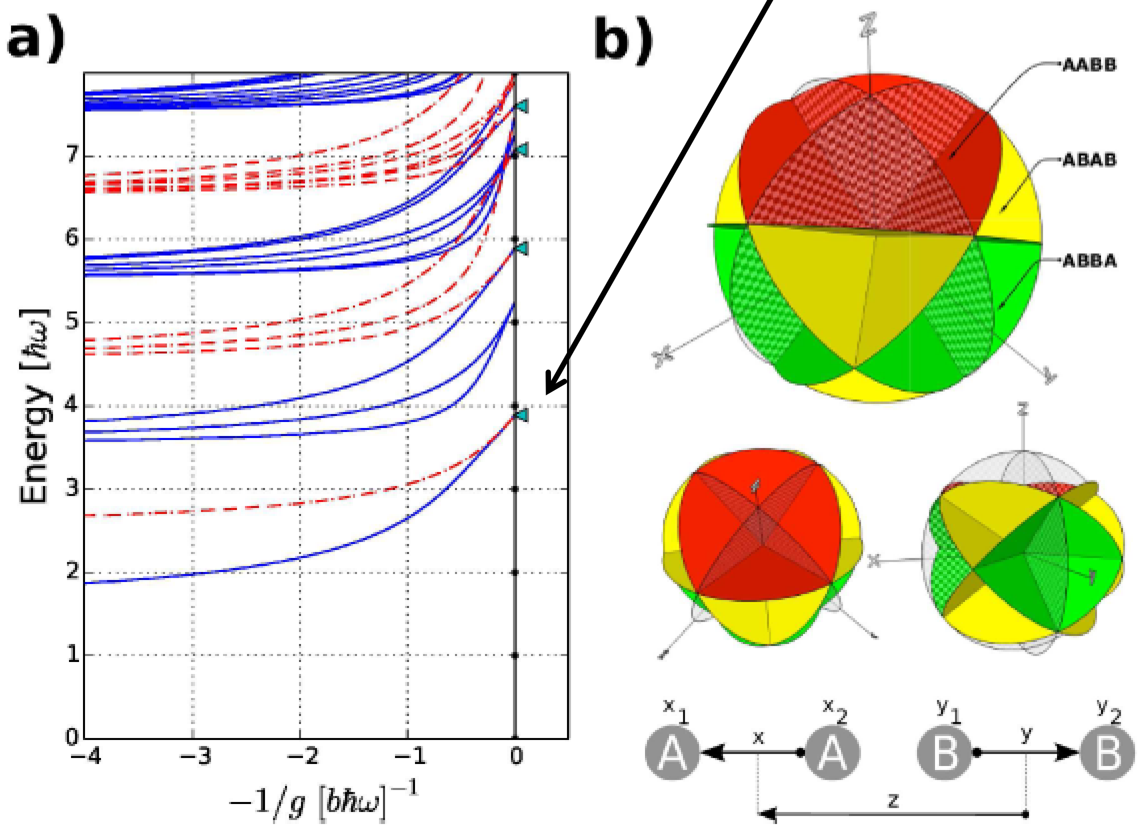


N.T. Zinner *et al.*, EPL **107**, 60003 (2014).

FOUR-BODY BOSONS

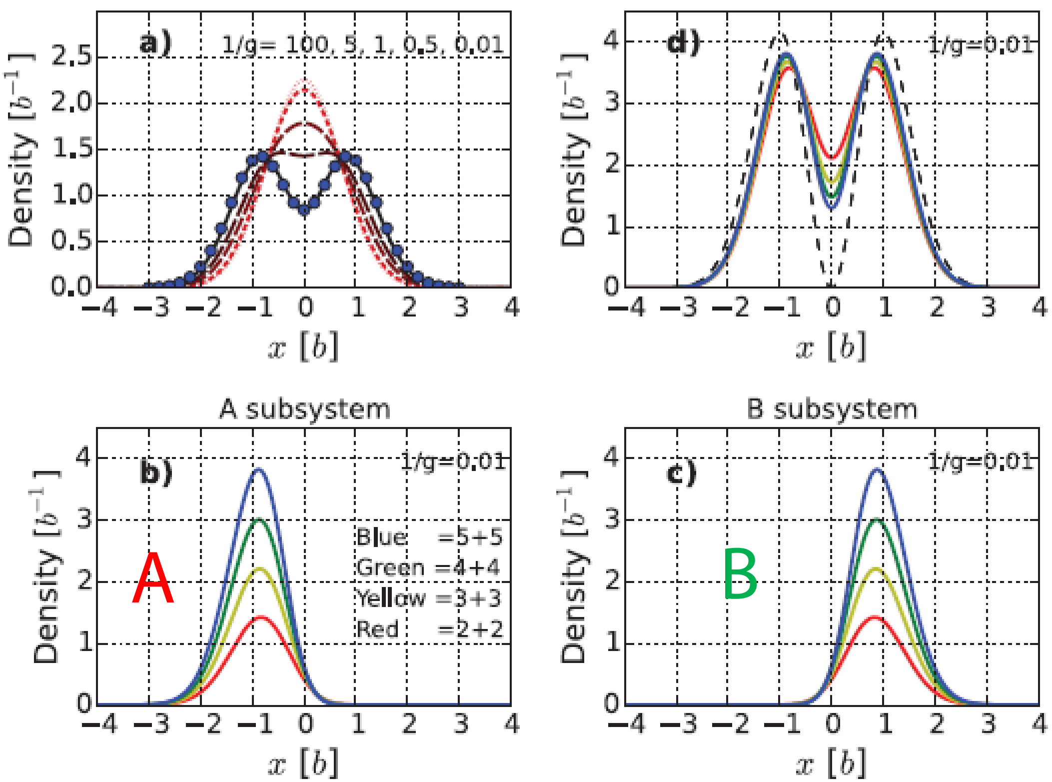
Ground state structure

AABB+BBAA



A.S. Dehkharghani *et al.*, arXiv:1409.4224 (2014), in review at Phys. Rev. X.

BALANCED N-BODY SYSTEM



Ground state structure for N=10

AAAAABBBBB+
BBBBBAAAAA

Taking linear combinations of the doubly degenerate ground state, we obtain perfectly separated densities.

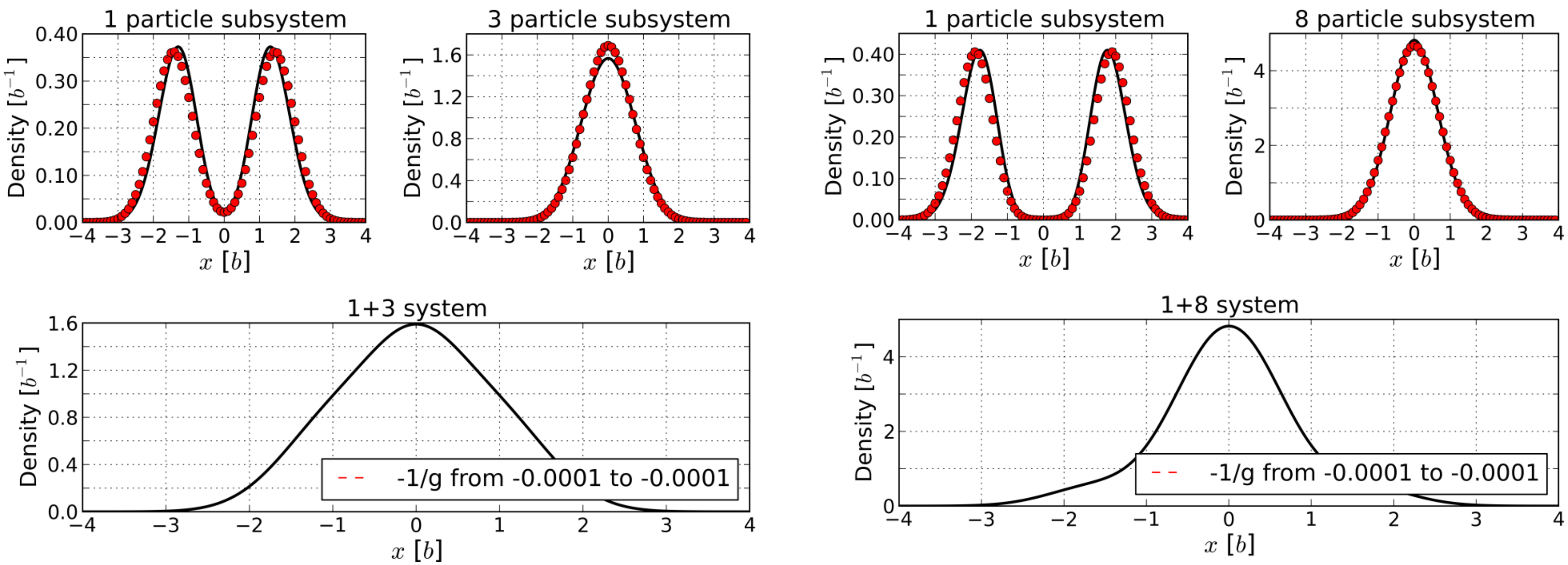
Confirms physical picture!

A.S. Dehkharghani *et al.*, arXiv:1409.4224 (2014), in review at Phys. Rev. X.

BOSE POLARONS

Ground state structure

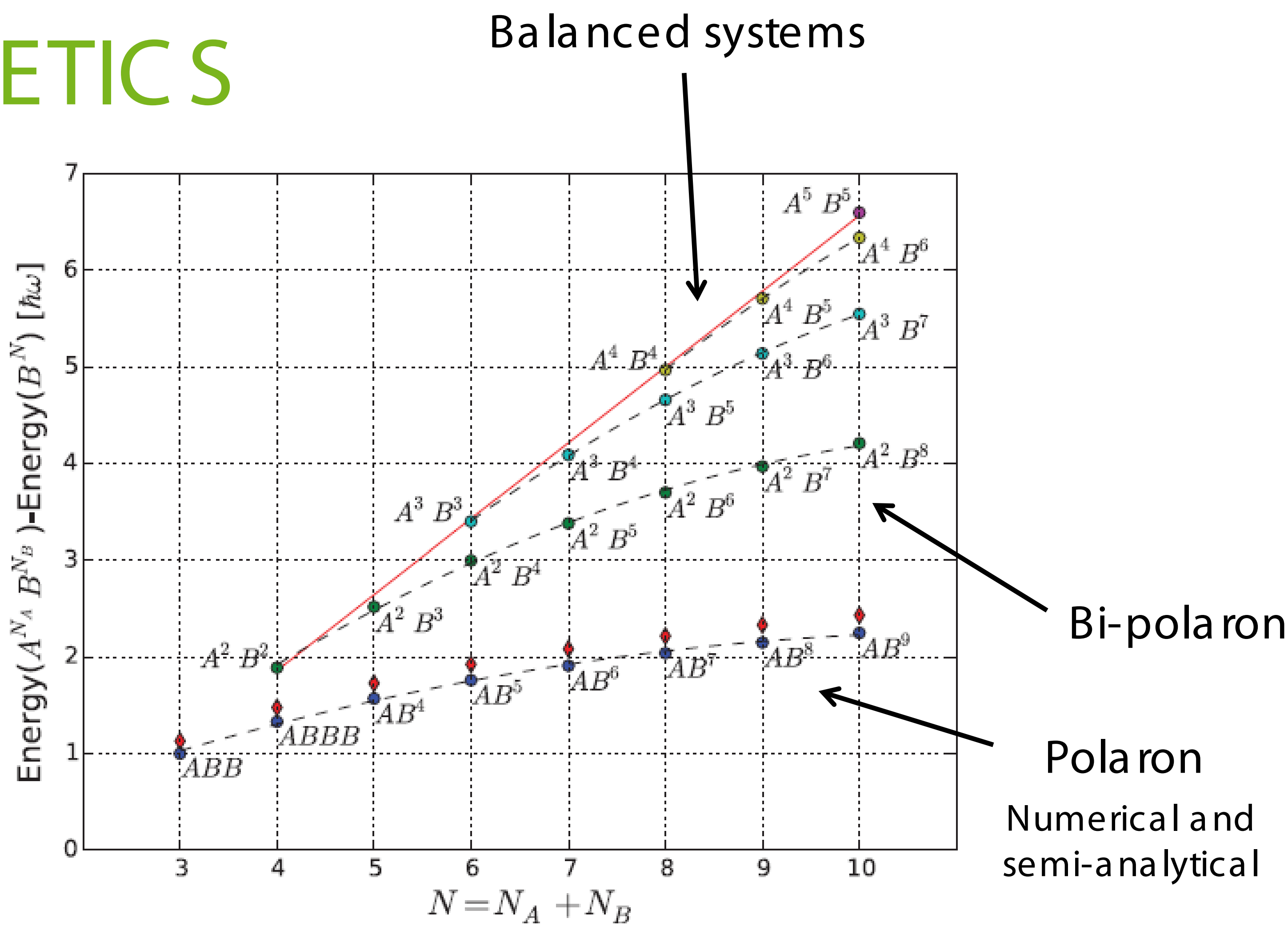
ABB...BB+BB...BBA



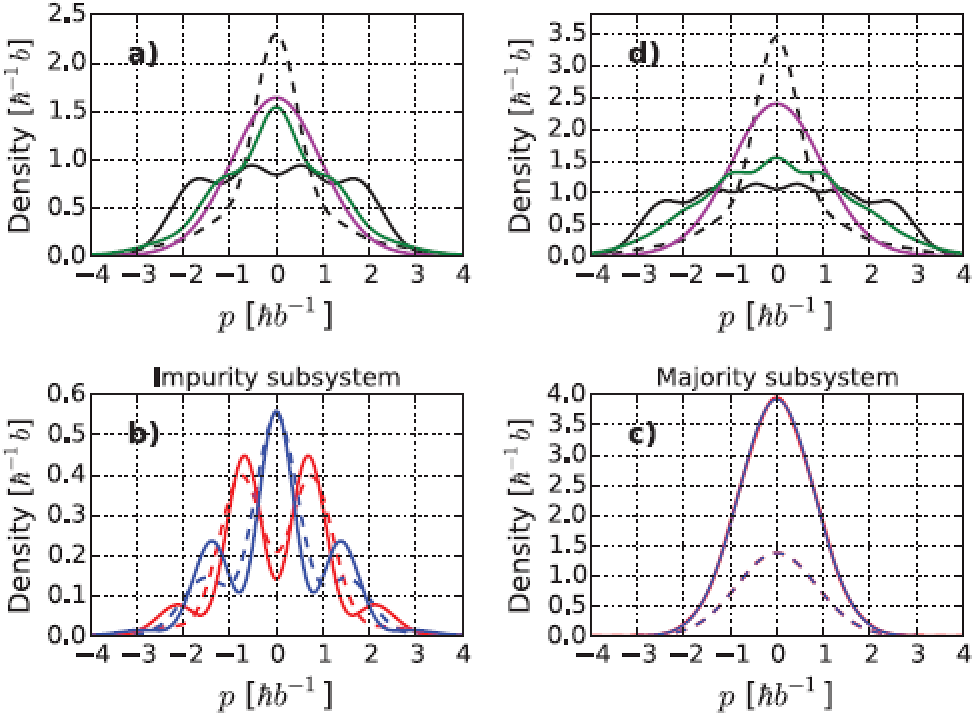
In the ground state, impurities will NEVER penetrate the majority component!

A.S. Dehkharghani *et al.*, arXiv:1409.4224 (2014), in review at Phys. Rev. X.

ENERGETICS


Balanced systems
Bi-polaron
Polaron
Numerical and
semi-analytical
Energy($A^{N_A} B^{N_B}$)-Energy(B^N) [$\hbar\omega$]
$N = N_A + N_B$
$A^5\ B^5$
$A^4\ B^6$
$A^4\ B^5$
$A^3\ B^7$
$A^4\ B^4$
$A^3\ B^6$
$A^3\ B^5$
$A^3\ B^4$
$A^2\ B^8$
$A^2\ B^7$
$A^3\ B^3$
$A^2\ B^6$
$A^2\ B^5$
$A^2\ B^4$
$A^2\ B^3$
$A^2\ B^2$
AB^9
AB^8
AB^7
AB^6
AB^5
AB^4
$ABBB$
ABB

MOMENTUM DISTRIBUTIONS



SPIN MODELS

We can map strongly interacting two-component 1D systems in a trap to a spin model of XXZ type and do ENGINEERING!

Nearest-neighbor interactions are tunable via external trap!

$$H_s = \sum_{i=1}^{N-1} \left[J_i \left(\boldsymbol{S}(i) \cdot \boldsymbol{S}(i+1) \right) - \frac{2J_i}{\kappa} \left(S_z(i) S_z(i+1) \right) \right]$$

$$g_{\uparrow\uparrow} = g_{\downarrow\downarrow} = \kappa g_{\uparrow\downarrow}$$

Note: It is **not** a lattice index! It is a particle index.

TABLE I: Effective Heisenberg spin models for strongly interacting atoms in 1D traps.

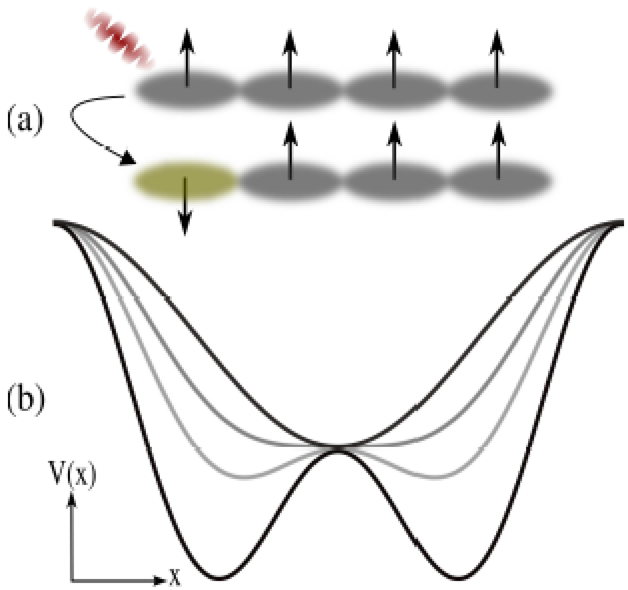
Spin-$\frac{1}{2}$ model	Constituents	κ
XXZ	bosons	$0 < \kappa < \infty$
XXX	bosons or fermions	$\kappa \to \infty$
XX	bosons	$\kappa = 2$

Confinement is taken into account **exactly**.

A.G. Volosniev *et al.*, arXiv:1408.3414 (2014).

STATE TRANSFER

Use trap to manipulate dynamics – example of quantum state transfer

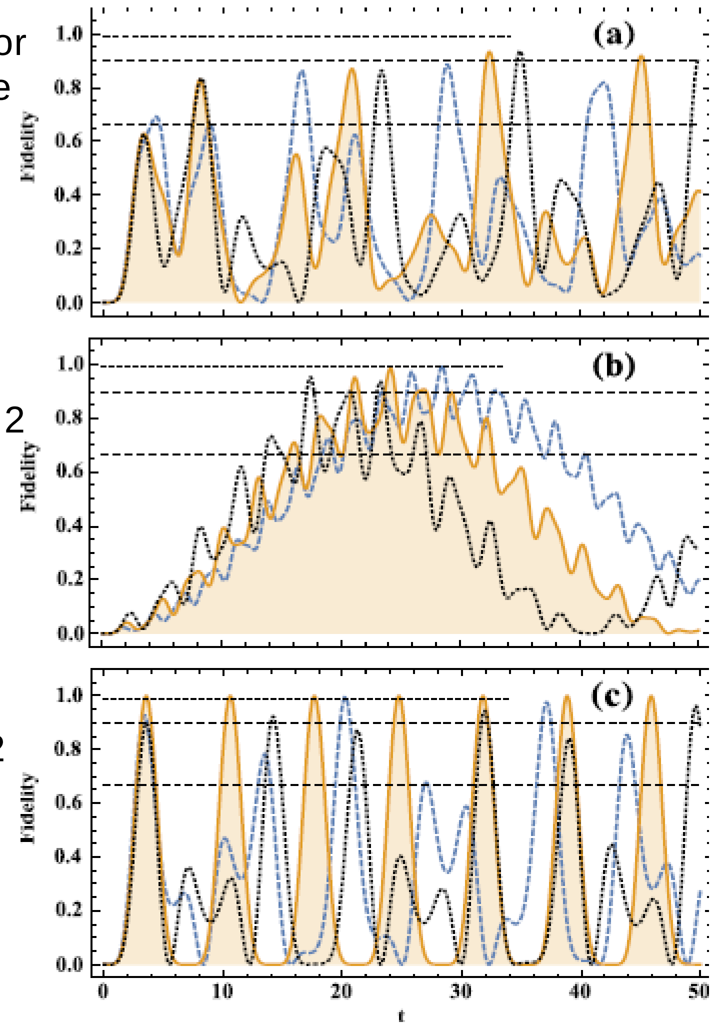


Fidelity of quantum state transfer

Fermions or hard-core bosons

Bosons kappa=1/2

Bosons kappa=2



A.G. Volosniev *et al.*, arXiv:1408.3414 (2014).

MAIN MESSAGES

- Complete theory goes beyond Bose-Fermi mapping
- Must connect states to eigenstates in the spectrum
- ‘Magnetic’ correlations are accessible
- Good agreement with experimental data
- Fermions and bosons can be VERY different even in the hard-core limit!
- Engineering of ferro- and antiferromagnetic states!
- Wave functions and not energies are the most important objects!

ACKNOWLEDGEMENTS

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- Jonathan Lindgren, graduate student (Chalmers)
- Christian Forssén and Jimmy Rotureau (Chalmers)
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Thank you for your attention!