Detector Networks

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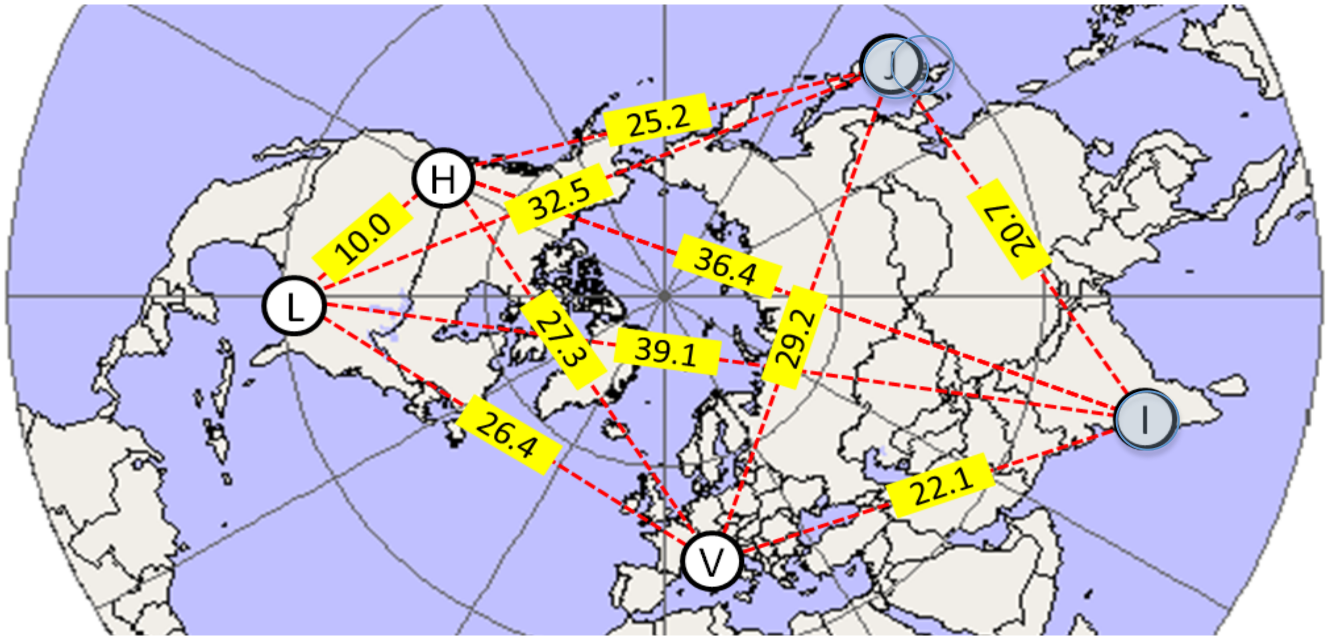


- Network response to a GW signal
- Network plane, Dominant Polarization Frame (DPF)
- Network properties: (sensitivity, acceptance, alignment, index)
- What can we learn just looking at the network GW response?
- Do we need that many detectors?
- Is it close to or far from our “dream” network?

Detector response to a GW signal

- **Antenna patterns**

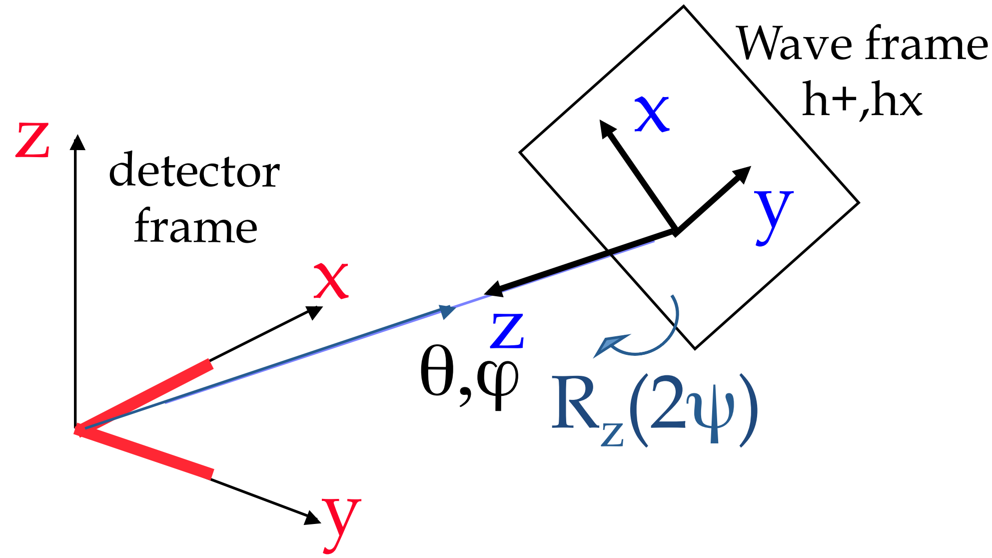
$$F = \begin{bmatrix} \cos(2\psi) & \sin(2\psi) \\ -\sin(2\psi) & \cos(2\psi) \end{bmatrix} \begin{bmatrix} F_+[\theta,\phi] \\ F_\times[\theta,\phi] \end{bmatrix}$$

$$F_+[\theta,\phi] = \tfrac{1}{2}(1+\cos^2\theta)\cos 2\varphi$$

$$F_\times[\theta,\phi] = \cos\theta \sin 2\varphi$$

- **Sampled GW signal**

$$h[i] = \begin{bmatrix} \cos(2\psi) & \sin(2\psi) \\ -\sin(2\psi) & \cos(2\psi) \end{bmatrix} \begin{bmatrix} h_+[i] \\ h_\times[i] \end{bmatrix}$$



- **Sampled detector response**

$$\xi[i] = F_+ h_+[i] + F_\times h_\times[i] = F^T \cdot h[i]$$

- **Direction to the source θ,φ and polarization angle Ψ define relative orientation of the detector and wave frames.**
- **Rotation of the wave frame $R_z(2\Psi)$ induces transformations both for F and h, but ξ is INVARIANT**

In the analysis we have freedom to select any Ψ we like.

Whitened Network Response

$$\vec{\xi}[i] = [\vec{f}_+^T[i], \vec{f}_\times^T[i]] \begin{bmatrix} h_+[i] \\ h_\times[i] \end{bmatrix} = f[i] \cdot \mathrm{h[i]}$$

- **Noise scaled network antenna patterns**
  - **in general time-frequency dependent**
  - **calculated for each TF data sample i characterized by noise PSD estimator S[i]**

$$\vec{f}_+[i] = \frac{F_{1+}(\theta,\phi,\psi)}{\sqrt{S_1[i]}}, \ldots, \frac{F_{K+}(\theta,\phi,\psi)}{\sqrt{S_K[i]}} = |f_+|\vec{e}_+$$

$$\vec{f}_\times[i] = \frac{F_{1\times}(\theta,\phi,\psi)}{\sqrt{S_1[i]}}, \ldots, \frac{F_{K\times}(\theta,\phi,\psi)}{\sqrt{S_K[i]}} = |f_\times|\vec{e}_\times$$

- **Dominant polarization wave frame:**

$$\vec{f}_+(\psi) \cdot \vec{f}_\times(\psi) = 0 \qquad \left|\vec{f}_+(\psi)\right| \geq \left|\vec{f}_\times(\psi)\right|$$

Klimenko et al, PRD 72, 122002 (2005)

Network response to a GW event

- Consider a network event consisting of I TF samples

$$\begin{bmatrix} \xi[1] \\ \xi[2] \\ \dots \\ \xi[I] \end{bmatrix} = \begin{bmatrix} f[1] & 0 & .. & 0 \\ 0 & f[2] & .. & 0 \\ .. & .. & .. & .. \\ 0 & 0 & .. & f[I] \end{bmatrix} \begin{bmatrix} h[1] \\ h[2] \\ \dots \\ h[I] \end{bmatrix}$$

$$\Xi = F\ H$$

  - Ξ – network response to a GW event
  - F – event network matrix
  - H – GW event amplitudes

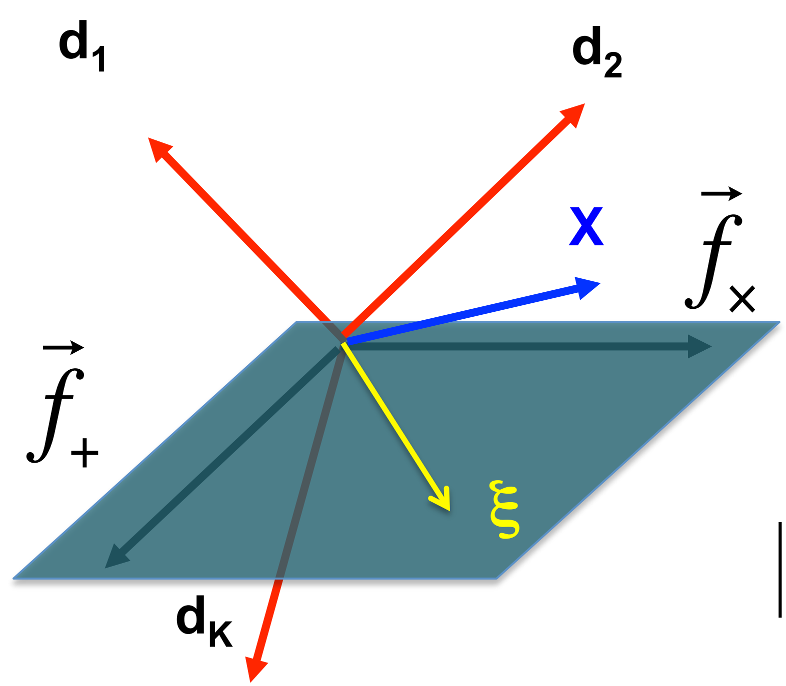
- Network data stream X (Lecture 4: How to find H from noisy data?)

$$X = F\ H + N$$

  - N - network noise

Network Plane

- Vectors $\vec{f}_+, \vec{f}_\times$ define network plane in the space $\{d_1, d_2, ..d_K\}$
- GW response $\vec{\xi} = (\xi_1, \ldots, \xi_K)$ is always in the network plane
- Noisy response X can be outside of the network plane

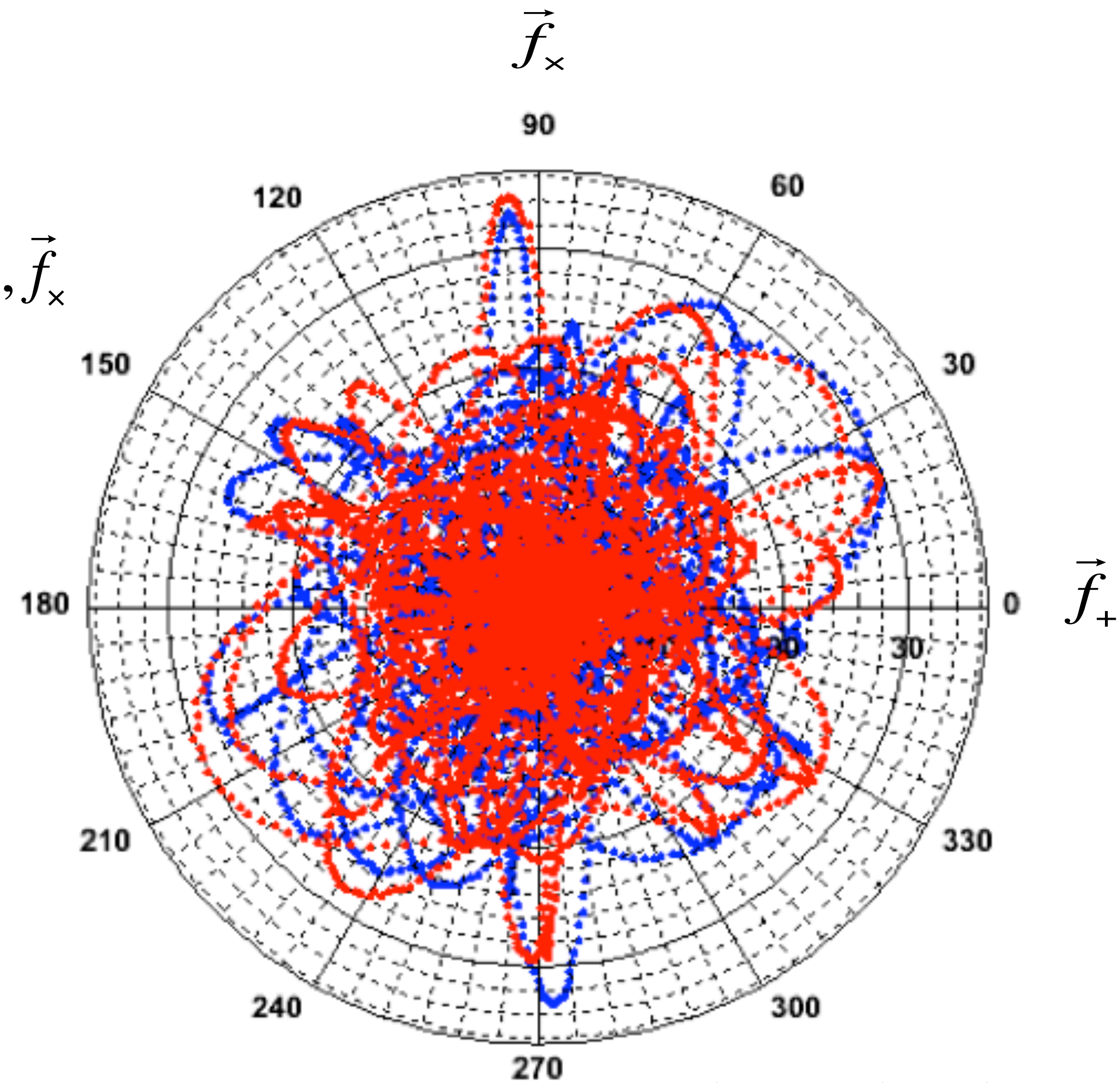


$$\left|\vec{f}_+(\psi)\right| \geq \left|\vec{f}_\times(\psi)\right|$$

Polargrams

- **Polargrams show evolution of the response vectors** $\vec{\xi}$ **in the network plane** $\vec{f}_+, \vec{f}_\times$ **(polarization state)**

- **This polargram shows evolution of simulated GW signal with random polarization: blue – 0-phase response, red – 90-phase response (quadrature) – defined on page 14**



Vedovato, Klimenko

Fundamental Network Parameters

- **Network sensitivity** $S_{net} = \left(\sum_k S_k^{-1}\right)^{-1}$
- **Network acceptance** $F = \sqrt{\left(|f_+|^2 + |f_\times|^2\right) S_{net}}$
- **Network alignment** $\mathrm{A} = \dfrac{|f_\times|}{|f_+|}$
- **Network/response index** $\mathrm{I}_+ = \left(\sum_k e_+^4[k]\right)^{-1}, \quad \mathrm{I}_\times = \left(\sum_k e_x^4[k]\right)^{-1}$
- Network parameters are frequency dependent
- The utility of network parameters is described below

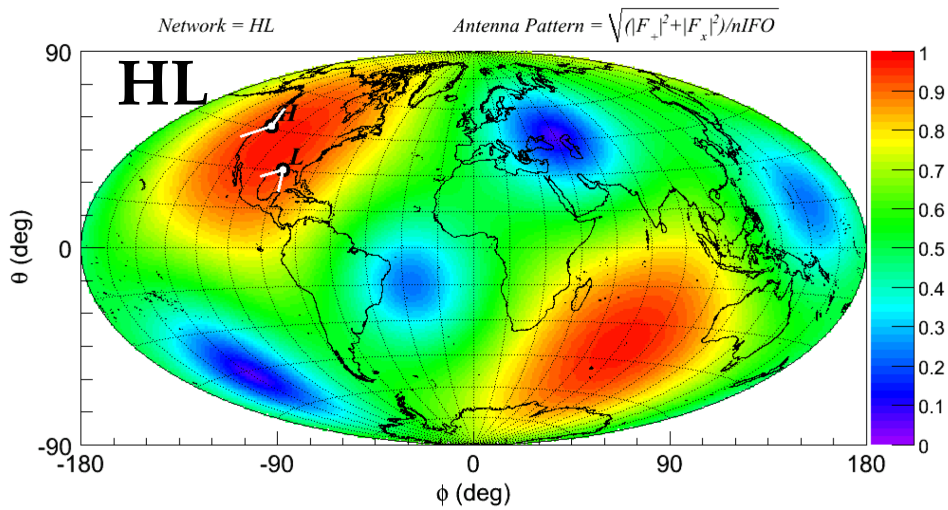
Klimenko, et al

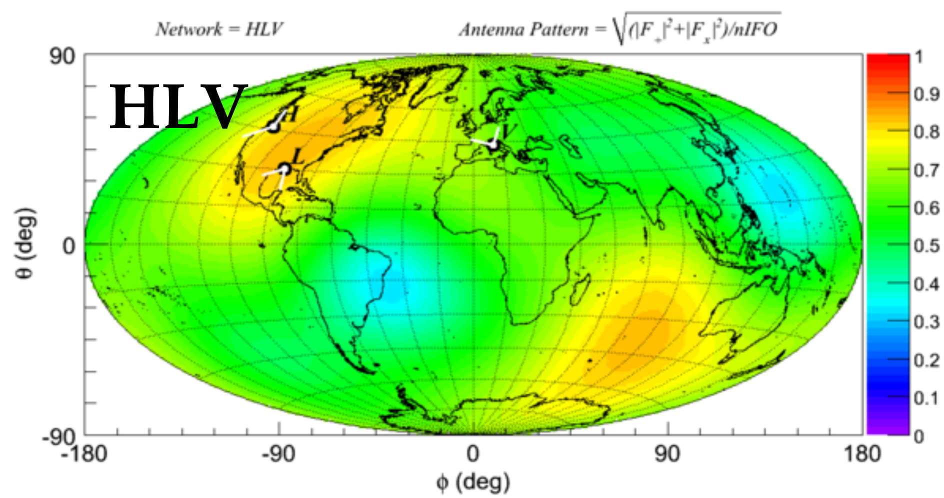
Network Acceptance

$$F(\omega) = \sqrt{\left(|f_+|^2 + |f_\times|^2\right) S_{net}}$$

Antenna sensitivity w.r.t
a network of
omni-directional detectors
(100% acceptance)
sky average <F>~0.5

here and later for calculation of antenna patterns we assume equal sensitivity of all detectors – actual acceptance are frequency dependent





Network SNR

$$\rho_{net} = 2\sqrt{\sum_k \int_0^\infty \left[\left| \vec{f}_{+k} h_+(f) \right|^2 + \left| f_{\times k} h_\times(f) \right|^2 \right] df}$$

- **GW hrss amplitude** $$h_{rss} = \sqrt{\int [h_+^2(t) + h_\times^2(t)]\, dt}$$

- **Population average SNR**
  - **assume** $\bar{h}_{+rss} \approx \bar{h}_{\times rss}$
  - **assume 1-side S(*f*)=const around the characteristic signal frequency *f*$_o$**

$$\bar{\rho}_{net} \approx 2F \sqrt{\int_0^\infty \frac{|h_+(f)|^2 + |h_\times(f)|^2}{S_{net}(f)} df}$$

$$\bar{\rho}_{net} \approx F\rho_o \approx \frac{F \cdot h_{rss}}{\sqrt{S_{net}}}$$

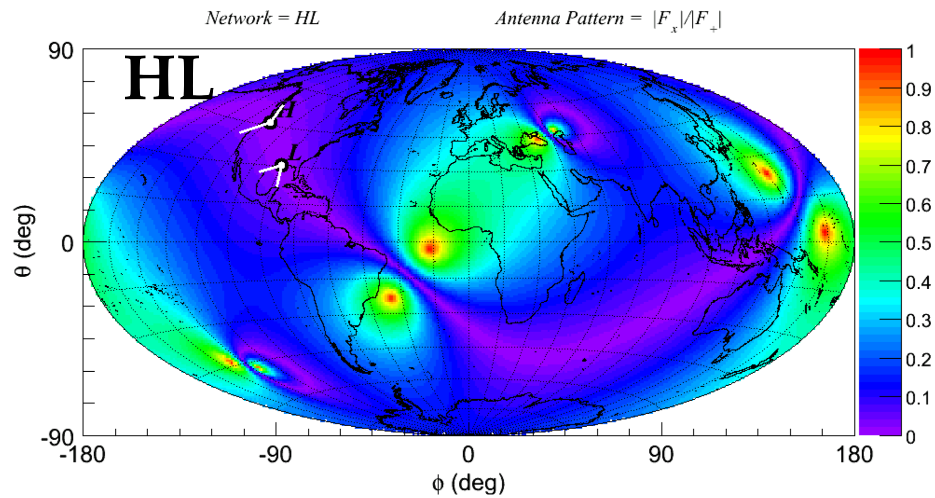
Schutz, *CQG* **28** 125023(2011)
Klimenko, et al PRD 83, 102001 (2011)

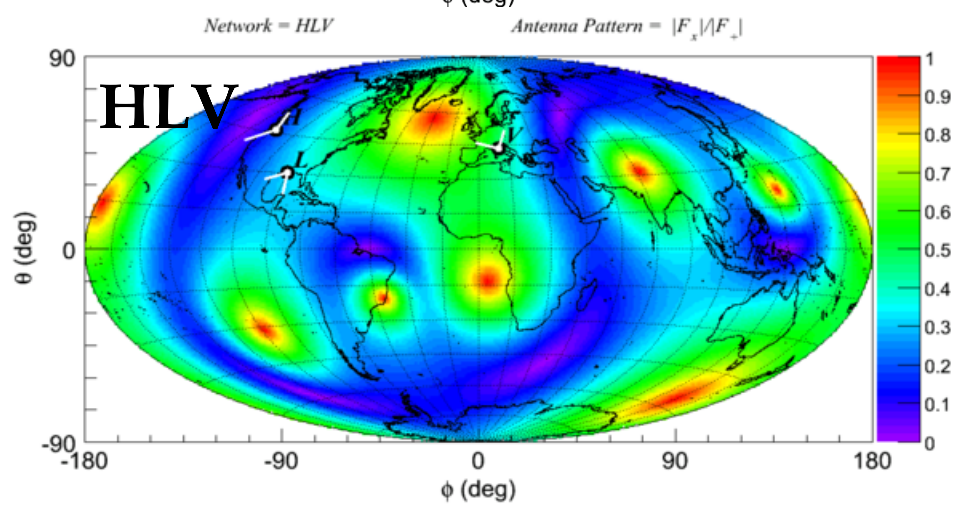
Network Alignment

$$A = \frac{|f_\times|}{|f_+|}$$

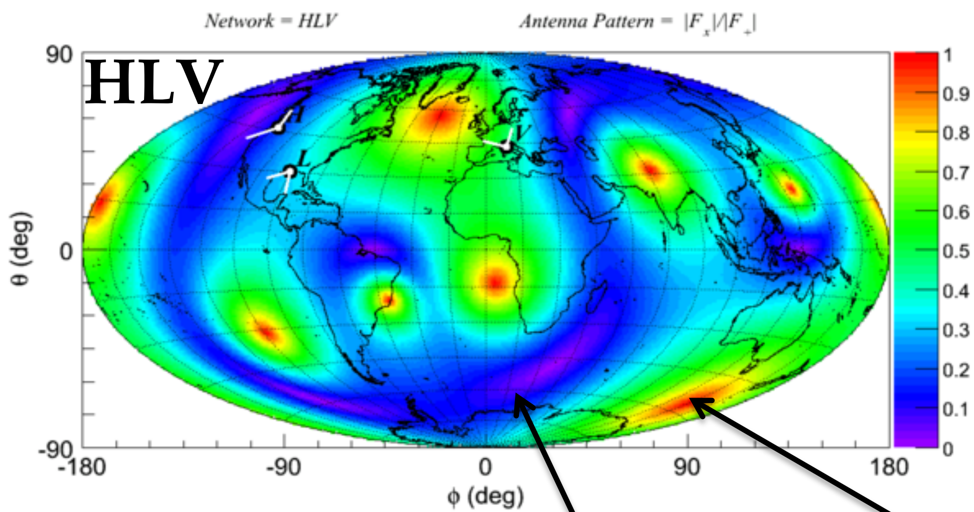
- tells how well the second polarization is detected

- For co-aligned detectors A=0 – detect only one GW component
  - any incoming GW signal looks like a linearly polarized wave with fixed polarization angle
  - network can not distinguish polarization state of incoming wave
- A – fraction of total network SNR due to the second component





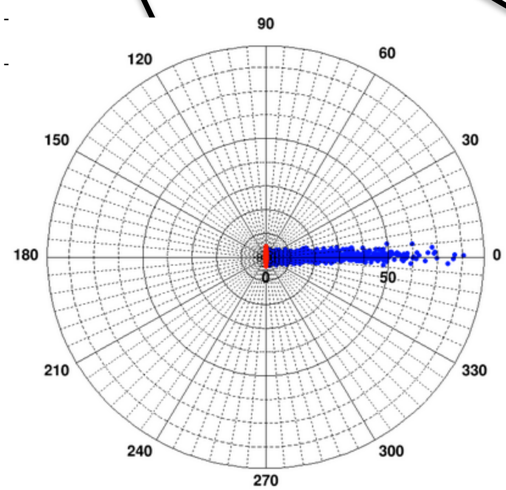
Capturing polarization state



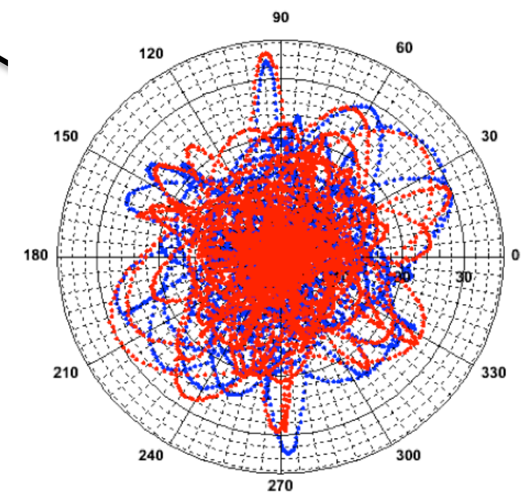
- GW polarization state is captured as a pattern of sampled responses on the network plane

$$\xi[i] \approx \vec{f}_+ h_+[i]$$

$$\left|\vec{f}_+\right| >> \left|\vec{f}_\times\right|$$



$$\xi[i] = \vec{f}_+ h_+[i] + \vec{f}_\times h_\times[i]$$



$$\left|\vec{f}_+\right| \approx \left|\vec{f}_\times\right|$$

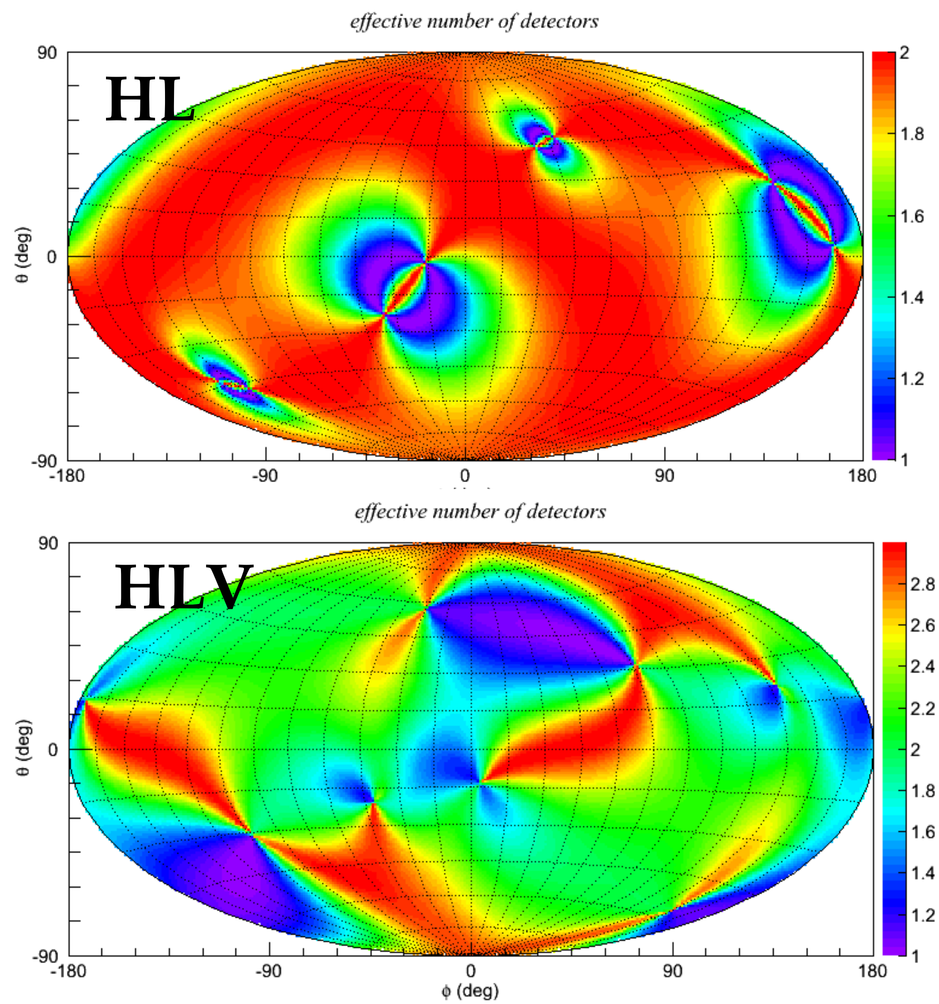
Full alignment coverage is important for reconstruction of gravitational wave polarization state

Network Index

$$I_+ = \sum_k e_+^4[k]$$

I_+^{-1} - effective number of detectors

- Given network of K detectors, depending on sky location, not all K detectors participate in the measurement - some detectors are spectators
- $1/I_+$ - effective number of detectors potentially available for measurement: distributed between 1 and K
- Detection and reconstruction greatly depend on the sky location



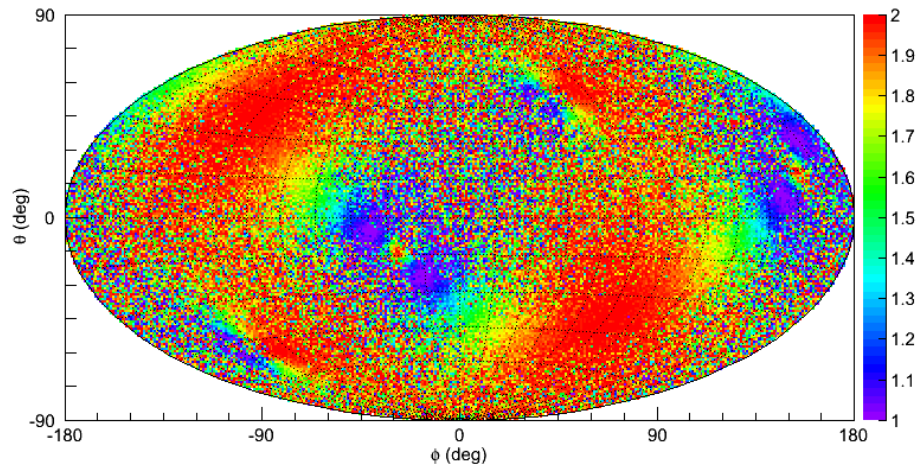
Response Index

$$I_r = \sum_k n^4[k]$$

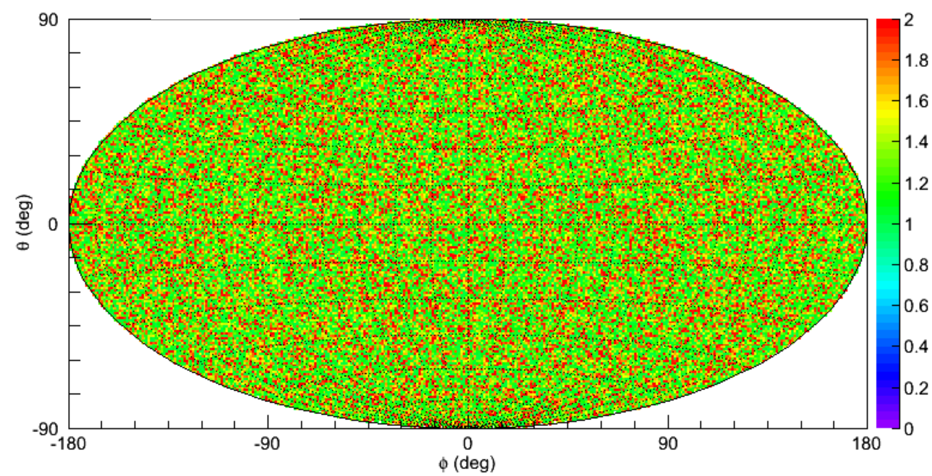
$$\vec{n} = \vec{\xi}/|\vec{\xi}|$$

- $1/I_r$ - effective number of detectors contributing to total network SNR: distributed between 1 and K
- For GW signals response index correlates with network index
- For noise and glitches there is no correlation
- Describes how similar (coherent) are responses in individual detectors
- Great tool to distinguish signal from glitches

HL: signal with random polarization

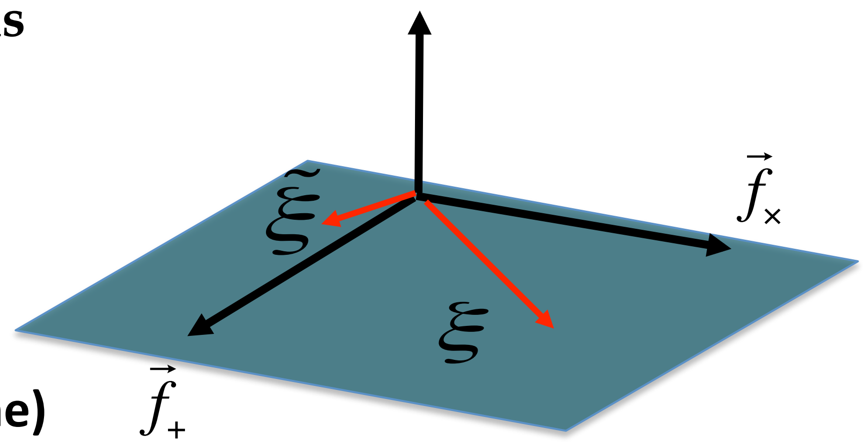


HL: random noise



Some Magic: Dual Stream Phase Transform

- **Dual data stream:** x and $\tilde{x}$ - quadrature
  - **quadrature data stream contains the same information as x**
  - **network response can be presented as pairs of vectors** $\vec{\xi}, \vec{\tilde{\xi}}$



- **Phase transform**
  - **Apply phase transform to projections (don't care about projections out of plane)**

$$\xi = \xi' \cos(\lambda) + \tilde{\xi}' \sin(\lambda)$$

$$\tilde{\xi} = \tilde{\xi}' \cos(\lambda) - \xi' \sin(\lambda)$$

- **With appropriate phase transformation the polarization pattern is revealed**

The Pattern

- **Phase Transform**

$$\cos(\lambda) \propto \left(\xi' \cdot f_+\right) / |f_+|^2$$

$$\sin(\lambda) \propto \left(\tilde{\xi}' \cdot f_+\right) / |f_+|^2$$

Parameterization of response

$$\vec{\xi}' = \vec{\mathrm{F}}_+(\Psi)\,\mathrm{q} + e\,\vec{\mathrm{F}}_\times(\Psi)\,\mathrm{Q}$$

$$\vec{\tilde{\xi}}' = -\vec{\mathrm{F}}_+(\Psi)\,\mathrm{Q} + e\,\vec{\mathrm{F}}_\times(\Psi)\,\mathrm{q}$$

- **The Pattern:** $\vec{\xi} = \vec{\xi}_+ + \vec{\xi}_\times \;,\; \vec{\tilde{\xi}}$

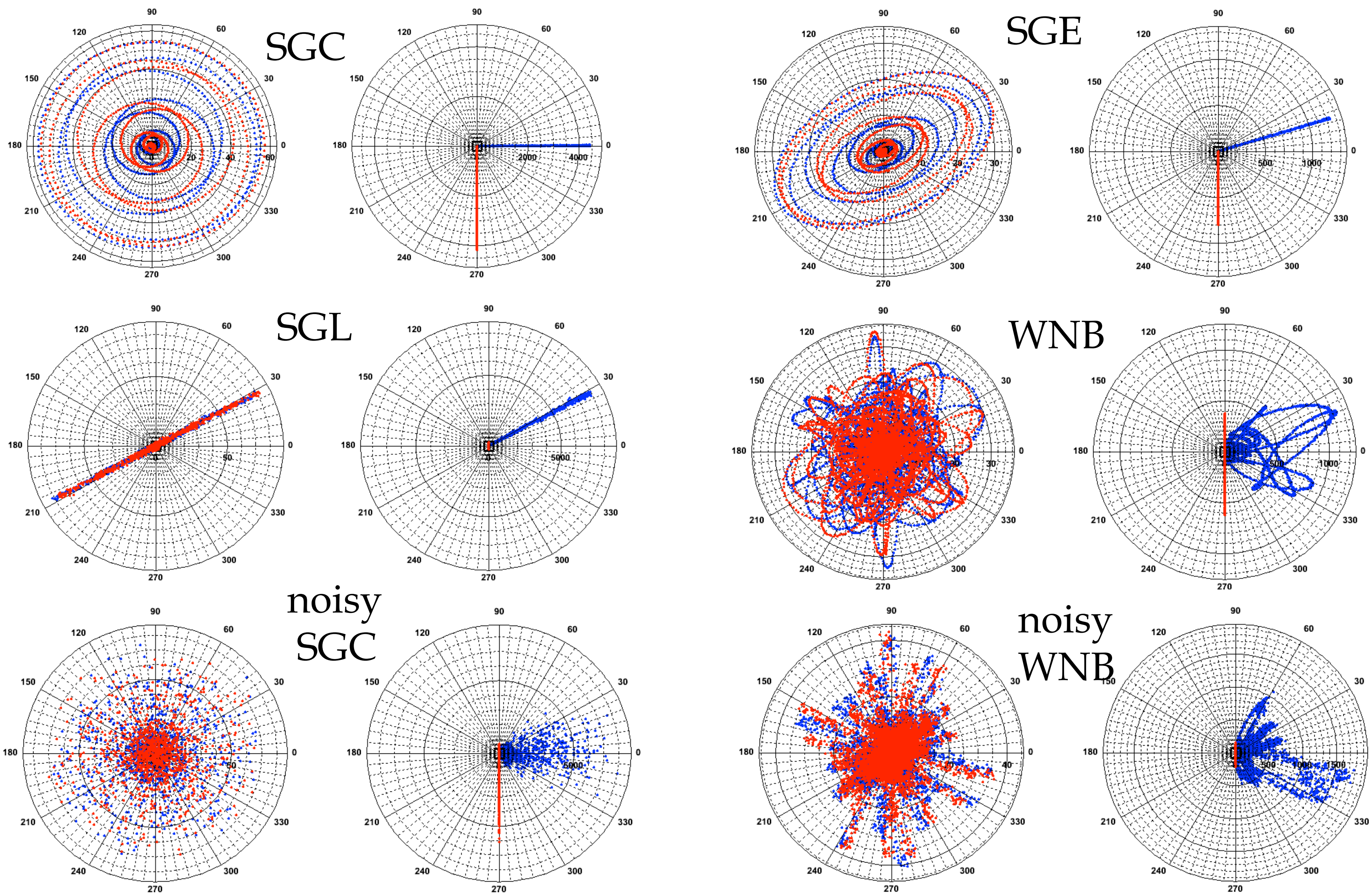
$$\vec{\xi}_+ = \vec{f}_+ \;\{(1+e^2) + (1-e^2)\;\cos[2(\psi - \Psi)]\}\;(q^2 + Q^2)/2$$

$$\vec{\xi}_\times = \vec{f}_\times \;(e^2 - 1)\;\sin[2(\psi - \Psi)]\;(q^2 + Q^2)/2$$

$$\vec{\tilde{\xi}} = \vec{f}_\times \; e\;(q^2 + Q^2)$$

- **Wave parameterization:**
  - **Ψ – DPF angle, ψ – polarization angle,**
  - **q,Q – wave quadrature amplitudes ,**
  - **e – ellipticity (equivalent to inclination angle for CBC sources)**

Polarization pattern



Parameter estimation for dummies

- **How to learn something about a generic GW transient?**
  - **Do TF-transform of detector data**
  - **Identify excess power (loud) data samples**
  - **Produce "The Pattern"**

$$\vec{\xi}_{+} = \vec{f}_{+}\ \{(1+e^2)+(1-e^2)\ \cos[2(\psi-\Psi)]\}\ (q^2+Q^2)/2$$

$$\vec{\xi}_{\times} = \vec{f}_{\times}\ (e^2-1)\ \sin[2(\psi-\Psi)]\ (q^2+Q^2)/2$$

$$\vec{\tilde{\xi}} = \vec{f}_{\times}\ e\ (q^2+Q^2)$$

  - **Find and their errors by fitting the pattern**
  - **Caveats:**
    - ✓ **need to know (or measure) sky location**
    - ✓ **need networks with full alignment coverage**

$$\xi_{\times} = \tilde{\xi} = 0 \text{ when } |f_{\times}| = 0$$

Binaries as Standard Sirens

- What do we need to know to find the luminosity distance D_L?

Red-shifted chirp mass: analysis of binary's TF evolution

Direction to the binary: source localization with networks of GW detectors

$$\mathcal{M} = m_1^{3/5} m_2^{3/5} / (m_1 + m_2)^{1/5}$$

$$h_+ = \frac{2[(1+z)\mathcal{M}]^{5/3}}{D_L} [\pi f(t)]^{2/3} \left[1 + (\hat{L} \cdot \hat{n})^2\right] \cos[\Phi(t)]$$

$$h_\times = \frac{4[(1+z)\mathcal{M}]^{5/3}}{D_L} [\pi f(t)]^{2/3} \left[\hat{L} \cdot \hat{n}\right] \sin[\Phi(t)] \ .$$

$D_L(z)$ degeneracy: find electro-magnetic counterpart with telescopes

source orientation: reconstruction of the wave's polarization state to resolve *iota*-D_L digeneracy

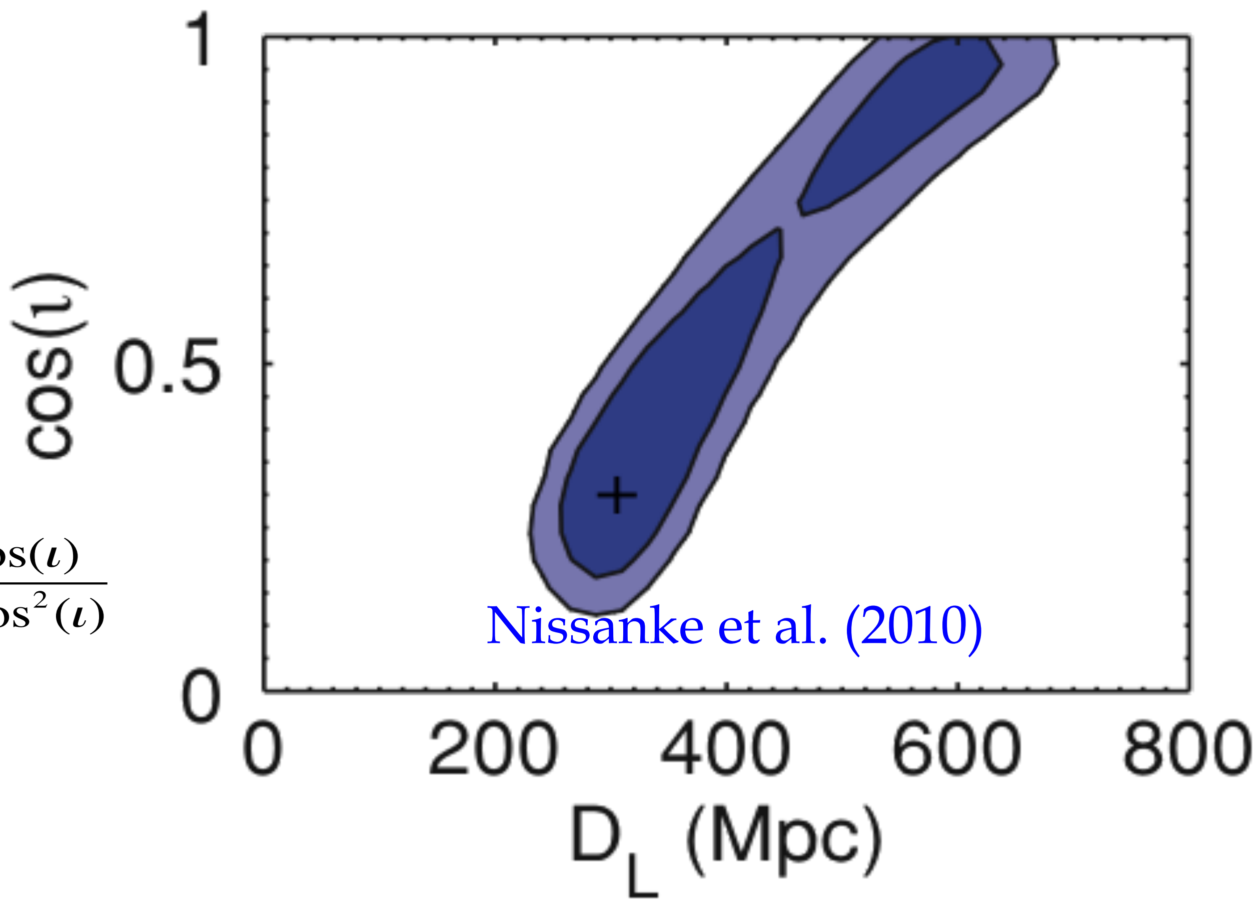
- Full alignment is critical for polarization and greatly improves sky localization

CQG, 20 (2003) ApJ.725,2010

iota – distance degeneracy

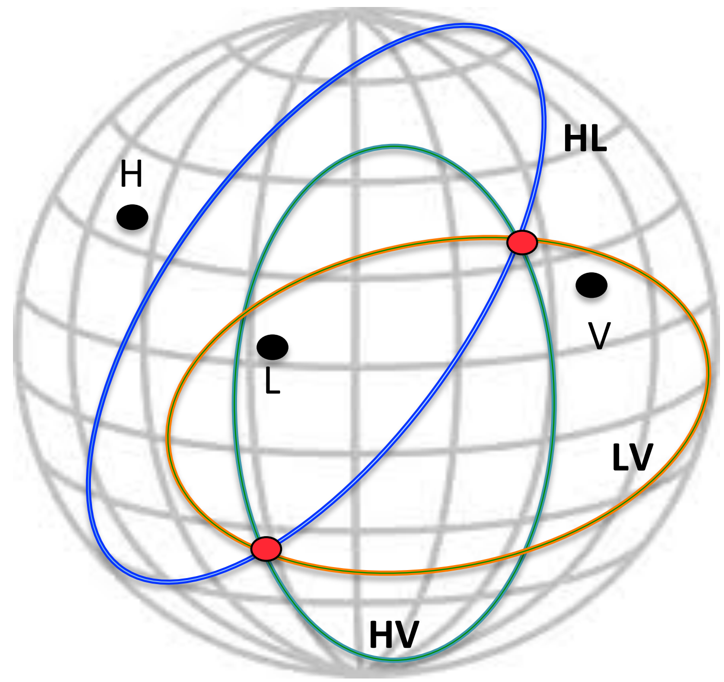
- measured parameter ellipticity is related to the CBC source inclination angle *iota*.
- independent measurement of e resolves *iota* –distance degeneracy

$$e = 1 - \frac{2\cos(\iota)}{1+\cos^2(\iota)}$$



Source Localization

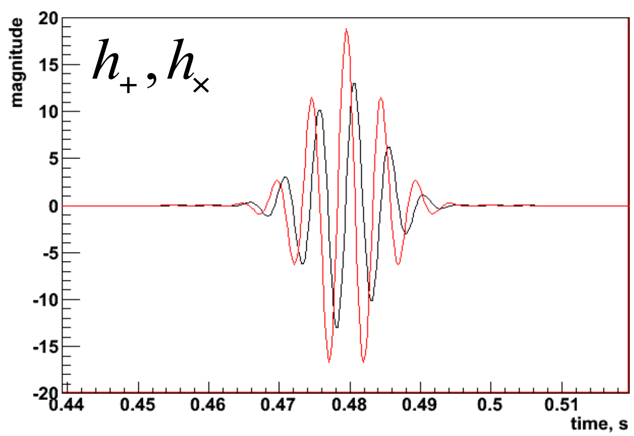
- All skyloc algorithms use a combination of two basic methods
  - ➢ measure time of flight with 2 or more spatially separated detector sites – reconstruct ToF rings
  - ➢ resolve degeneracy along the rings by using variability of antenna patterns
- Localization is greatly improved for 3 and more detectors – find intersection of rings at constant time delay for detector pairs
- Network index coverage is poor for 3 detectors: only for a fraction of events all 3 rings can measured → more detectors



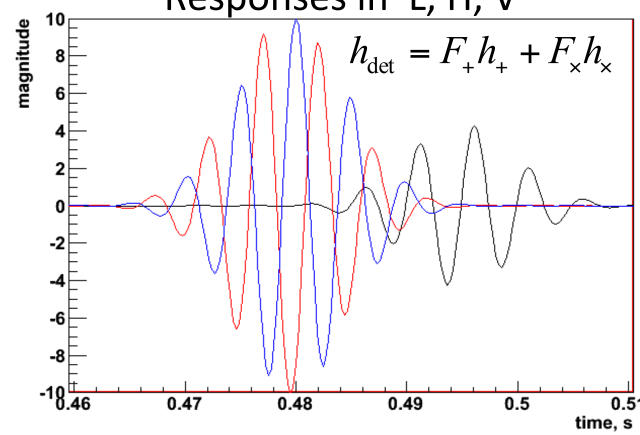
Wen 2009
Fairhurst 2009
Klimenko et al 2009

Triangulation

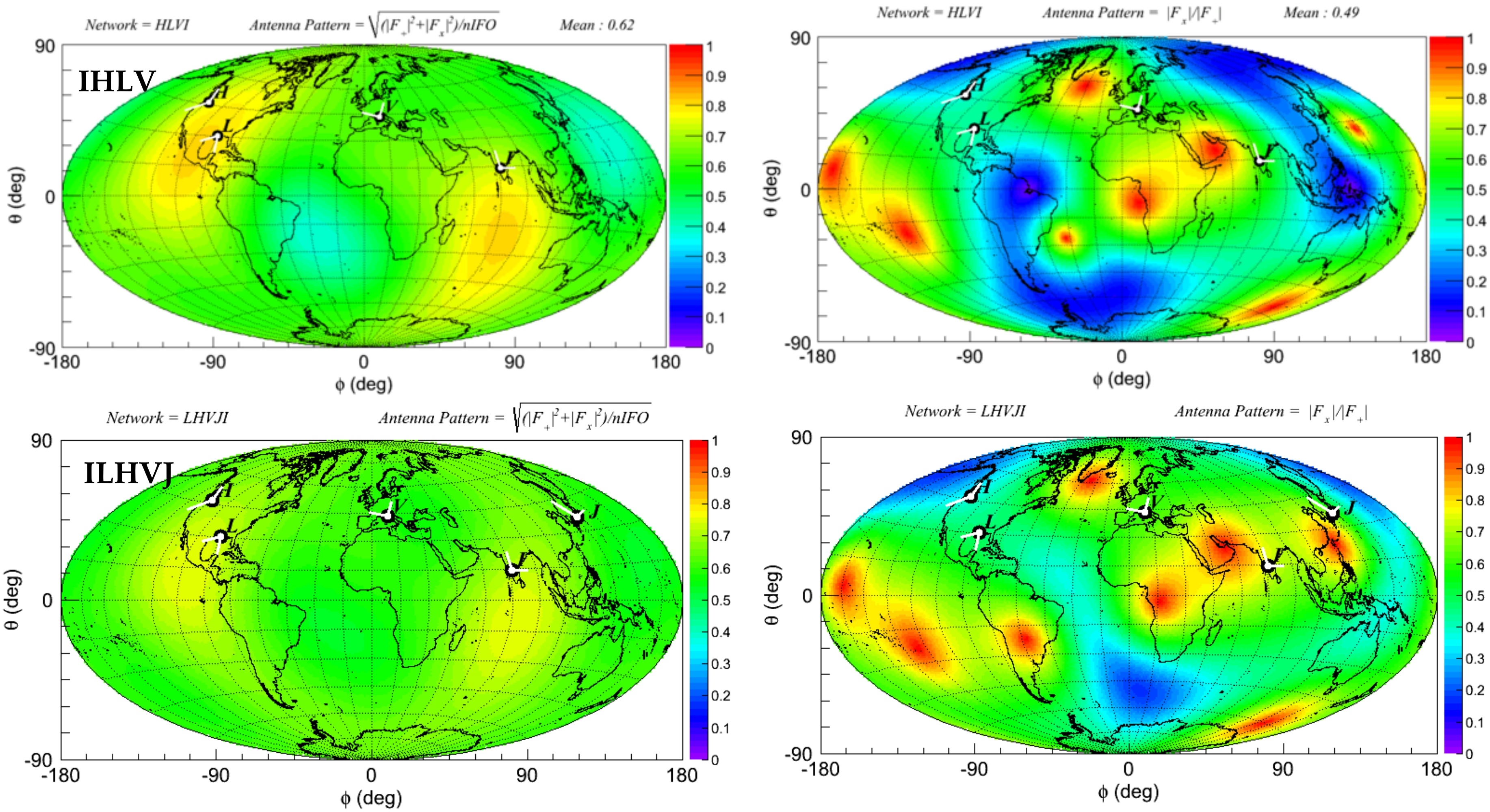
- How to measure ToF? Time shift detectors until they are "synchronized".
- What synchronize means? Correlate detector data with the reference waveform
  - find max correlation <data|reference>
- Template searches: template is the reference waveform
  - find max <data|template>: can be wrong when Nature and theory disagree
- Burst searches: reference is the waveform in the other detector: $\langle \xi[i] \mid \xi[j] \rangle$
  - $\max \sum_{i \neq j} \langle \xi[i] \mid \xi[j] \rangle$ - unbiased ToF only for linear waves (quadrature $\tilde{\xi} = 0$)
  - In general unbiased estimator is provided by $\max \sum_{i \neq j} \left\{ \langle \xi[i] \mid \xi[j] \rangle + \langle \tilde{\xi}[i] \mid \tilde{\xi}[j] \rangle \right\}$
  - Again, for better sky localization need more detectors to measure $\tilde{\xi}$



Responses in L, H, V

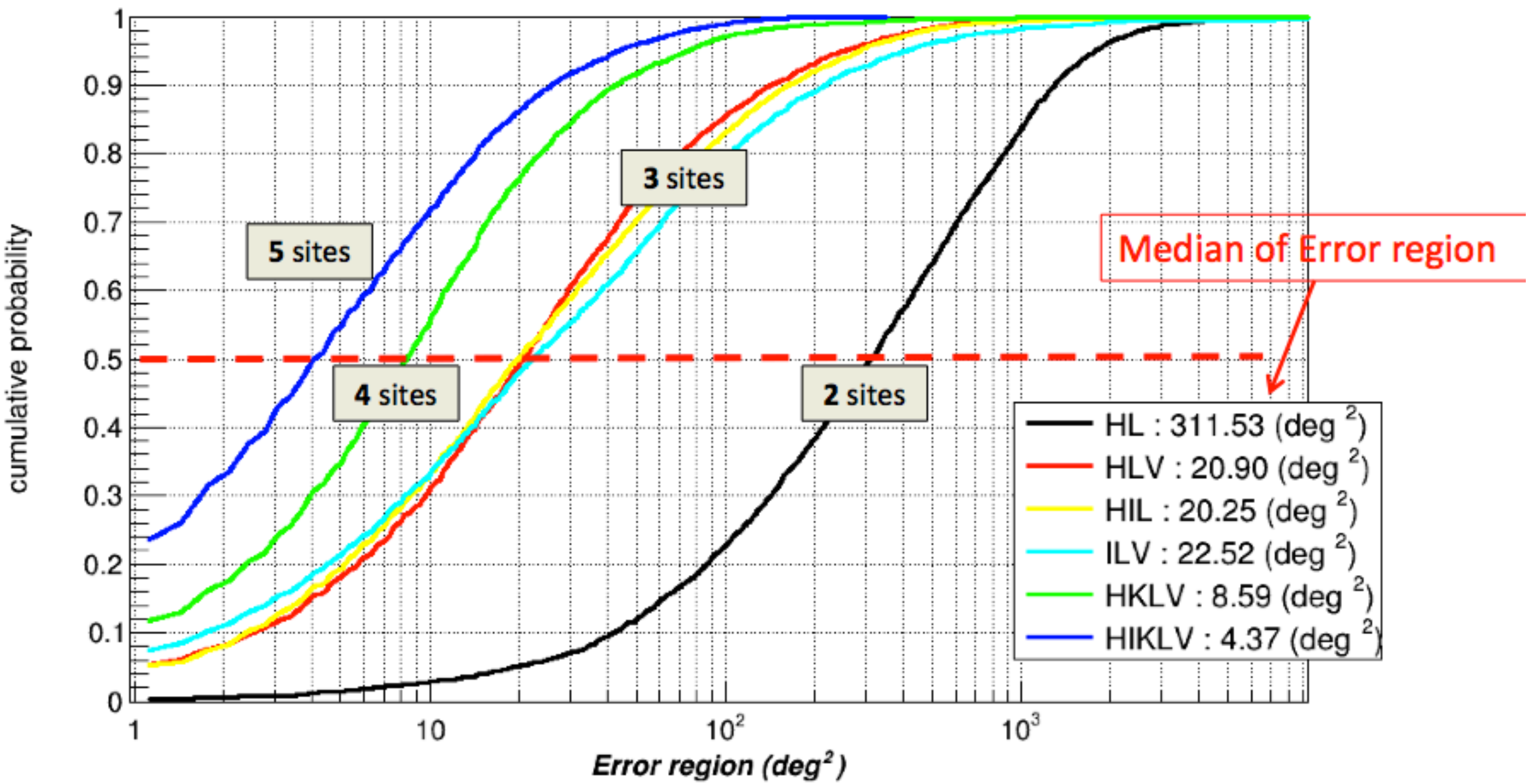


coverage with 4/5 site networks



- Sky coverage gradually increase (still some gaps). The number of 5D events decreases as DC^5 where DC is the duty cycle. For DC=0.8, approximately 70% of events will be detected with either 4D or 5D network.

Sky localization with advanced networks



- Simulation for a population of 15-25 Mo binary black holes.
- 4D or 5D networks <10 sq. deg. resolution (@50%) can be achieved.

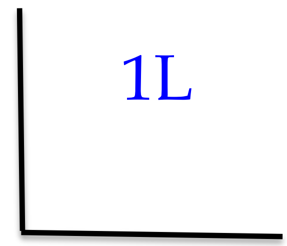
Quick Observations

- Just looking at the network parameters we can learn a lot without even doing a complicated GW analysis
  - Size of the network (number of detectors) and locations matter: multiple detectors improve coverage of the sky (acceptance & alignment)
  - average acceptance is around 0.5
  - overall network strain sensitivity improves as more detectors are added. Detectors with low sensitivity have marginal contribution to the network SNR
  - Even sensitive detectors not always participate in the measurement. Depending on sky location they can be spectators, effectively reducing the network size
  - the wave polarization can be directly observed as a pattern in the network plane.
  - The phase transform can reveal interpretable polarization pattern. Polarization parameters can be measured independent from the other source parameters
  - existing LH and LHV networks have very limited alignment coverage – for most of the sky the polarization state of a moderate SNR event can not be measured → greatly affects source reconstruction and sky localization
- Network sensitivity is a primary concern now. But as soon as there are first detections, the focus will shift to improve network acceptance, alignment and index as they greatly affect the network reconstruction capabilities→ how to improve them?

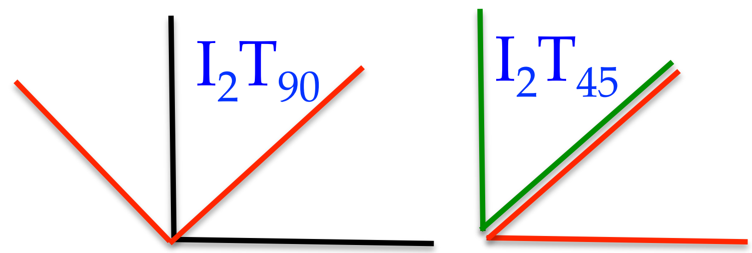
Invariant site topology

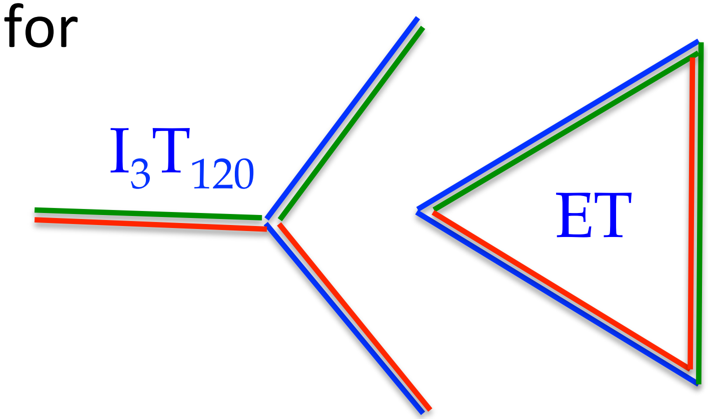
How to increase polarization coverage and improve GW signal reconstruction?

Use **invariant topology** when site's antenna sensitivity does not depend on the global orientation of detector arms in the site plane

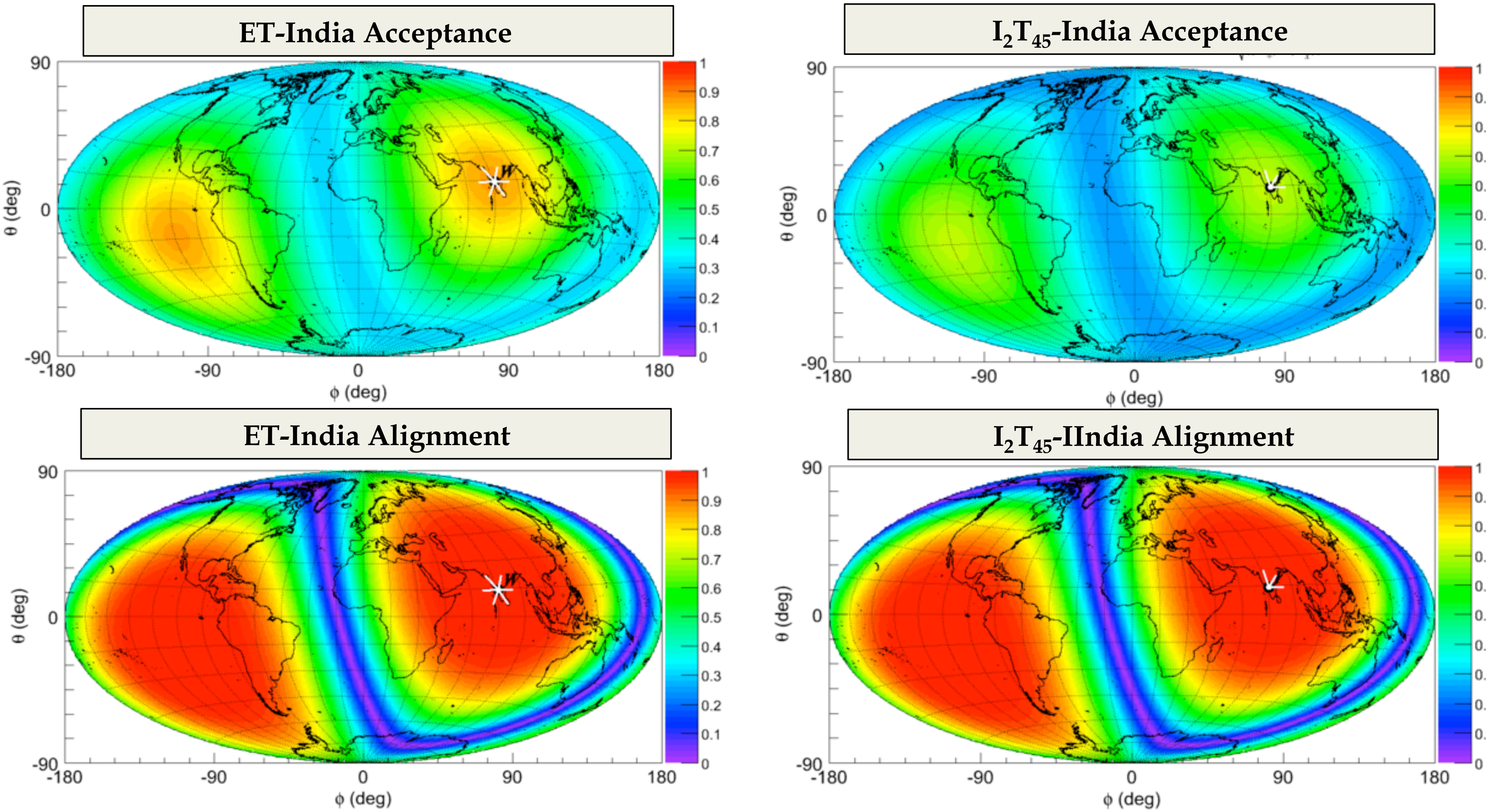


- one L-shape (1L) detector – not invariant
- Einstein Telescope (ET) - invariant
- Interferometric Telescopes (IT):
  there are many invariant topologies
  - 2 detectors (I_2T) - possible upgrade for L-site: build one/two more arms
  - 3 detectors (I_3T/ET)
- Networks built with IT sites is a way to achieve sub-degree sky resolution





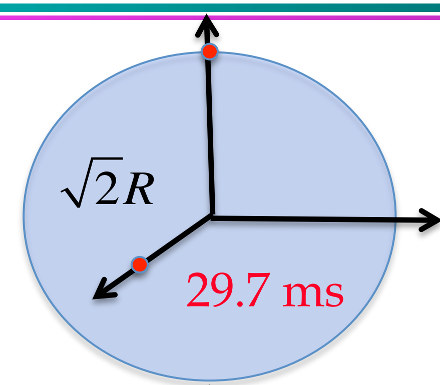
Why telescopes?



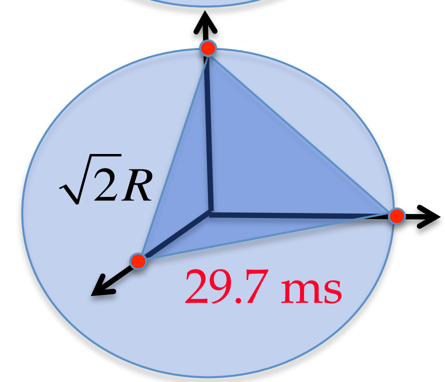
- **IT site is like a stationary telescope with large FOV**
- **ET is equivalent to I_2T_{90} and 1.5x more sensitive than I_2T_{45}**

Dream networks

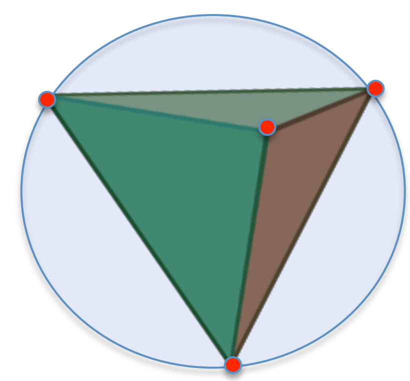
- Networks with the best antenna coverage & and time-of-flight between detectors
- 2-site networks
  - Cartesian2 (2 site at the Cartesian frame axes)



- 3-site networks
  - Cartesian3 (3 site at the Cartesian frame axes)
  - JLV&JHV sites are very close to Cartesian3



- 4-site networks
  - Tetrahedron.
  - If extending JLV/JHV sites, closest to tetrahedron is a site either in Australia or Argentina (excluding the most optimal location at South Pole)

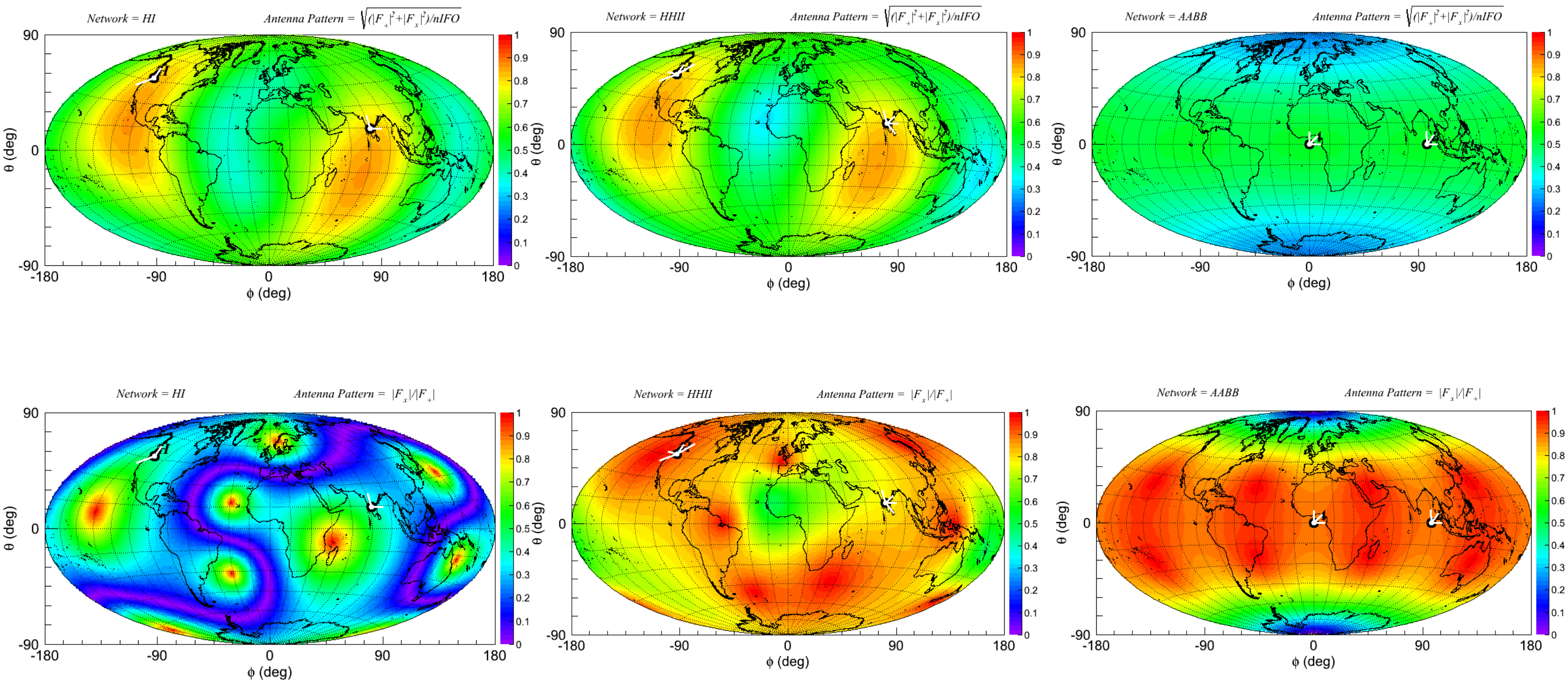


- What do we gain? How far are existing and planned network sites from the optimal?

Coverage with 2 IT sites

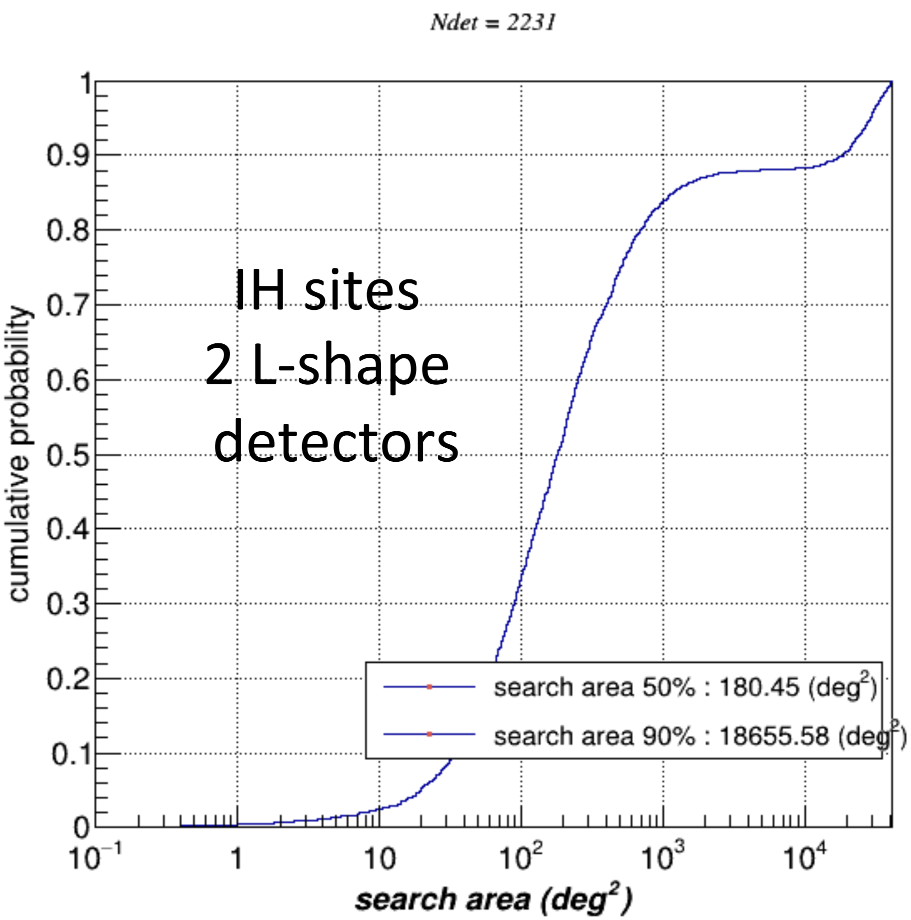
- **IT detectors provide much better alignment coverage**
  - **next slide shows how it translates in better reconstruction**
- **IH IT site coverage is comparable with the perfect Cartesian2**

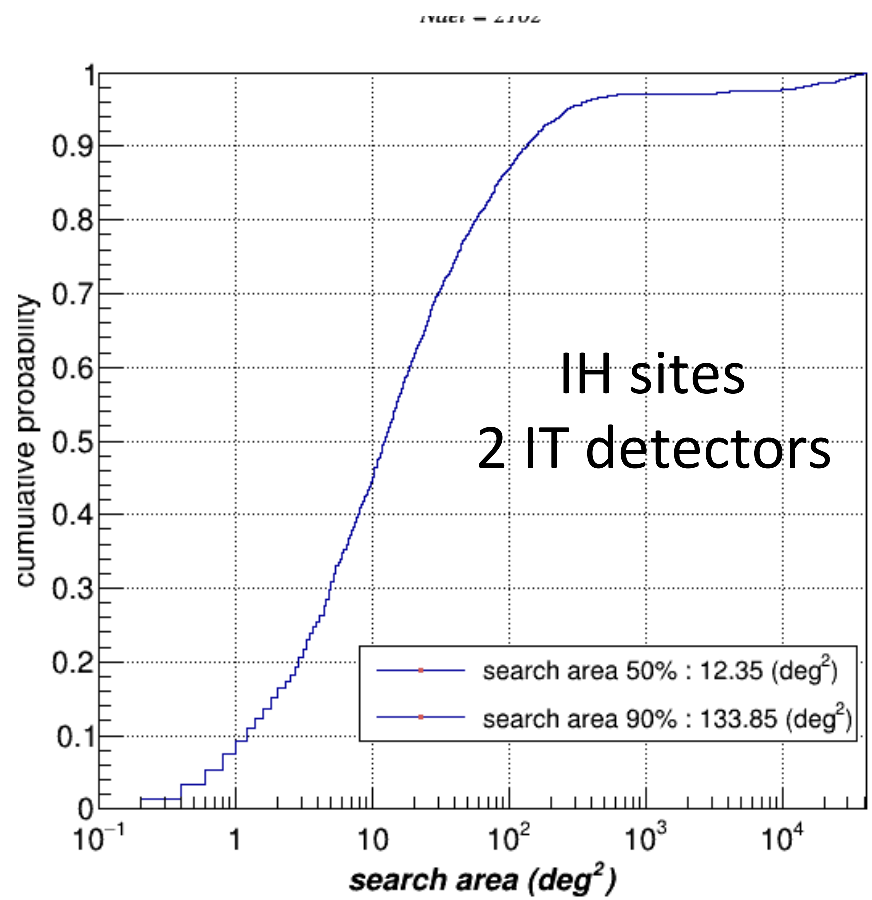
IH sites 2 L detectors | IH sites 2 IT detectors | Cartesian2



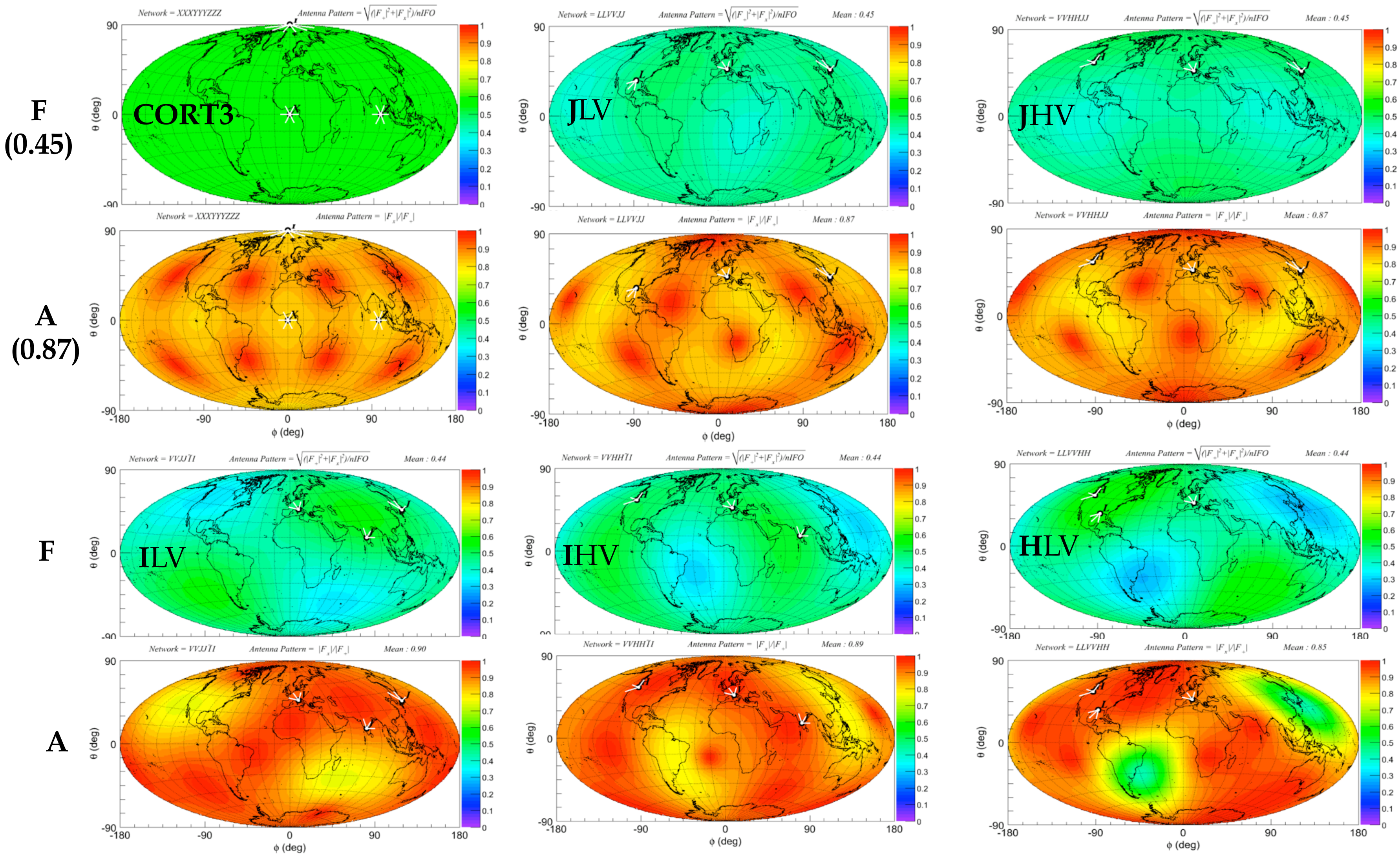
Localization with 2D network

- **Any 2-site network with L-shaped detectors have poor localization**
- **What if we have IT detectors at H and I sites? – huge improvement!**
- **Expensive (add 1 or 2 more arms), but not as developing a new site**



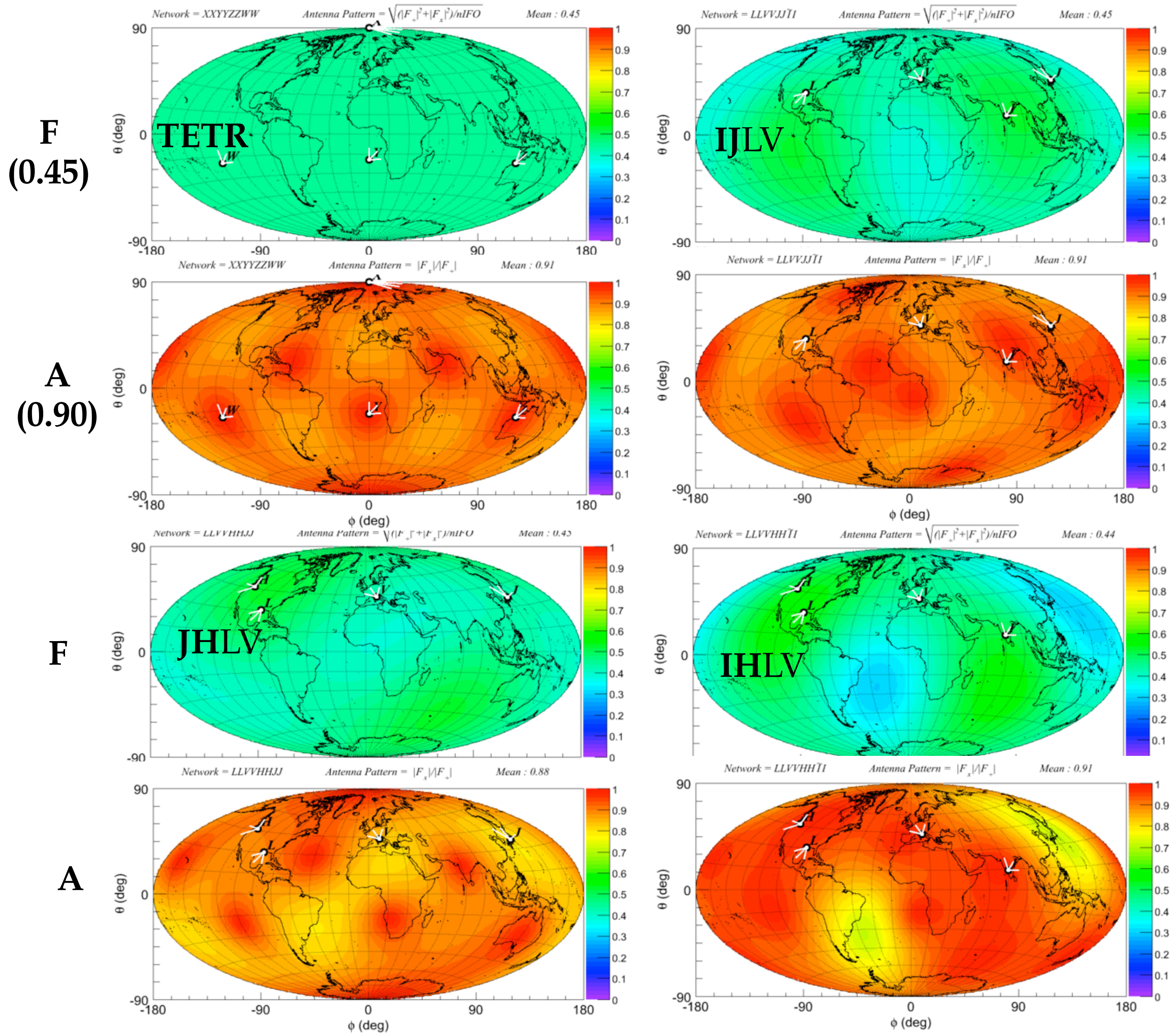


Coverage with 3 IT-site networks



- JLV & JHV are close to optimal. IJH, IHV, ILV, HLV are less optimal but anyway provide much better coverage than 1L-topology networks

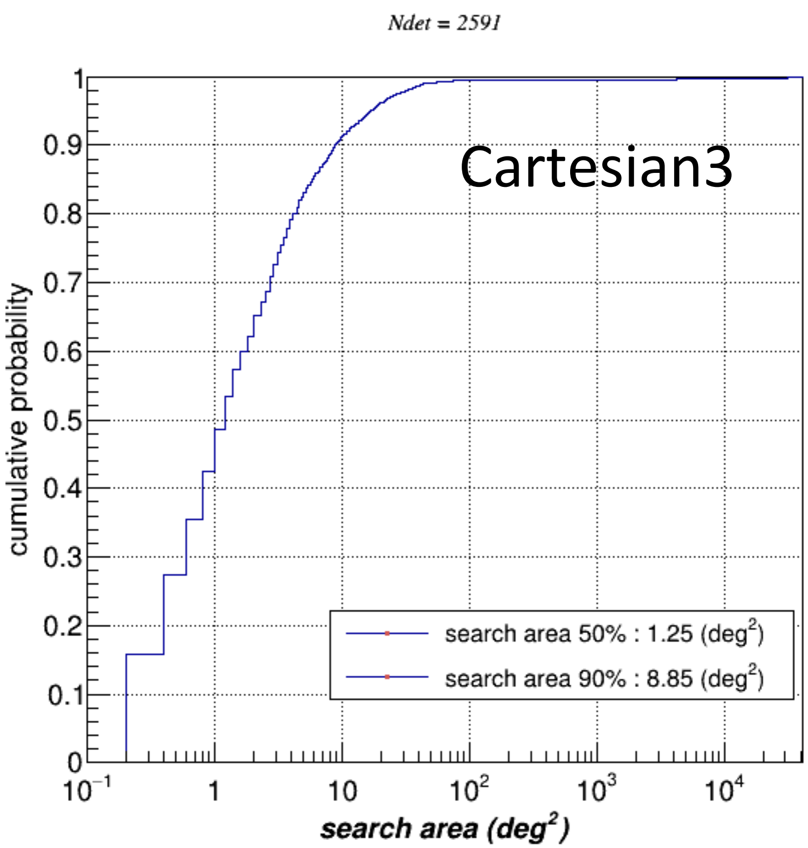
coverage with 4 IT-site networks

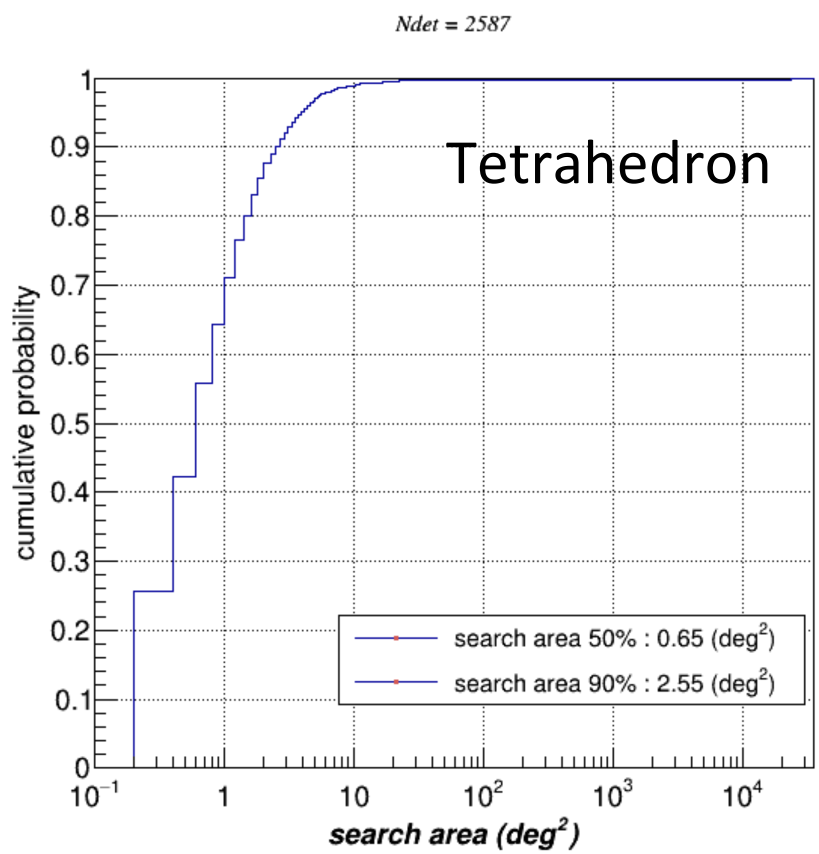


- Excellent coverage close to tetrahedron

Performance of the Dream Network

- Sub-degree sky localization is achievable (more compatible with telescopes)
- Expect comparable performance if few existing sites are furnished with IT detectors.





Summary

- advanced (2G) network targets first detection and will be increasing in size in the next few years.
- Science output of GW searches greatly depends on network configurations and their potential to capture astrophysics
- GW networks and their response can be characterized with few figures of merit describing the strain and antenna sensitivities (acceptance, alignment and network index)
- Direct (without relaying on source model) measurement of the wave polarization and source sky location is possible with several GW detectors.
- Locations of existing (H,L,V) and planned (I,J) sites are close to optimal, however, advanced networks (particularly early) suffer from the sky alignment under-coverage
- IT site topology significantly improves reconstruction and could be a viable upgrade for 2G network after first detections. However, 2G upgrade will greatly depend on what astrophysical landscape we see.

Reading Material

- LIGO/Virgo publications on burst searches: https://www.lsc-group.phys.uwm.edu/ppcomm/Papers.html
- Guersel,Tinto, PRD 40 v12,1989
  - reconstruction of GW signal for a network of three misaligned detectors
- Likelihood analysis: Flanagan, Hughes, PRD57 4577 (1998)
  - likelihood analysis for a network of misaligned detectors
- Two detector paradox: Mohanty et al, CQG 21 S1831 (2004)
  - state a problem within standard likelihood analysis
- Semi-coherent burst search. Klimenko S and Mitselmakher CQG 21 S1819 (2004)
- Constraint likelihood: Klimenko et al, PRD 72, 122002 (2005)
  - address problem of ill-conditioned network response matrix (rank deficiency)
  - first introduction of likelihood constraints/regulators
- Penalized likelihood: Mohanty et al, CQG 23 4799 (2006).
- Rank deficiency of network matrix: Rakhmanov, CQG 23 S673 (2006)
- GW signal consistency: Chatterji et al, PRD 74 082005(2006)
- Coherent Burst search: S. Klimenko et al., Class. Quantum Grav. 25, 114029 (2008)
- Sky localization with advanced network. S. Klimenko et al. PRD 83, 102001 (2011).
- Three figures of merit..., B.Schutz, *CQG* **28** 125023(2011)