Subtracted Geometries

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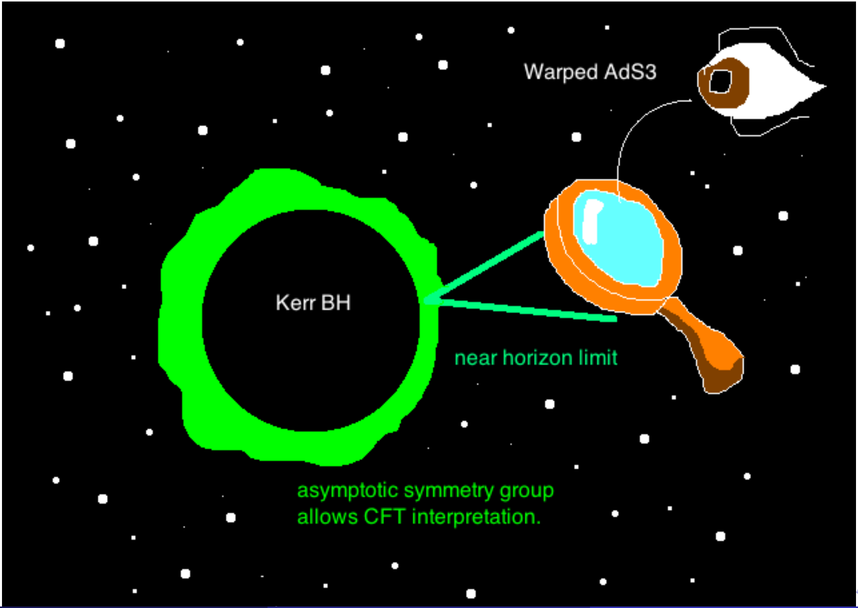
Introduction

Extremal Kerr/CFT



Extremal Kerr/CFT

Work of M. Guica, T. Hartman, W. Song and A. Strominger (2008).
The Kerr line element is

$$
\begin{aligned}
ds^2 &= -\frac{\Delta}{\rho}^2 (d\hat{t} - a\sin^2\hat{\theta})^2 + \frac{\sin^2\theta}{\rho^2}\left((\hat{r}^2 + a^2)d\hat{\phi} - ad\hat{t}\right)^2 + \\
&+ \frac{\rho^2}{\Delta} d\hat{r}^2 + \rho^2 d\theta^2,
\end{aligned}
$$

with

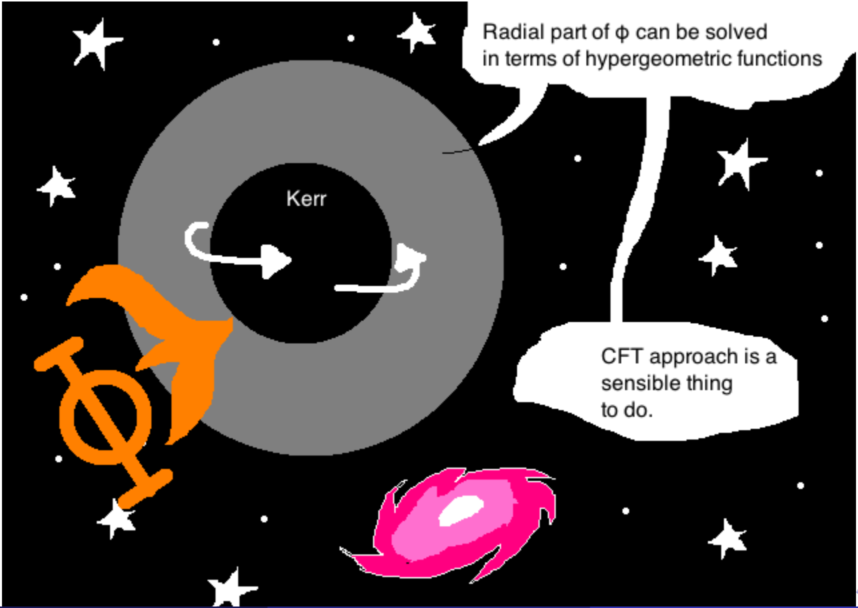
$$
\Delta \equiv \hat{r}^2 - 2Mr + a^2, \quad \rho^2 \equiv \hat{r}^2 + a^2\cos^2\theta
$$

and the NHEK line element is

$$
ds^2 = 2J\Omega^2(\theta)\left(-(1+r^2)d\tau^2 + \frac{dr^2}{1+r^2} + d\theta^2 + \Lambda(\theta)^2(d\varphi + rd\tau)^2\right)
$$

This allows a CFT interpretation.

Kerr/CFT


Radial part of ϕ can be solved in terms of hypergeometric functions
Kerr
CFT approach is a sensible thing to do.

Kerr/CFT

Work of A. Castro, A. Maloney and A. Strominger (2010)

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\Phi) = 0.$$

Say $\Phi(t,r,\theta,\phi) = e^{-i\omega t+im\phi}R(r)S(\theta)$ and get

$$\Big[\partial_r\nabla\partial_r + \frac{(2Mr_+\omega - am)^2}{(r-r_+)(r_+r_-)} - \frac{(2Mr_-\omega - am)^2}{(r-r_+)(r_+r_-)} + (r^2+2M(r+2M))\omega^2\Big]R(r) = K_\ell R(r).$$

asking $\omega M \ll 1$, we can discard the last term for the $r < M$ region, and solve the equation as an hypergeometric function.

Subtracted Geometry

Rotating Black Hole

Rotating and Charged Black Hole.

Rotating and Charged BTZ Black Hole.

Subtracted Geometry

Proposed by M. Cvetic, M. Guica and Z. Saleem (2013). The subtracted geometry line element is

$$ds^2 = -e^{2U}(dt + \omega_3)^2 + e^{-2U} ds_3^2$$

with

$$\begin{aligned} ds_3^2 &= \frac{\Delta_2}{\Delta} dr^2 + \Delta_2 d\theta^2 + \Delta \sin^2\theta d\phi^2, \\ \Delta &= r^2 - 2mr + a^2,\ \Delta_2 = \Delta - a^2 \sin^2\theta. \end{aligned}$$

The uplift works as

$$\begin{aligned} ds_5^2 &= f^2(dz + A^0)^2 + f^{-1} ds_4^2 \\ &= ds_{BTZ} + S^2. \end{aligned}$$

First Law of Thermodynamics

First Law of Thermodynamics

The original Kerr first law is satisfied by

$$dM_{Kerr} = \frac{\kappa}{8\pi} dA + \Omega dJ$$

now

$$dM = \frac{\kappa}{8\pi} dA + \Omega dJ + \Phi^i_E dQ^i + \Phi^i_M dP^i$$

this is not easy because asymptotically

$$ds^2 = -dt^2 \rho^{3/2} + \frac{d\rho^2}{\rho^{3/2}} + d\theta^2 \rho^{1/2} + \sqrt{\rho} \sin^2 \theta d\phi^2$$

and this makes fail

$$M \propto \int \sqrt{\gamma^{(2)}} n_\mu \sigma_\nu \nabla^\mu \chi^\nu_{(t)}, \; J \propto \int \sqrt{\gamma^{(2)}} n_\mu \sigma_\nu \nabla^\mu \chi^\nu_{(\phi)}$$

Summary

(1) Extremal Kerr has a well stablished CFT dual. Entropy can be computed.
(2) Non extremal Kerr shows some signs of duality. Entropy can be computed, but why?
(3) Subtracted geometries try to give an explanation of this.
(4)Still more work is needed to establish a formal thermodynamics of this geometries.