Area terms in Entanglement Entropy

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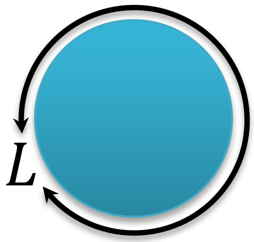
Entanglement Entropy in QFT

… for a region of size $\sim L$



area

$$S = \mu L^{d-2} + \text{subleading } (UV\ divergent) \text{ shape dependent terms}$$

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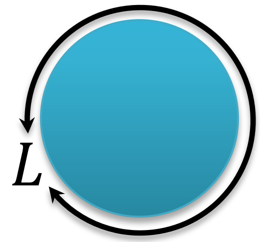

universal

$C \times \log(\epsilon/L)$

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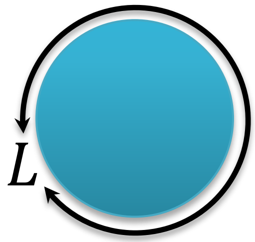

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We also have universal terms (k_0 or k'_0) inside the area term if the theory has some scale m !

if the theory has some scale (mass m)

universal

$$\mu = \left(\frac{k_{d-2}}{\epsilon^{d-2}} + k_{d-3} \frac{m}{\epsilon^{d-2}} + \ \dots \ + \ k_0 \, m^{d-2} \log(m\epsilon) + k'_0 m^{d-2} \right)$$

Result

Statement: We propose a formula for calculating the universal part of the area term

$$\mu^{\text{univ.}} = \frac{-\pi}{d(d-1)(d-2)} \int d^d x \, x^2 \, \langle 0|\Theta(0)\Theta(x)|0\rangle$$

arXiv:1007.0993
Wilczek, Hertzberg

$$\frac{m^{d-2}\,\Gamma(1-d/2)}{3\,\pi^{d/2-1}\,2^d}$$

$d =$ even $\qquad d =$ odd

$$\frac{(-1)^{(d-1)/2}\,m^{d-2}}{12\,(2\pi)^{(d-3)/2}\,(d-2)!!}$$

$$\frac{(-1)^{d/2-1}\,m^{d-2}\log(m\epsilon)}{3\,\pi^{d/2-1}\,2^{d-1}\left(\frac{d}{2}-1\right)!}$$

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Relation to c-theorem in d=2

$$\mu\, L^{d-2} \underset{d\to 2}{\longrightarrow} \mu_{d=2} = \frac{-\pi}{2}\left(\int d^2x\, x^2 \langle 0|\Theta(0)\Theta(x)|0\rangle\right)\log(m\,\epsilon)$$

$\Delta c/3\pi$ (Zamolodchikov)

$$\mu_{d=2} = \frac{\Delta c}{6}\log(m\,\epsilon)$$

$$S_{UV} = \frac{c_{UV}}{3}\log\left(\frac{R}{\epsilon}\right) + k^{UV}$$

$$S_{IR} = \frac{c_{IR}}{3}\log\left(\frac{R}{\epsilon}\right) + k^{IR}$$

$$\Delta S = \frac{\Delta c}{3}\log(m\epsilon)$$

= universal piece

Informal ideas about stablishing the relation

Adler Zee formula 1982 (quantum correction to Newton constant)

$$\Delta\left(\frac{1}{4G}\right) = -\frac{\pi}{d(d-1)(d-2)} \int d^d x\, x^2 \langle 0|\Theta(0)\Theta(x)|0\rangle + \frac{4\pi}{d-2}\left\langle \frac{\delta\Theta}{\delta R} \right\rangle$$

Black Hole entropy $\quad S_{BH} = \dfrac{Area}{4G}$

$$\boxed{S_{EE} = \mu\, Area + \ldots}$$

$\mu =$ renormalization of the area term between UV and IR

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A better proof of the formula involves:

- $\delta S = \delta\langle H\rangle$ variation of the mass of the theory
- Spectral decomposition of the two point correlation function of the trace of the stress tensor.

Conclusions

disregarding other details (because of time):

$$\mu \overset{\text{univ.}}{=} \frac{-\pi}{d(d-1)(d-2)} \int d^d x \, x^2 \, \langle 0|\Theta(0)\Theta(x)|0\rangle$$

thanks