

Hawkmoths: pollinators that are herbivores

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ANNALS OF
BOTANY
Founded 1887

Reproductive biology of *Datura wrightii*: the benefits of a herbivorous pollinator

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THE SYSTEM

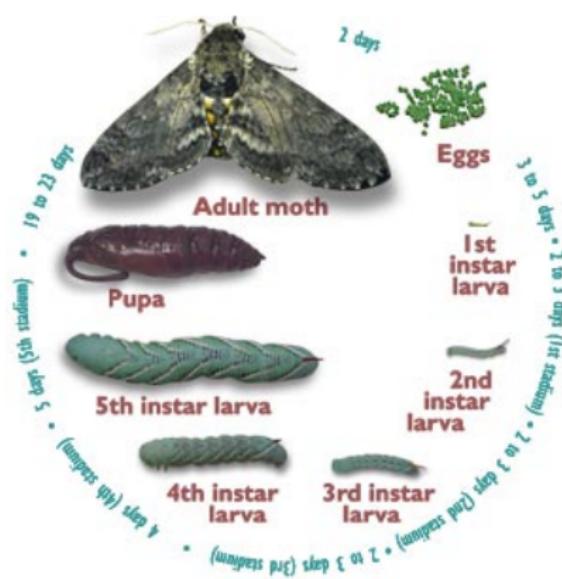


Figure: Hawkmoth Lifecycle

THE SYSTEM

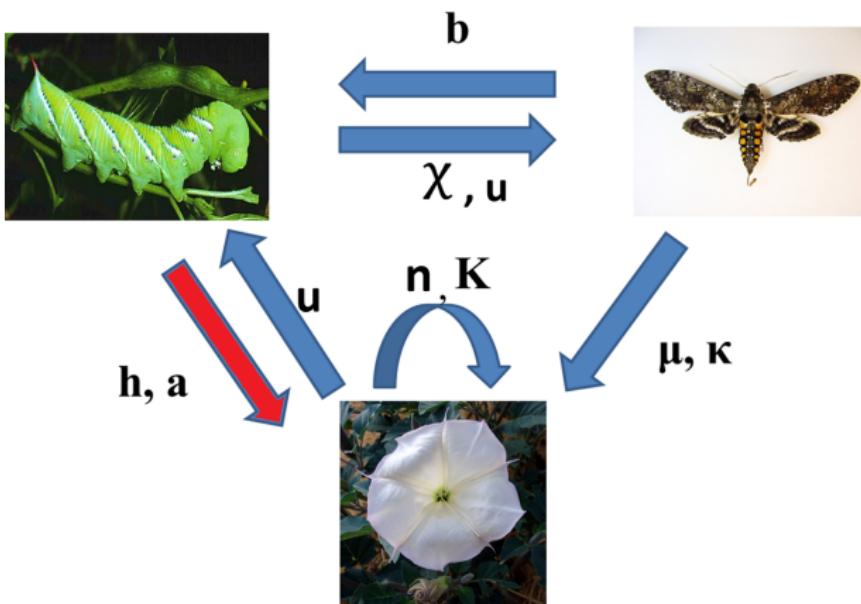


Figure: Model Diagram

BIOLOGICAL FACTS

- ▶ *Datura wrightii* (P) is highly self-compatible
- ▶ Plant leaves are the only food of *Manduca sexta* larvae (M_L)
- ▶ But *M. sexta* adults (M_A) don't depend exclusively on P
- ▶ Pollinated flowers by M_A produce more fruit and seeds
- ▶ The floral visitation component of the moth-plant interaction is mutualistic

OBJECTIVES

Identify the key biological aspects of the system.

Create a minimal model that describes the population dynamics of the system $P-M_L-M_A$.

Can this model predict coexistence of a plant and a pollinator that is also its herbivore?

SYSTEM OF EQUATIONS

$$\begin{aligned}\frac{dP}{dt} &= \left(n + \frac{\kappa M_A}{1 + \mu M_A}\right) P \left(1 - \frac{P}{K}\right) - \frac{a M_L P}{1 + ahP} \\ \frac{dM_L}{dt} &= b M_A - m_L M_L - \frac{\chi M_L P}{1 + uP} \\ \frac{dM_A}{dt} &= \frac{\chi M_L P}{1 + uP} - m_A M_A\end{aligned}$$

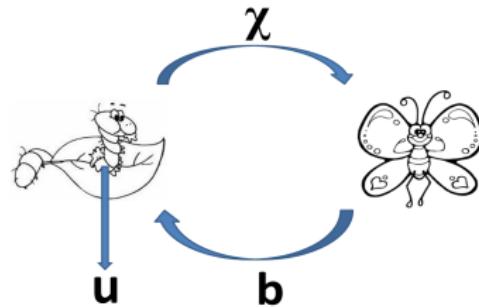
SYSTEM OF EQUATIONS

Setting some parameters:

$$\frac{dP}{dt} = \left(1 + \frac{3M_A}{1+M_A}\right) P (1-P) - \frac{M_L P}{1+0.3P}$$

$$\frac{dM_L}{dt} = bM_A - 0.2M_L - \frac{\chi M_L P}{1+uP}$$

$$\frac{dM_A}{dt} = \frac{\chi M_L P}{1+uP} - 0.2M_A$$



ANALYTICAL ANALYSIS

Now we search for equilibria:

$$\frac{dP}{dt} = \frac{dM_A}{dt} = \frac{dM_L}{dt} = 0$$

$$P^* = \frac{1}{\frac{\chi(b-m_A)}{m_L m_A} - u}$$

$$M_L^* = \left(n + \frac{3M_A^*}{1+M_A^*} \right) (1+hP^*)(1-P^*)$$

$$M_A^* = \frac{\chi P^* M_L^*}{m_A (1+uP^*)}$$

ANALYTICAL ANALYSIS

P solution:

Gives a condition in order to get a biologically relevant equilibrium:

$$P^* = \frac{1}{\frac{\chi(b-m_A)}{m_L m_A} - u} \Rightarrow \frac{u}{\chi} < \frac{b-m_A}{m_L m_A}$$

ANALYTICAL ANALYSIS

```
In [3]: s[t][1]
Out[3]: sqrt(152587890625.0*b**8*x**8 - 48828125000.0*b**7*u*x**7 - 366210937500.0*b**7*x**8 - 48828125000.0*b**6*h*x**7 + 6835937500.0*b**6*u**2*x**6 + 97656250000.0*b**6*u*x**7 + 439453125000.0*b**5*x**8 + 4882812500.0*b**6*x**7 + 9765625000.0*b**5*h*x**6 + 13671875000.0*b**5*h*x**7 + 1953125000.0*b**5*h*x**6 - 5468750000.0*b**5*u**3*x**5 - 11132812500.0*b**5*u**2*x**6 - 91796875000.0*b**5*u*x**7 - 9765625000.0*b**5*u*x**6 - 2880859375000.0*b**5*x**8 - 13671875000.0*b**5*x**7 + 156250000.0*b**4*h*x**6 - 78125000.0*b**4*h*x**2*x**6 - 1914062500.0*b**4*h*x**6 - 12500000.0*b**4*h*x**5 - 10242187500.0*b**4*x**7 - 156250000.0*b**4*x**6 - 12500000.0*b**3*h*x**5 - 12500000.0*b**3*h*x**4 + 109863281250.0*b**4*x**8 + 10742187500.0*b**4*x**7 - 31250000.0*b**3*h*x**4 + 108000000.0*b**3*h*x**5 + 1875000.0*b**3*h*x**6 + 12500000.0*b**3*h*x**5 - 875000.0*b**3*u**5*x**3 - 26562500.0*b**3*u**4*x**7 - 36875000.0*b**3*u**3*x**5 - 3125000.0*b**3*u**2*x**4 - 3015625000.0*b**3*u**2*x**5 - 10000000.0*b**3*u**2*x**6 - 13671875000.0*b**3*u**2*x**7 - 1140625000.0*b**3*u**2*x**6 - 12500000.0*b**3*u**2*x**5 - 25195312500.0*b**3*x**8 - 3986250000.0*b**3*x**7 - 125000000.0*b**3*x**6 + 2500000.0*b**2*h**2*x**2*x**4 + 7500000.0*b**2*h**2*x**2*x**5 + 500000.0*b**2*x**4 + 37500000.0*b**2*h**2*x**6 + 75000000.0*b**2*h**2*x**5 + 2500000.0*b**2*h**2*x**4 - 625000.0*b**2*h**2*x**3 - 375000.0*b**2*h**2*x**4 - 50000.0*b**2*h**2*x**3 + 41250000.0*b**2*h**2*x**5 - 2125000.0*b**2*h**2*x**4 - 303125000.0*b**2*h**2*x**6 - 375000.0*b**2*h**2*x**5 - 50000.0*b**2*h**2*x**4 - 742187500.0*b**2*h**2*x**7 - 165625000.0*b**2*h**2*x**6 - 7500000.0*b**2*h**2*x**5 + 17500.0*b**2*x**6 - 60000.0*b**2*x**5*x**3 + 9625000.0*b**2*x**4*x**4 + 62500.0*b**2*x**4*x**3 + 96250000.0*b**2*x**3*x**5 + 2375000.0*b**2*x**2*x**4 + 623437500.0*b**2*x**2*x**6 + 41250000.0*b**2*x**2*x**5 + 250000.0*b**2*x**2*x**4 + 2281250000.0*b**2*x**7 + 303125000.0*b**2*x**6 + 7500000.0*b**2*x**5*x**5 + 3437500000.0*b**2*x**8 + 742187500.0*b**2*x**7 + 37500000.0*b**2*x**6 - 100000.0*b**2*x**2*x**8 - 4 - 1500000.0*b**2*h**2*x**5*x**5 - 200000.0*b**2*h**2*x**4*x**4 - 500000.0*b**2*h**2*x**4*x**6 - 1500000.0*b**2*h**2*x**4*x**5 + 100000.0*b**2*h**2*x**4*x**4 + 500.0*b**2*h**2*x**5*x**2 + 25000.0*b**2*h**2*x**4*x**3 + 500.0*b**2*h**2*x**4*x**2 + 575000.0*b**2*h**2*x**3*x**4 + 20000.0*b**2*h**2*x**3*x**3 + 7000000.0*b**2*h**2*x**5 + 625000.0*b**2*h**2*x**4*x**4 + 37812500.0*b**2*h**2*x**6 + 7000000.0*b**2*h**2*x**5 + 200000.0*b**2*h**2*x**4*x**4 + 71875000.0*b**2*h**2*x**7 - 21562500.0*b**2*h**2*x**6 + 700.0*b**2*u**7*x**2 - 150000.0*b**2*u**6*x**2 - 135000.0*b**2*u**5*x**3 - 500.0*b**2*u**5*x**2 - 1537500.0*b**2*u**4*x**4 - 250.0*b**2*u**4*x**3 - 12375000.0*b**2*u**3*x**5 - 5750000.0*b**2*u**3*x**4 - 66562500.0*b**2*u**2*x**6 - 7000000.0*b**2*u**2*x**5 - 100000.0*b**2*u**2*x**4 - 203125000.0*b**2*u**2*x**7 - 37812500.0*b**2*u**2*x**6 - 1500000.0*b**2*u**2*x**5 - 257812500.0*b**2*x**8 - 71875000.0*b**2*x**7 - 5000000.0*b**2*x**6 + 100.0.0*h**2*x**2*x**4 + 100000.0*h**2*x**2*x**5 + 200000.0*h**2*x**2*x**4 + 250000.0*h**2*x**2*x**6 + 160000.0*h**2*x**2*x**5 + 10000.0*h**2*x**2*x**4 - 100.0.0*h**2*x**2*x**3 - 2500.0*h**2*x**2*x**2 - 450000.0*h**2*x**2*x**1 - 45000.0*h**2*x**2*x**0 - 20000.0*h**2*x**2*x**0 - 2812500.0*h**2*x**7 - 1062500.0*h**2*x**6 - 100000.0*h**2*x**5 + u**8 + 4.0.0*u**7*x**7 + 800.0*u**6*x**2 + 10000.0*u**5*x**3 + 100.0*u**5*x**2 + 91250.0*u**4*x**4 + 2500.0*u**4*x**3 + 625000.0*u**3*x**5 + 45000.0*u**3*x**4 + 2875000.0*u**2*x**6 + 4250000.0*u**2*x**5 + 10000.0*u**2*x**4 + 7500000.0*u**2*x**3 + 1812500.0*u**2*x**2 + 100000.0*u**2*x**1 + 93750.0*b**2*x**4 + 250.0*b**2*x**3 + 8203125.0*x**8 + 2812500.0*x**7 + 250000.0*x**6)/(156250.0*b**3*x**4 - 12500.0*b**2*x**3 - 93750.0*b**2*x**4 + 250.0*b**2*x**2 + 5000.0*b**2*x**3 - 18750.0*b**2*x**4 - 50.0*u**2*x**2 - 500.0*u**2*x**3 - 1250.0*u**2*x**4) - 0.02*(398625.0*b**4*x**4 - 62500.0*b**3*u*x**3 - 62500.0*b**3*x**4 - 12500.0*b**2*x**4 + 12500.0*b**2*x**3 + 500.0*b**2*x**2 + 5000.0*b**2*x**3 - 100.0*b**2*x**2 - 100.0*b**2*x**3 - 2000.0*b**2*x**2 - 17500.0*b**2*x**3 - 500.0*b**2*x**2 - 5000.0*b**2*x**3 - 100.0*b**2*x**2 - 500.0*h**2*x**3 - 100.0*h**2*x**2 + u**4 + 20.0*u**3*x + 250.0*u**2*x**2 + 1500.0*u**2*x**3 + 100.0*u**2*x**2 + 3125.0*x**4 + 500.0*x**3)/(x**2*(5.0*b - 1.0)*(25.0*b*x - u - 5.0*x)**2)
```

Figure: M_L Solution

ANALYTICAL ANALYSIS

```

In [2]: s[1][0]
Out[2]: 25.0*x**2*(5.0*b - 1.0)*(sqrt(152587890625.0*b**8*x**8 - 48828125000.0*b**7*u*x**7 - 366210937500.0*b**7*x**8 - 48828125000.0*b**6*u*x**7 + 6835937500.0*b**6*u*x**2*x**6 + 97656250000.0*b**6*u*x**7 + 439453125000.0*b**6*x**8 + 4882812500.0*b**6*x**7 + 976562500.0*b**5*h*x**6 + 13671875000.0*b**5*h*x**7 + 195312500.0*b**5*h*x**6 - 546875000.0*b**5*u*x**5 - 11132812500.0*b**5*u*x**2*x**6 - 9176875000.0*b**5*u*x**7 - 976562500.0*b**5*u*x**6 - 288805937500.0*b**5*x**8 - 13671875000.0*b**5*x**7 + 156250000.0*b**4*h*x**6 - 78125000.0*b**4*h*x**7 - 1914062500.0*b**4*h*x**5 - 31250000.0*b**4*h*x**5 - 10742187500.0*b**4*h*x**7 - 828312500.0*b**4*h*x**6 + 76 + 27343750.0*b**4*u*x**4 + 703125000.0*b**4*u*x**3*x**5 + 7968750000.0*b**4*u*x**2*x**6 + 781250000.0*b**4*u*x**5 + 4687500000.0*b**4*u*x**4 + 1914062500.0*b**4*u*x**6 + 109863281250.0*b**4*x**8 + 10742187500.0*b**4*x**7 + 156250000.0*b**4*x**6 - 12500000.0*b**3*h*x**5 - 125000000.0*b**3*h*x**2*x**6 - 12500000.0*b**3*h*x**2*x**5 + 3125000.0*b**3*h*x**4 + 100000000.0*b**3*h*x**7 + 578125000.0*b**3*h*x**6 + 12500000.0*b**3*h*x**5 - 875000.0*b**3*x**5*x**3 - 26562500.0*b**3*x**4*x**4 - 36875000.0*b**3*x**3*x**3 - 3125000.0*b**3*x**3*x**4 + 301562500.0*b**3*x**2*x**6 - 100000000.0*b**3*x**2*x**5 - 13671875000.0*b**3*x**7 - 1140625000.0*b**3*x**6 - 12500000.0*b**3*x**5 - 25195312500.0*b**3*x**8 - 3906250000.0*b**3*x**7 - 125000000.0*b**3*x**6 + 250000.0*b**2*h*x**2*x**4 + 750000.0*b**2*h*x**2*x**3*x**6 + 7500000.0*b**2*h*x**2*x**5 + 250000.0*b**2*h*x**2*x**4 - 62500.0*b**2*h*x**3*x**3 - 2375000.0*b**2*h*x**2*x**4*x**4 - 50000.0*b**2*h*x**2*x**3*x**3 - 4215000.0*b**2*h*x**2*x**5 - 2125000.0*b**2*h*x**2*x**4*x**4 - 3750000.0*b**2*h*x**2*x**5 - 50000.0*b**2*h*x**2*x**4*x**4 - 742187500.0*b**2*h*x**7 - 15625000.0*b**2*h*x**6 - 7500000.0*b**2*h*x**5 - 17500.0*b**2*x**6*x**2 + 60000.0*b**2*x**5*x**3 - 9625000.0*b**2*x**4*x**4 + 62500.0*b**2*x**4*x**3 + 6250000.0*b**2*x**2*x**5 + 24150000.0*b**2*x**2*x**4 + 250000.0*b**2*x**2*x**3*x**4 + 363125000.0*b**2*x**2*x**2*x**6 - 150000000.0*b**2*x**2*x**5 - 363125000.0*b**2*x**2*x**4 + 3437500000.0*b**2*x**2*x**7 + 742187500.0*b**2*x**2*x**6 - 37500000.0*b**2*x**2*x**5 - 100000.0*b**h*x**2*x**4 - 1500000.0*b**h*x**2*x**3*x**5 - 200000.0*b**h*x**2*x**4*x**4 - 5000000.0*b**h*x**2*x**3*x**6 - 1500000.0*b**h*x**2*x**5 - 100000.0*b**h*x**2*x**4 + 500.0*b**h*x**5*x**2 + 25000.0*b**h*x**4*x**3 + 500.0*b**h*x**4*x**2 + 575000.0*b**h*x**3*x**4 + 20000.0*b**h*x**3*x**3 + 7000000.0*b**h*x**2*x**4 - 37812500.0*b**h*x**2*x**5 + 7600000.0*b**h*x**2*x**6 + 20000.0*b**h*x**2*x**7 + 71875000.0*b**h*x**7*x**7 + 21562500.0*b**h*x**6*x**5 + 1500000.0*b**h*x**5*x**5 - 200.0*b**h*x**7*x**7 - 7500.0*b**h*x**6*x**2 - 135000.0*b**h*x**5*x**3 - 500.0*b**h*x**5*x**2 - 1537500.0*b**h*x**4*x**4 - 25000.0*b**h*x**3*x**3 - 12375000.0*b**h*x**2*x**2 - 575000.0*b**h*x**3*x**4 - 66562500.0*b**h*x**2*x**6 - 7000000.0*b**h*x**5*x**5 - 100000.0*b**h*x**2*x**4 - 203125000.0*b**h*x**7 - 37812500.0*b**h*x**6 - 150000.0*b**h*x**5*x**5 - 257812500.0*b**h*x**8 - 71875000.0*b**h*x**7 - 500000.0*b**h*x**6 + 10000.0*h**2*x**2*x**4 + 100000.0*h**2*x**2*x**5 + 250000.0*h**2*x**2*x**6 + 100000.0*h**2*x**2*x**7 + 100000.0*h**2*x**2*x**8 - 100.0*h**2*x**5*x**2 - 2500.0*h**2*x**4*x**3 - 100.0*h**2*x**4*x**2 - 45000.0*h**2*x**3*x**4 - 2000.0*h**2*x**3*x**3 - 425000.0*h**2*x**2*x**4 - 1812500.0*h**2*x**6 - 45000.0*h**2*x**5 - 28000.0*h**2*x**4 - 2812500.0*h**2*x**7 - 1062500.0*h**2*x**6 - 100000.0*h**2*x**5 + u**8 + 40.0*u**7*x + 800.0*u**6*x**2 + 10000.0*u**5*x**3 + 100.0*u**5*x**2 + 91250.0*u**4*x**4 + 2500.0*u**4*x**3 + 625000.0*u**3*x**5 + 450000.0*u**3*x**4 + 2875000.0*u**2*x**6 + 425000.0*u**2*x**5 + 10000.0*u**2*x**4 - 26000.0*u**2*x**3*x**4 + 1812500.0*u**2*x**7 - 1526250.0*b**3*x**4*x**4 + 12500.0*b**2*x**3*x**5 + 8203125.0*x**8 + 2812500.0*x**7 + 250000.0*x**6)/(156250.0*b**3*x**4*x**4 + 12500.0*b**2*x**3*x**3 - 93750.0*b**2*x**2*x**4 + 250.0*b**2*x**2*x**2*x**2 + 5000.0*b**2*x**3*x**3 - 625000.0*b**2*x**3*x**4 - 12500.0*b**2*x**2*x**3*x**3 + 3750.0*b**2*x**2*x**2*x**2 + 62500.0*b**2*x**2*x**3*x**2 - 100.0*b**2*x**3*x**3 - 28000.0*b**2*x**2*x**2*x**2 - 17500.0*b**2*x**2*x**3 - 500.0*b**2*x**2*x**2*x**2 - 50000.0*b**2*x**3*x**4 - 5000.0*b**2*x**3*x**3 - 100.0*h**2*x**2*x**2 - 500.0*h**2*x**3*x**3 - 100.0*h**2*x**2*x**2 + u**4 + 20.0*u**3*x**3 + 250.0*u**2*x**2*x**2 + 1500.0*u*x**3 + 100.0*u*x**2*x**2 + 3125.0*x**4 + 500.0*x**3)/(x**2*(5.0*b - 1.0)*(25.0*b*x - u - 5.0*x)**2))/((25.0*b*x - u - 5.0*x)**2)

```

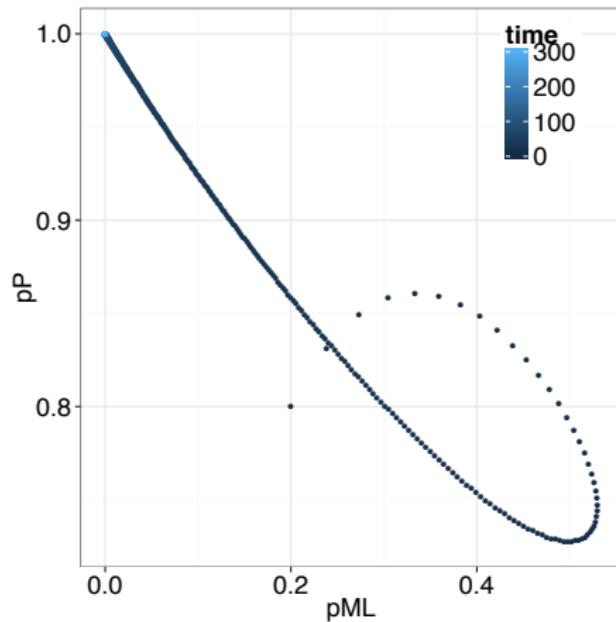
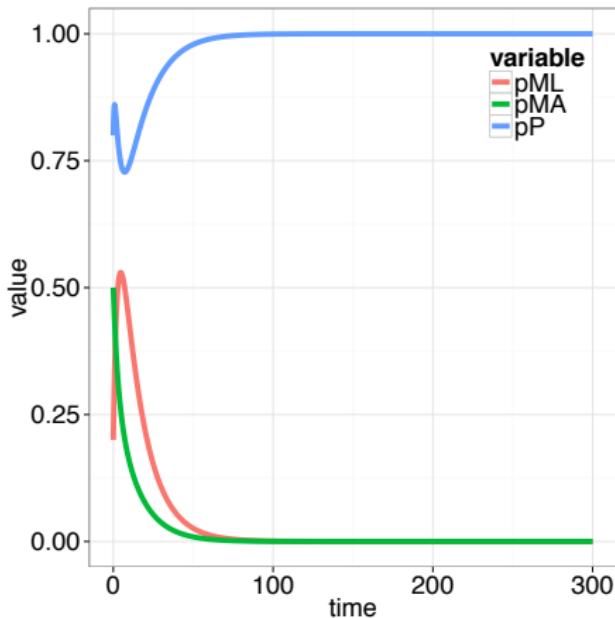
Figure: M_A Solution

ANALYTICAL ANALYSIS

$$J = \begin{pmatrix} -P^*(1 + \frac{3M_A^*}{1+M_A^*} + \frac{0.3M_L^*}{(1+hP^*)^2}) & \frac{-P^*}{1+hP^*} & \frac{3P^*(1-P^*)}{(1+M_A^*)^2} \\ \frac{-\chi M_L^*}{(1+uP^*)^2} & -0.2 - \frac{\chi P^*}{1+uP^*} & b \\ \frac{\chi M_L^*}{(1+uP^*)^2} & \frac{\chi P^*}{1+uP^*} & -0.2 \end{pmatrix}$$

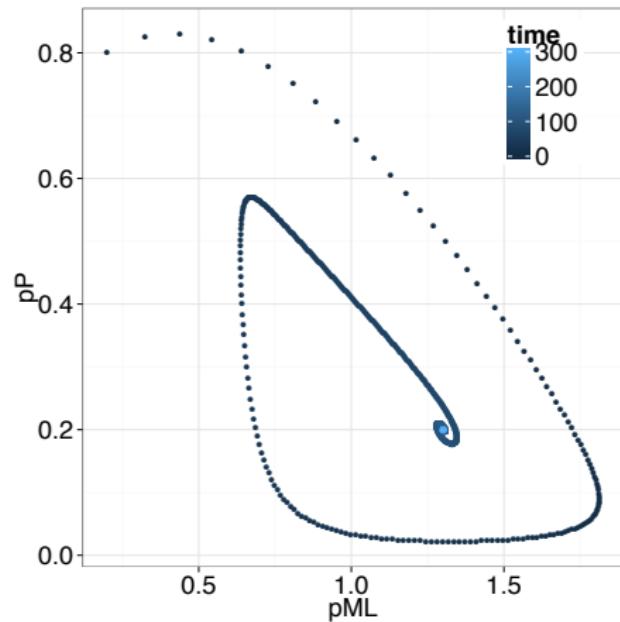
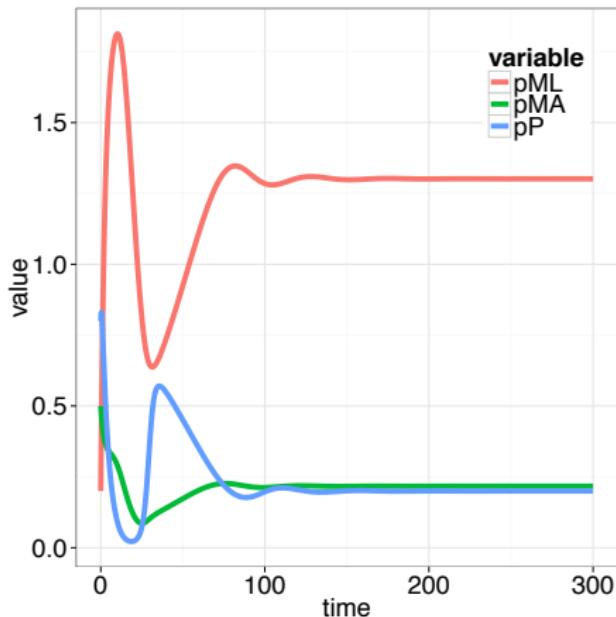
GRAPHICS

Setting $b = 0.5, u = 10$ (real part of eigenvalues negative):



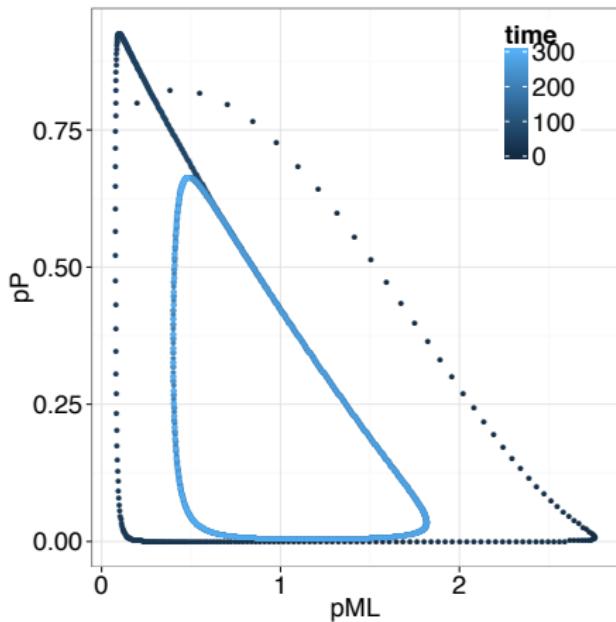
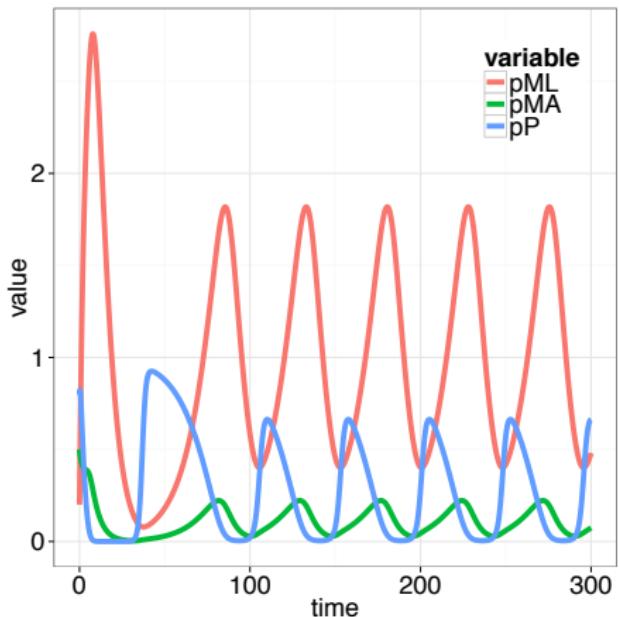
GRAPHICS

Setting $b = 1.4, u = 10$ (real part of eigenvalues negative):

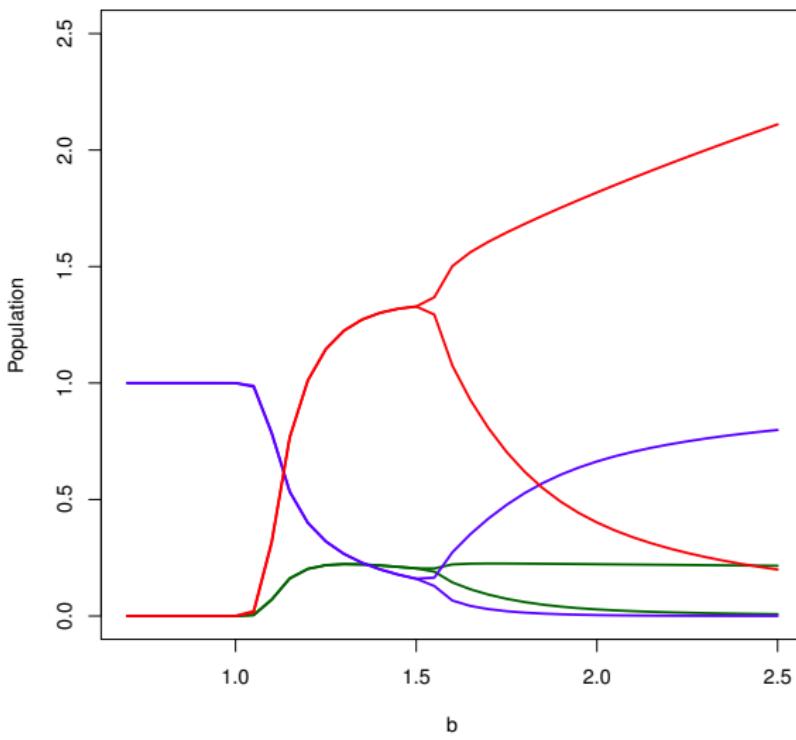


GRAPHICS

Setting $b = 2.0, u = 10$ (at least one eigenvalue positive):

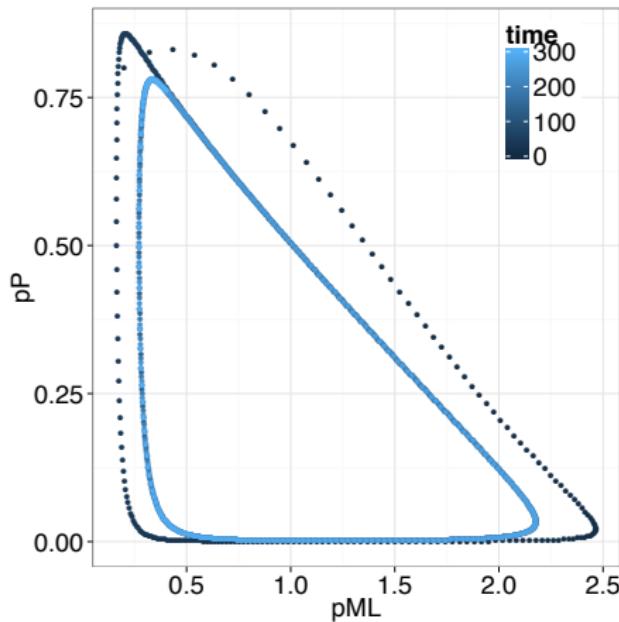
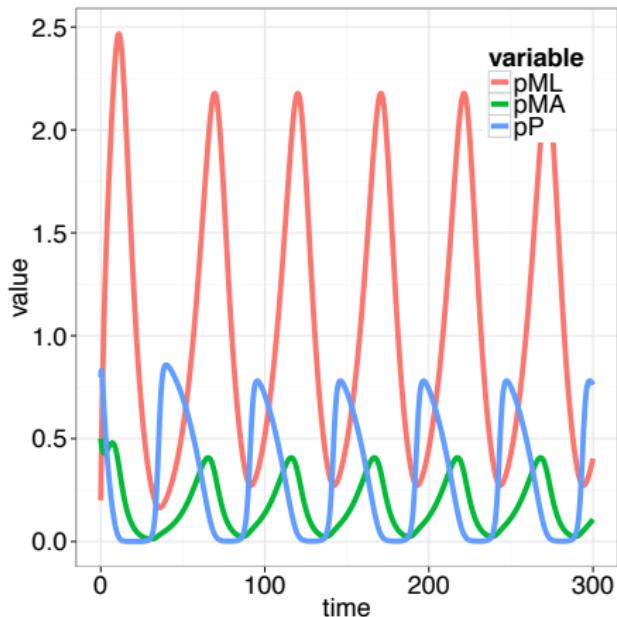


BIFURCATION DIAGRAM FOR b



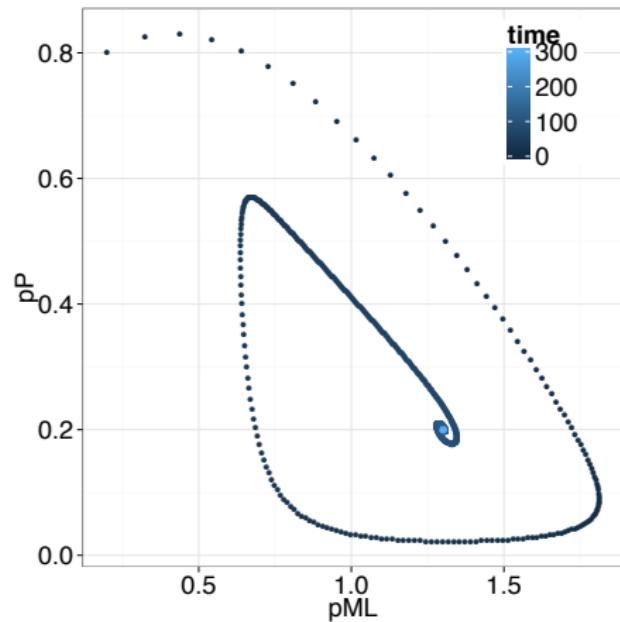
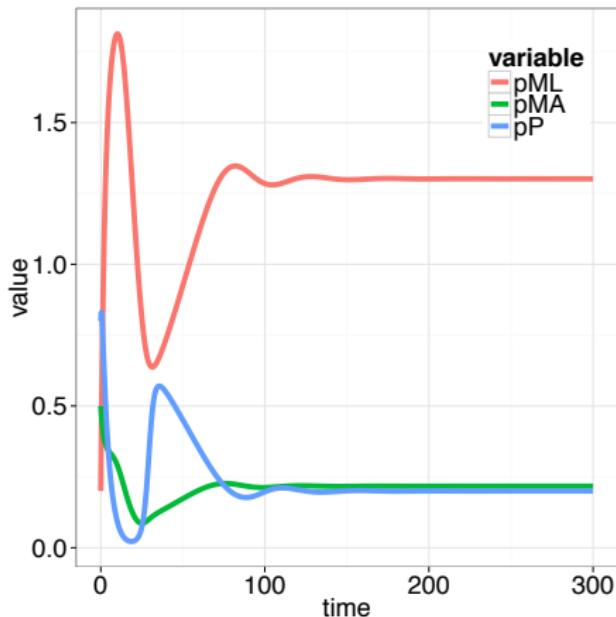
GRAPHICS

Setting $b = 1.4$, $u = 5$ (at least one eigenvalue positive):



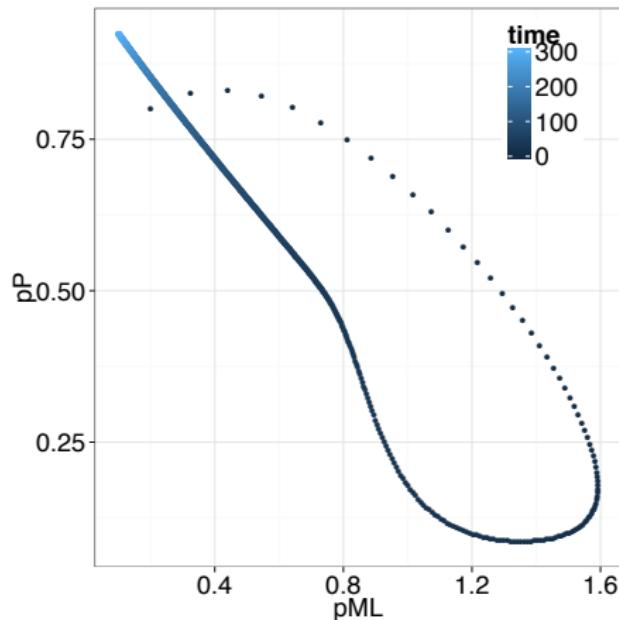
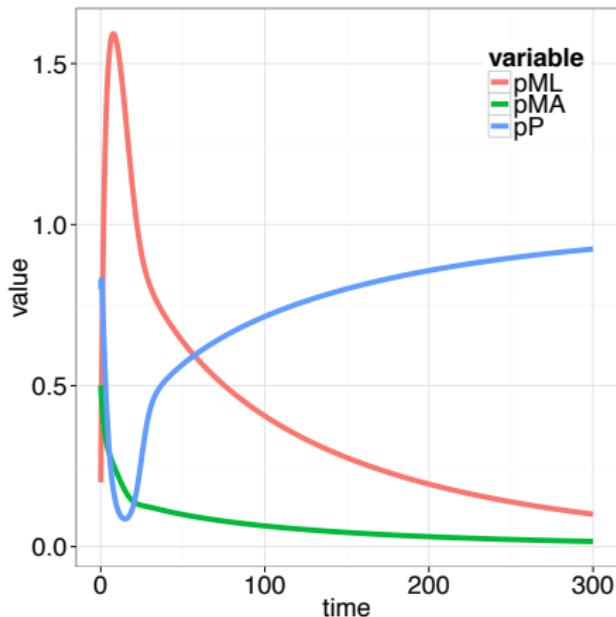
GRAPHICS

Setting $b = 1.4, u = 10$ (real part of eigenvalues negative):

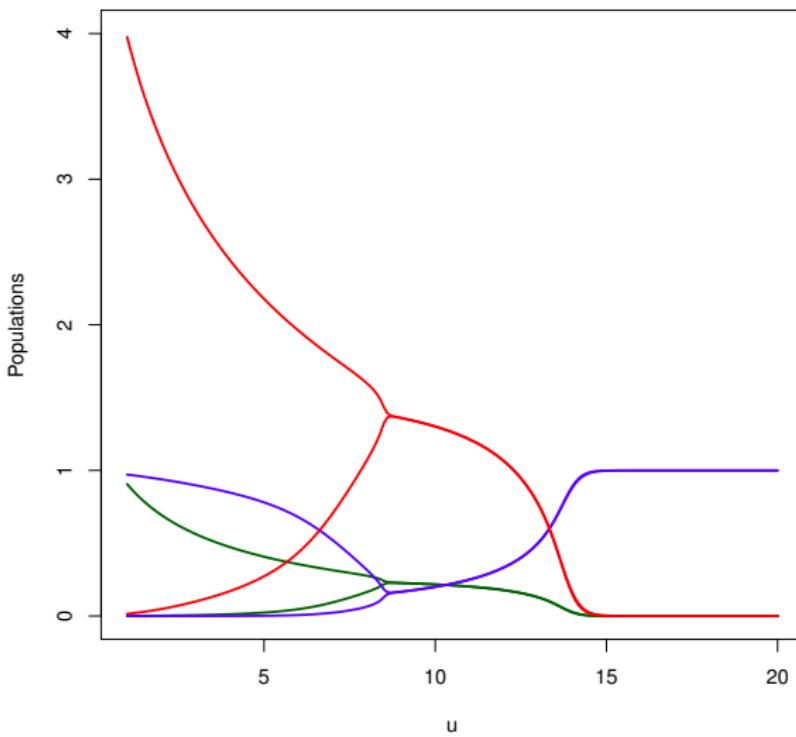


GRAPHICS

Setting $b = 1.4$, $u = 14$ (real part of eigenvalues negative):



BIFURCATION DIAGRAM FOR u



CONCLUSIONS

- ▶ This model can describe the population dynamics of the system P - M_L - M_A
- ▶ We identified three parameters (b, u, χ) that drive different dynamics of the system:
 - ▶ Extinction of the moth
 - ▶ Coexistence
 - ▶ Oscillations

FUTURE STEPS

- ▶ Evaluate the effects of K varying with M_A instead of being a fixed value
- ▶ Rescale the variables
- ▶ Make an exhaustive exploration of parameter values' variation

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