Non-minimal Couplings in Randall-Sundrum Scenarios.

Geova Maciel de Alencar

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Content

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Background solution

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The question is: are the fields confined to the brane?

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$$\hat{O}_{4d}\phi(x) = -m^2\phi(x)$$

and

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The solutions of the second equation above therefore give us the allowed masses in the visible brane. The best way to study this equation is to transform it in a Schroedinger -like equation

$$\left(-\frac{d^2}{dz^2} + U(z)\right)\overline{\psi}(z) = m^2\overline{\psi}(z), \tag{2}$$

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$$\left(-\frac{d^2}{dz^2} + U(z)\right)\overline{\psi}(z) = m^2\overline{\psi}(z), \tag{2}$$

$$\int \overline{\psi}^2 dz = finite$$

which is like the square integrable condition of quantum mechanics.

Fields

gravity:

$$U(z) = \frac{3}{2}A'' + \frac{9}{4}A'^2 = \frac{15}{4}\frac{1}{|z|+1} + 3\delta(z) \tag{3}$$

with $\psi = (k|z|+1)^{-\frac{3}{2}}$, which is localized.

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$$U(z) = \frac{1}{2}A'' + \frac{1}{4}A'^2 = \frac{3}{4}\frac{1}{|z|+1} + \delta(z)$$

with $\psi = (k|z|+1)^{-\frac{1}{2}}$Not localized!

Dilaton

Dilaton

$$S = \int d^5x \sqrt{-G}e^{-\lambda\pi}[F_{M_1M_2}F^{M_1M_2}], \qquad (4)$$

where $F_{M_1M_2} = \partial_{[M_1}A_{M_2]}$ is the field strength of the gauge field.

Scalar Fields

Scalar Fields

$$S = \int d^5x \sqrt{-G}[f(\phi)F_{M_1M_2}F^{M_1M_2} + \phi'^2 - V(\phi))], \qquad (5)$$

Quasi-localized models

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A boundary term was added by Dvali et al

$$S = \int d^5x \sqrt{-G}[F_{M_1M_2}F^{M_1M_2} + \frac{1}{m}\delta(z)F_{\mu\nu}F^{\mu\nu}], \qquad (6)$$

Localization is attained just for $m \to \infty$.

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and we get identities like

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Splinting the field as

$$A_T^\mu = (\delta_\nu^\mu - \frac{\partial^\mu \partial_\nu}{\Box})A^\nu, A_L^\mu = \frac{\partial^\mu \partial_\nu}{\Box}A^\nu$$

we get a massless gauge invariant field decoupled from Φ.

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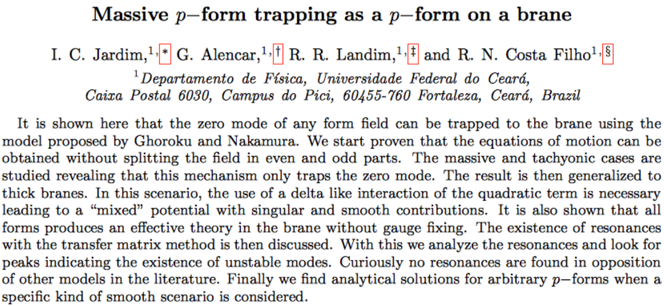
with $\psi = (k|z| + 1)^{-\frac{1}{2}-M^2}$!Localized? The boundary is fixes $M = 0$!To modify the boundary condition Ghoroku introduced in the action a boundary term $c\delta(z)X^2$. A precise relation between c and M was found.



Massive $p-$form trapping as a $p-$form on a brane

I. C. Jardim,[1,*] G. Alencar,[1,†] R. R. Landim,[1,‡] and R. N. Costa Filho[1,§]
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It is shown here that the zero mode of any form field can be trapped to the brane using the model proposed by Ghoroku and Nakamura. We start proven that the equations of motion can be obtained without splitting the field in even and odd parts. The massive and tachyonic cases are studied revealing that this mechanism only traps the zero mode. The result is then generalized to thick branes. In this scenario, the use of a delta like interaction of the quadratic term is necessary leading to a "mixed" potential with singular and smooth contributions. It is also shown that all forms produces an effective theory in the brane without gauge fixing. The existence of resonances with the transfer matrix method is then discussed. With this we analyze the resonances and look for peaks indicating the existence of unstable modes. Curiously no resonances are found in opposition of other models in the literature. Finally we find analytical solutions for arbitrary $p-$forms when a specific kind of smooth scenario is considered.

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$$R = -4(2A'' + 3A'^2)e^{-2A} = 16k\delta(z) - 20k^2$$

This could provides an smooth version just by using A in all places.

Gauge field case

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$$S = \int d^5x \sqrt{-G}[F_{M_1M_2}F^{M_1M_2} + \gamma_1 RA^1], \tag{8}$$

repeating the procedure we get

$$U = \frac{1}{2}A'' + \frac{1}{4}A'^2 - \gamma Re^{2A}$$

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repeating the procedure we get

$$U = \frac{1}{2}A'' + \frac{1}{4}A'^2 - \gamma Re^{2A}$$

if we choose $\gamma = 1/16$ we get

$$U = A'' + A'^2$$

with $\psi = e^A$. Therefore for any A recovering RS for large z our solution is localized!!

Geometrical coupling paper

Gauge Field Localization on the Brane Through Geometrical Coupling

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(Dated: October 14, 2014)

In this paper we consider a geometrical Yukawa coupling as a solution to the problem of gauge field localization. We show that upon dimensional reduction the vector field component of the field is localized but the scalar component (A_5) is not. We show this for any smooth version of the Randall-Sundrum model. The covariant version of the model with geometrical coupling simplify the generalization to smooth versions generated by topological defects. This kind of model has been considered some time ago, but there it has been introduced two free parameters in order to get a localized solution which satisfy the boundary conditions: a mass term in five dimensions and a coupling with the brane. The boundary condition fix one of them and the model is left with one free parameter M. First we show that by considering a Yukawa coupling with the Ricci scalar it is possible to unify these two parameter in just one fixed by the boundary condition. With this we get a consistent model with no free parameters and the mass term can be interpreted as a coupling to the cosmological constant.

PACS numbers: 64.60.ah, 64.60.al, 89.75.Da

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Gauge Field Emergence from Kalb-Ramond Localization

G. Alencar[a],[*] R. R. Landim[a],[†] M. O. Tahim[b],[‡] and R.N. Costa Filho[a][§]
[a] *Departamento de Física, Universidade Federal do Ceará, 60451-970 Fortaleza, Ceará, Brazil and*
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(Dated: December 23, 2014)

A new mechanism, valid for any smooth version of the Randall-Sundrum model, of getting localized massless vector field on the brane is described here. This is obtained by dimensional reduction of a five dimension massive two form, or Kalb-Ramond field, giving a Kalb-Ramond and an emergent vector field in four dimensions. A geometrical coupling with the Ricci scalar is proposed and the coupling constant is fixed such that the components of the fields are localized. The solution is obtained by decomposing the fields in transversal and longitudinal parts and showing that this give decoupled equations of motion for the transverse vector and KR fields in four dimensions. We also prove some identities satisfied by the transverse components of the fields. With this is possible to fix the coupling constant in a way that a localized zero mode for both components on the brane is obtained. Then, all the above results are generalized to the massive $p-$form field. It is also shown that in general an effective p and $(p-1)-$forms can not be localized on the brane and we have to sort one of them to localize. Therefore, we can not have a vector and a scalar field localized by dimensional reduction of the five dimensional vector field. In fact we find the expression $p = (d-1)/2$ which determines what forms will give rise to both fields localized. For $D = 5$, as expected, this is valid only for the KR field.

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ELKO spinor case

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Comment on "Localization of 5D Elko Spinors on Minkowski Branes"

I. C. Jardim,[1,*] G. Alencar,[1,†] R. R. Landim,[1,‡] and R. N. Costa Filho[1,§]

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We show that the statement in Yu-Xiao Liu, Xiang-Nan Zhou, Ke Yang and Feng-Wei Chen [Phys. Rev. D 86 , 064012 (2012)] that the zero mode of ELKO spinor is localized in some thin brane scenarios is not correct. This reopens the problem of localization of ELKO spinors.

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Solutions to the problem of ELKO spinor localization in brane models

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[1] *Departamento de Física, Universidade Federal do Ceará, Caixa Postal 6030, Campus do Pici, 60455-760 Fortaleza, Ceará, Brazil*

In this paper we present two different solutions to the problem of zero mode localization of ELKO spinor. In a recent paper the present authors reopened this problem since the solution presented before did not satisfy the boundary condition at the origin. The first solution is given by the introduction of a mass term and by coupling the spinor with the brane through a delta function. The second solution is reached by a Yukawa geometrical coupling with the Ricci scalar. This two models changes consistently the the boundary condition at infinity and at the origin. For the case of Geometrical coupling we are able to show that the zero mode is localized for any smooth version of the RS model.

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$$S = -\frac{1}{4e_5^2}\int d^5x\sqrt{-g}\left[TrF^a_{MN}F^{MNa} + \beta\delta(z)TrF^a_{\mu\nu}F^{\mu\nu a}\right] - \frac{1}{2}\int d^5x\sqrt{-g}M^2(z)TrA^a_M A^{Ma} \quad (9)$$

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They found β and the parameters of M such that the gauge invariance is recovered in 4D.

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The model is covariant but still "quasi-localized". Is it possible to localize AND keep gauge invariance with general A?

A propose

$$
\begin{aligned}
S &= -\frac{1}{4e_5^2}\int d^5x\,\sqrt{-g}\left[TrF^a_{MN}F^{MNa} + \gamma_1\Delta^{AB}{}_{CD}TrF^a_{AB}F^{CDa}\right] \\
&- \frac{\gamma_2}{2}\int d^5x\,\sqrt{-g}RTrA^a_M A^{Ma}, \qquad (12)
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Non-minimal couplings in Randall-Sundrum Scenarios

G. Alencar[a],[*] R. R. Landim[a],[†] C. R. Muniz[b],[‡] and R.N. Costa Filho[a§]
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[b] *Universidade Estadual do Ceará, Faculdade de Educação,*
Ciências e Letras do Sertão Central - R. Epitácio Pessoa, 2554, 63.900-000 Quixadá, Ceará, Brazil.
(Dated: February 5, 2015)

In this paper we propose a new model to solve the problem of Yang-Mills localization in Randall-Sundrum scenarios without the introduction of other fields or new degrees of freedom. The model is based only in non-minimal couplings with the gravity field. We show that two non-minimal couplings are necessary, one with the field strength and the other with a mass term. Despite the loosing of five dimensional gauge invariance by the mass term a massless gauge field is obtained over the brane. To obtain this, we need of a fine tuning of the two parameters introduced through the couplings. The fine tuning is obtained by imposing the boundary conditions and to guarantee non-abelian gauge invariance in four dimensions. With this we are left with no free parameters and the model is completely determined. The model also provides analytical solutions to the linearized equations for the zero mode and for a general warp factor.

PACS numbers: 64.60.ah, 64.60.al, 89.75.Da