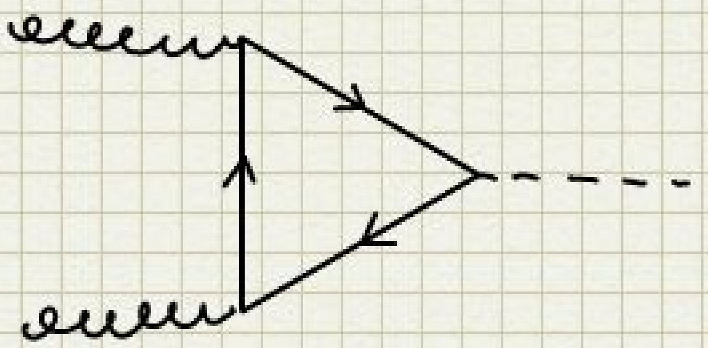
Gluon Fusion

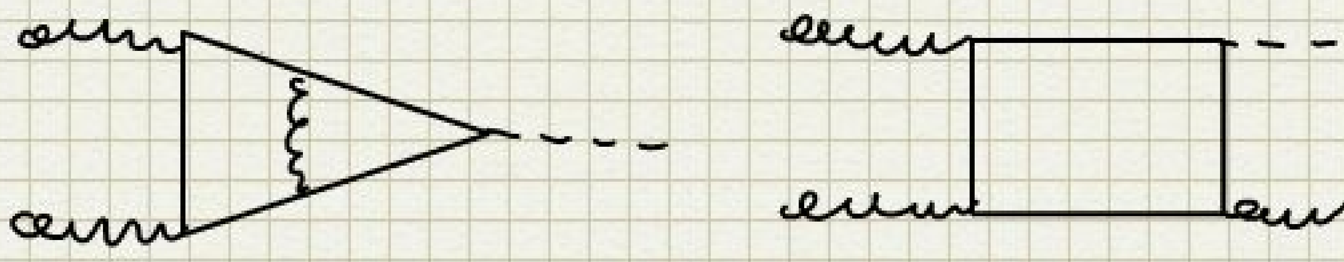
The gluon fusion process is loop induced



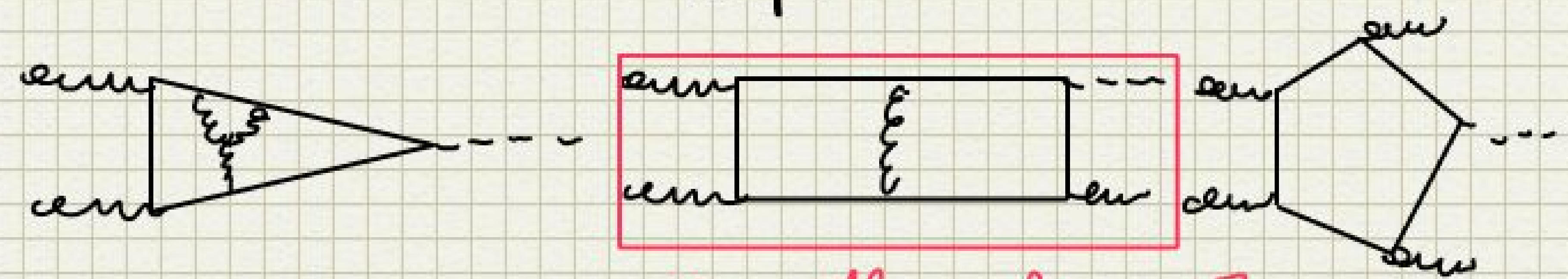
$\rightsquigarrow$ Perturbative series is "shifted"

LO $\to$ 1-loop

NLO $\to$ 2-loops



NNLO $\to$ 3-loops



Currently unknown!

> We only know how to compute ggF up to NLO exactly in the SM!

$\rightsquigarrow$ Try to make a clever approximation:
If we consider all other quarks massless, and integrate over the final state, the total cross section depends on

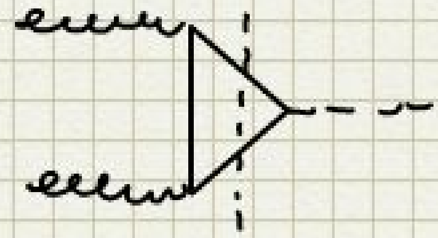
$$\hat{\sigma}_{gg}\left(\alpha_s(\mu_R^2), \underbrace{\frac{m_H^2}{\hat{s}}}_{= z}, \frac{m_H^2}{m_t^2}, \underbrace{\frac{m_H^2}{\mu_F^2}, \frac{\mu_R^2}{\mu_F^2}}\right)$$

will suppress these in the following. Or simply put $\mu_R = \mu_F = m_H$.

$$= \sum_{l=2}^{\infty} \alpha_s^l \, \hat{\sigma}_{gg}^{(l)}\left(z, \frac{m_H^2}{m_t^2}\right)$$

$$m_H^2 \leqslant 4 m_t^2$$

$$(125)^2 \qquad 4(175)^2$$



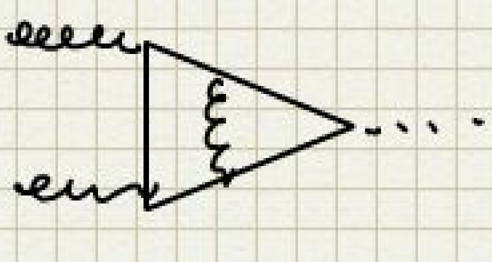
$\rightsquigarrow$ Below the $t\bar{t}$ threshold we can expand in m_H^2/m_t^2

$$\rightsquigarrow \sum_{l=2}^{\infty} \sum_{k=2}^{\infty} \alpha_s^l \left(\frac{m_H^2}{m_t^2}\right)^k \underbrace{\hat{\sigma}_{gg}^{(l,k)}(z)}_{\text{contains still } \log \frac{m_H^2}{m_t^2}}$$

$\rightsquigarrow$ Such an expansion is the realm of EFTs:

$$\mathcal{L}_{EFT} = \underbrace{\mathcal{L}_{QCD,5}}_{\text{= QCD with 5 massless flavors}} - \frac{C}{4v} H G^a_{\mu\nu} G^{\mu\nu}_a + \underbrace{\mathcal{O}\left(\frac{1}{v^2}\right)}_{\dim \geqslant 6}$$

* $m_t \sim y_t v$
* C = Wilson coefficient. Captures $\log \frac{m_H^2}{m_t^2}$ corrections, plus



$$C = -\frac{\alpha_s}{3\pi}\left\{ 1 + \frac{11}{4}\frac{\alpha_s}{\pi} + \left(\frac{\alpha_s}{\pi}\right)^2 \left[\frac{19}{16}\log\frac{\mu^2}{m_t^2} + \frac{2777}{288} + N_F\left(\frac{1}{3}\log\frac{\mu^2}{m_t^2} - \frac{67}{96}\right)\right] + \mathcal{O}(\alpha_s^3) \right\}$$

$\rightsquigarrow$ EFT computation reproduces order-by-order $1/m_t$ expansion.

$\rightsquigarrow$ LO in $1/m_t$ (dim = 5)

<u>Question:</u> How well does this work?

* $m_H^2/m_t^2 \sim 0{,}5$ $\rightsquigarrow$ not that small
* If we integrate over additional radiation, get contributions from hard emissions:

$$\hat{s} \sim m_t^2 \gg m_H^2$$

Show luminosity plot

$\rightsquigarrow$ Total cross section is dominated by $z \to 1$ region where

$$m_t^2 \gg \hat{s} \sim m_H^2$$

How can we test approx?

1) we know NLO exactly!

$\rightsquigarrow$ compare

Show Harlander plot

2) $1/m_t$ terms known up to $\mathcal{O}(1/m_t^{10})$ @ NNLO

$\rightsquigarrow$ tiny!

$\rightsquigarrow$ leading term is enough.

State of the art for inclusive xsec:

* LO and NLO : exact
* NNLO : large m_t limit $\oplus$ $1/m_t$ corrections (tiny!)
* N^3LO : large-m_t limit.
  - Why do N^3LO? (1)
  - Is large-m_t limit good enough? (2)

@(1): Scale variation

@(2):

$$\hat\sigma^{LO} + \hat\sigma^{NLO} \checkmark$$

$$+ \alpha_s^4 \left[\frac{\hat\sigma^{(4,2)}}{m_t^2} + \boxed{\frac{\hat\sigma^{(4,3)}}{m_t^3} + \frac{\hat\sigma^{(4,4)}}{m_t^4} \ldots} \right]$$

tiny!

$$+ \alpha_s^5 \left[\boxed{\frac{\hat\sigma^{(5,2)}}{m_t^2}} + \ldots \right]$$

most relevant

N^3LO Pheno

Discuss slides

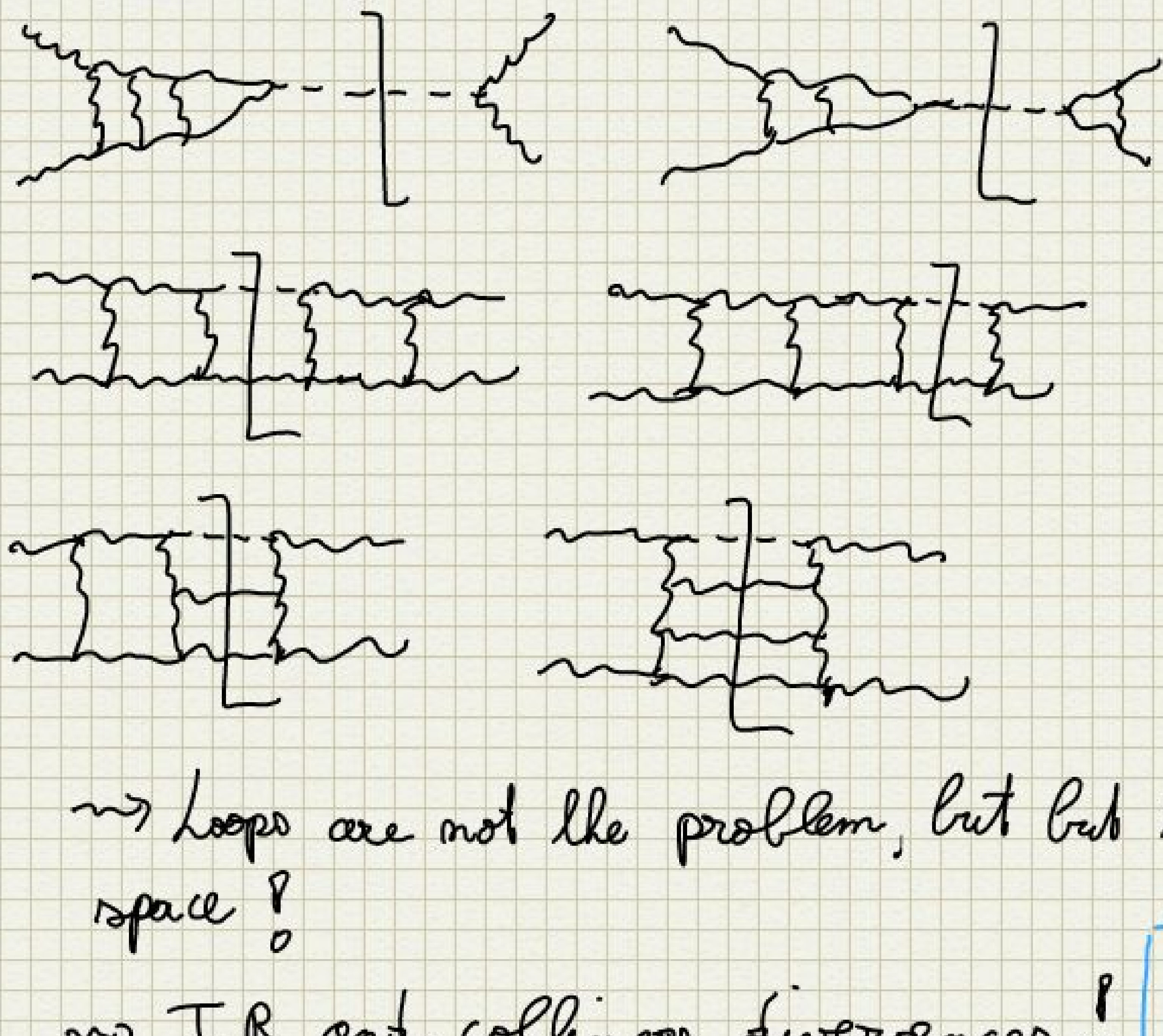
Uplot: * Scale uncertainty ~ 2% @ N^3LO

* $m_H/2$ seems a good scale choice

⇝ series looks convergent.

How to do N^3LO computation in a nutshell:

Contributions:



⇝ Loops are not the problem, but ~~but~~ the phase space!

⇝ IR and collinear divergences! [See Gavin's lecture]

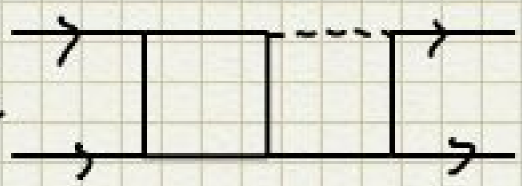
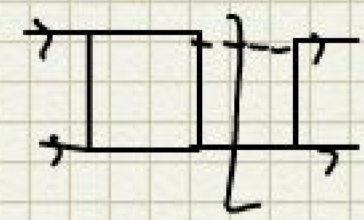
→ Sum of all contributions is finite, but individual pieces are not!

→ Perform phase space in D dimensions!

Can make phase-space integrals acessible to loop technology?

$\rightsquigarrow$ Reverse unitarity!

Idea: Optical theorem:

Im  $= \int d\Phi$ 

"loop" $\longleftrightarrow$ "Phase space"

$\rightsquigarrow$ Loop technology is applicable to PS integrals as well [See Johannes's lecture]

$\rightsquigarrow$ IBP, with additional rule

$$\left(\frac{1}{q^2}\right)_c^n = 0 \quad \text{if } n \leq 0$$

[numerators do not give rise to discs]

$\rightsquigarrow$ Phase space master integrals

$\rightsquigarrow$ differential equations

$\exists$ two scales, $\hat{s}$ and m_H^2

$\longrightarrow$ fct of 1 variable z!

Differential equations:

m_H^2 only enters through PS constraint

$$\frac{\partial}{\partial m_H^2}\,\delta(p_H^2 - m_H^2) \quad \sim \quad \underbrace{\delta'(p_H^2 - m_H^2)}_{=?}$$

$$\Big\downarrow \text{R.U.}$$

$$\frac{\partial}{\partial m_H^2}\,\frac{1}{p_H^2 - m_H^2} = \frac{2}{(p_H^2 - m_H^2)^2}$$ IBP reduction to PS masters

$\rightsquigarrow$ PS masters satisfy system of DEs

$$\frac{\partial}{\partial z}\vec{I} = A(z,\varepsilon)\,\vec{I}$$

@ N^3LO: # MIs > 1000 !

+ Boundaries !

* Boundaries from $z \to 1$ limit (threshold)
* Solve DEs as series around $z = 1$

$\rightsquigarrow$ PDFs peak at $z = 1$.

$$I_k = \sum_{m=0}^{6}\sum_{n=0}^{\infty} \underbrace{(1-z)^{-1-m\varepsilon}} c_{kmn}\,(1-z)^n$$

N.B.: Pole at $z = 1$

$\rightsquigarrow$ soft singularity !

$$(1-z)^{-1+\varepsilon} = \frac{1}{\varepsilon}\,\delta(1-z) + \left[\frac{1}{1-z}\right]_+ + \varepsilon\left[\frac{\log(1-z)}{1-z}\right]_+ + \dots$$

$\rightsquigarrow$ Obtain partonic cross section as expansion around threshold.

$\boxed{\text{Discuss convergence plot}}$

Uncertainties on the cross section

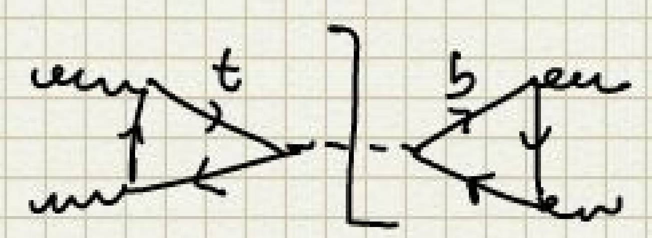
Scale uncertainty @ N^3LO : ~ 2%

⇝ Need to be carefull about what uncertainty we assign to result.

* PDF and α_s (1)
* EW (2)
* Top bottom (3)
* Higher orders (4)

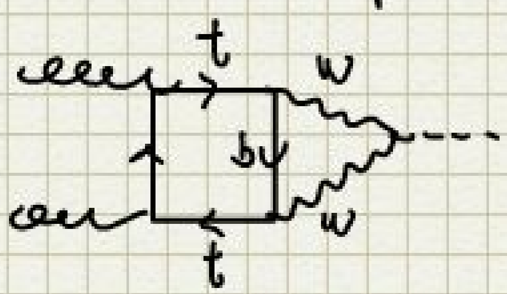
+ more !
cf. Wilson coefficient

(3) unknown @ 2 loops.



etc.

(2) Two-loop EW diagrams are known



etc ⇝ NLO EW

⇝ +5%

Issue: How to combine with QCD?

* 2 schemes :

Factorisable / Multiplicative | non-fact / Additive

$$\sigma = \sigma^{LO}(1+\delta_{QCD})(1+\delta_{EW}) = \sigma^{LO}(1+\delta_{QCD}+\delta_{EW}$$

$$+\mathcal{O}(\alpha_s\alpha_{EW}) \qquad \mathcal{O}(\alpha_s\alpha_{EW}) \quad +\mathcal{O}(\alpha_s\alpha_{EW}))$$

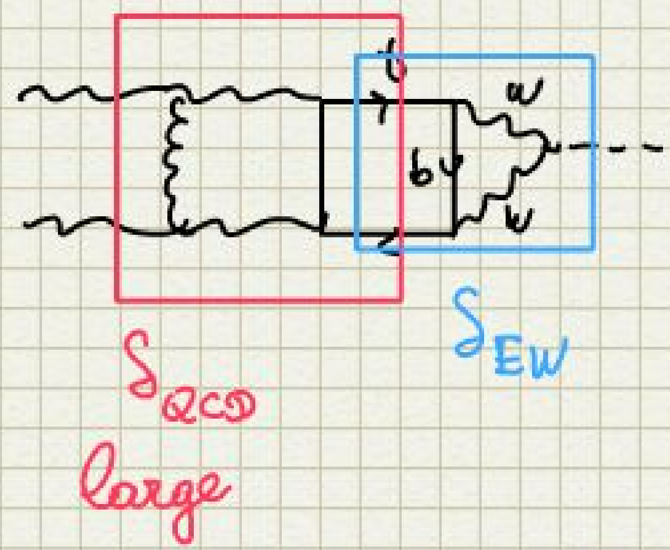
$$= \sigma^{LO}(1+\delta_{QCD}+\delta_{EW}+\underbrace{\delta_{QCD}\delta_{EW}}_{\text{large}})$$

* Fact scheme seems preferred

→ First results for $\alpha_s \alpha_{EW}$ DY

→ Partial results for $\alpha_s \alpha_{EW}$ ggF

→ "Intuition"



(1) $\boxed{\text{Discuss PDF slides}}$

(4) Look at threshold resummation.

Threshold resummation

We already said that the cross section is dominated by the region $z \to 1$.

⇝ large logs at each order in perturbation theory:

$$\hat{\sigma}^{(\ell)}_{gg}(z) \sim \alpha_s^{\ell} \left\{ a\,\delta(1-z) + \sum_{k=0}^{2\ell-1} b_k \left[\frac{\log^k(1-z)}{1-z} \right]_+ \right\} + \mathcal{O}(1-z)^0$$

these large logs can be resummed, but in Mellin-space!

$$\hat{\sigma}_{gg}(N) = \int_0^1 dz \, z^{N-1} \hat{\sigma}_{gg}(z)$$

$$\hat{\sigma}_{gg}(z) = \int_{C-i\infty}^{C+i\infty} \frac{dN}{2\pi i} z^{-N} \hat{\sigma}_{gg}(N)$$ where contour is such that all poles are to the right.

N.B: Mellin transform turns convolution into an ordinary product!

$$(f \otimes g)(z) \longrightarrow f(N) g(N)$$

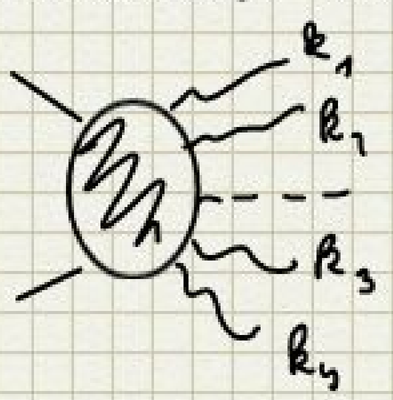
$$\frac{\sigma_{gg}}{\tau}(\tau) = \left[\mathcal{L}_{gg} \otimes \frac{\hat{\sigma}_{gg}}{z} \right](\tau)$$

$$\Rightarrow \sigma_{gg}(N-1) = \mathcal{L}_{gg}(N) \, \hat{\sigma}_{gg}(N-1)$$

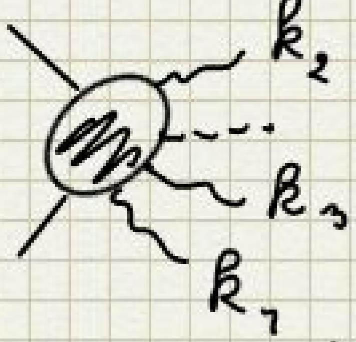
Why Mellin space?

$\rightsquigarrow$ Will look at QED case!

Consider emission of soft gluons.



$\sim \mathrm{Eik}(k_1) \times$



$\sim \mathrm{Eik}(k_1) \, \mathrm{Eik}(k_2)$

$\vdots$

$\sim \mathrm{Eik}(k_1) \dots \mathrm{Eik}(k_4)$

$$\Rightarrow \mathcal{M}_n(H + n \text{ soft gluons}) = \frac{1}{n!} \prod_{i=1}^{n} \mathcal{M}_1(H + 1 \text{ soft gluon } k_i)$$

But phase space does <u>not</u> factor

$$d\Phi_{1+n} \sim \delta(z - z_1 \dots z_n)$$

$$z_i \sim \frac{1 - E_1 - \dots - E_i}{1 - E_1 - \dots - E_{i-1}}$$

[<u>Details</u>: In CoM of initial state, plus $k_i = \frac{1}{2}\sqrt{\hat{s}}\,E_i\,\beta_i$, on shell constraint for H is

$$0 = m_H^2 - \hat{s} + \hat{s}\left(\sum_{i=1}^{n} E_i\right) = \hat{s}\left[z - \left(1 - \sum_{i=1}^{n} E_i\right)\right]$$

and

$$1 - \sum_{i=1}^{n} E_i = z_n \left(1 - \sum_{i=1}^{n-1} E_i\right)$$

$$= z_n \dots z_1$$

]

But Phase space factors in Mellin space!

$$\delta(z - z_1 \dots z_n) \rightsquigarrow \int_0^1 dz\, z^{N-1}\, \delta(z - z_1 \dots z_n)$$

$$= z_1^{N-1} \dots z_n^{N-1}$$

$$\hat{\sigma}(z) \sim \delta(1-z) + \sum_{n=1}^{\infty} \int_0^1 dz_1 \dots dz_n\, \delta(z - z_1 \dots z_n)\, \frac{1}{n!} M_1(z_1) \dots M_1(z_n)$$

$$\rightsquigarrow \hat{\sigma}(N) = 1 + \sum_{n=1}^{\infty} \int_0^1 dz_1 \dots dz_n\, z_1^{N-1} \dots z_n^{N-1}\, \frac{1}{n!} M_1(z_1) \dots M_1(z_n)$$

$$= 1 + \sum_{n=1}^{\infty} \frac{1}{n!} \left[M_1(N)\right]^n$$

$$= \exp M_1(N)$$

Exponentiation only happens in the soft limit where $z \to 1$

$\rightsquigarrow$ corresponds to 'large N', $N \to \infty$

More precisely

$$\int_0^1 dz\, z^{N-1} \left[\frac{\log^k(1-z)}{1-z}\right]_+ = \frac{(-1)^k}{k+1} \log^{k+1} N + \mathcal{O}\left(\frac{1}{N}\right)$$

Putting it all together, we obtain

$$\hat{\sigma}_{gg}(N) = g_0(\alpha_s) \exp\left[\frac{1}{\alpha_s} g_1(\lambda) + g_2(\lambda) + \alpha_s g_3(\lambda) + \dots\right] + \mathcal{O}(1/N)$$

* g_0 contains hard corrections (δ-fct, constants)
* exponent is exact in $\lambda \equiv \beta_0 \alpha_s \log N$
* g_i are known up to $i = 1 \dots 4$ (except 4-loop cusp, and g_0 is known up to N^3LO.
* Structure of the exponent: $L = \log N$

$\frac{1}{\alpha_s} g_1(\lambda) \rightsquigarrow LL$

$= g_{12} \alpha_s L^2 + g_{13} \alpha_s^2 L^3 + g_{14} \alpha_s^3 L^4 + \dots$

$g_2(\lambda) \rightsquigarrow NLL$

$= g_{21} \alpha_s L + g_{22} \alpha_s^2 L^2 + g_{23} \alpha_s^3 L^3 + \dots$

$\alpha_s g_3(\lambda) \rightsquigarrow NNLL$

$= g_{31} \alpha_s^2 L + g_{32} \alpha_s^3 L^2 + g_{33} \alpha_s^4 L^3 + \dots$

N. B: There is technical issue when going from Mellin space back to z-space.

$\boxed{\text{Show result plot}}$

Caveat! 1) Can we use scale variation as a measure of uncertainty?

2) This is 1 particular flavor of resummation.
$\rightsquigarrow$ cf. SCET.

3) Be aware that there are uncertainties!

* What to exponentiate

$$g_0(\alpha_s) \exp\left[\frac{1}{\alpha_s} \sum_{\ell=1}^{\infty} g^{(\ell)}(\lambda)\right]$$

$$= \tilde{g}_0(\alpha_s) \exp\left[\frac{1}{\alpha_s} \sum_{\ell=1}^{\infty} \tilde{g}^{(\ell)}(\lambda)\right]$$

with $\tilde{g}_0 = g_0 \, e^{f(\alpha_s)}$ $\quad \tilde{g}^{(\ell)}(\lambda) = g^{(\ell)}(\lambda) - f^{(\ell)}$

$$f(\alpha_s) = \sum_{\ell=0}^{\infty} \alpha_s^{\ell} f^{(\ell)}$$

$\rightsquigarrow$ effect : exponentiation of non Log-terms.

$\rightsquigarrow$ σ_{gg} unchanged to all orders, but affected if we truncate.

* We can add term of order $1/N$, without destroying the log accuracy!

E.g.:

$$\log(1+N) = \log N + \mathcal{O}(1/N)$$

$$\frac{N+1}{N}\log N = \log N + \mathcal{O}(1/N)$$

$$\psi(N) = \log N + \mathcal{O}(1/N)$$

Could use any of these in the exponent!

$\boxed{\text{Show Marzani comparison plot}}$

The limits of the large-m_t limit

So far: only inclusive xsec.

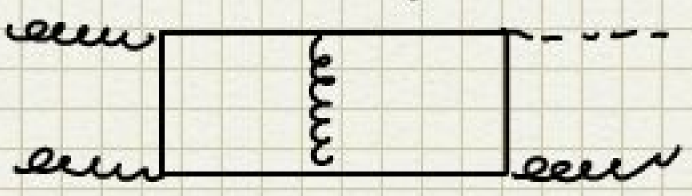
By now we know H and H+j @ NNLO fully differentially.

$\boxed{\text{Show plots}}$

Advantage: arbitrary cuts on final state.

H+j is only known at NNLO in the large-m_t limit.

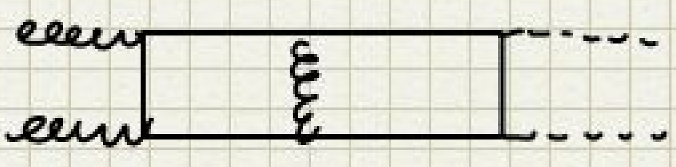
→ In the exact theory, we do not even know NLO!



not known!

High - p_T tail only known to LO !

Similar conclusions hold for HH



For diHiggs, we do not even know the total xsec @ NLO

$\rightsquigarrow$ HH system not necessarily at rest.

Current approach:

- Generate NLO in EFT
- Reweight virtuals by exact LO
- Reweight reals by exact H+j@LO.