In search of general relativity

or: How I learned to stop worrying and understood that not everything is what it seems to be

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Context and ADM decomposition

Context: quantum gravity (QG)

- A standalone theory of gravity in $3+1$ dimensions, valid at all scales.
- Particular candidate: **Hořava-Lifshitz gravity**[a]:
  - Built from anisotropic UV fixed point, defined s.t. $[g_N] = 1$,
  - EFT point of view is then applicable,
  - "IR limit" is different from Einstein-Hilbert action.

$\Longrightarrow$ searching for GR in this limit.

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Ingredients: ADM decomposition

- 3+1 decomposition of the metric,

$$g_{ij} \equiv {}^{(4)} g_{ij}, \qquad N \equiv \left(-{}^{(4)}g^{00}\right)^{-1/2}, \qquad N_i \equiv {}^{(4)} g_{0i}.$$

- All time derivatives encoded in the extrinsic curvature,

$$K_{ij} = \frac{1}{2N}\left(\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i\right).$$

GR revisited

Action + Wheeler-DeWitt metric

- The $3+1$ version of the Einstein-Hilbert action,

$$S = \int dt \int d^3x \sqrt{g} N \left(K_{ij}K^{ij} - K^2 + R - 2\Lambda\right)$$
$$= \int dt \int d^3x \sqrt{g} N \left(K_{ij}G^{ijkl}K_{kl} + R - 2\Lambda\right),$$

- G^{ijkl} is the Wheeler-DeWitt metric,

$$G^{ijkl} = \frac{1}{2}\left(g^{ik}g^{jl} + g^{il}g^{jk}\right) - g^{ij}g^{jk},$$

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Total Hamiltonian and constraints

- Performing the Legendre transformation yields the Hamiltonian,

$$H = \int d^3x \left(N\mathcal{H} + N^i\mathcal{H}_i + \alpha\phi + \alpha^i\phi_i\right)$$

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- $\mathcal{H}$ and $\mathcal{H}_i$ denote the Hamiltonian and momentum constraints,

$$\mathcal{H} = \frac{\pi^{ij} G_{ijkl} \pi^{kl}}{\sqrt{g}} - \sqrt{g} \left(R - 2\Lambda \right).$$

- Imposing $(\dot{\phi}, \dot{\phi}_i) = (0, 0)$ yields $(\mathcal{H}, \mathcal{H}_i) \approx (0, 0)$ and no new constraints arise.

The λ-R model

Action + generalized Wheeler-DeWitt metric

- Breaking Diff($\mathcal{M}$), a new dimensionless coupling appears,

$$S = \int dt \int d^3x \sqrt{g} N \left(K_{ij} K^{ij} - \lambda K^2 + R - 2\Lambda \right)$$
$$= \int dt \int d^3x \sqrt{g} N \left(K_{ij} G_\lambda^{ijkl} K_{kl} + R - 2\Lambda \right),$$

- G_λ^{ijkl} is the generalized Wheeler-DeWitt metric,

$$G_\lambda^{ijkl} = \frac{1}{2} \left(g^{ik} g^{jl} + g^{il} g^{jk} \right) - \lambda g^{ij} g^{jk},$$

Total Hamiltonian and constraints

- Performing the Legendre transformation yields the Hamiltonian,

$$H = \int d^3x \left(N \mathcal{H}_\lambda + N^i \mathcal{H}_i + \alpha \phi + \alpha^i \phi_i \right)$$

- $\mathcal{H}_i$ and its algebra remain unchanged, with λ present in $\mathcal{H}_\lambda$,

$$\mathcal{H}_\lambda = \frac{\pi^{ij} G^\lambda_{ijkl} \pi^{kl}}{\sqrt{g}} - \sqrt{g} \left(R - 2\Lambda \right).$$

- While $(\mathcal{H}_\lambda, \mathcal{H}_i) \approx (0, 0)$, $\dot{\mathcal{H}}_\lambda \approx 0$ is not trivial.

The tertiary constraint

Tertiary constraint $\sim$ gauge fixing

- General solution: $\nabla_i \pi \approx 0$.
- Asymptotically flat spaces: $\pi = 0$[a]
    - GR in the maximal slicing gauge ($\pi = 0$) is recovered.
- For compact spaces: $\frac{\pi}{\sqrt{g}} = a(t)$ (CMC gauge condition),
    - at first glance, the λ-dependance remains.

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More on the compact case

- Time preservation of tertiary constraint yield two more equations:
  - λ-dependent N (denoted by $\mathcal{A} \approx 0$) and α fixing equations.
- 1st and 2nd class classification of constraints yields 2 d.o.f.,
  - 2nd class constraints: $\left(\mathcal{A}, \phi, \mathcal{H}_\lambda, \pi - a\sqrt{g}\right)$
  - 1st class constraints: $(\phi_i, \mathcal{H}_i)$.

Outlook and references

- Originally, we worked with $\dot{a} = 0$, new results soon with fully general $a(t) = \frac{1}{V} \int d^3x \pi$.
  - Using York's conformal methods, it seems to be possible to prove equivalence with CMC general relativity for $\lambda > 1/3$,
  - Proof of Solution of Lichnerowicz-York does not seem to generalize for $\lambda < 1/3$.
- Would also be interesting to:
  - Check spaces with different boundary conditions,
  - Include (possibly as a perturbation) the term $\nabla_i \log N \nabla^i \log N$.

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- P. Hořava: arXiv:0901.3775v2 [hep-th];
- D. Giulini, and C. Kiefer: http://arxiv.org/abs/gr-qc/9405040;
- J. Bellorin, and A. Restuccia: http://arxiv.org/abs/1004.0055;
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Thank you!

Don't make a sound: they're not dead, just sleeping.