Topological entanglement entropy, ground state degeneracy and holography

in collaboration with Andrei Parnachev

Nick Poovuttikul

Lorentz Institute for theoretical physics, Leiden University

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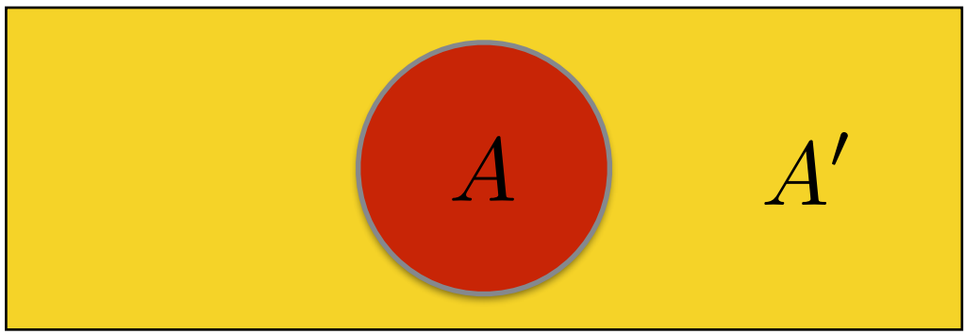
EE in topological field theory

- In topological field theory, the entanglement entropy for a disc region contain a topological term!
  (I'm living in 2+1d field theory)



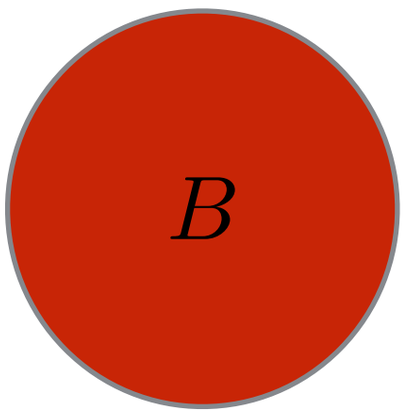
$$S_A = \alpha R - \gamma$$

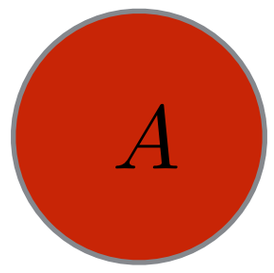
Levin & Wen '05

Kitaev & Preskill '05

Dong, Fradkin, Leigh & Nowling '08

- Long-ranged entangled !
  Mutual information $I(A:B)$ only depends only on γ





Computation of TEE

- It can be shown that γ can be written as partition function of a sphere

Witten ‘89

Kitaev & Preskill ‘05

Levin & Wen ‘05

$$\gamma = -\log \mathcal{Z}(S^3)$$

in topologically trivial gapped system $\gamma = 0$

- Order parameter? Smoking gun test?
- Lots of work in CMT

- We are interested in a relation between γ and zero temperature entropy on $\Sigma_g \times S^1$

Relation in $U(1)_k$ and $SU(N)_k$

- Case (i) : $U(1)_k$ i.e. electron gas in magnetic field

$$\boxed{2g\gamma = S_g}$$

e.g. Wen ‘89

- Case (ii) : $SU(N)_k$ with $N \gg k \gg 1$

$$\boxed{2(g-1)\gamma = S_g}$$

$$\gamma = \frac{k^2}{2}\log N$$

Verlinde ‘89

Camperi, Levstein & Zemba ‘90

Isidro, Nunes& Schnitzer ‘95

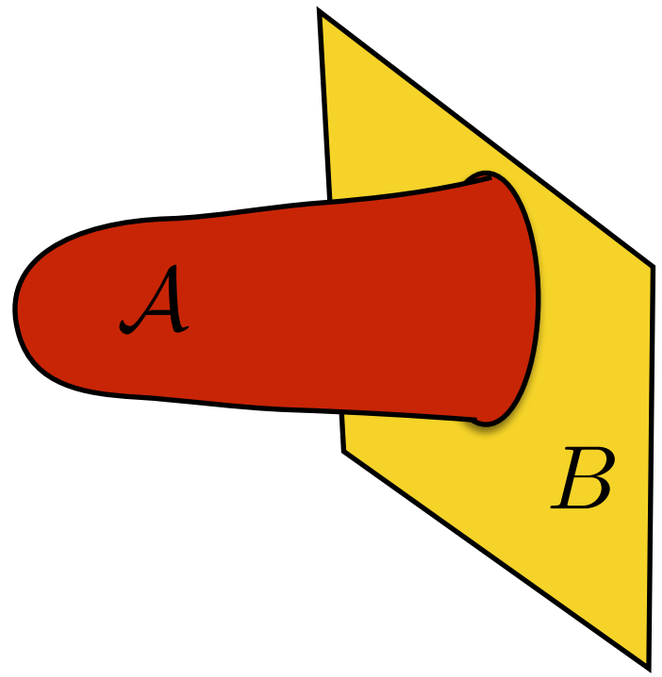
In holography, can one see….?

- Non-zero γ ?
  - There are many gapped holographic systems.
  - Can any of them has long-ranged entanglement ?
  - γ is subleading in large N but can we add correction?

- Simple relation between γ and S_g ?

Holographic entanglement entropy (1)

- In the field theory dual to AdS_{d+1} the entanglement entropy is given by the area of the minimal surface



$$S_A = \frac{\text{Area}(\mathcal{A})}{4G}$$

- In this case, where the gravity is a simple Einstein-Hilbert, the area of $\mathcal{A}$ can be computed analytically

$$S_A = \alpha_1 R + \alpha_0 \qquad \text{for odd } d$$

Ryu-Takayanagi ‘06

Holographic entanglement entropy (2)

- It can be extend to higher derivative gravity e.g. Gauss-Bonnet gravity described by an action

$$I = \frac{1}{16\pi G}\int d^d x\sqrt{-g}\left(R - \Lambda + \frac{\lambda L^2}{2}(R^2 + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu})\right)$$

the entanglement entropy is found by minimising the following functional

$$S_A = \frac{1}{4G}\int d^{d-2}y\sqrt{\hat{h}}(1 + \lambda L^2\hat{R}) + \text{boundary term}$$

- To find S_g, set the boundary to be $\Sigma_g \times S^1$ and compute the “thermal” entropy using Wald's formula

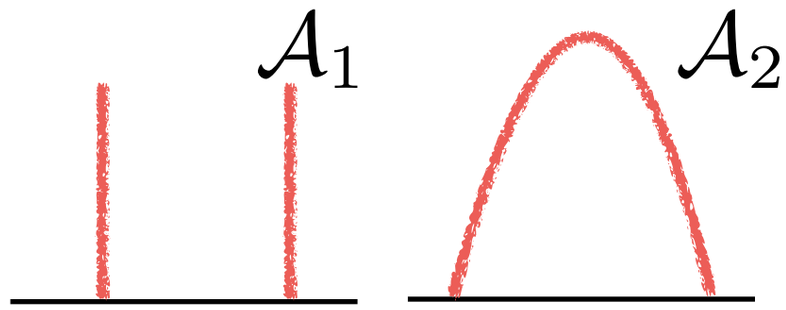
de Boer, Kulaxizi & Parnachev ‘11

Hung, Myers & Smolkin ‘11

Non zero γ & Relation between S_A & S_g

- Let's look at the Gauss-Bonnet term in both entropies

$$S_A = \frac{1}{4G_N}\left(\alpha_1 R + \alpha_0 + 2\pi\lambda L^2 \chi(\mathcal{A})\right)$$



$$S_g = \frac{1}{4G_N}\left(\int d^2y\sqrt{h} + 4\pi\lambda L^2(1-g)\right)$$

when the unwanted terms form these two surfaces vanish, we have relation like those in CS

$$2(g-1)\gamma = S_g$$

- Is there a model that has these properties ? YES!

Discussion

- Non-zero topological entanglement entropy can be realised in some gapped holographic systems

  - Maybe using this as a smoking gun test?

- Seems like non-locality is captured by GB term

- How on Earth there is a relation like those in CS ?

- Field theory origin of the nonzero entropy in $\Sigma_g \times S^1$ and the origin of 4d Gauss-Bonnet term ?

- Extensions to probe brane systems?

Thank you!

Examples of gapped holographic model

- For a simple gapped system (compact KK circle, hard wall), minimum surface becomes cylinder
  - No constant term from the area
  - Euler characteristic is also zero

Pakman & Parnachev 0805.1819

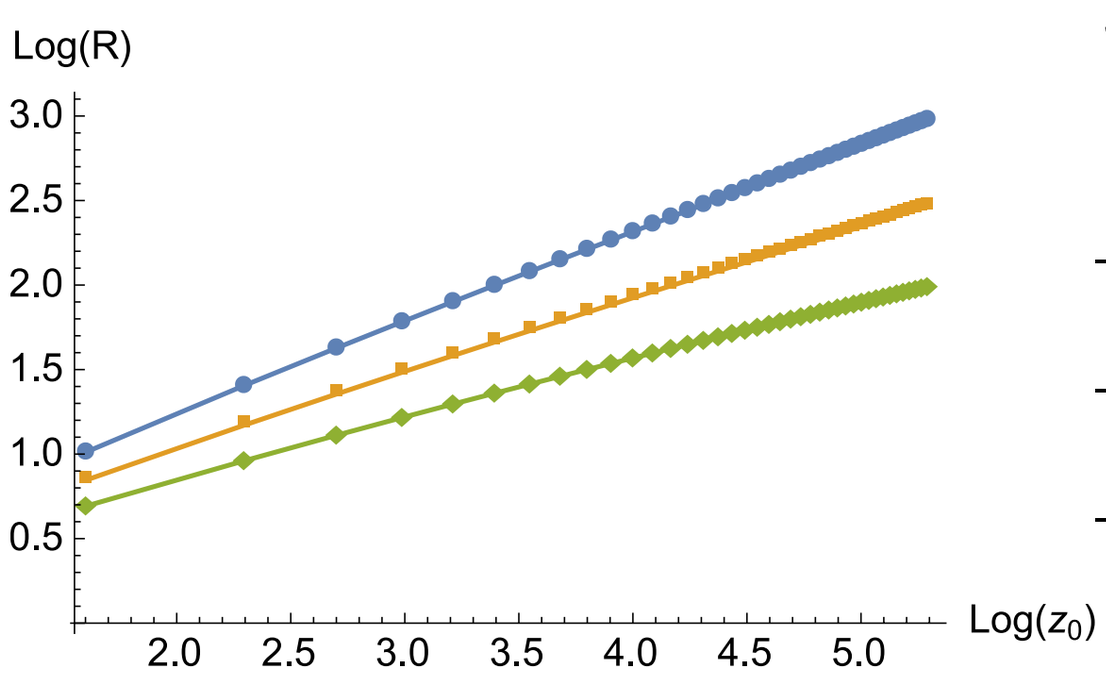
- Let's make the gap "softer"

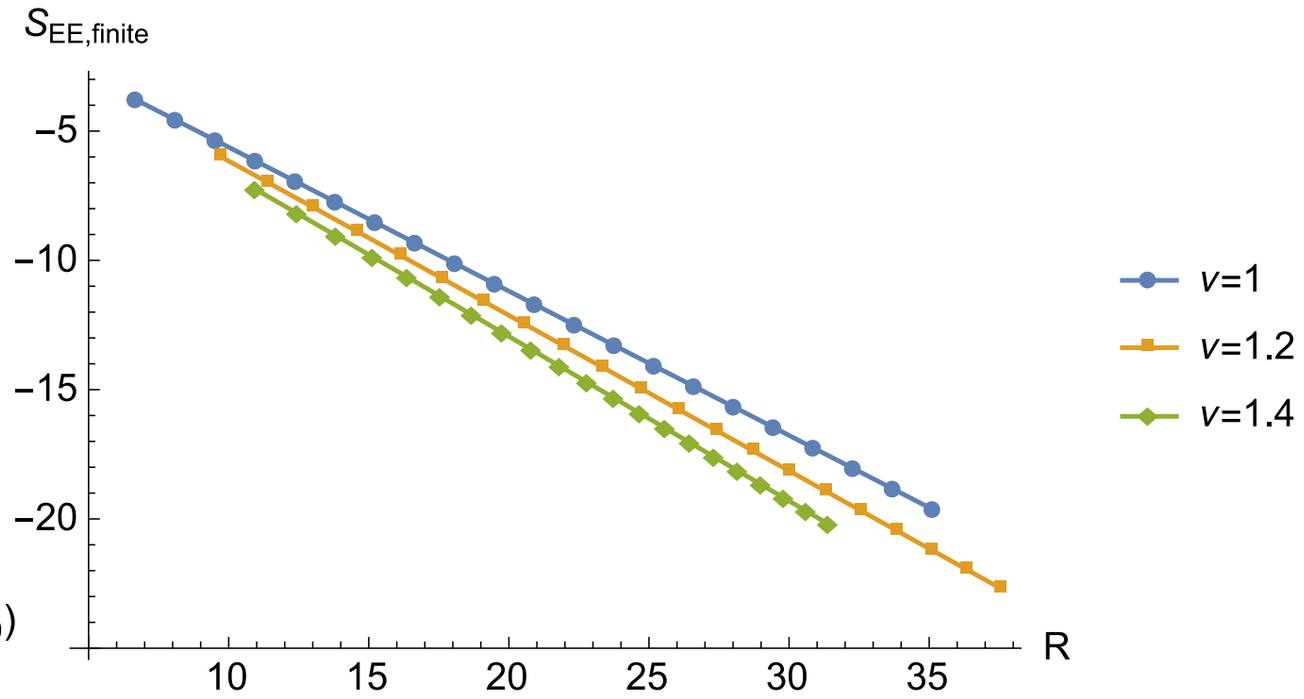
$$ds^2 = \frac{e^{-(\mu z)^\nu}}{z^2}\left(dz^2 - dt^2 + dx^2 + dy^2\right)$$

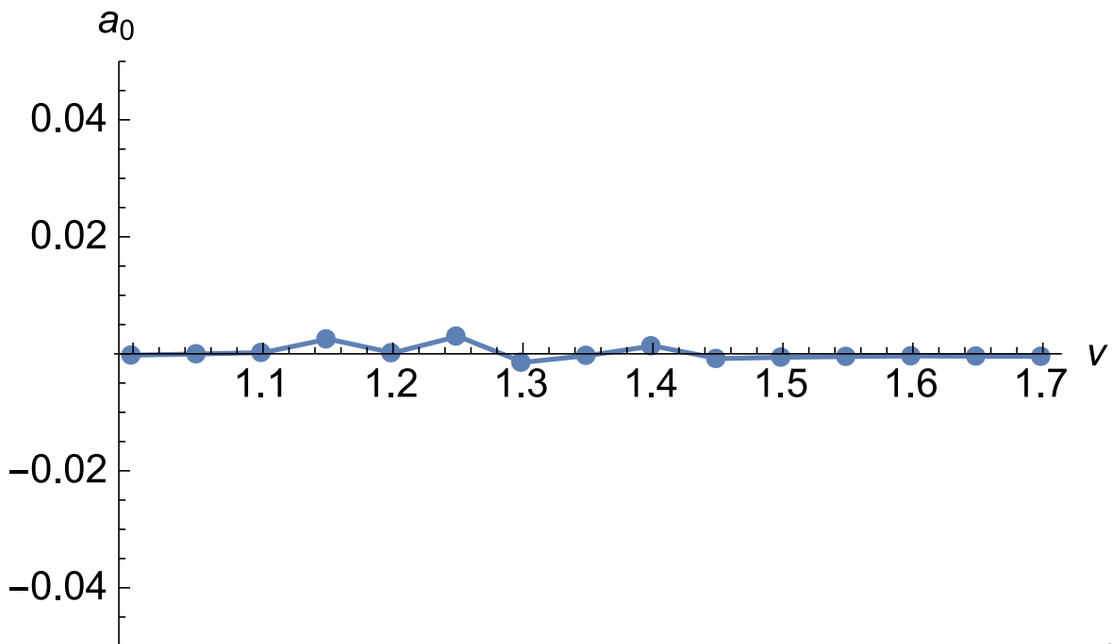
No solution with cylinder topology for $\nu < 2$!
and the system is still gapped for $\nu \geq 1$

Karch, Katz, Son & Stephanov 0602229
Falkowski & Perez-Victoria 0806.1737
Batell, Gherghetta & Sword 0808.3977

No constant term from the area ?







- Fit the entanglement entropy in the the form

$$S_{EE} = \alpha_1 R + \alpha_0 + \frac{\alpha_{-1}}{R}$$