2d-4d Correspondences

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Class $\mathcal{S}$ Theories

Class $\mathcal{S}$ theories are 4d super Yang-Mills theories constructed in M-theory:

- $N$ M5 branes wrap a Riemann surface $C_g$ of genus $g$ and spacetime
- $n$ transverse M5s intersect the $N$ M5s creating $n$ punctures
Example: $N = 2$

At **low energies** and **small area** two M5 branes describe an $\mathcal{N} = 2$ SCFT characterized by $C_{n,g}$: Gaiotto[09]

- Number of $SU(2)$ **gauge groups** determined as $n - 3 + 3g$
- Number of $SU(2)$ **flavour symmetry groups** determined as $n$
- Seiberg-Witten curve is a double cover of $C_{n,g}$
Original correspondence finds equivalence\(^1\) between:

Alday, Gaiotto, Tachikawa[09]

- Conformal blocks \(\mathcal{F}_{1...n}\) of the \(n\)-point correlator of **Liouville theory** on \(C_g\)

- Instanton partition function \(Z_{\text{inst}}\) of **4d gauge theory** associated to \(C_{n,g}\) on \(\mathbb{R}^4_{\epsilon_1, \epsilon_2}\) Nekrasov[02]

\(^1\)Up to some irrelevant constants
AGT Correspondence

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and

- Liouville three-point functions \(C_{123}\)

- Perturbative partition function \(Z_{\text{pert}}\)

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Specific Example

- **Liouville four-point function** on $C_0 = S^2$:

$$\langle V_1(\infty) V_2(1) V_3(q) V_4(0) \rangle = \int_0^\infty dp \, |q|^{2\Delta_p} C_{12p} C_{p34} F_{12}^{\text{p}3} (q) \overline{F}_{12}^{\text{p}3} (\bar{q})$$

- **Partition function** on $S^4$: Pestun[07]

$$Z_{S^4}(m_i, \tau) = \int da |q|^{2a^2} Z_{1-\text{loop}}(m_1, m_2, a) Z_{1-\text{loop}}(a, m_3, m_4) Z_{\text{inst}}(q) \overline{Z}_{\text{inst}}(\bar{q})$$

- **Crossing symmetry CFT** ‘proves’ **S-duality** of the gauge theory!
6d Perspective

How to understand the AGT correspondence?

- 6d $A_{N-1} (2, 0)$ interacting CFT
- Worldvolume theory of $N$ coincident M5 branes
- No Lagrangian description

$$\begin{align*}
\text{6d (2, 0) theory} & \quad \text{on } C \times S^4 \\
\text{2d Liouville theory} & \quad \text{on } C \\
\text{4d } \mathcal{N} = 2 \text{ theory } \mathcal{T}[C] & \quad \text{on } S^4
\end{align*}$$
Observations and Questions

- Both the $S^4$ theory and recently found $S^2 \times S^2$ theory correspond to CFTs with central charge $c = 26$, related to 2d quantum gravity! Bawane, Bonelli, Ronzani, Tanzini [14]

- Is there any sense in integrating over the gauge coupling? Cheng, Dijkgraaf, Vafa [10]
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- Why does Liouville appear? In particular: could it have a gravitational origin?

- More precisely: the Liouville/Toda central charge was reproduced from the anomaly-polynomial of the $\mathcal{N} = (2, 0)$ theory. Is there a relation between anomaly inflow and Liouville? Alday, Benini, Tachikawa [09]