Exercises - Excursion sets

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1. Compute and plot the Press-Schechter distribution

$$f_{\mathrm{PS}}(s) = \frac{\mathrm{d}}{\mathrm{d}s} \int_b^\infty \mathrm{d}\delta \, p(\delta) \tag{1}$$

   for a generic scale dependent barrier $b(s)$, assuming that δ is a zero-mean Gaussian process of variance $\langle \delta^2 \rangle = s$. Remember that a diffusive Gaussian process obeys the Fokker-Plank equation

$$\frac{\partial}{\partial s} p(\delta) = \frac{1}{2} \frac{\partial^2}{\partial \delta^2} p(\delta) \, . \tag{2}$$

2. A diffusive process for which each step is an independent stochastic variable is said to be a Markov process. For a constant barrier $b = \delta_c$, the probability that a Markovian random walk reaches a given value at scale s whithout having touched the barrier for any $S < s$ is

$$\Pi(\delta) = p(\delta) - p(\delta - 2\delta_c) \, . \tag{3}$$

   Do you remember why? Why is it crucial that the process is Markovian? How does using $\Pi(\delta)$ instead of $p(\delta)$ change $f_{\mathrm{PS}}(s)$

3. The upcrossing probability $f_{\mathrm{up}}(s)$ can be defined as

$$f_{\mathrm{up}}(s) = \int_{b'}^\infty \mathrm{d}v \, (v - b') \, p(b, v) \, , \tag{4}$$

   where $p(b, v)$ is the Gaussian joint pdf that $\delta = b$ and $\delta \equiv= \mathrm{d}\delta/\mathrm{d}s = v$. Compute $f_{\mathrm{up}}(s)$. In which limit does it reduce to $f_{\mathrm{PS}}(s)$?

4. Which of the above three results still depends on Cosmology?