PROBLEMS WITH HOT FRW COSMOLOGY

(Take a look at the original paper: A. Guth PRD 23, 2 (1981) 347)

Problems: fine tuning of initial conditions, not experimental inconsistencies

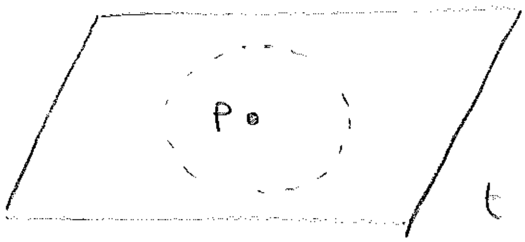
Do we have to care?

Horizon problem: homogeneity vs particle horizon

$$ds^2 = -dt^2 + a^2(t)\left(dx^2 + dy^2 + dz^2\right) \qquad \text{Flat FRW}$$

Matter dominance: $a(t) \propto t^{2/3}$ MD

Radiation dominance: $a(t) \propto t^{1/2}$ RD



$t = \text{const}$

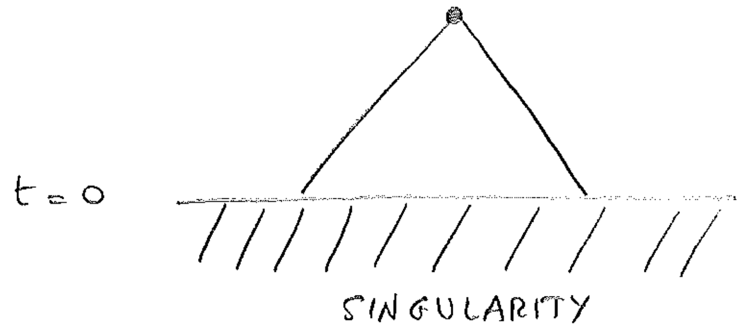
Which comoving observers could communicate with P?

Conformal time: $\eta = \int \frac{dt}{a(t)}$ $\qquad a(t)\, d\eta = a(t)$

$$ds^2 = a^2(\eta)\left[-d\eta^2 + dx^2 + dy^2 + dz^2\right]$$

Conformally flat metric

Light rays, $ds^2 = 0$, do not care about $a(\eta)$



Both in RD and MD
the integral converges
in the past

(I used flat FRW, but anyway curvature becomes irrelevant in the past)

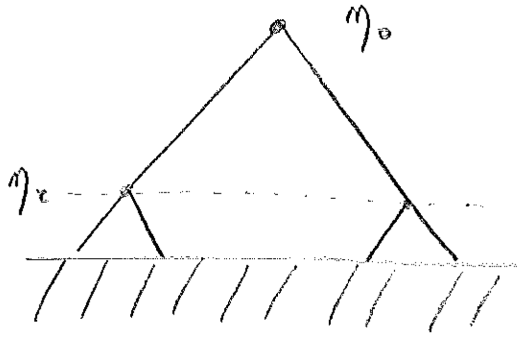
Let us calculate it:

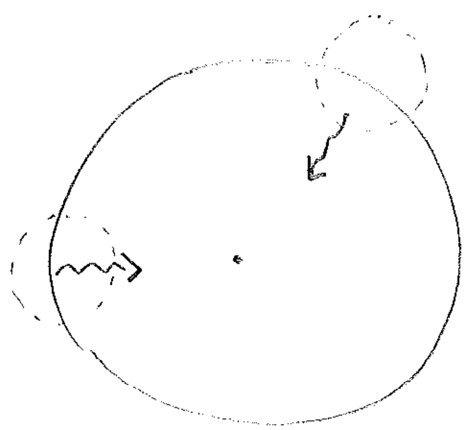
$$\text{MD} \qquad a = a_0 t^{2/3} \qquad H = \frac{\dot{a}}{a} = \frac{2}{3} t^{-1} \qquad \text{Hubble rate}$$

$$d_{HOR} = a \int_0^t \frac{d\tilde{t}}{a_0 \tilde{t}^{2/3}} = a \frac{3 t^{1/3}}{a_0} = 3t = 2H^{-1}$$

(Obviously if the integral converges I always get $\sim H^{-1}$
E.g. in RD I get $d_{HOR} = H^{-1}$)

Let us look at recombination. The Universe becomes neutral and photons free stream: a real causality diagram





(CMB is very clean as photons do not interact anymore, otherwise I always see things which are ~ causal contact)

Why is CMB so homogeneous? It comes from points that were never in causal contact!

How many uncorrelated spots?

$$\left(\begin{array}{lll} MD & a \propto t^{2/3} & \eta = \int \frac{dt}{a} \propto t^{1/3} \implies a \propto \eta^2 \\ RD & a \propto t^{1/2} & \eta = \int \frac{dt}{a} \propto t^{1/2} \implies a \propto \eta \end{array}\right)$$

$$\frac{\eta_r}{\eta_0} = \sqrt{\frac{a_r}{a_0}} \sim \frac{1}{\sqrt{1+z_{rec}}} \sim \frac{1}{\sqrt{1100}}$$

few degrees (which is obviously the CMB peak)

Even worse if we go further back: $T \sim 10^{17}$ GeV initial conditions below Planck era

Very roughly using RD

$$\frac{\eta_{Planck}}{\eta_0} \sim \frac{a_{Planck}}{a_0} \sim \frac{10^{-4}\,eV}{10^{17}\,GeV} \sim 10^{-30}$$

(rough, I should use entropy conservation + MD phase)

Our present Universe is composed by $\sim 10^{30}$ boxes which are disconnected at the Planck era!

I have to choose carefully these 10^{30} initial conditions to give rise to our Universe, which is so homogeneous

Planck abracadabra

As we do not know anything about quantum gravity, maybe everything is solved there (locality breaks down...)

→ But at the Planck era the separation between the horizon and present Universe is maximal...

→ Inflation will be a way to solve the problems with physics which is under control

Nowadays the focus is more on the perturbations and predictions of inflation. I cannot "prove" inflation using FRW problems

Particle horizon VS Hubble radius (or "horizon" unfortunately)

$$d_{HOR} = a \int_0^t \frac{d\tilde{t}}{a(\tilde{t})}$$

we would like to make this distance longer and longer

$$d_{HOR} = a \int_0^a \frac{d\tilde{a}}{\tilde{a}\,\dot{a}(\tilde{a})}$$

The convergence of the integral in the past depends on

$$\ddot{a} \gtrless 0$$

In particular for $\ddot{a} > 0$ the integral diverges!

MD, RD confuse two physically ≠ scales

- H^{-1} Timescale of evolution. Whether I can neglect or not the expansion in a given process
  Quantity defined at a given time

  $K/a \lessgtr H$ a wavelength inside / outside H behaves very differently

- d_{HOR} Distance travelled by a photon since the beginning
  Global quantity

$d_{HOR} \simeq H^{-1}$ in MD, RD (or any decelerated phase)

Inflation completely separates the two concepts

$$\boxed{\text{INFLATION} = \ddot{a} > 0}$$

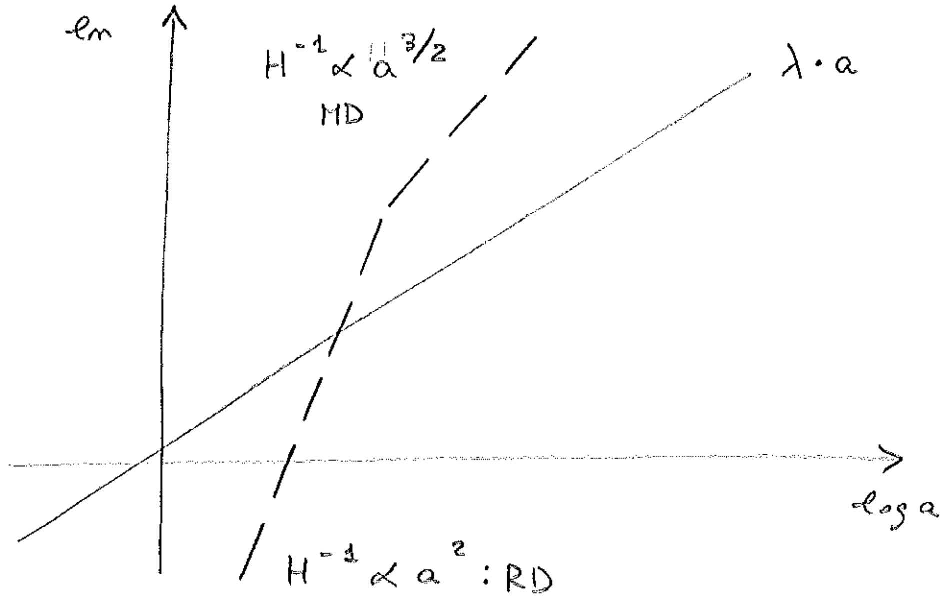
With this definition we are inflating, of course I am talking about an early phase

Inside and outside Hubble (or "horizon")

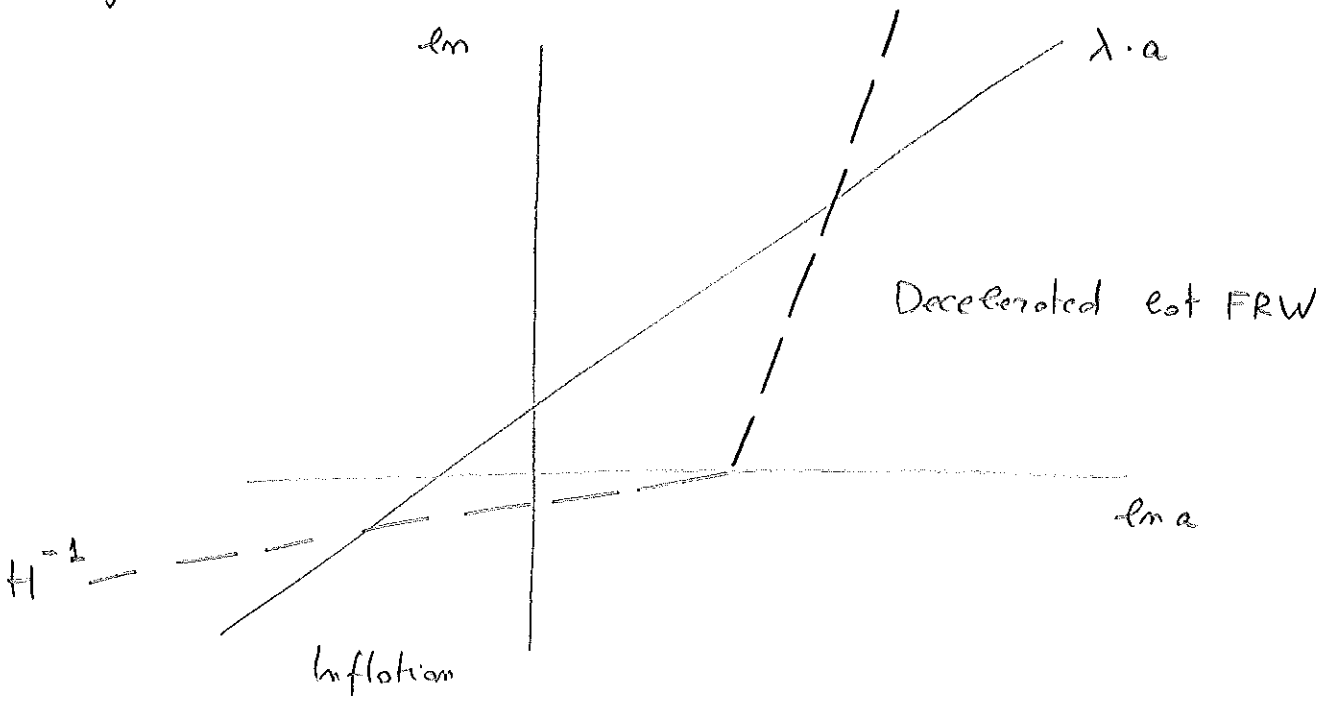
$$\frac{k}{aH} \quad \begin{array}{lll} \gg & \text{inside } H & \sim \text{Minkowski} \\ \ll & \text{outside } H & \sim \text{dominated by the expansion} \end{array}$$

$$\frac{k}{a\,\frac{\dot a}{a}} \quad \begin{array}{ll} \ddot a < 0 & \text{Modes come in} \\ \ddot a > 0 & \text{Modes go out} \end{array}$$

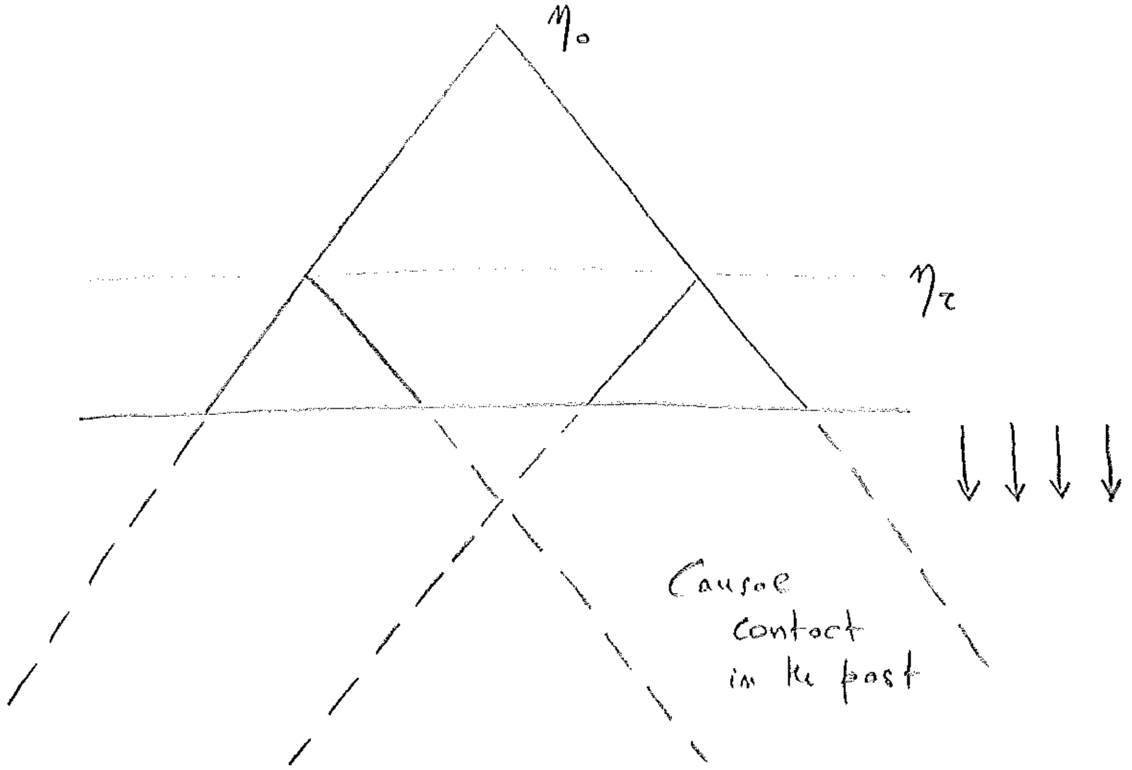
$\ddot a$ again



We see again the horizon problem in Fourier space: the initial condition for each mode must be set way out of H^{-1}.
Why we do not see crazy things entering all the time?



In the CMB diagram in conformal time:



Curvature problem

Friedmann equation: $H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho$

Spatial curvature enters in the Einstein equation like a source red-shifting as a^{-2}. More and more important wrt matter/radiation (before dark energy, it was the curvature that decided the fate of the Universe)

As the curvature is irrelevant nowadays, it must have been __very__ irrelevant in the past

$$\rho_{CR} \equiv \frac{3H^2}{8\pi G} \qquad \Omega(t) = \frac{\rho}{\rho_{CR}} = \frac{k}{(aH)^2} + 1$$

k has a ≠ meaning, but anyway it is the same ratio as before

RD: $|\Omega - 1|_0 \propto \left(\frac{a_0}{a_i}\right)^2 |\Omega - 1|_i$

The density must be very close to critical at the beginning of RD

As $|\Omega - 1| \propto \frac{1}{\dot{a}^2}$ $\quad |\Omega - 1| \to 0$ during $\ddot{a} > 0$

During inflation $|\Omega - 1| \to 0$. Avoiding tuning we expect very small $(\Omega - 1)_0$. <u>General "prediction" of inflation</u>

Now (2012): $|\Omega - 1|_0 \lesssim 10^{-2}$

Future : $|\Omega - 1|_0 \lesssim 10^{-3}; 10^{-4}$

(Of course one can get to 10^{-5}, which is the approximation to which we can talk of curved FRW)

(Notice curvature has a typical wavelength, so the previous analyses of exiting / entering is exactly the same as solving the curvature problem)

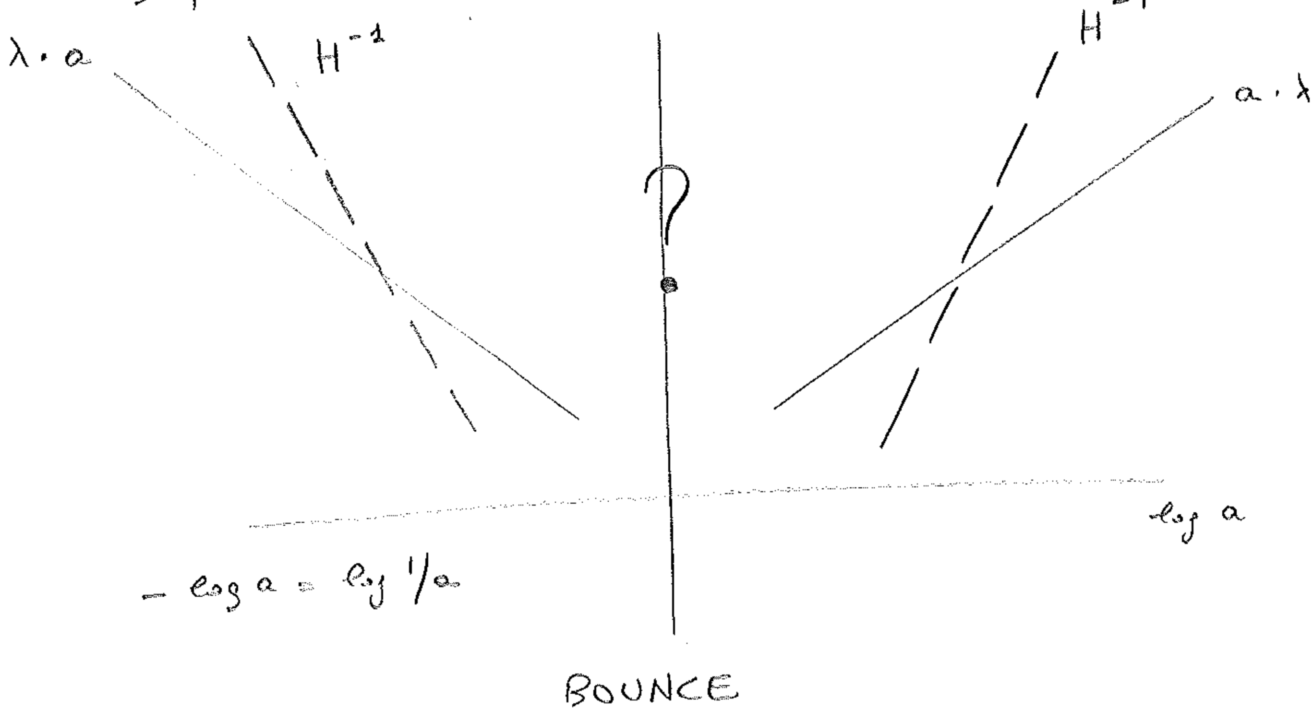
In other words: if M_P is the typical scale why an FRW (even assuming homogeneity) should last so long?

Notice that M_P exists only with $\hbar \neq 0$ ...

Bouncing models

Usefull to compare inflation with bouncing models in terms of kinematics of the modes

Before the standard cosmology a phase of contraction



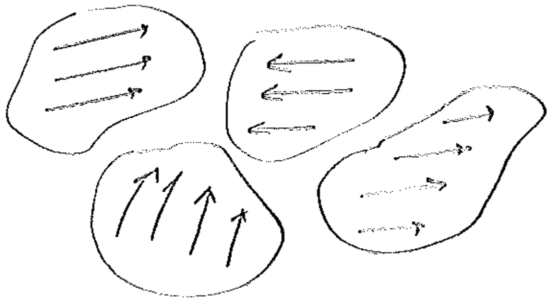
- Modes leave the horizon during the accelerated (standard) contraction
- What happens at the bounce? Naively $H \to 0$ to flip sign and all modes come back. But the hope is that this happens in a very short time so that modes have no time to evolve

Not compelling, but a logical possibility!

Monopole problem :

another classic, but less compelling, motivation for inflation

GUT phase transition: $G \longrightarrow SM$ Scale $M \simeq 10^{16}$ GeV



By Hubble argument, I expect at least 1 monopole per Hubble volume: by causality the gauge configuration is unrelated out of H^{-1}

$$n_{MONOP.} \simeq H^3 \simeq \left(\frac{M^2}{M_P}\right)^3 \quad \text{while} \quad n_\gamma \simeq T^3 \simeq M^3$$

$$\frac{n_{MONOP}}{n_\gamma} \simeq \left(\frac{M}{M_P}\right)^3 \simeq 10^{-9}$$

Experimental limits are $< 10^{-39}$ monopole/photon !

But it could be that there is no GUT and monopole to start with ...

How to obtain $\ddot{a} > 0$

- $$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) > 0$$

"Gravity becomes repulsive" actually nothing happens to gravity, it is just a change $T_{\mu\nu}$

This inequality violates the strong energy condition (SEC)

$$\left(T_{ab} - \frac{1}{2} g_{ab} T\right) t^a t^b \geq 0 \qquad \forall t \text{ time-like}$$

which, for comoving observers, gives $\rho + 3p \geq 0$

- Vacuum energy has $p = -\rho$ $\quad \Lambda$ gives $\ddot{a} > 0$

Not so exotic nowadays as we are now accelerating!

The complete solution is de Sitter space: $ds^2 = -dt^2 + e^{2Ht} d\vec{x}^2$

Maximally symmetric space with isometry $SO(4;1)$

- But we want to have de Sitter with a transition to a standard hot FRW cosmology: we want de Sitter with a clock!

- Notice that $\ddot{a} > 0$ is very different from $\dot{H} > 0$

$$\dot{H} = \frac{\ddot{a}}{a} - H^2$$

The energy density is always decreasing in an expanding Universe (or at most constant)

$$\dot{H} = -4\pi G(\rho + p)$$

I would need a violation of the null energy condition (NEC): $\quad T_{\mu\nu} n^\mu n^\nu \geq 0 \quad \forall n^\mu$ null

(Bouncing models require $\dot{H} > 0$ at the bounce to flip sign, violation of the NEC)

How inflation addresses these problems

One has to check in an explicit model but we can discuss few qualitative points

- Event horizon:

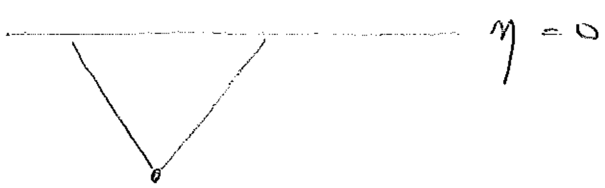
Now this integral converges in the future

$$a(t) \int \frac{da}{a\,\dot{a}(a)} = r_e(t)$$

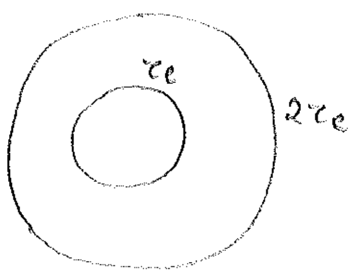
A particle can influence (if inflation lasts forever) only a limited portion of comoving coordinates

For example dS space in conformal coordinates reads

$$ds^2 = \frac{1}{H^2 \eta^2}\left(-d\eta^2 + d\vec{x}^2\right) \qquad \eta \in (-\infty ; 0)$$



This implies that if I start with a homogeneous region of radius $2r_e$, the internal ring cannot be reached by inhomogeneities outside



As $\frac{k}{aH} \to 0$ this region becomes very large compared to the Hubble radius

$\Longrightarrow$ Enough to have a good spot somewhere

- Gradients are stretched out of Hubble

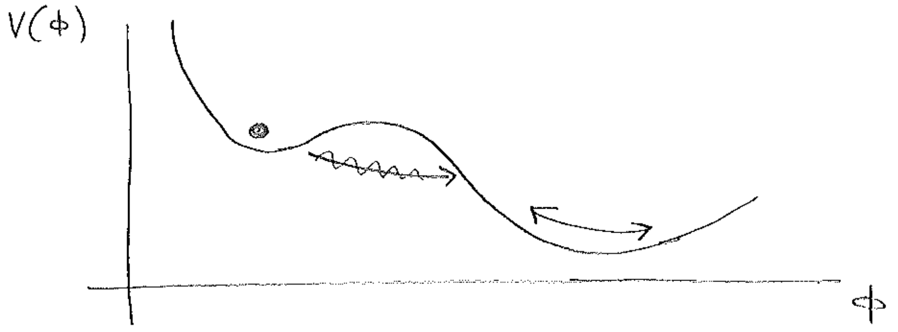
- For homogeneous (but anisotropic) cosmologies there are exact results in GR: anisotropies die off during inflation in all Bianchi models

Wald PRD 28 (1983) 2118

Kitada, Maeda PRD 45 (1992) 1416

"Cosmic no hair theorem" (?)

Old inflation   Original proposal by A. Guth '81



Metastable minimum: can I solve my problems and then tunnel?

The decay happens non-perturbatively through bubble nucleation

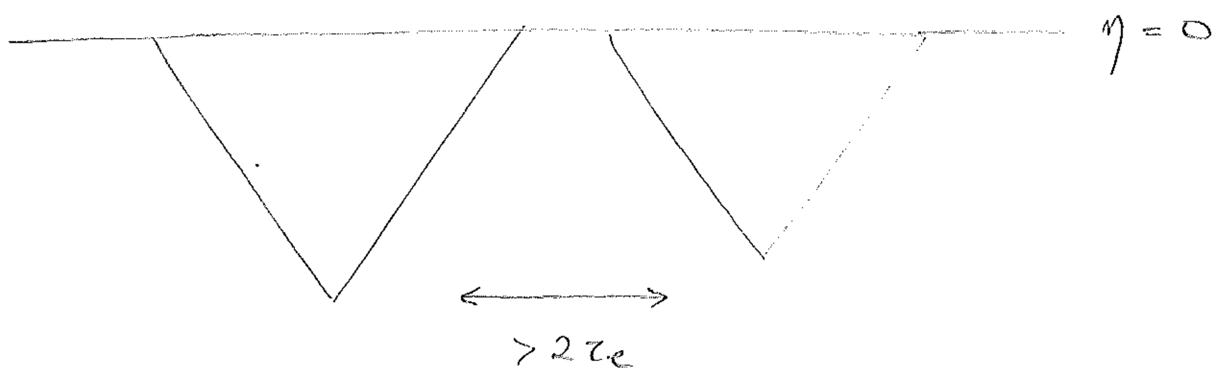
Γ is the probability per unit time and volume

Two different regimes:

$\Gamma \gg H^4$ Bubble percolates and you can reach the new phase in a time $\ll H^{-1}$. No inflation

$\Gamma \ll H^4$ You start inflating but the bubbles of the new phase cannot find each other. No new phase

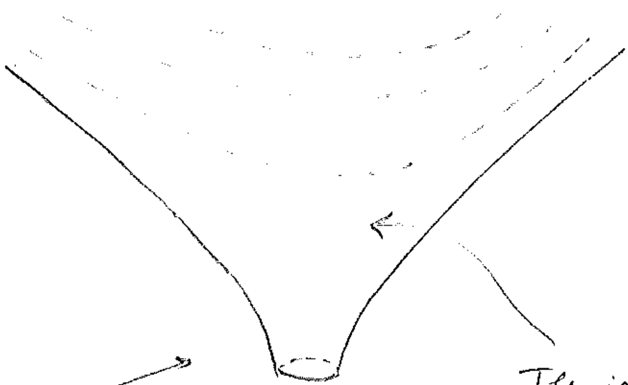
Not enough to wait as the Universe expands. Bubbles separated by > event horizon will never coalesce



But maybe I can live *inside* the bubble!

It is homogeneous, energy is given by scalar oscillations ...

(Notice the thin wall approximation where I end up in the minimum is misleading: usually I have residual energy in the bubble. Mukhanov's book?!)



The radius of the bubble is always $\lesssim H^{-1}_{\text{inflation}}$

The inside of the bubble can be obtained by analytic continuation of the instanton

It is an open Universe with spatial curvature equal to the one of the bubble: you are immediately curvature dominated!

<u>But</u> nothing prevents to have an episode like this in our past, followed by slow-roll inflation

Landscape ...

How to obtain "graceful exit"?

For old inflation dynamics: Guth, Weinberg NPB 212 (1983) 321
Coleman, de Luccia PRD 21,12 (1980) 3305

SLOW-ROLL INFLATION

Very profound idea:

Take the Lagrangian for a (minimally coupled scalar):

$$S = \int d^4x \, \sqrt{-g} \left[\frac{M_P^2}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi - V(\phi) \right]$$

$$M_P^2 = (8\pi G)^{-1}$$

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}} = \partial_\mu \phi \, \partial_\nu \phi + g_{\mu\nu} \left[-\frac{1}{2} g^{\alpha\beta} \partial_\alpha \phi \, \partial_\beta \phi - V(\phi) \right]$$

$$\left(\delta \sqrt{-g} = -\frac{1}{2} \sqrt{-g} \, g_{\mu\nu} \, \delta g^{\mu\nu} \right)$$

Homogeneous solution: $\phi(t)$

$$\rho = T_{00} = \frac{1}{2} \dot{\phi}^2 + V(\phi) \qquad (\rho + p) U_\mu U_\nu + p \, g_{\mu\nu} = T_{\mu\nu}$$

$$p = a^{-2} T_{ii} = \frac{1}{2} \dot{\phi}^2 - V(\phi)$$

We want to be close to dS. Kinetic energy $\ll$ potential energy

<u>Slow-roll</u>: $\quad \dfrac{\dot{\phi}^2}{2} \ll V(\phi)$ $\quad$ (for many Hubble times)

Nothing changes if I consider

$\longrightarrow$ Non-minimal coupling : $f(\phi)R$

$\longrightarrow$ Coupling with matter : Ψ_{SM} coupled to $g_{\mu\nu}\, h(\phi)$

$\longrightarrow$ Take $f(R)$ theory

See exercises

Besides Friedmann equations I have the EoM of scalar :

$$\ddot{\phi} + \underbrace{3H\dot{\phi}}_{\text{Hubble friction}} + V'(\phi) = 0$$

(Friction : in the absence of a potential, the kinetic energy of the scalar red-shifts

$\nabla^{\mu} T_{\mu\nu} = 0 \qquad \dot{\rho} + 3H(\rho + p) = 0$

Kinetic : $\rho = p \qquad \rho \propto a^{-6}$

Indeed $\partial_t (a^3 \dot{\phi}) = 0 \qquad \frac{\dot{\phi}^2}{2} \propto a^{-6}$)

$$\begin{cases} \ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 \\ H^2 = \dfrac{1}{3M_P^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right) \end{cases}$$

I expect, for sufficiently small tilt, to get rid of excessive $\dot{\phi}$ and be dragged to $\sim$ CC solution

Simple example : $V = \frac{1}{2} m^2 \phi^2$ The correct one ?

$$\begin{cases} H^2 = \frac{1}{3M_P^2} \left(\frac{1}{2}\dot\phi^2 + \frac{1}{2} m^2 \phi^2 \right) \\ \ddot\phi + \frac{\sqrt{3/2}}{M_P} \left(\dot\phi^2 + m^2 \phi^2 \right)^{1/2} \dot\phi + m^2 \phi = 0 \end{cases}$$

No explicit time dependence $\dot\phi(\phi)$ $\quad \ddot\phi = \frac{d\dot\phi}{d\phi} \dot\phi$

$$\frac{d\dot\phi}{d\phi} = -\frac{1}{\dot\phi} \left[\sqrt{3/2} \, \frac{\dot\phi}{M_P} \left(\dot\phi^2 + m^2\phi^2 \right)^{1/2} + m^2 \phi \right]$$

Focus on $\phi \gg M_P$ (!)

- $\dot\phi^2 \gg m^2 \phi^2$: kinetic domination. Opposite to what we are interested in

EOM reduces to $\frac{d\dot\phi}{d\phi} = -\frac{\sqrt{3/2}}{M_P} |\dot\phi|$

$$\dot\phi \propto e^{-\frac{\sqrt{3/2}}{M_P}\phi}$$

$\dot\phi$ redshifts exponentially in the direction of motion

Kinetic energy dies off much faster than exponential (exp vs power) and we leave kinetic domin.

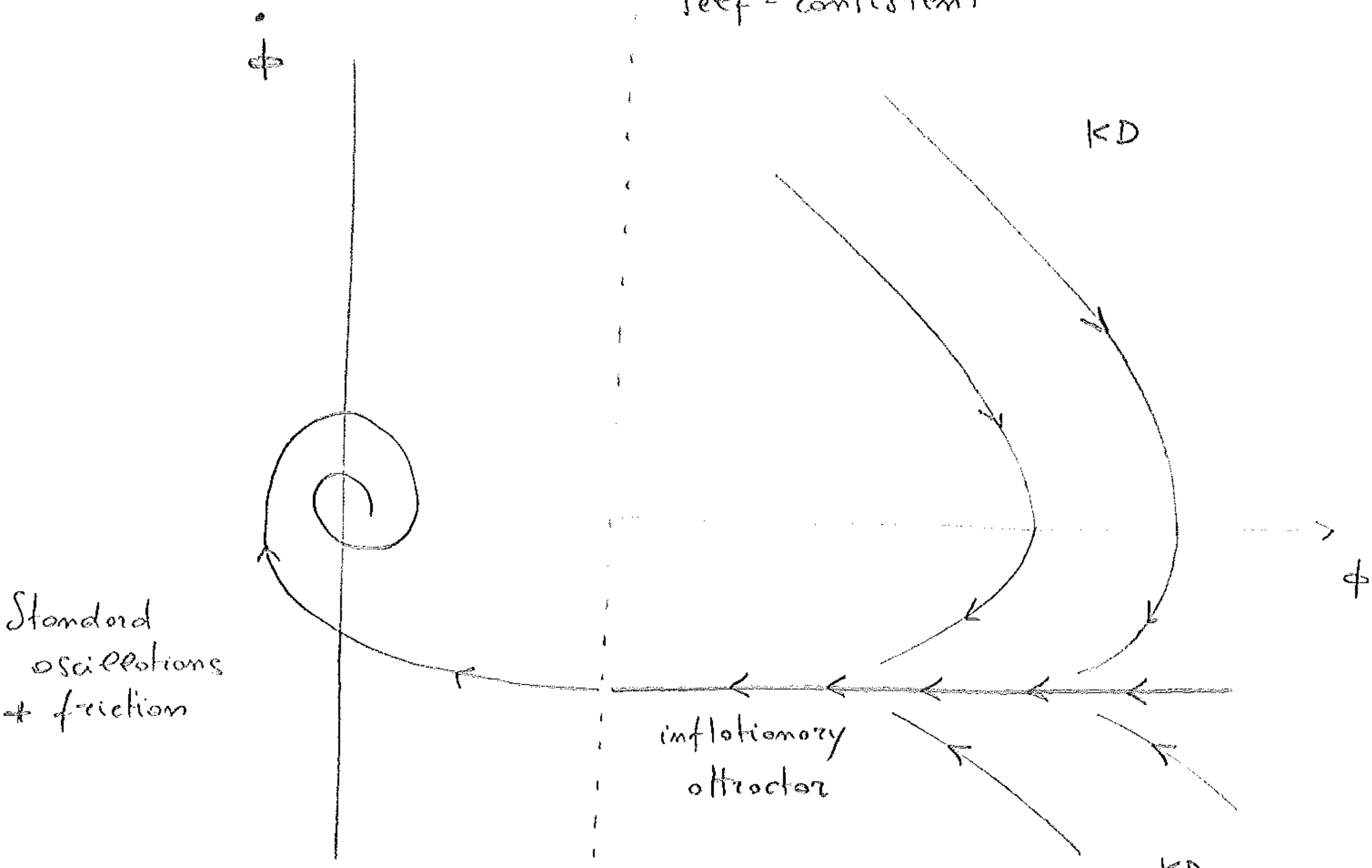
- Inflationary attractor : $\dot\phi^2 \ll m^2 \phi^2$ $\quad \phi \gg M_P$

Assume $\frac{d\dot\phi}{d\phi} = 0$

$$\frac{d\dot\phi}{d\phi} \propto \frac{\sqrt{3/2}}{M_P} \dot\phi \, m \phi + m^2 \phi \implies \dot\phi = -\sqrt{\frac{2}{3}} \, m M_P$$

During this phase : $\frac{\dot{\phi}^2}{2} \simeq m^2 M_P^2 \ll \frac{1}{2} m^2 \phi^2 = V$

self-consistent



$\phi \simeq M_P$
kinetic and potential energy are similar and inflation ends

$$\phi = \phi_f + \sqrt{\frac{2}{3}}\, m M_P \, |t - t_f|$$

As $H \simeq \frac{m\phi}{M_P}$ ϕ moves by M_P in an Hubble time. Thus $\phi_f \simeq M_P$ is soon negligible

$$\left(\frac{\dot{a}}{a}\right)^2 \simeq \frac{1}{3M_P^2}\,\frac{2}{3}\,\frac{1}{2}\, m^4 M_P^2 \,|t-t_f|^2 = \frac{1}{9} m^4 |t-t_f|^2$$

$$a(t) = e^{\frac{m^2}{6}|t-t_f|^2}$$

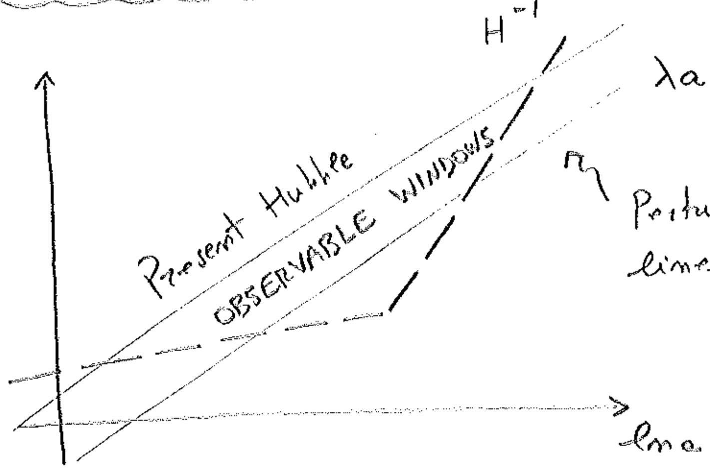
$$N = \frac{m^2}{6} |t - t_f|^2 \qquad \text{Number of e-folds}$$

- Usually we do not an explicit solution for $a(t)$, but I can approximate it as an exponential expansion

$$H(t) = \frac{m^2}{3} |t - t_f| \qquad \frac{\dot{H}}{H^2} \simeq \frac{1}{m^2 |t-t_f|^2} \simeq \frac{1}{N}$$

N Hubble times before the end, H is $\sim$ constant up to $\frac{1}{N}$ corrections

Kinematic and the observable window



Perturbations are very non-linear and difficult to study

We can roughly calculate the number of necessary e-folds comparing H_0^{-1} with the Hubble radius at the end of inflation

Rough: RD up to now + reheating at 10^{15} GeV

$$\frac{H_{end}^{-1}\, a_0/a_e}{H_0^{-1}} \sim \frac{\eta_e}{\eta_0} \sim \frac{a_e}{a_0} \sim \frac{T_0}{T_e} \sim \frac{10^{-4}\, eV}{10^{15}\, GeV} \simeq 10^{-28}$$

During inflation $H \simeq$ const, while $a \propto e^{Ht}$

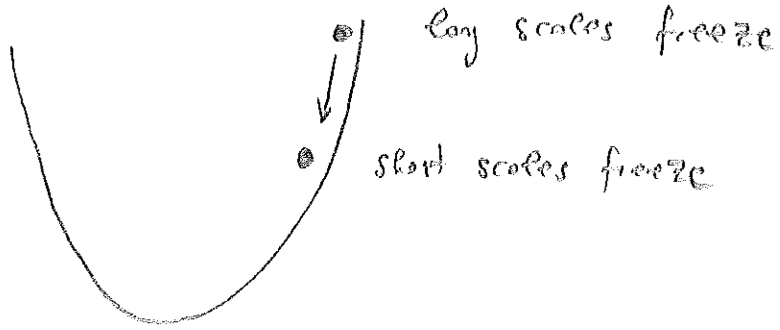
At least $N = \log 10^{28} \simeq 64$

+ window $\log \left(\frac{10^3 \text{ Mpc}}{0.1 \text{ Mpc}} \right) \simeq 9$

- Required # of e-folds depends logarithmically on the scale of inflation + reheating temperature + how I glue together inflation and RD

  Experimentally we are only confident $T_{RH} \gtrsim 1$ MeV for nucleosynthesis

- In any model:



- No bound on the total number of e-folds

Graceful exit + reheating

Now we clearly have a graceful (i.e. smooth) exit from inflation

In the quadratic case we can follow the solution after it leaves the attractor

$$\dot{\phi}^2 + m^2\phi^2 = 6 M_P^2 H^2$$

$$\dot{\phi} = \sqrt{6}\, M_P H \,\mathrm{sen}\,\theta \qquad m\phi = \sqrt{6}\, M_P H \cos\theta$$

$$\theta = \mathrm{arctg}\,\frac{\dot{\phi}}{m\phi} \qquad \dot{\theta} = \frac{1}{1+\frac{\dot{\phi}^2}{(m\phi)^2}}\left[\frac{\ddot{\phi}}{m\phi} - \frac{\dot{\phi}^2}{m\phi^2}\right] =$$

$$= \frac{1}{1+\left(\frac{\dot{\phi}}{m\phi}\right)^2}\left[\frac{-3H\dot{\phi} - m^2\phi}{m\phi} - \frac{\dot{\phi}^2}{m\phi^2}\right] = -m - 3H\,\frac{\dot{\phi}/m\phi}{1+\left(\frac{\dot{\phi}}{m\phi}\right)^2}$$

$$= -m - \frac{3H}{2}\,\mathrm{sen}\,2\theta$$

while $\quad H^2 = \frac{1}{3M_P^2}\frac{1}{2}\left(\dot{\phi}^2 + m^2\phi^2\right)$

$$2H\dot{H} = \frac{1}{6M_P^2}\left(2\dot{\phi}\ddot{\phi} + 2m^2\phi\dot{\phi}\right) = \frac{\dot{\phi}}{3M_P^2}\left(-3H\dot{\phi}\right)$$

$$\begin{cases} \dot{H} = -3H^2\,\mathrm{sen}^2\theta \\ \dot{\theta} = -m - \frac{3}{2}H\,\mathrm{sen}\,2\theta \end{cases}$$

$H \simeq m$ at the end of inflation, but eventually $H \ll m$

$$\theta \simeq -mt$$

$$\dot{H} = -3H^2\,\mathrm{sen}^2 mt$$

$$H(t) = \frac{2}{3t}\left(1 - \frac{\mathrm{sen}\,2mt}{2mt}\right)^{-1} \qquad a(t) \propto t^{2/3}$$

for $mt \gg 1$
i.e. $H \ll m$

Indeed :

✓ Inflaton oscillations are just a coherent state of massive particles with zero velocity (homogeneous) : MD

✓ $\rho = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}m^2\phi^2$

$p = \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}m^2\phi^2$ $\quad\quad \langle p \rangle = 0$ over many periods

(Indeed the amplitude of oscillations $\propto \frac{1}{t} \propto a^{-3/2}$

$\Longrightarrow \rho \propto a^{-3}$)

Reheating

- Oscillations around the bottom of the potential are a coherent state with density

$$n_\phi = \frac{\rho_\phi}{m} = \frac{1}{2m}\left(\dot{\phi}^2 + m^2\phi^2\right) \simeq \frac{1}{2} m \Phi^2$$

where Φ is the amplitude

(For $m = 10^{13}$ GeV
$\Phi \simeq M_P$ I get $10^{92}\,\text{cm}^{-3}$!)

- Perturbative decay of the inflaton with decay rate Γ (into what? what are the couplings?)

The decay completes when $H \simeq \Gamma$

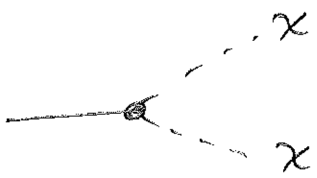
$$g_* T^4 \simeq H^2 M_P^2 \simeq \Gamma^2 M_P^2 \quad \Longrightarrow \quad T_{RH} \simeq \sqrt{M_P \Gamma}\, g_*^{-1/4}$$

If the decay is very fast RD starts with $H_{RD}^{max} \simeq H_{inflation}$, otherwise I may have a long period of oscillation

Preheating

Coherent effects are usually important and the treatment above is not correct

$$\mathcal{L} \supset \left(\frac{m_\chi^2}{2} - g\phi \right) \chi^2$$



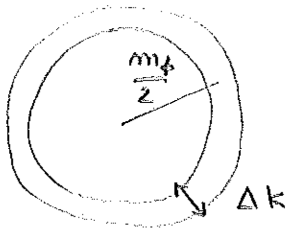
Example to show that the perturbative calculation can be wrong

$$\Gamma = \frac{g^2}{8\pi m_\phi}$$

$$\left(\frac{m_\phi}{2}\right)^2 = k^2 + \underbrace{m_\chi^2 - 2g\Phi \cos mt}_{\text{Assume} \ll m_\phi}$$

χ particles are produced in a narrow range of k and Bose-enhancement becomes relevant

$$\Delta k \simeq \frac{4g\Phi}{m}$$



Phase space density

$$\hat{n}_{k=\frac{m}{2}} \simeq \frac{n_\chi}{(4\pi k_0^2 \Delta k)/(2\pi)^3} = \frac{2\pi^2 n_\chi}{m g \Phi} = \frac{\pi^2 \Phi}{g} \frac{n_\chi}{m_\phi}$$

Bose condensation is relevant when

$$n_\chi > n_\phi \cdot \frac{g}{\pi^2 \Phi}$$

To have $n_\chi < n_\phi$

$$\frac{g\Phi}{\pi^2 \Phi^2} < \frac{m_\phi^2}{\pi^2 \Phi^2} \sim 10^{-12}$$

Coherent, non-perturbative production

(Preheating starts with: Linde, Kofman, Starobinsky
hep-th/9405187)

- Very <u>complicated</u> and model dependent
- In most cases <u>unobservable</u>

Predictions of inflation will be (non-linearly) insensitive to what happens when modes are out of H^{-1}

Insensitive to many interesting things: which particles?
Phase transitions?

- Everything originates from a very <u>out-of-equilibrium</u> process. Quite $\neq$ from standard hot FRW cosmology

Good place for <u>baryogenesis</u> for example

General potential: slow-roll approximation

For a general potential the analysis is more complicated, but everything simplifies in the slow-roll regime

$$\begin{cases} \ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 \\ H^2 = \frac{1}{3M_p^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right) \end{cases} \xrightarrow{SR} \begin{cases} \dot{\phi} \simeq -\frac{V'}{3H} \\ H^2 \simeq \frac{V}{3M_p^2} \end{cases}$$

Slow-roll parameters

- $\frac{1}{2}\dot{\phi}^2 \ll V \qquad \frac{V'^2}{H^2} \ll V$

$$\epsilon = \frac{1}{2} M_p^2 \left(\frac{V'}{V}\right)^2 \ll 1 \qquad \text{Also} \quad \frac{\dot{H}}{H^2} = -\epsilon$$

Parametrizes the departure from de Sitter

$$\dot{H}: \quad 2H\dot{H} = \frac{1}{3M_p^2}\left(\frac{1}{2}\dot{\phi}^2 + V\right)^{\cdot} = \frac{1}{3M_p^2}\left(\dot{\phi}\ddot{\phi} + V'\dot{\phi}\right) = -\frac{H\dot{\phi}^2}{M_p^2}$$

$$\frac{\dot{H}}{H^2} \simeq -\frac{1}{2}\frac{\dot{\phi}^2 M_p^{-2}}{\frac{V}{3}M_p^{-2}} = -\frac{3}{2}\frac{\dot{\phi}^2}{V} = -\frac{\cancel{3}}{2}\frac{V'^2}{\cancel{3}3H^2}\frac{1}{V} = -\frac{1}{2}\frac{V'^2}{V^2}M_p^2 = -\epsilon$$

- $\ddot{\phi} \simeq \left(-\frac{V'}{3H}\right)^{\cdot} \underset{\substack{\text{up to} \\ \epsilon \text{ corrections}}}{\simeq} \frac{V''}{H}\dot{\phi} \simeq \frac{V''V'}{H^2} \ll V'$ in EOM for ϕ

$$\Longrightarrow V'' \ll H^2$$

$$\eta \equiv M_p^2 \frac{V''}{V} \qquad (\text{notice} \gtrless 0)$$

$$\frac{\ddot{H}}{H\dot{H}} \underset{\uparrow}{=} \frac{1}{H}\frac{2\dot\phi\ddot\phi}{\dot\phi^2} = 2\epsilon - \frac{2}{3}\frac{V''}{H^2} = 2\epsilon - 2\eta$$

$$\dot{H} = -\frac{1}{2}\frac{\dot\phi^2}{M_P^2} \qquad \ddot\phi \simeq \left(-\frac{V'}{3H}\right)^{\cdot} = \frac{V''}{3H}\frac{V'}{3H} + \underbrace{\frac{V'}{3H^2}\dot{H}}_{\epsilon H \dot\phi}$$

So we are talking about the relative variation of H and $\dot{H}$ (which will be the only things appearing in EFTI)

- Slow-roll conditions are necessary, but not sufficient for inflation. I can always start with a large velocity... Anyway I expect there is an attractor as in the $m^2\phi^2$ case

- One can define <u>higher-order slow-roll parameters</u>, requiring $\epsilon, \eta \ll 1$ for many Hubble times

Not enough to have a flex point

E.g. $\delta\eta = M_P^2 \frac{\delta V''}{V}$ + terms $O(\epsilon, \eta)$

$$M_P^2 \frac{V''' \dot\phi/H}{V} = M_P^2 \frac{V''' V'}{V} H^{-2} = M_P^4 \frac{V' V'''}{V^2} \equiv \xi^2$$

and so on

Quartic: $\quad M_P^4 \frac{V' V^{(4)} \cdot V'/H^2}{V^2} \ll 1$

$$V^{(4)} \ll \frac{H^2}{M_P^2}\frac{1}{\epsilon} \sim 10^{-10}$$

Precisely this combination is fixed by the power spectrum

<u>Tremendously weakly coupled!</u>

Very rough "classification" of slow-roll models

- Nothing happens at the end of inflation

$$V \propto \phi^n \qquad \epsilon ; \eta \sim \left(\frac{M_P}{\phi}\right)^2$$

$$\epsilon \sim \eta \sim \frac{1}{N} \qquad \Delta\phi \gg M_P$$

$\left(\frac{1}{\sqrt{\epsilon}} \frac{H}{M_P} \right.$ is fixed by experiments to $\left. 10^{-5} \right)$

H is large, $\Delta\phi$ is super Planckian : high energy models

As ϵ is not very small GWs are observable

$\left(\frac{H}{M_P} \right.$ gives GWs contribution $\left. \right)$

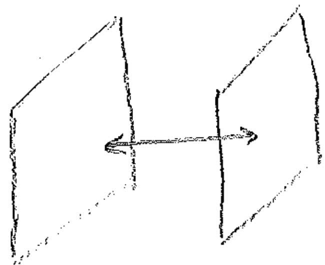
- Some kind of phase transition at the end of inflation

Hybrid models:

$$V = \frac{1}{2} m^2 \phi^2 + \frac{1}{4} \lambda (\psi^2 - M^2)^2 + \frac{1}{2} \lambda' \psi^2 \phi^2$$

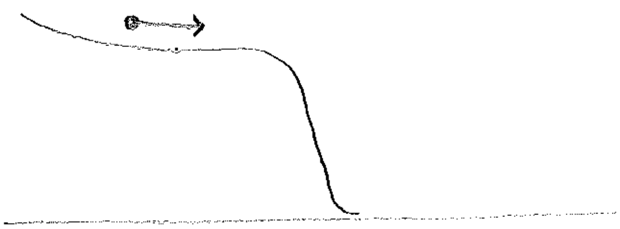
ϕ rolls and triggers some kind of phase transition

Brane inflation:



Inflaton is the interbrane separation
At the end they annihilate + create matter

Some kind of plateau in energy



V_0 $\qquad \epsilon \lll 1$

$$\Delta\phi \simeq \dot{\phi}/H \simeq V'/H^2 \simeq \frac{V'}{V} M_P^2 \simeq M_P\sqrt{\epsilon}$$

$$\epsilon \lll 1 \implies \Delta\phi \ll M_P \iff \text{No GWs}$$
$$\implies H \lll 10^{13}\ \text{GeV}$$

<u>Low-energy inflation</u>

Notice that it does not mean $|\eta| \lll 1$

$$\eta = \frac{V''}{V} M_P^2 \qquad \sqrt{\epsilon} = \frac{V'}{V} M_P$$

$$\frac{\sqrt{\epsilon}}{\eta} \simeq \frac{1}{M_P}\frac{V'}{V''} \ll 1 \qquad \text{for sub Planckian models}$$

Both these cases have been argued to be generic / extremely tuned

- $\Delta\phi \gg M_P$ is silly !?
- $\epsilon \lll 1$ is silly !?

The point is <u>vital for GW detection</u>!

Why is it so difficult to find an inflaton

1) Can we have $\Delta\phi > M_P$?

2) Can we have a sufficiently flat potential?

- About (1) notice that when we say $\phi > M_P$ we still have $\rho << M_P^4$

  E.g. $V = \frac{1}{2} m^2 \varphi^2$ I still have a large number of e-folds before losing control of GR

- About (2). Even for subPlanckian model you have to face UV physics

$$V = V_0(\varphi) + V_0(\varphi)\frac{\varphi^2}{M_P^2} + \cdots$$

For superPlanckian models, this is not even an expansion ...

This generates $\eta \sim 1$. So I have to control M_P-suppressed operators.

UV-sensitivity of inflation

Many years in BSM physics taught us that there are two ways to keep a potential flat (like the mass of the Higgs)

- shift-symmetry: $\phi \longrightarrow \phi + c$
- Supersymmetry

Approximate shift symmetry (PNGB) Natural inflation

$$V = \Lambda^4 \cos(\phi/f)$$

Slow-roll requires $f \gg M_P$ to inflate

- Perturbatively there is no problem
- Non-perturbatively I do not expect gravity to respect any global symmetry (wormholes...) But you can use a gauge symmetry
- Not easy to realize in string theory (essentially I cannot make a coupling small as this also increases M_P).

  Recent axion monodromy McAllister, Silverstein, Westphal 0808.0706

Supersymmetry

which must be promoted to SUGRA

$$V_F = e^{K/M_P^2}\left[K^{\varphi\bar\varphi} D_\varphi W \overline{D_\varphi W} - \frac{3}{M_P^2}|W|^2\right]$$

$$D_\varphi W = \partial_\varphi W + M_P^{-2}(\partial_\varphi K)W$$

$$K^{\varphi\bar\varphi}\partial_\mu\varphi\partial_\mu\bar\varphi \qquad e^{K^{\varphi\bar\varphi}|\varphi|^2}\left[\quad V \quad\right]$$

$\rightsquigarrow \eta \sim 1$ η-problem in SUGRA