Lectures on Monte Carlo Event Generators

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1. **Introduction**
2. **Parton showering**
3. **Improving parton showers with fixed-order matrix elements**
4. **Soft physics, multiparton interactions and hadronisation.**

Many previous lectures can be found at
`http://users.phys.psu.edu/∼cteq` and `montecarlonet.org`.
Further references at the end of the slides.

Part 1: Introduction

a) Why do we need Event Generators?

b) Event generation at hadron colliders.

How will we find what is out there?

Know what we want to look for...

Know what we're facing...

Assess if there is a realistic chance with our current experiments
...and check before building a new experiment.

How will we find what is out there?

Know what we want to look for...

Missing E_T and jets (a.k.a. classical SUSY)?

Compressed masses?

Dark sectors? New bound states?

Know what we're facing...

Assess if there is a realistic chance with our current experiments

...and check before building a new experiment.

Know what we want to look for...

Missing E_T and jets (a.k.a. classical SUSY)?

Compressed masses?

Dark sectors? New bound states?

Know what we're facing...

QCD,

QCD,

QCD.

Assess if there is a realistic chance with our current experiments
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How will we find what is out there?

Know what we want to look for...

Missing E_T and jets (a.k.a. classical SUSY)?

Compressed masses?

Dark sectors? New bound states?

Know what we're facing...

QCD,

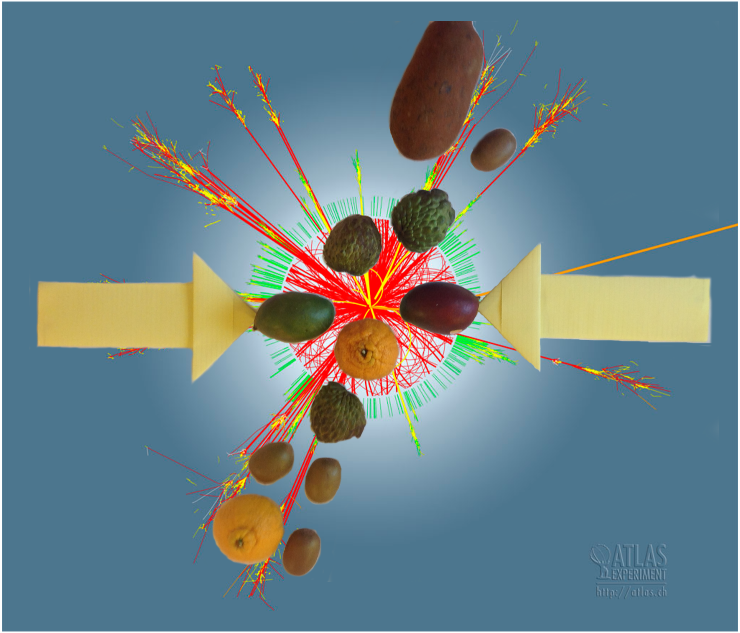
QCD,

QCD.

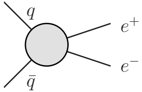
Assess if there is a realistic chance with our current experiments
...and check before building a new experiment.

We need an accurate representation of "known" and "unknown" physics that feels like data!
$\Longrightarrow$ **Event generators**

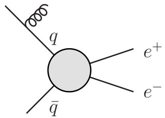
New physics signals



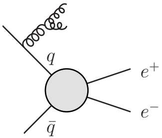
Event generation: Start from hard process



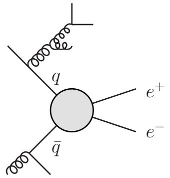
...and emit gluons from incoming partons



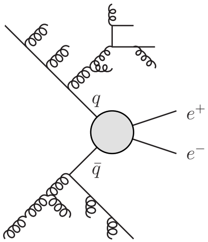
...or outgoing partons



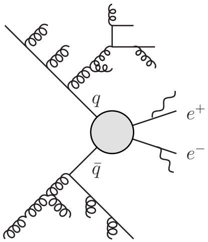
...or split gluons into quarks



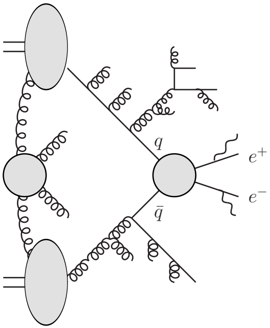
...and how to do this arbitrarily often



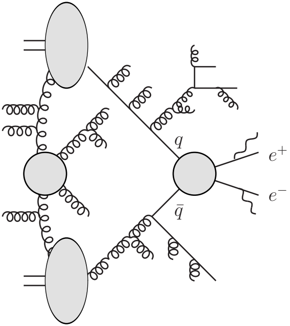
...and emit photons from charged fermions



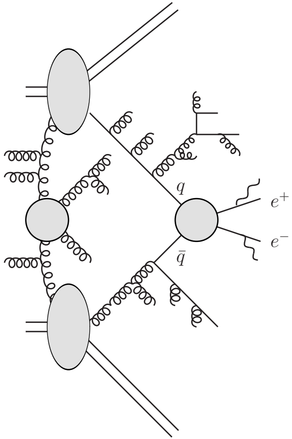
...and include multiple interactions between composite protons



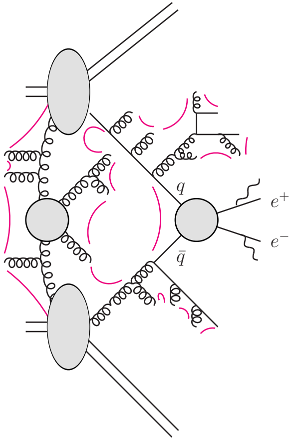
...which again produce more radiation



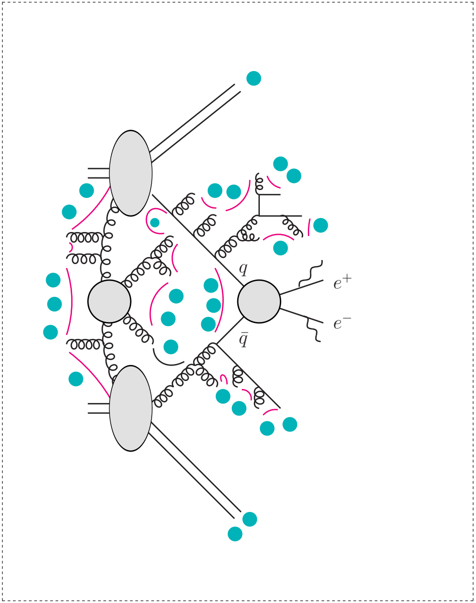
...and add beam remnants to form a colourless state



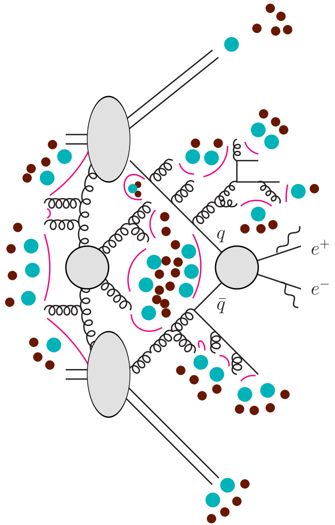
...and form strings (colour flux tubes)



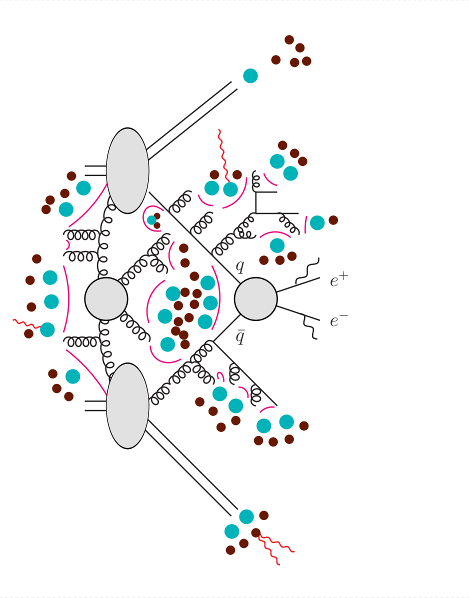
...and produce hadrons from strings and remnants



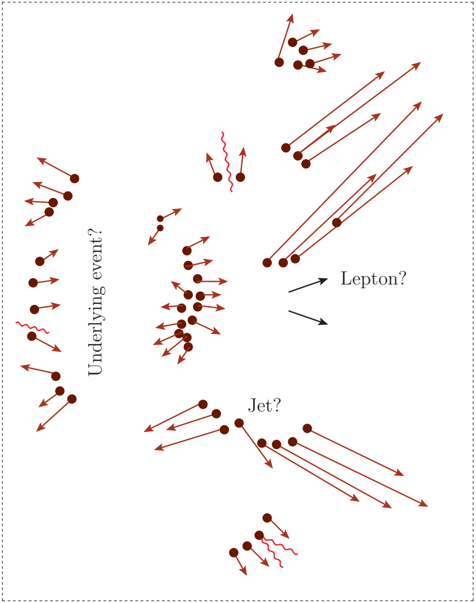
...and decay the excited hadrons



...which can again involve photons



And the detector records this...



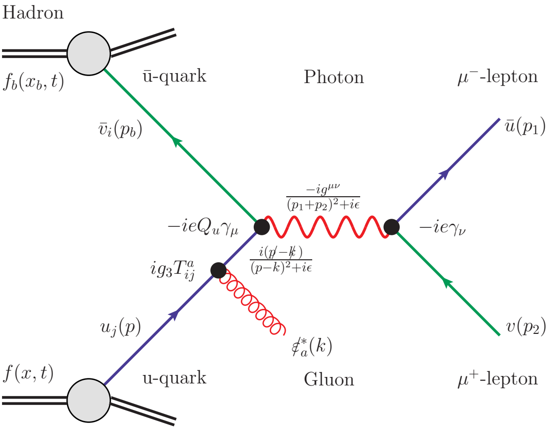
The three commonly used General Purpose Event Generators are

HERWIG	PYTHIA	SHERPA
Basic ME generator	Basic ME generator	Mature ME generator
Angular ordered $\tilde{q}$ shower and $p_\perp$-ordered CS dipole shower	$p_\perp$-ordered dipoles with ME-corrections, VINCIA antenna shower	$p_\perp$-ordered CS dipole shower, ANTS antenna shower
YFS multipole QED MPI afterburner	QED from shower Interleaved MPI	YFS multipole QED MPI afterburner
Cluster hadronisation	String hadronisation	Cluster hadronisation

(Warning: No purists in this game. Every theorist has to learn and compromise)

Part 2: Parton showering

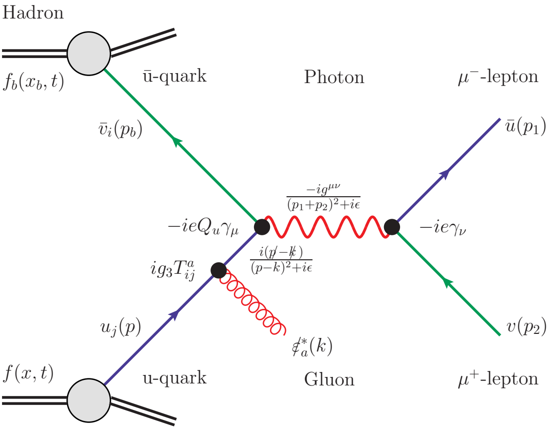
a) Factorisation and logarithms
b) Picturing QCD calculations
c) From probabilities to parton showers
d) Parton shower details



The hadronic cross section is

$$d\sigma(\mathsf{pp} \to \mu^+\mu^-\mathsf{g} + X) = dx dx_b f(x,t) f_b(x_b,t) d\hat{\sigma} \quad , \quad d\hat{\sigma} = \frac{\left|\mathcal{M}(\mathsf{u\bar{u}} \to \mu^+\mu^-\mathsf{g})\right|^2 d\Phi_{n+1}}{4\sqrt{(pp_b)^2}}$$

Factorisation: Divide and conquer

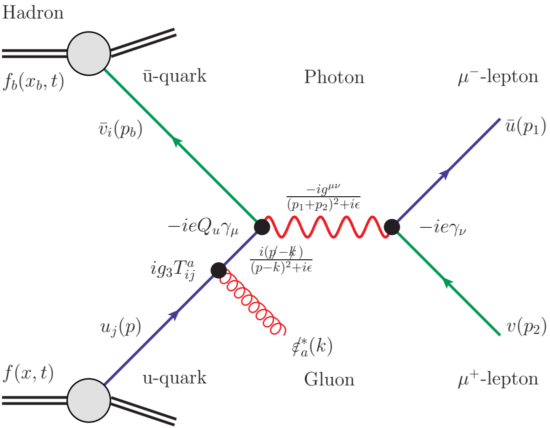


$E_{(p-k)} \approx zE_p$ and small gluon $p_\perp \Rightarrow$ Interal quark almost on-shell. Then:

$$\frac{i(\not p - \not k)}{(p-k)^2} \approx \frac{u(p_a)\bar{u}(p_a)}{p_a^2} \quad , \quad d\Phi_{n+1} \approx d\Phi_n \frac{d\phi dz dp_\perp^2}{4(2\pi)^3(1-z)} \quad , \quad \frac{1}{4\sqrt{(pp_b)^2}} \approx \frac{z}{4\sqrt{(p_a p_b)^2}}$$

$\Longrightarrow$ Matrix element, phase space integration and flux factors factorise!

Factorisation: Divide and conquer



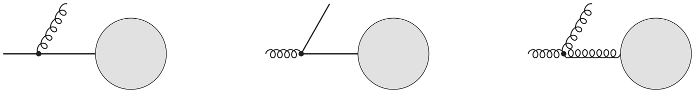
Matrix element, phase space integration and flux factors factorise:

$$d\sigma(\mathrm{pp}\to\mu^+\mu^-\mathrm{g}+X)=d\sigma(\mathrm{pp}\to\mu^+\mu^-+X)\int\frac{dp_\perp^2}{p_\perp^2}\frac{dz}{z}\frac{\alpha_s}{2\pi}C_F\frac{f(\frac{x_a}{z},t)}{f_a(x_a,t)}\frac{1+z^2}{1-z}$$

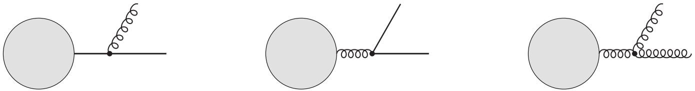
Every cross section containing an additional collinear gluon can be factorised as

$$d\sigma(\mathrm{pp} \to Y + \mathrm{g} + X) = d\sigma(\mathrm{pp} \to Y + X) \int \frac{dp_\perp^2}{p_\perp^2} \frac{dz}{z} \frac{\alpha_s}{2\pi} \frac{f(\frac{x_a}{z}, t)}{f_a(x_a, t)} P(z)$$

with the splitting kernels $P(z)$



$$P_{qq} = C_F \frac{1+z^2}{1-z} \qquad P_{qg} = T_R[z^2 + (1-z)^2] \qquad P_{gg} = C_A \frac{(1-z(1-z))^2}{z(1-z)}$$



This is independent of the process $\mathrm{pp} \to Y + X$!

$\Longrightarrow$ Multi-parton cross sections can be approximated by “dressing up” low-multiplicity results with many collinear partons!

Emission probabilities

The splitting kernels

... are independent of the "hard" scattering;

... have a probabilistic interpretation:

$$\int_{p^2_{\perp min}}^{p^2_{\perp max}} \frac{dp^2_\perp}{p^2_\perp} \int_{z_{min}}^{z_{max}} dz \frac{\alpha_s}{2\pi} P(z) \equiv$$

aa

Probability of emitting a gluon with momentum fraction $1 - z \in [z_{min}, z_{max}]$ and transverse momentum $p_\perp \in [p_{\perp min}, p_{\perp max}]$.

Also, note

$$\frac{dp^2_\perp}{p^2_\perp} = \frac{dQ^2}{Q^2} = \frac{d\Theta^2}{\Theta^2} = \frac{d\rho}{\rho} \quad (\text{for} \quad \rho = f(z) p^2_\perp)$$

$\Longrightarrow$ Many variables can be used to characterise the collinear limit!

...and note that we've put the z-range $[z_{min}, z_{max}]$. The lower limit z_{min} comes from the constraint $\frac{x_a}{z} < 1$, the upper limit when conserving 4-momentum.

Emission probabilities

Integrating the splitting probability, we get

$$\int_{p^2_{\perp min}}^{p^2_{\perp max}} \frac{dp^2_\perp}{p^2_\perp} \int_{z_{min}}^{z_{max}} dz \frac{\alpha_s}{2\pi} P(z) \approx \int_{p^2_{\perp min}}^{p^2_{\perp max}} \frac{dp^2_\perp}{p^2_\perp} \int_{z_{min}}^{z_{max}} dz \frac{\alpha_s}{2\pi} \frac{2C_{F/A}}{(1-z)}$$

$$\approx \alpha_s \ln\left(\frac{p^2_{\perp max}}{p^2_{\perp min}}\right) \ln\left(\frac{z_{max}}{z_{min}}\right)$$

More generally, we can write

$$d\sigma(\mathrm{pp} \to Y + \mathrm{g} + X) = d\sigma(\mathrm{pp} \to Y + X) \otimes \left(\alpha_s c_2 L^2 + \alpha_s c_1 L + \alpha_s c_0\right)$$

with $L = \ln\left(Q^2/p^2_{\perp min}\right)$, $Q^2 = \mathcal{O}(p^2_{\perp max})$, $p^2_{\perp min} = \mathcal{O}(\Lambda_{\mathrm{QCD}})$.

Even more generally

$$d\sigma(\mathrm{pp} \to Y + n\mathrm{g}) = d\sigma(\mathrm{pp} \to Y) \otimes \alpha_s^n \left(c_{2n} L^{2n} + c_{2n-1} L^{2n-1} + \cdots + c_0\right)$$

$$d\sigma(\mathrm{pp} \to Y + n\mathrm{g}) \approx d\sigma(\mathrm{pp} \to Y) \alpha_s^n c_{2n} L^{2n}$$

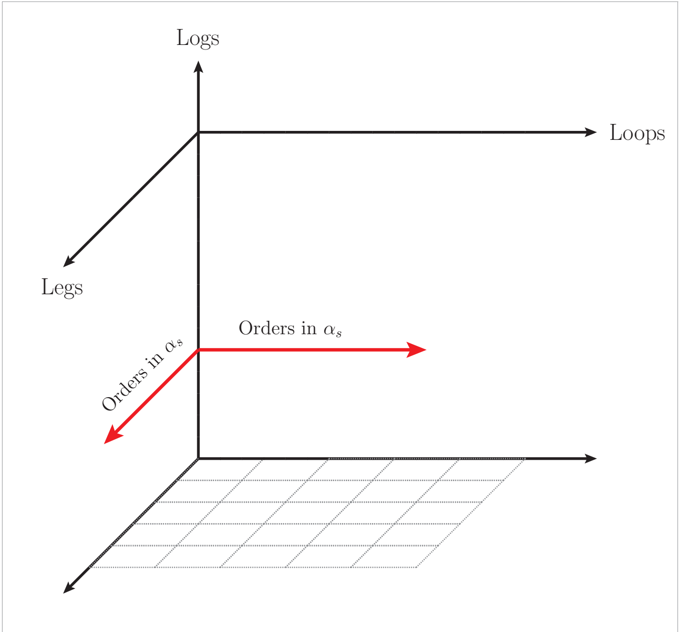
$\Rightarrow$ Multi-parton cross sections can be approximated by leading (double) log.
$\Rightarrow$ Comes from "dressing" low-multiplicity states with many collinear partons!

We found

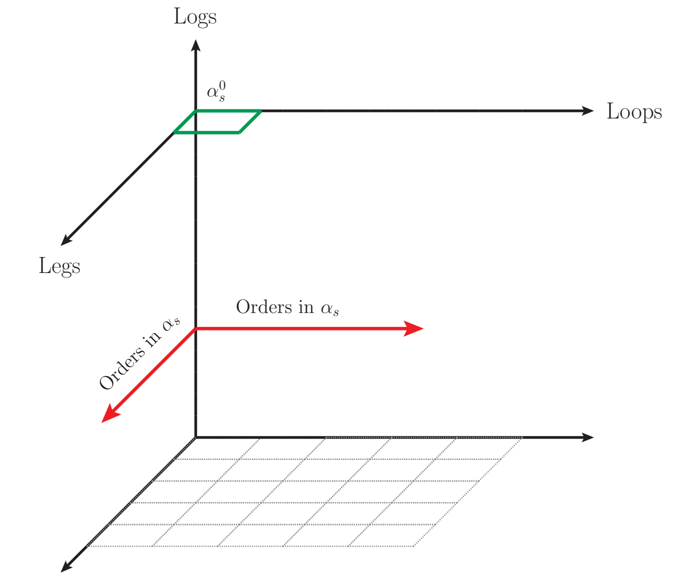
$$d\sigma(\mathsf{pp} \to Y + n\mathsf{g}) \approx d\sigma(\mathsf{pp} \to Y)\alpha_s^n c_{2n} L^{2n}$$

We can illustrate this logarithmic structure with a "legs-and-logs" plot.

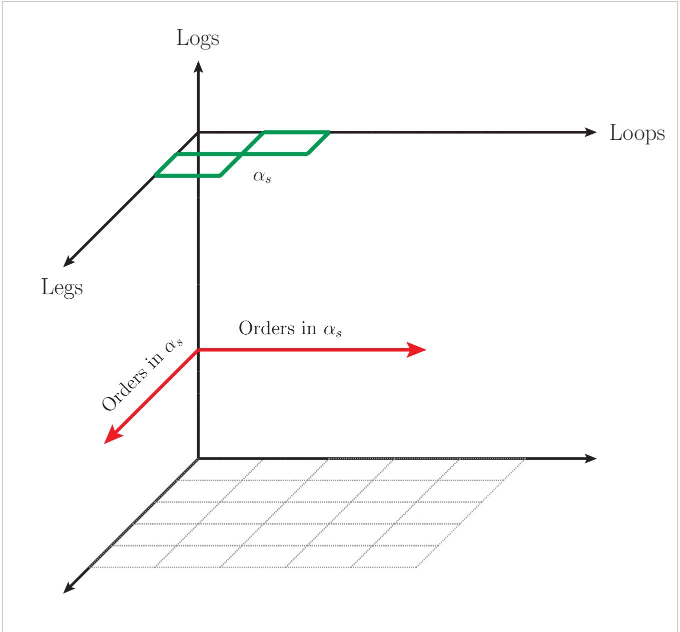
Symbolic figures for QCD calculations: α_s-orders fill diagonals



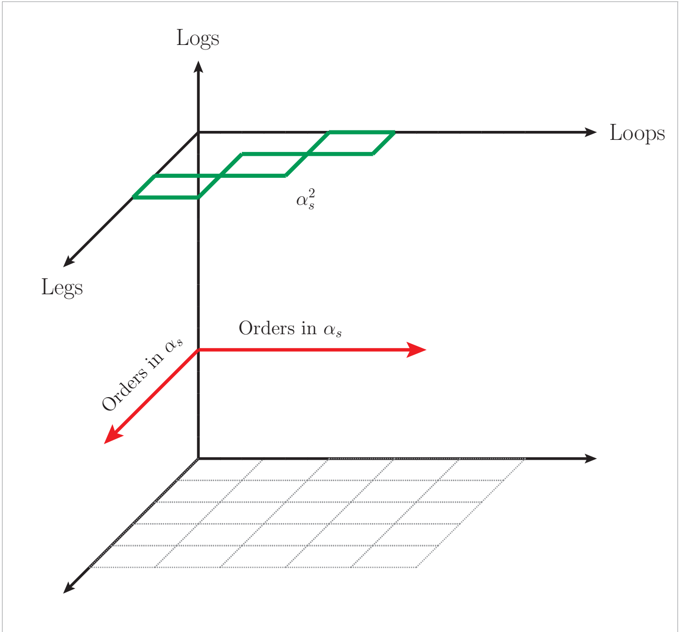
Symbolic figures for QCD calculations: α_s-orders fill diagonals: LO



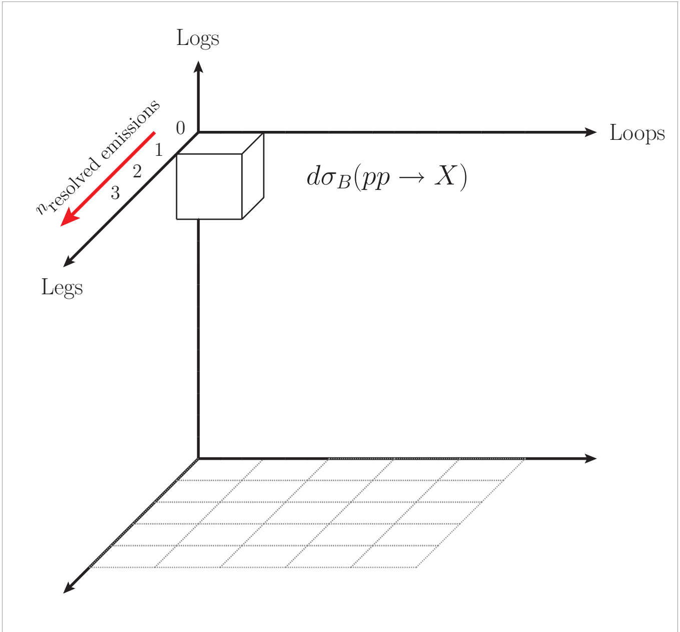
Symbolic figures for QCD calculations: α_s-orders fill diagonals: NLO



Symbolic figures for QCD calculations: α_s-orders fill diagonals: NNLO



Symbolic figures for QCD calculations: Tree-level terms fill towers



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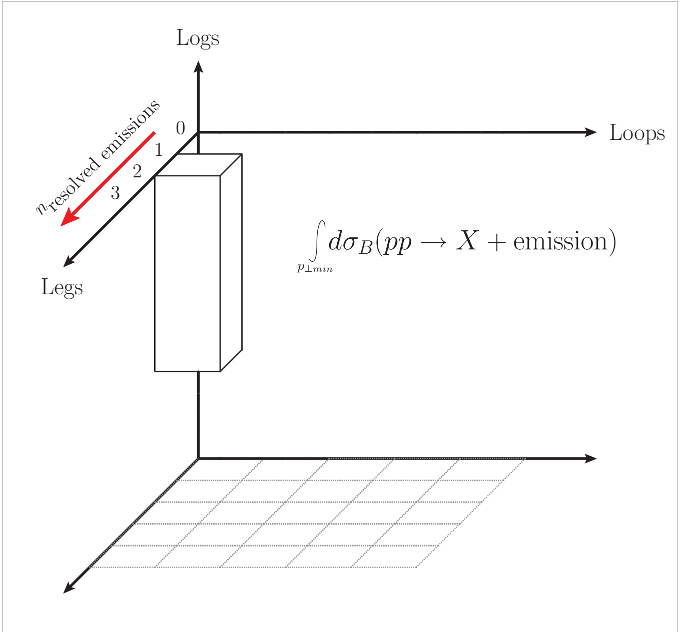
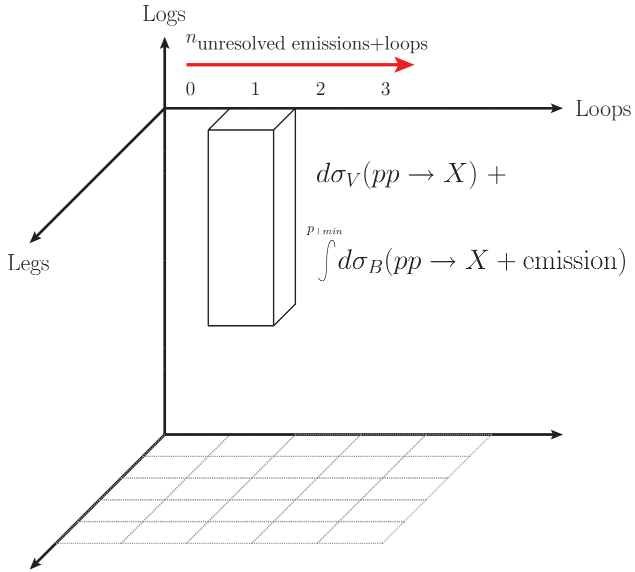
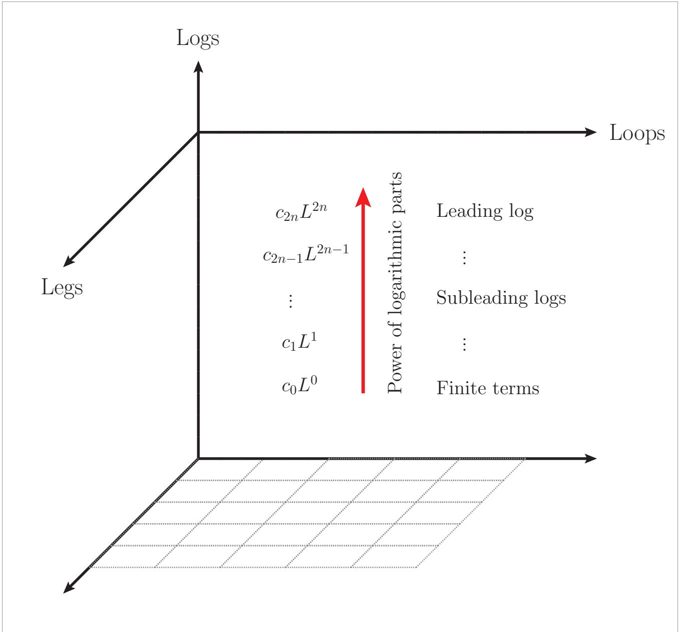


Symbolic figures for QCD calculations: Virtual corrections fill towers



Symbolic figures for QCD calculations: Towers are composed of logs



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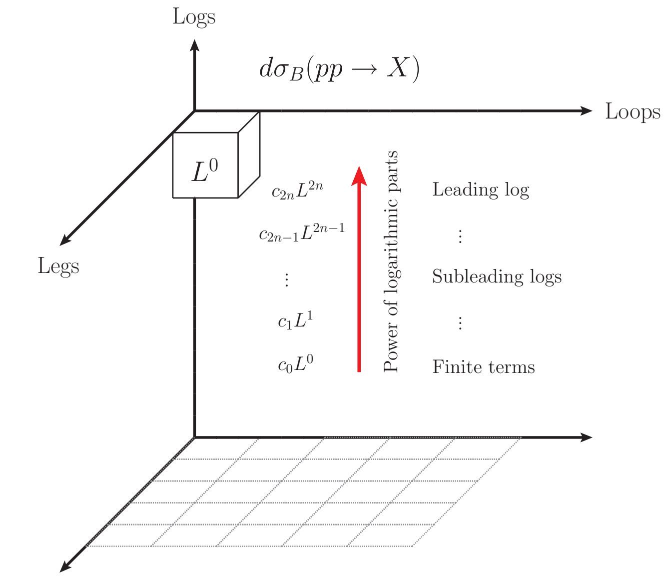

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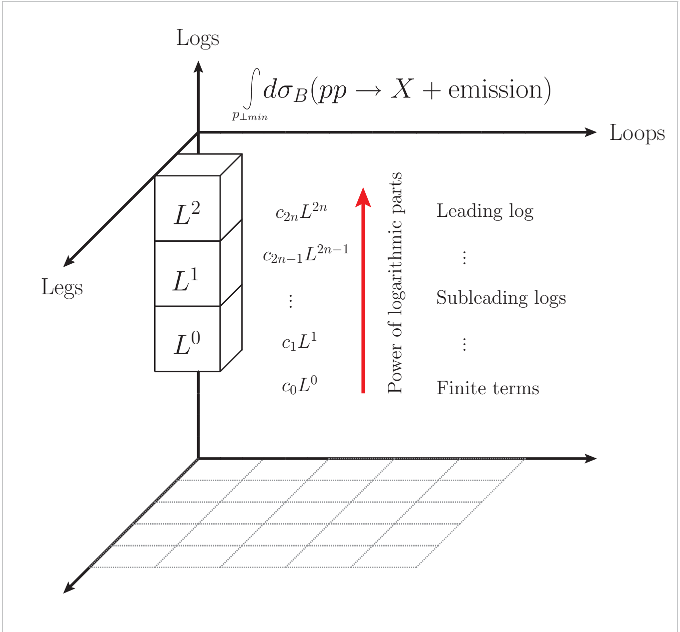


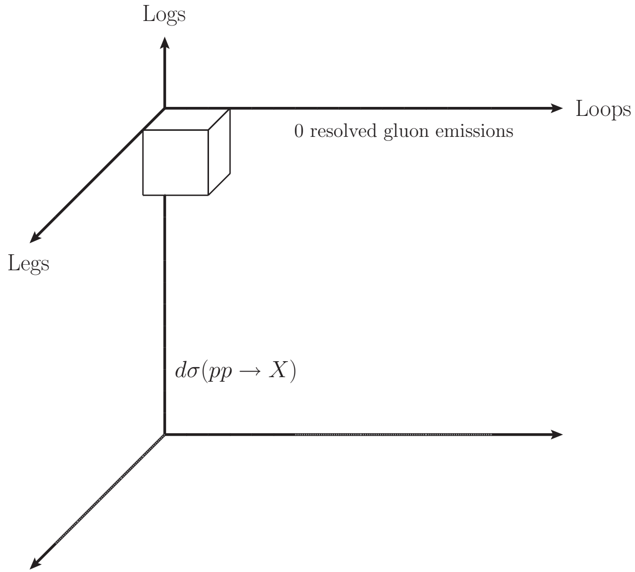
So far, we had $d\sigma(\mathrm{pp} \to Y + n\mathrm{g}) \approx d\sigma(\mathrm{pp} \to Y)\alpha_s^n c_{2n} L^{2n}$.

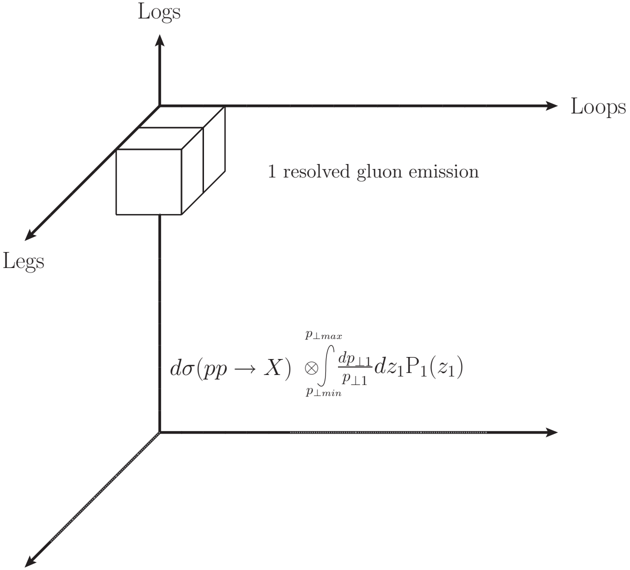
- (Multiple) gluon emission give the largest contribution to this multi-parton cross section.
- A more careful analysis shows: The dominant contributions to the cross section are produced by ordered emissions

$$\rho_0 > \rho_1 > \rho_2 > \ldots$$

Idea: Let's approximate the multi-parton cross section by multiplying splitting probabilities!

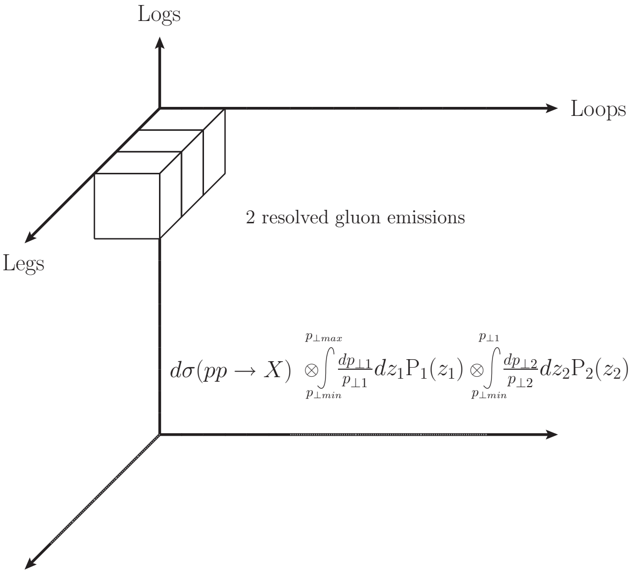
Iterating the collinear approximation: Hard process





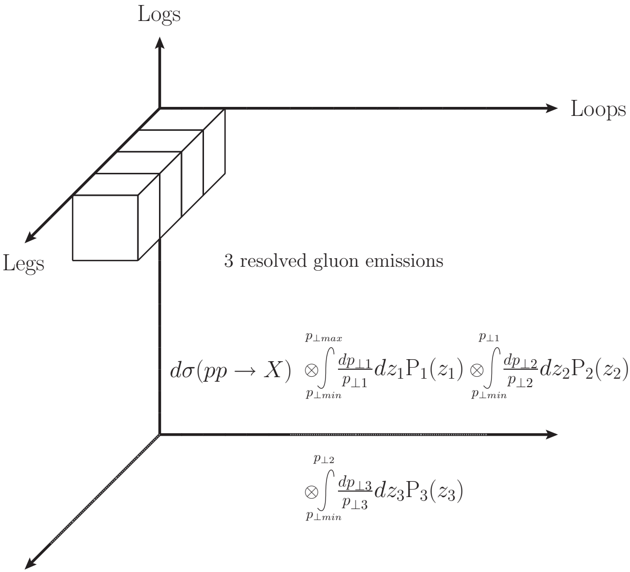
$$d\sigma(pp \to X) \otimes \int_{p_{\perp min}}^{p_{\perp max}} \frac{dp_{\perp 1}}{p_{\perp 1}} dz_1 \mathrm{P}_1(z_1)$$

Iterating the collinear approximation: Two emissions



$$d\sigma(pp \to X) \otimes \int_{p_{\perp min}}^{p_{\perp max}} \frac{dp_{\perp 1}}{p_{\perp 1}} dz_1 \mathrm{P}_1(z_1) \otimes \int_{p_{\perp min}}^{p_{\perp 1}} \frac{dp_{\perp 2}}{p_{\perp 2}} dz_2 \mathrm{P}_2(z_2)$$

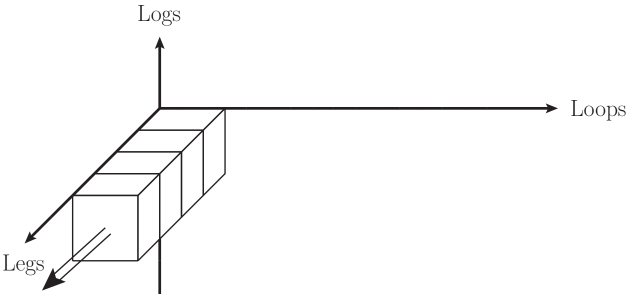
Iterating the collinear approximation: Three emissions



$$d\sigma(pp \to X) \otimes \int_{p_{\perp min}}^{p_{\perp max}} \frac{dp_{\perp 1}}{p_{\perp 1}} dz_1 \mathrm{P}_1(z_1) \otimes \int_{p_{\perp min}}^{p_{\perp 1}} \frac{dp_{\perp 2}}{p_{\perp 2}} dz_2 \mathrm{P}_2(z_2)$$

$$\otimes \int_{p_{\perp min}}^{p_{\perp 2}} \frac{dp_{\perp 3}}{p_{\perp 3}} dz_3 \mathrm{P}_3(z_3)$$

Iterating the collinear approximation: Infinitely many emissions



Infinitely many resolved gluon emissions

$$d\sigma(pp \to X) \otimes \int_{p_{\perp min}}^{p_{\perp max}} \frac{dp_{\perp 1}}{p_{\perp 1}} dz_1 \mathrm{P}_1(z_1) \otimes \int_{p_{\perp min}}^{p_{\perp 1}} \frac{dp_{\perp 2}}{p_{\perp 2}} dz_2 \mathrm{P}_2(z_2)$$

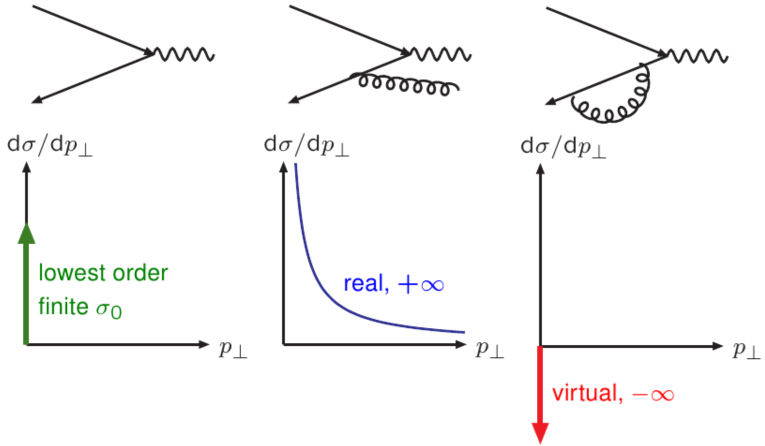
$$\otimes \int_{p_{\perp min}}^{p_{\perp 2}} \frac{dp_{\perp 3}}{p_{\perp 3}} dz_3 \mathrm{P}_3(z_3) \otimes \quad \cdots$$

Note that $d\sigma(\mathrm{pp} \to Y)\alpha_s^n c_{2n} L^{2n}$ is divergent as $p_{\perp min} \to 0$.

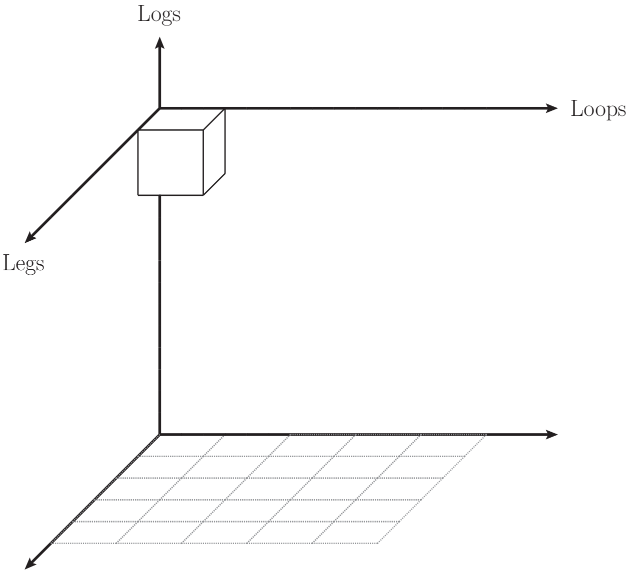
$\Longrightarrow$ To give a sensible approximation of the multi-parton cross section, we need to do more than just multiply splitting probabilities!

Pen-and-paper: Add Born + Real + Virtual

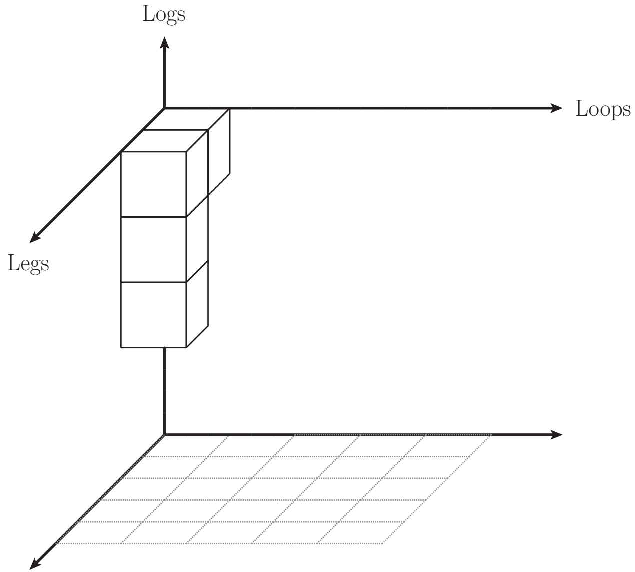
$$\langle\mathcal{O}\rangle^{\text{NLO}} = \int \mathrm{B}_n\mathcal{O}(\Phi_n)d\Phi_n + \int \mathrm{B}_{n+1}\mathcal{O}(\Phi_n)d\Phi_{n+1} + \int \mathrm{V}_n\mathcal{O}_n(\Phi_n)d\Phi_n$$



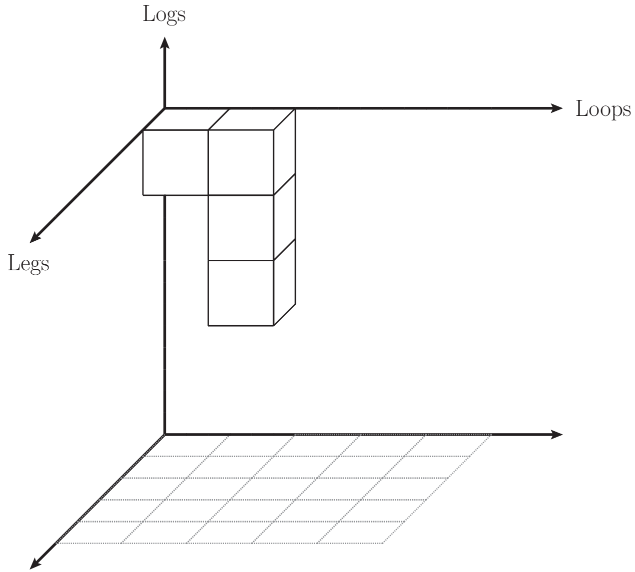
The KLN theorem: Lowest order is finite



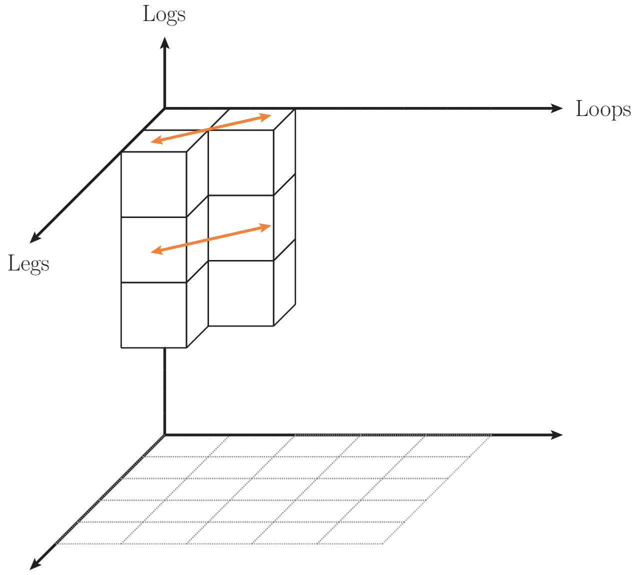
The KLN theorem: Real emissions diverge



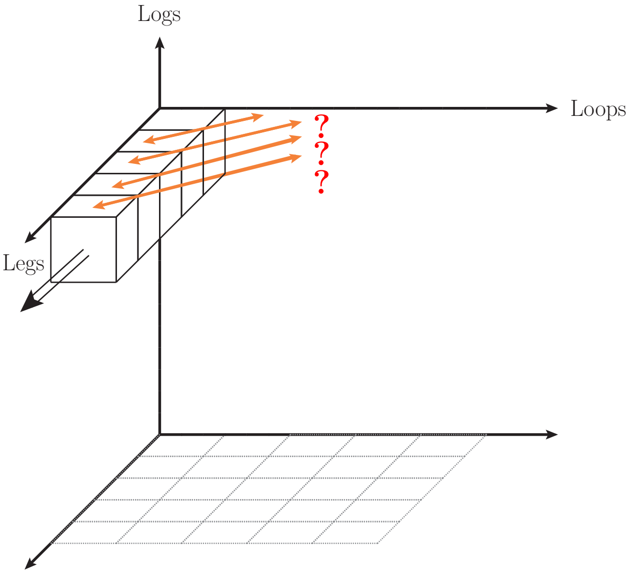
The KLN theorem: Virtual corrections diverge



The KLN theorem: Virtual + Real is finite ...because all logs cancel!



Can we cancel the product of splittings with all-order virtual corrections?



$\Longrightarrow$ To give a sensible approximation of the multi-parton cross section, we also need (approximate all-order) virtual corrections!

Approximate all-order virtual corrections form a Sudakov form factor

$$\begin{aligned}\Pi(\rho_0, \rho_{min}) &= \exp\left(-\int_{\rho_{min}}^{\rho_0} \frac{d\rho}{\rho} \int dz \frac{\alpha_s}{2\pi} P(z)\right) \\ &= 1 - \int_{\rho_{min}}^{\rho_0} \frac{d\rho_1}{\rho_1} \int_{z_{min}}^{z_0} dz_1 \frac{\alpha_s}{2\pi} P_1(z_1) \\ &\quad + \int_{\rho_{min}}^{\rho_0} \frac{d\rho_1}{\rho_1} \int dz_1 \frac{\alpha_s}{2\pi} P_1(z_1) \int_{\rho_{min}}^{\rho_1} \frac{d\rho_1}{\rho_1} \int dz_2 \frac{\alpha_s}{2\pi} P_2(z_2) \ + \ldots\end{aligned}$$

But how do we get there?

$\Longrightarrow$ Probabilities!

Taking probabilities seriously

We have already found:

$$\frac{\delta p_\perp^2}{p_\perp^2}\int_{z_1}^{z_0} dz \frac{\alpha_s}{2\pi} P(z) \equiv$$

Probability of an emission with $1 - z \in [z_1, z_0]$ and $p_\perp^2$ in the range $[p_{\perp min}^2, p_{\perp min}^2 + \delta p_\perp^2]$.

Then the probability of no emission is

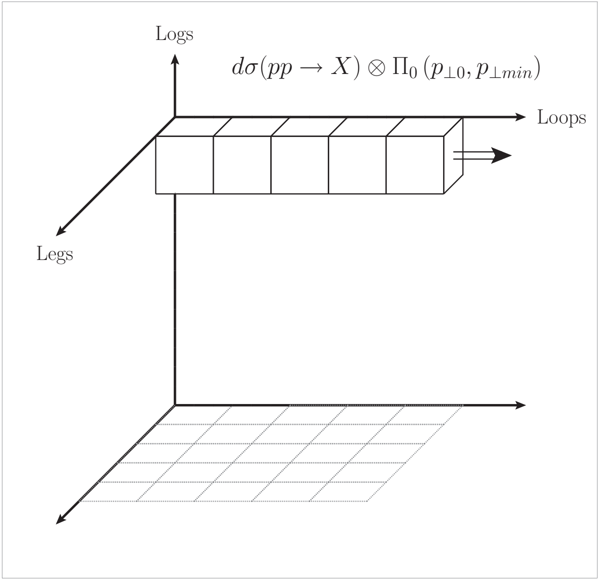
$$1 - \frac{\delta p_\perp^2}{p_\perp^2}\int_{z_1}^{z_0} dz \frac{\alpha_s}{2\pi} P(z)$$

or, if $\delta p_\perp^2$ is divided into n parts, and the no-emission probabilities are independent

$$\left[1 - \frac{\delta p_\perp^2/n}{p_\perp^2}\int_{z_1}^{z_0} dz \frac{\alpha_s}{2\pi} P(z)\right]^n \underset{n\to\infty}{\rightarrow} \exp\left(-\int_{p_{\perp min}^2}^{p_{\perp min}^2+\delta p_\perp^2} \frac{dp_\perp^2}{p_\perp^2}\int_{z_1}^{z_0} dz \frac{\alpha_s}{2\pi} P(z)\right)$$

The Sudakov factor is the probability of no resolvable emission in the range $[p_{\perp min}^2, p_{\perp min}^2 + \delta p_\perp^2]$, where resolvable means $1 - z \in [z_1, z_0]$.

The no-emission probability introduces all-order virtual corrections.
These do not change the number of legs $\Rightarrow$ Fill rows.



Branching probabilities

We have already found:

$$\int_{\rho_{min}}^{\rho_0} \frac{d\rho}{\rho} \int_{z_1}^{z_0} dz \frac{\alpha_s}{2\pi} P(z) \quad \equiv$$

Probability of a resolvable emission with $p_\perp^2$ in the range $[\rho_{min}, \rho_0^2]$.

$$\exp\left(- \int_{\rho_{min}}^{\rho_0} \frac{d\rho}{\rho} \int_{z_1}^{z_0} dz \frac{\alpha_s}{2\pi} P(z) \right) \equiv$$

Probability of no resolvable emission with $p_\perp^2$ in the range $[\rho_{min}, \rho_0]$.

We can construct an all-legs and all-loops result with probabilities only!
$\Longrightarrow$ Ideal for numerical iteration with random numbers.
$\Longrightarrow$ Monte Carlo parton showers.

So far, we only added gluons. It is possible to add photon emission, $g \to q\bar{q}$, $\gamma \to q\bar{q}$ etc. in the same way.
Also, we have not defined an ordering. Any ordering in ρ is allowed if $d\rho/\rho = dp_\perp^2/p_\perp^2$ (holds for angle, virtuality, $p_\perp$)

An algorithm to produce multiple emissions

0. Construct a state with no emissions (easy!).
1. Begin algorithm at a "largest $p_\perp$" ρ_{max} (evolution parameter).
2. Propose a new state with an emission at $\rho < \rho_{max}$.
3. Decide if the new state should be constructed according to the splitting function probability. If yes, construct the new state (need to conserve momentum in this step!)
4. Set $\rho_{max} = \rho$. Start from 1. (possibly with a new input state).

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4. Set $\rho_{max} = \rho$. Start from 1. (possibly with a new input state).

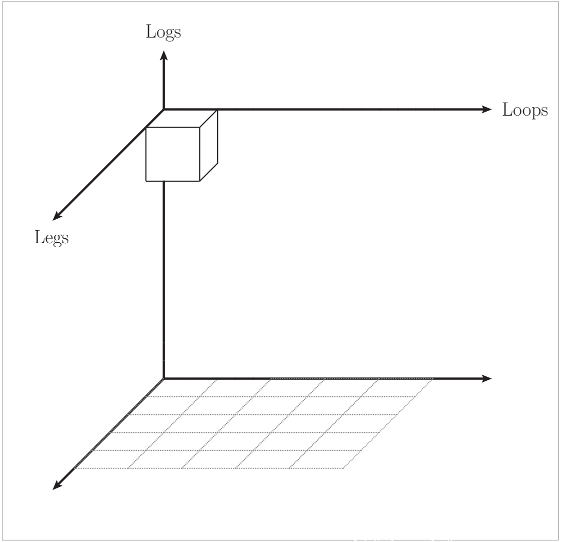
When the "$p_\perp$" is decreased by $\delta\rho$, there are two possibilities:

- ◇ The algorithm produced an emission at scale ρ.
- ◇ The algorithm did not produce an emission.

P(No emission above ρ_{min}) + P(No emission above ρ)$\times$ P(One emission at ρ)

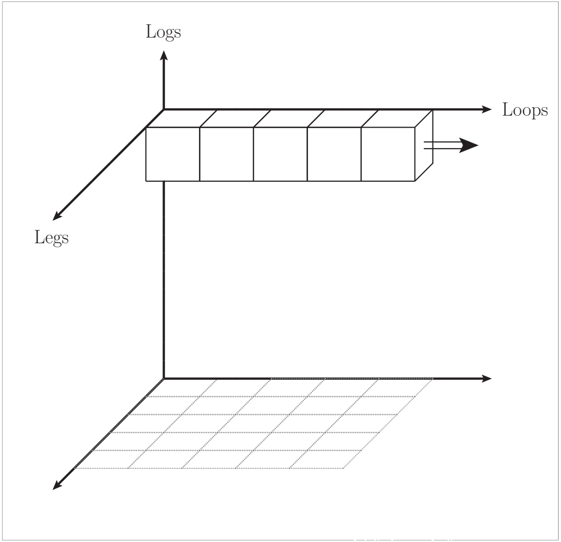
$$= d\sigma \otimes \Pi_0(\rho_0, \rho_{min})\ \mathcal{O}_0 \qquad + \ d\sigma \otimes \int^{\rho_0} \frac{d\rho}{\rho} \int^{z_0} dz \frac{\alpha_s}{2\pi} P(z)\ \Pi_0(\rho_0, \rho)\ \mathcal{O}_1$$

Parton shower: Fixed order input



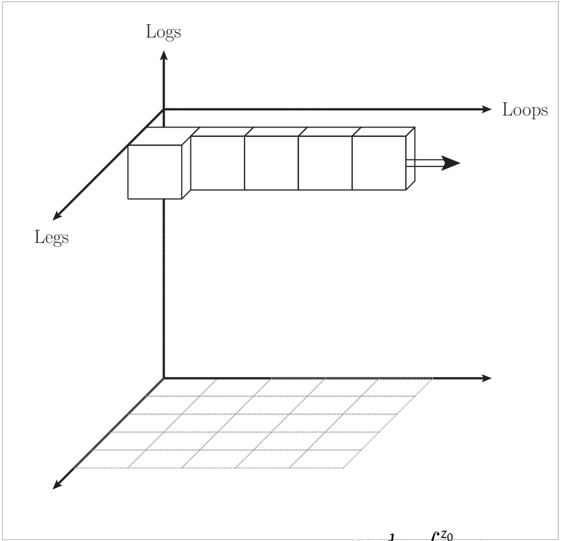
$d\sigma_B(pp \to X)$ $\mathcal{O}_0$

Parton shower: No emission



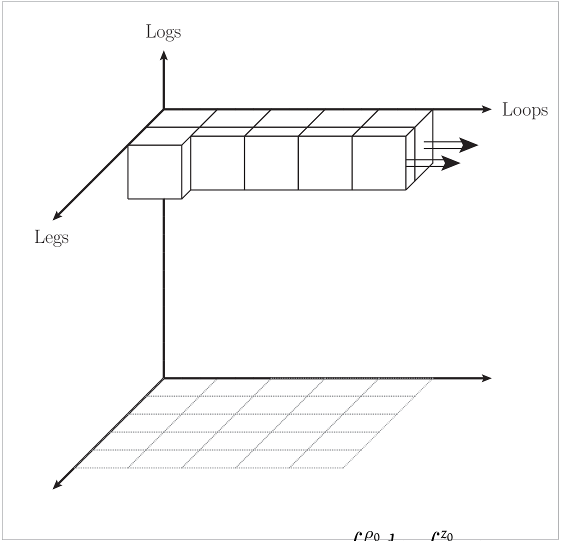
$d\sigma_B(pp \to X) \otimes \Pi_0(\rho_0, \rho_{min})$ $\mathcal{O}_0$

Parton shower: One emission at ρ



$$d\sigma_B(pp \to X) \otimes \quad \frac{d\rho}{\rho} \int_{z_1}^{z_0} dz \frac{\alpha_s}{2\pi} P(z)\ \Pi_0(\rho_0, \rho)\ \mathcal{O}_1$$

Parton shower: No or one emission



$$d\sigma_B(pp \to X) \otimes \Pi_0(\rho_0, \rho_{min})\ \mathcal{O}_0\ +\ d\sigma_B(pp \to X) \otimes \int_{\rho_{min}}^{\rho_0} \frac{d\rho}{\rho} \int_{z_1}^{z_0} dz \frac{\alpha_s}{2\pi} P(z)\ \Pi_0(\rho_0, \rho)\ \mathcal{O}_1$$

Each of these cross sections is finite because of Sudakov suppression:

$$d\sigma_B(pp \to X) \otimes \int_{\rho_{min}}^{\rho_0} \frac{d\rho}{\rho} \int_{z_1}^{z_0} dz \frac{\alpha_s}{2\pi} P(z)\ \Pi_0(\rho_0, \rho)\ \mathcal{O}_1 \underset{\rho_{min} \to 0}{\longrightarrow} \text{finite}$$

Now remember that we derived the no-emission probability from

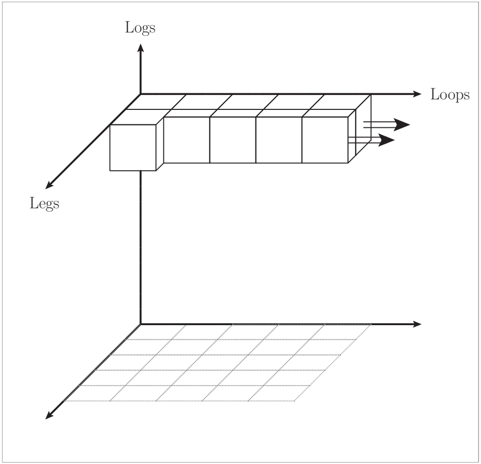
$$P_{\text{emission}} + P_{\text{no emission}} = 1$$

$\Longrightarrow$ The PS never changes the cross section, it only changes shapes.
This is called parton shower unitarity.

Unitarity means that parton showers define how the *inclusive* cross section is sliced up into *exclusive* cross sections:

$$\begin{aligned}
\sigma_{0\ or\ more\ jets} &= \sigma_{exactly\ 0\ jets} + \sigma_{1\ or\ more\ jets} \\
&= \sigma_{exactly\ 0\ jets} + \sigma_{exactly\ 1\ jet} + \sigma_{2\ or\ more\ jets}
\end{aligned}$$

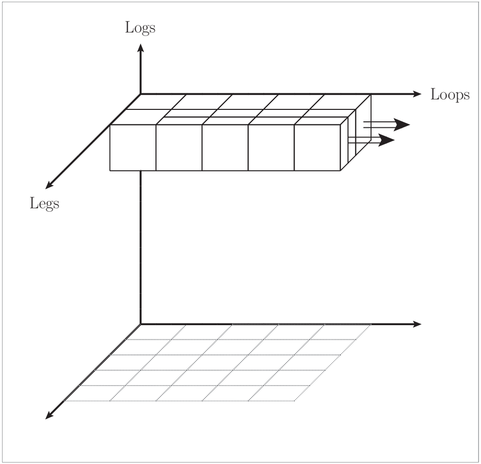
Parton shower example



PS generates

- no emission
- or one emission at ρ_1

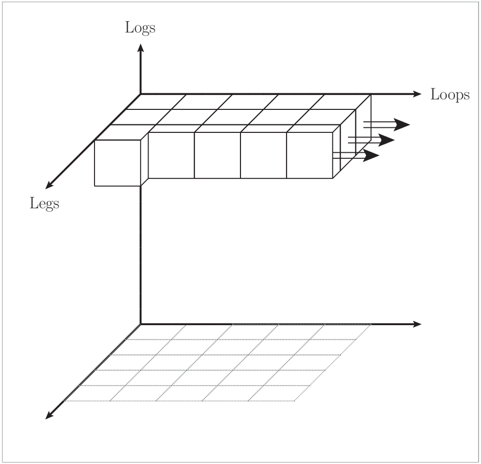
Parton shower example



PS generates

- no emission
- or one emission at ρ_1 and no further emission

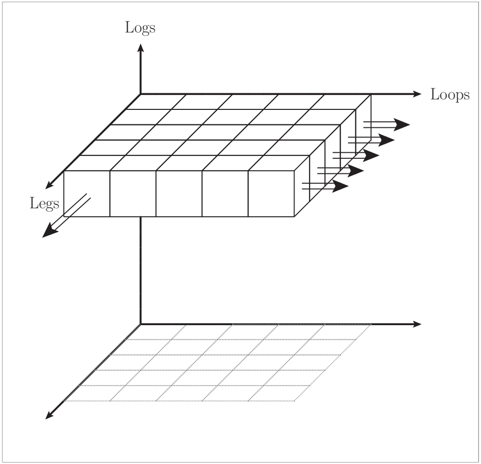
Parton shower example



PS generates

- no emission
- or one emission at ρ_1 and no further emission
- or one emission at ρ_1 and one at ρ_2

Parton shower example



PS generates

- no emission
- or one emission at ρ_1 and no further emission
- or one emission at ρ_1 and one at ρ_2
- and so on for arbirtary many emissions

Remember:

$$\Pi(\rho_0, \rho_1) = \exp\left(-\int_{\rho_1}^{\rho_0} \frac{d\rho}{\rho} \int_{z_1}^{z_0} dz \frac{\alpha_s}{2\pi} P(z)\right) \equiv$$

Probability of no resolvable emission with evolution scale in the range $[\rho_1, \rho_0]$.

$$\frac{d\rho}{\rho} \int_{z_1}^{z_0} dz \frac{\alpha_s}{2\pi} P(z) \Pi(\rho_0, \rho) \quad \equiv$$

Probability of a exactly one resolvable emission, with evolution scale ρ.

We will often call the no-emission probability "Sudakov (form) factor". The quark Sudakov form factor for a massless quark can be calculated analytically in QCD. For $q_\perp \to 0$, it reads

$$\Delta(\rho_0, \rho_1) = \exp\left(-\int_{q_\perp^2}^{Q^2} \frac{dp_\perp^2}{p_\perp^2} \frac{\alpha_s}{2\pi} C_F \left[\ln\left(\frac{Q^2}{p_\perp^2}\right) - \frac{3}{2} + \mathcal{O}\left(\frac{p_\perp^2}{Q^2}\right) \right] \right) \quad (1)$$

a) Assume that the parton shower splitting kernel is $P(z) = C_F \frac{1+z^2}{1-z}$, and that $z_1 = a_1 \frac{p_\perp}{Q} + a_2 \frac{p_\perp^2}{Q^2}$, $0 < z_1 < 1$ and $z_0 = 1 - z_1$. Write the no-emission probability, for $\frac{p_\perp}{Q} \to 0$, in the form of eq. (1). (Hint: Rewrite $P(z)$ so that you can clearly identify which term gives the logarithm and which term gives the constant) What are the phase space boundary z_0, z_1 necessary to match eq. (1)?

b) Now assume the splitting kernel $P(z, p_\perp^2) = C_F \frac{2(1-z)}{(1-z)^2 + p_\perp^2/Q^2} - (1+z)$. What is the form of z_0, z_1 now?

Which phase space is larger?

We have quietly dropped PDFs before. Keeping the PDFs, we would have arrived at

No-emission probability:

$$\Pi(\rho_0,\rho_1) = \exp\left(-\int_{\rho_1}^{\rho_0}\frac{d\rho}{\rho}\int_{z_1}^{z_0}\frac{dz}{z}\,\frac{\alpha_s}{2\pi}\,\frac{f_1(\frac{x}{z},\rho)}{f_0(x,\rho)}P(z)\right)$$

Probability of an emission with $x_{new} = \frac{x}{z}$ at evolution scale ρ:

$$\frac{d\rho}{\rho}\int_{z_1}^{z_0}\frac{dz}{z}\,\frac{\alpha_s}{2\pi}\,\frac{f_1(\frac{x}{z},\rho)}{f_0(x,\rho)}P(z)\Pi(\rho_0,\rho)$$

Note

$$\frac{d\ln\Pi}{d\ln\rho} = \int_{z_1}^{z_0}\frac{dz}{z}\,\frac{\alpha_s}{2\pi}\,\frac{f_1(\frac{x}{z},\rho)}{f_0(x,\rho)}P(z)$$

$\Rightarrow$ PDFs are crucial for radiating off an initial state parton.

Backward evolution

Remember: PDFs evolve according to the DGLAP equation, from small virtuality Q^2 to larger virtuality Q_0^2. PDFs are small at large Q_0^2.

Should parton showers do the same?

It would be **very** unlikely to "hit" a resonance (i.e. a Higgs or Z-boson propagator) in a narrow virtuality window at large Q_0^2.
$\Longrightarrow$ Simulating high-scale physics would be nearly impossible!

$\Longrightarrow$ Instead, reformulate DGLAP to evolve from large Q_0^2 and small x to smaller Q^2 and larger x/z.
$\Longrightarrow$ Backwards evolution.

DGLAP	: Sums up all emissions by evolving from Q^2 to Q_0^2
Backward evolution:	Performs all emissions (that had previously been summed up) by evolving from Q_0^2 to Q^2.

Review

Achievements so far:

- Found a way to approximate (one of) the largest contributions to a n-parton cross section: the collinear approximation
  ...and devised a probabilistic algorithm to produce this result.
- The parton shower produces finite results by introducing all-order (resummed) virtual corrections.
- We know how to treat emissions off final and initial state partons.

To get there, we needed

- To derive emission and no-emission probabilities.
- Find a prescription for momentum conservation - otherwise, we cannot iterate the procedure.
- We had to define an evolution scale ρ to reproduce the dominant terms.

But...

- Momentum conservation can be implemented in many different ways.
- The evolution scale ρ can be defined freely, as long as $d\rho/\rho = dp_\perp^2/p_\perp^2$.
  This e.g. allows (relative) angle, virtuality, $p_\perp^2$ ...

Choosing an ordering variable: Double-counting and hardness

Backward evolution in the initial state means evolving from a "hard process" at large momentum transfer to smaller momentum transfers.

The hard process is the "starting point" of the radiation cascade.

We want to start from an "exact" result, i.e. a good description of the inclusive cross section with n partons, and produce approximate higher order corrections.

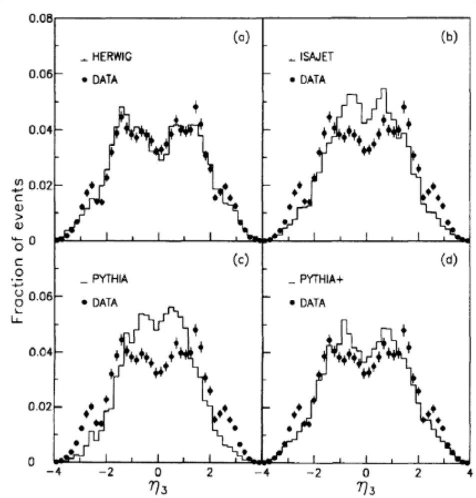
If the evolution scale is defined such that after some emissions, a "harder" process is generated, then the exact starting point is obscured, and we cannot do backward evolution.

$\Longrightarrow$ Initial state showers suggest to use a "hardness" ordering, i.e. where large momentum transfers happen early in the cascade (e.g. Q^2 or $p_\perp^2$).

Choosing an ordering variable: Is virtuality ordering safe?

Ordering:
PYTHIA: Virtuality,
HERWIG: Something else.

$\Longrightarrow$ Something is missing.



$\Longrightarrow$ Virtuality ordering did not capture the physics!
$\Longrightarrow$ Missing another important ingredient!

When trying to find an approximation of additional gluon emissions, we found that the largest contribution to $Q_i(p_i + k) \to Q_i'(p_i) + g(k)$ arose from an on-shell propagator

$$u(p_i)\not{\varepsilon}\,\frac{(\not{p}_i + \not{k})}{(p_i + k)^2} = u(p_i)\,\frac{p_i \varepsilon}{2 p_i k} = u(p_i)\,\frac{p_i \varepsilon}{(1-z) E_{Q_i}^2 (1 - \cos\Theta_{Q_i g})}$$

Apart from collinear divergence $\Theta_{Q_i g} \to 0$, there is also a soft divergence $z \to 1$.

$\Longrightarrow$ We were missing the soft piece before!

For $z \to 1$, already the amplitudes universally factorise. Thus, upon squaring

$$d\sigma_{n+1} = d\sigma_n \int \frac{dw}{w}\,\frac{d\Omega}{2\pi}\,\frac{\alpha_s}{2\pi} \sum_{ij} C_{ij} W_{ij}$$

$$\text{with} \quad W_{ij} = \frac{1 - \cos\Theta_{Q_i Q_j}}{(1 - \cos\Theta_{Q_i g})(1 - \cos\Theta_{Q_j g})}$$

$\Longrightarrow$ QM interference between gluon emission off partons Q_i and Q_j!

How can soft emissions be independent?

How can soft emissions be independent?
Let us write

$$W_{ij} = W_{ij}^1 + W_{ij}^2 \quad \text{with} \quad W_{ij}^i = \frac{1}{2}\left(W_{ij} + \frac{1}{(1-\cos\Theta_{Q_i g})} - \frac{1}{(1-\cos\Theta_{Q_j g})} \right)$$

Then, after integrating over the azimuthal angle, we get

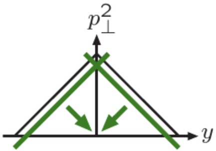
$$\int \frac{d\phi_{Q_i g}}{2\pi} W_{ij}^i = \begin{cases} \frac{1}{(1-\cos\Theta_{Q_i g})} & \text{for} \quad \Theta_{Q_i g} < \Theta_{Q_i Q_j} \\ 0 & \text{else} \end{cases}$$

Soft emissions are independent if ordered in emission angle!
Another (opening cone) argument shows: $p_\perp$-ordered final state emissions are okay as well.

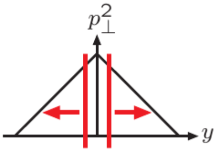
HERWIG had angular ordering in the CDF plot. Color coherence necessary to describe data! But angle does not define hardness!

We found: Hardness ordering (Q^2, $p_\perp^2$) motivated by ISR, Θ ordering by soft limit. Both mutually exclusive!

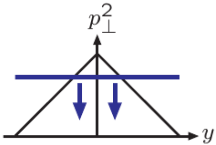
Q^2



$E^2\Theta^2$



$p_\perp^2$



Virtuality

- Defines hardness, as necessary in ISR.
- No coherence. Additional vetoes necessary.

Angle

- Does not define hardness. Additional vetoes necessary.
- Coherence by construction.

$p_\perp$

- Defines hardness, as necessary in ISR.
- Coherence in FSR. ISR not clear.

Is it hopeless? No! $\Longrightarrow$ Dipole/antenna showers.

In the soft limit, we found

$$d\sigma_{n+1} = d\sigma_n \int \frac{dw}{w} \frac{d\Omega}{2\pi} \frac{\alpha_s}{2\pi} \sum_{ij} C_{ij} W_{ij}$$

and after writing

$$W_{ij} = W_{ij}^1 + W_{ij}^2 \quad \text{with} \quad W_{ij}^i = \frac{1}{2} \left(W_{ij} + \frac{1}{(1 - \cos\Theta_{Q_i g})} - \frac{1}{(1 - \cos\Theta_{Q_j g})} \right)$$

derived angular ordering.

But we could have directly used W_{ij} as splitting probability ($\equiv$ QCD antenna), or partitioned cleverly ($\equiv$ QCD dipole).

Both antennae and dipoles can be inferred from NLO subtraction methods. This means they come with a well-defined phase space mapping.

Energy-momentum conservation

We have stressed the importance of energy-momentum conservation, but not given a prescription.

NLO subtraction formalisms give a one-to-one correspondence

$$d\Phi_{n+1} = d\Phi_n d\hat{\Phi} = d\Phi_n J(\rho, z, \phi) d\rho dz \frac{d\phi}{2\pi}$$

which maps an on-shell n-particle phase space point unto an on-shell $n+1$-particle configuration. The $n+1$-particle is completely covered.

This can be achieved by

- aborbing the "recoil" of a $1 \to 2$ splitting with a spectator (dipoles).
- performing $2 \to 3$ splittings (antennae).

$\Rightarrow$ Modern showers are all built in this way!

Momentum conservation in each intermediate step is the major advantage compared to analytical tools. It also makes systematic step-by-step improvements possible ($\to$ next lecture).

Freedom in the recoil scheme is an uncertainty of exclusive prediction!

Running scales

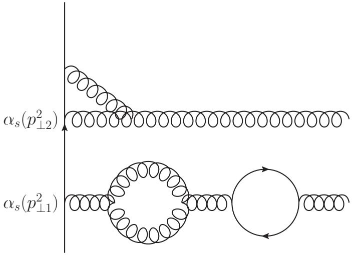
Until now, we have found:

- Parton showers generate the leading collinear logarithms. Angular ordering (or modern showers) include the soft limit as well.
- Local momentum conservation (formally beyond LL) is included.
- Initial state radiation requires PDF evaluations at dynamical scales (e.g. Q^2, $p_\perp^2$ of the branching).

Another important improvement is evaluation of α_s at dynamical scales $\alpha_s = \alpha_s(p_\perp^2)$.

This is known as Modified Leading Log Approximation. This resums dominant universal propagator corrections to all orders.

After this improvement, many more soft emissions are produced. The PS must ensure to avoid the Landau pole (e.g. $p_{\perp min} > \Lambda_{QCD}$).



Parton showers are usually part of event generator frameworks.
Commonly used event generators for LHC physics are

- **HERWIG++**: Improved angular ordered $\tilde{q}$ shower and $p_\perp$-ordered Catani-Seymour dipole shower.
- **PYTHIA 8**: $p_\perp$-ordered dipole shower based on DGLAP+ME-corrections, and VINCIA antenna shower as FSR plugin.
- **SHERPA** : $p_\perp$-ordered Catani-Seymour dipole shower, ANTS antenna shower

All three include QED radiation, EW effects, underlying event, diffractive modelling, hadronisation, higher-order improvements, hadron decays…

Other public QCD shower programs outside event generators include ARIADNE, CASCADE, DEDUCTOR, HERWIRI…

- QCD scattering cross sections factorise in the soft / collinear limits.
- The factorisation is universal, and can be viewed as probabilistic.
- The existence of emission and no-emission probabilities makes all-order (all-legs) numerical implementations possible.
- Parton showers require an ordering criterion. Hardness and angle are well-motivated, but not without pitfalls.
- Almost all modern showers are based on antennae or dipoles.
- With the inclusion of soft effects, momentum conservation and running scales, many (all-order) refinements are added.

...but parton showers only describe soft or collinear emissions!
We need to work harder to describe hard well-separated emissions!

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