TALES OF QUANTUM MECHANICS

A JOURNEY FROM FEW- TO MANY-BODY PHYSICS

NIKOLAJ THOMAS ZINNER
DEPARTMENT OF PHYSICS AND ASTRONOMY

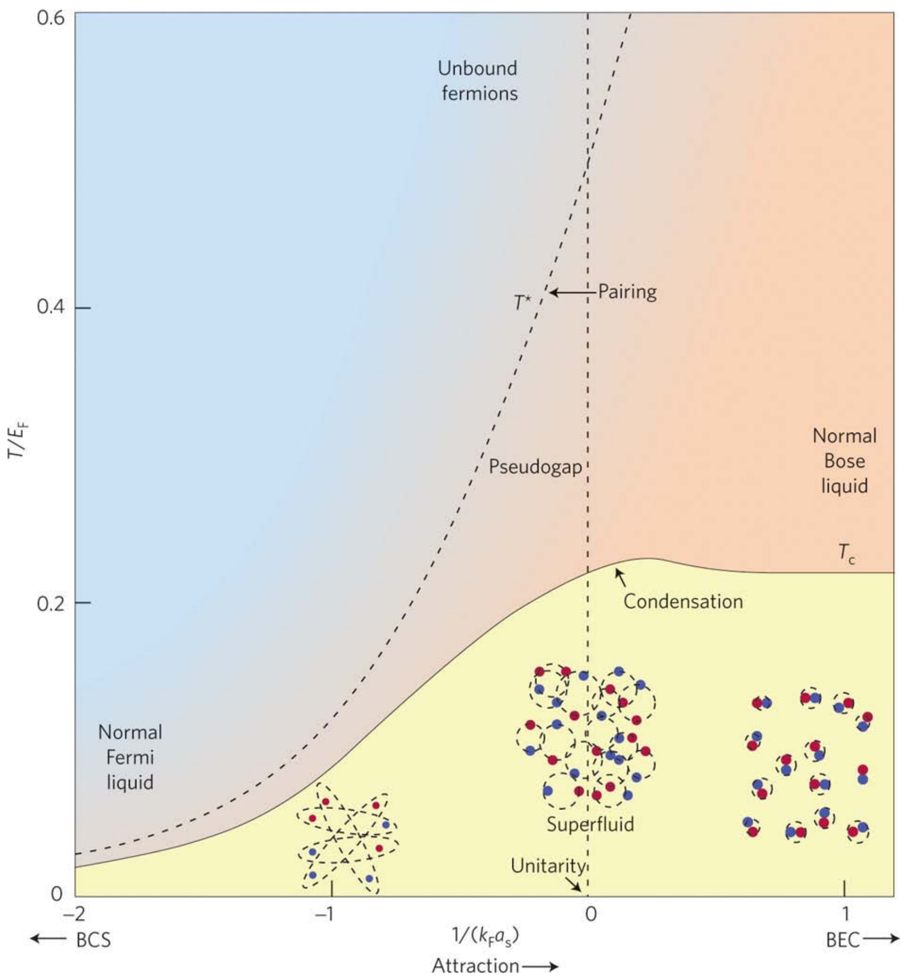
ICTP SAIFR International Centre for Theoretical Physics South American Institute for Fundamental Research

São Paulo International Schools on Theoretical Physics

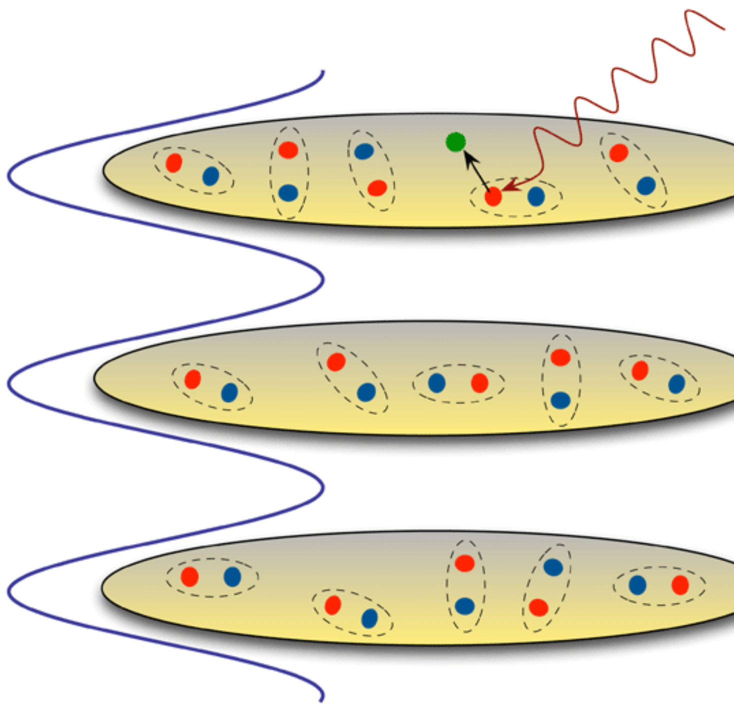
FAPESP ICTP unesp IFT

Mini-school on Few-body Physics

FROM FEW- TO MANY-BODY



Mohit Randeria, Nature Phys. **6**, 561 (2010)



Mohit Randeria, Physics **5**, 10 (2012)

COOPER PAIRS

Cooper considered a two-body problem with the restriction introduced by the Pauli principle

$$E \sim 2E_F - E_F \exp\left(\frac{1}{k_F a}\right), \;\; a \to 0^-$$

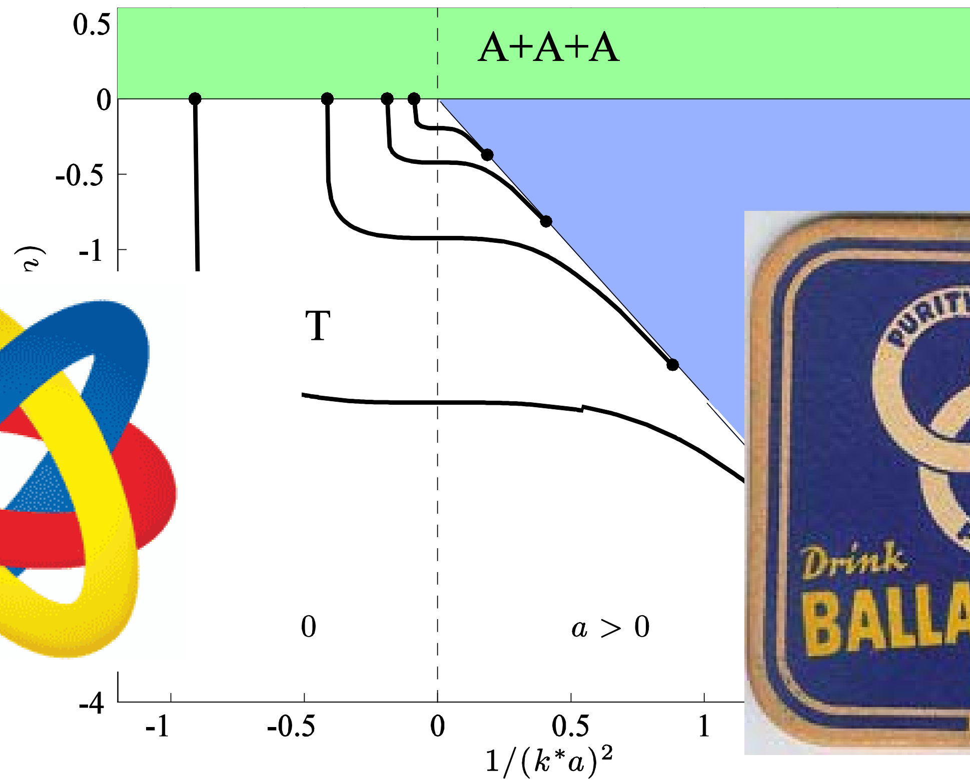
Concerns two spin components. In cold atoms there can be more than two. Either use several hyperfine states or use a mixture

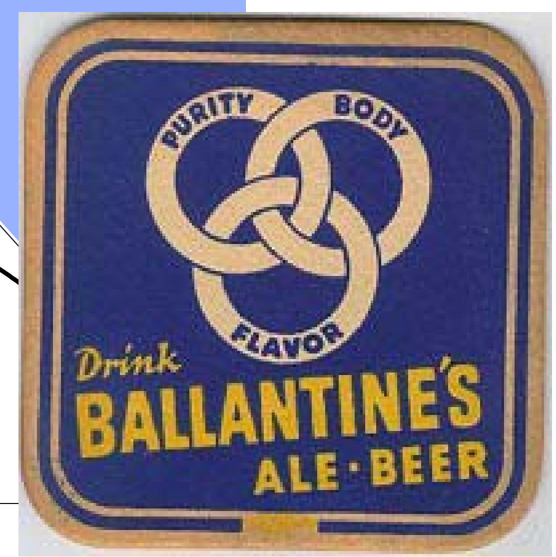
Is there an equivalent Cooper-like problem for three particles?

EFIMOV EFFECT MUST BE CONSIDERED

Vitaly Efimov 1970

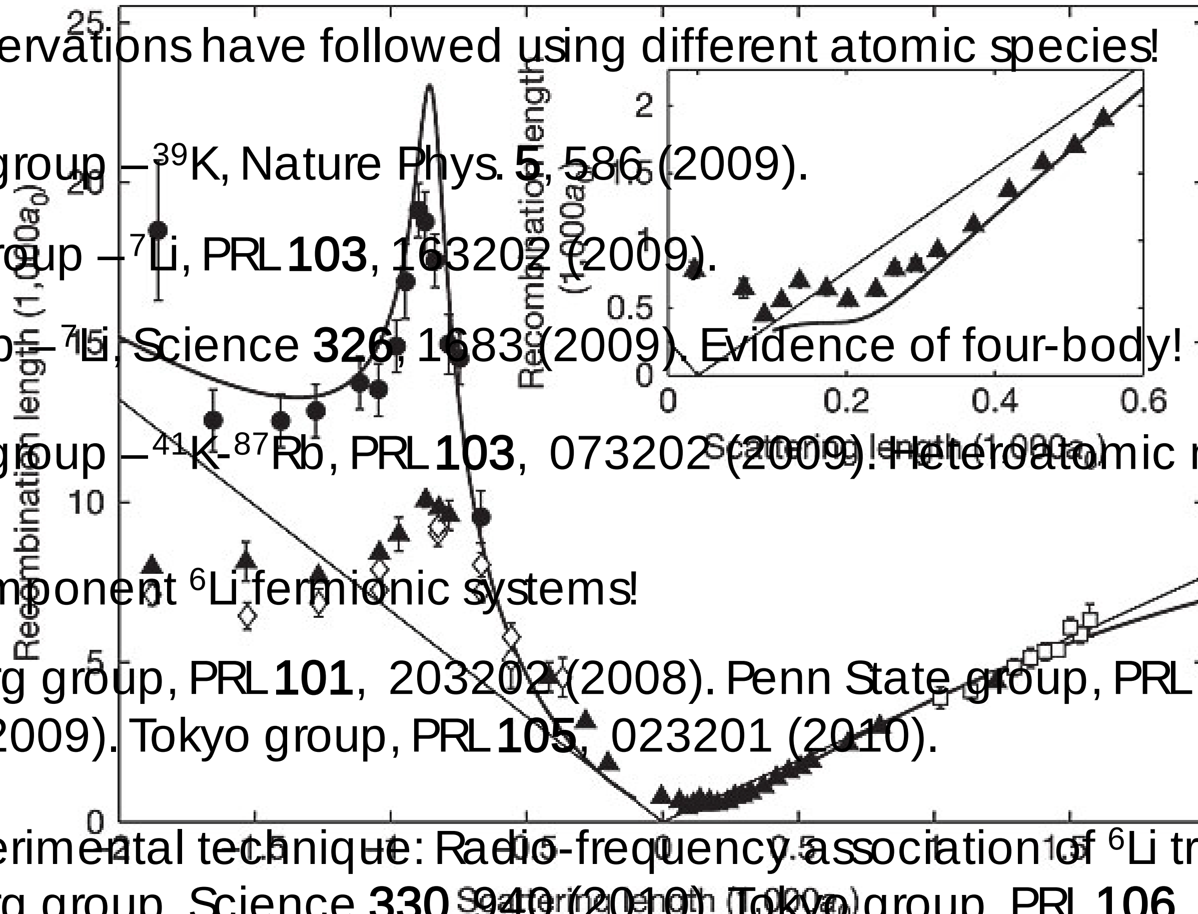
Identical bosons in 3D have an infinite ladder of three-body bound states when there is a two-body bound state at zero energy





ULTRACOLD ATOMS

Experimental observation – Grimm group. 133-Cesium Nature **440**, 315 (2006).



Many observations have followed using different atomic species!

Florence group – ^{39}K, Nature Phys. **5**, 586 (2009).

Bar Ilan group – ^{7}Li, PRL **103**, 163202 (2009).

Rice group – ^{7}Li, Science **326**, 1683 (2009). Evidence of four-body!

Florence group – ^{41}K-^{87}Rb, PRL **103**, 073202 (2009). Heteroatomic mixture!

Three-component ^{6}Li fermionic systems!

Heidelberg group, PRL **101**, 203202 (2008). Penn State group, PRL **102**, 165302 (2009). Tokyo group, PRL **105**, 023201 (2010).

New experimental technique: Radio-frequency association of ^{6}Li trimers. Heidelberg group, Science **330**, 940 (2010). Tokyo group, PRL **106**, 143201 (2011).

BACKGROUND EFFECTS?

External confinement

Non-universality

Finite temperature

Quantum degeneracy

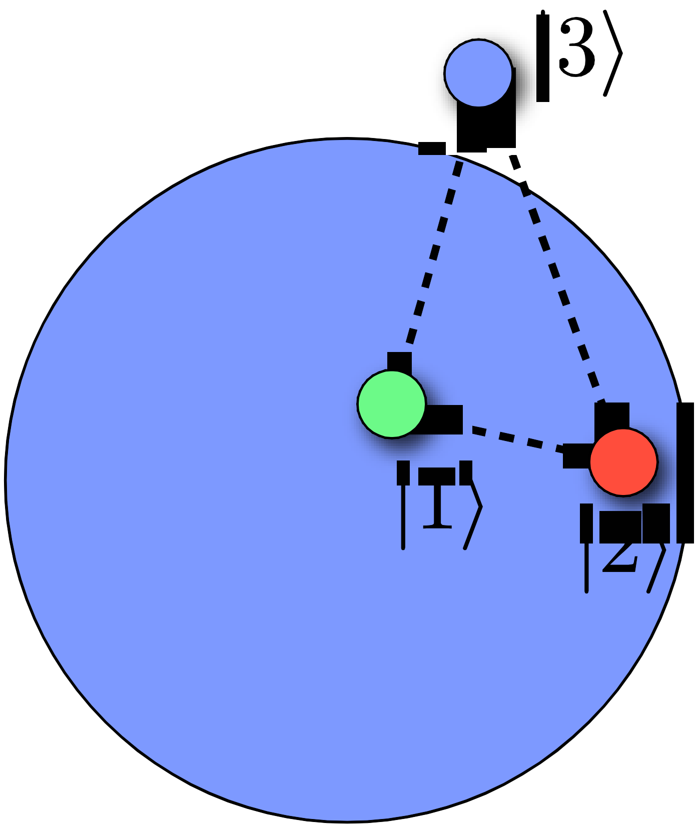
Condensed Bose or degenerate Fermi systems

REDUCTIONISM

Consider a single Fermi sea and two other particles

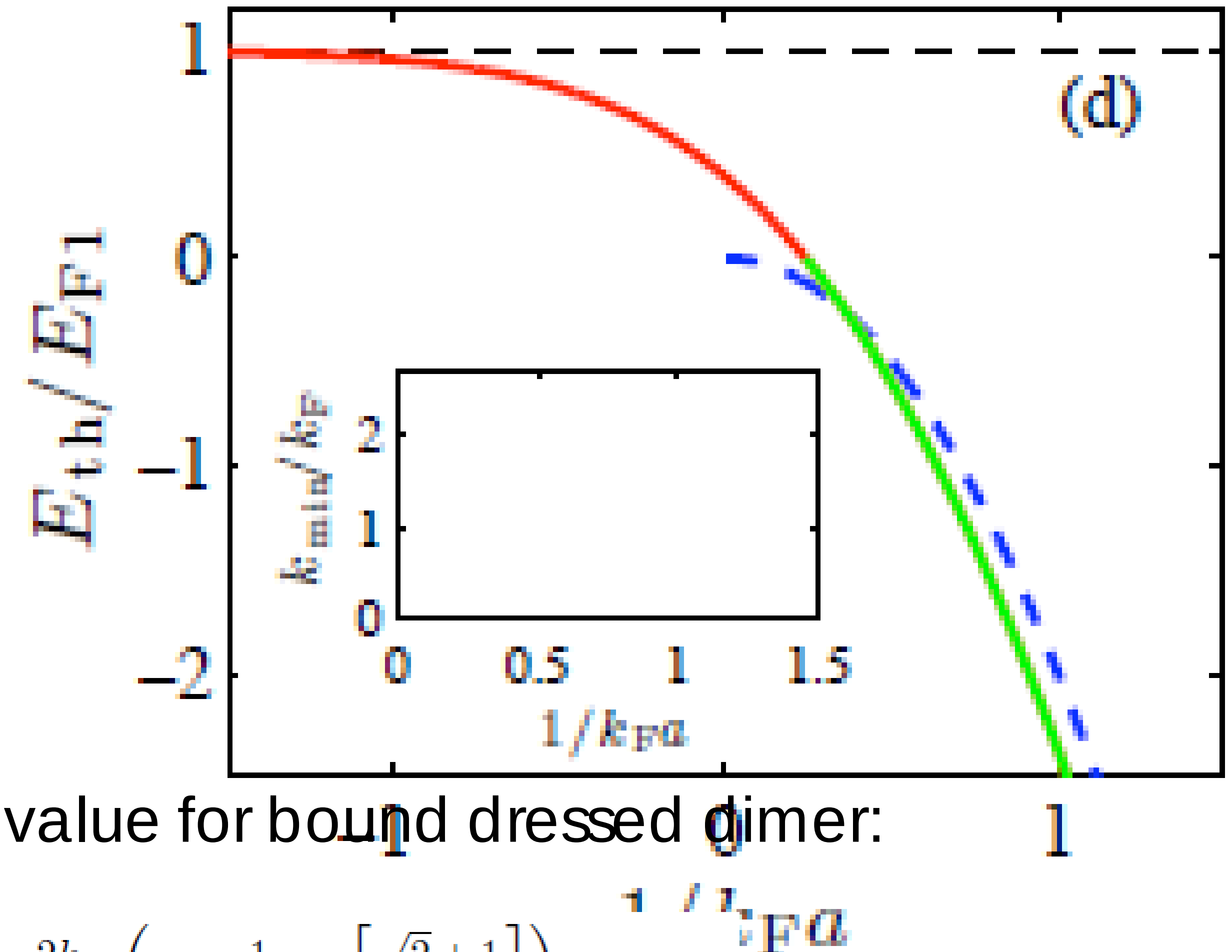
Pauli principle is simpler to handle in momentum space

Turns out the two-body physics is the same as for the Cooper pair problem



COOPER PAIR INSPIRATION

D(**q**,E)



$\overline{\overline{E + \mathrm{q}^2/4m}}$

$D(\mathrm{q}, I$

$\overline{+ i\epsilon}$

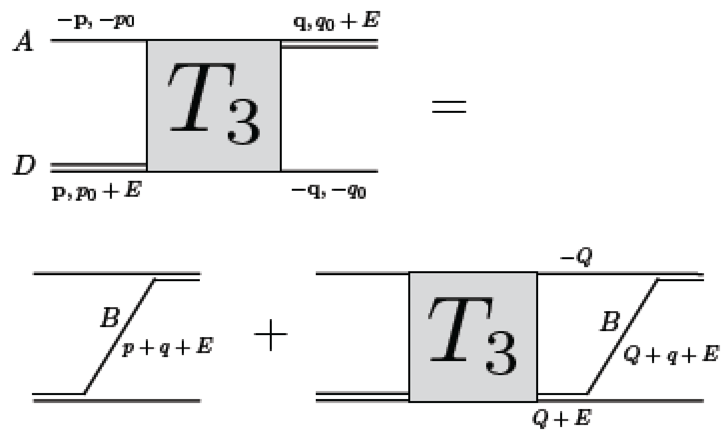
Critical value for bound dressed dimer:

$$\frac{1}{a_c} = \frac{2k_F}{\pi}\left(1 - \frac{1}{2^{3/2}} \ln\left[\frac{\sqrt{2}+1}{\sqrt{2}-1}\right]\right) = 0.24k_F$$

THREE-BODY PROBLEM

Momentum-space three-body equations

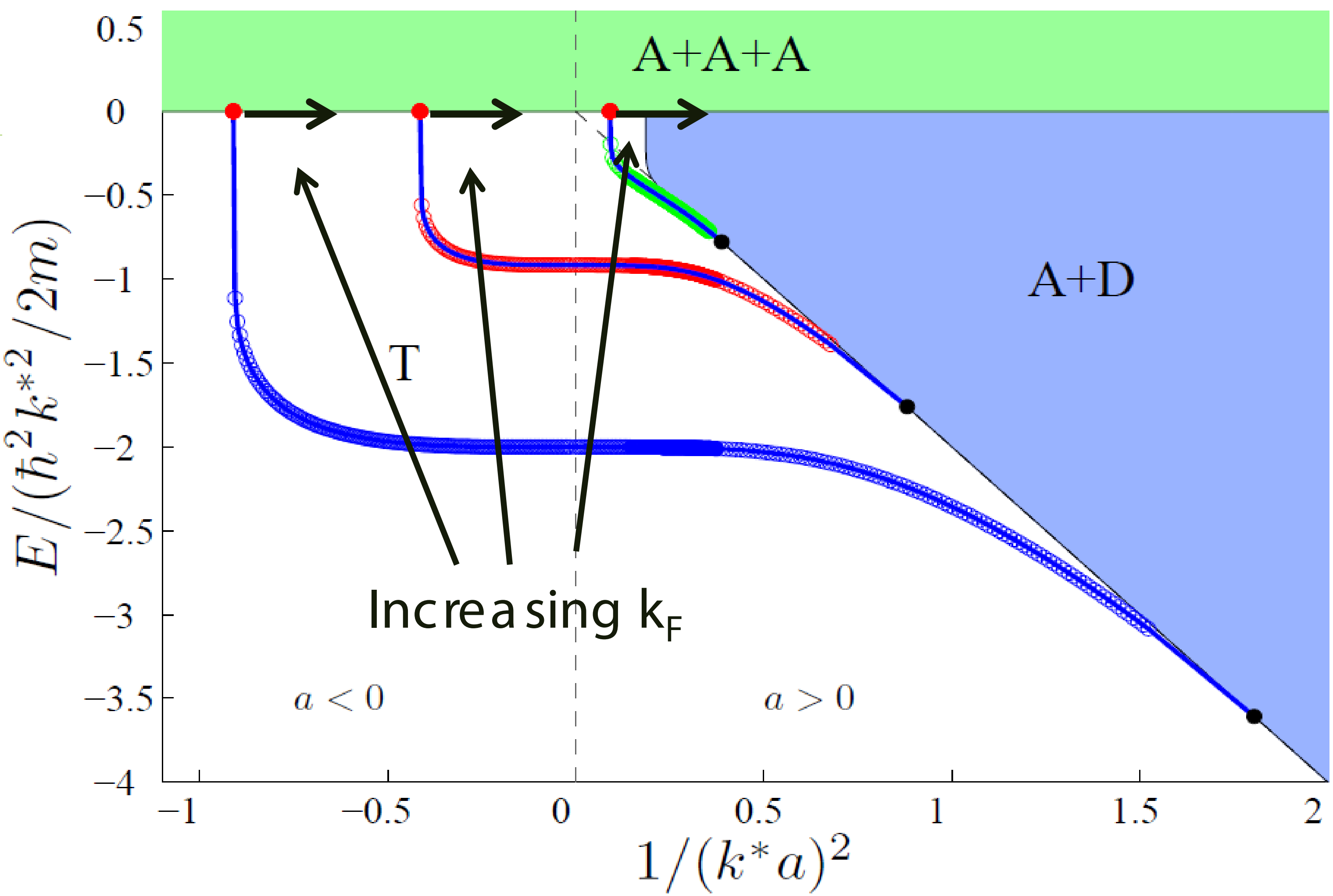
Skornyakov and Ter-Martirosian, Zh.Eksp. Teor. Fiz. **31**, 775 (1956).



Bound states:

$$B(p) = \frac{\hbar^2}{\pi m_{AB}} \int_0^\infty dq\, q^2 \left[K(q,p) - K(q,k_{\text{reg}})\right] \frac{B(q)}{-1/a_{AB} + \sqrt{\frac{m_{AB}}{m_{AD}} q^2 - \frac{2m_{AB}E}{\hbar^2}} - i0+}$$

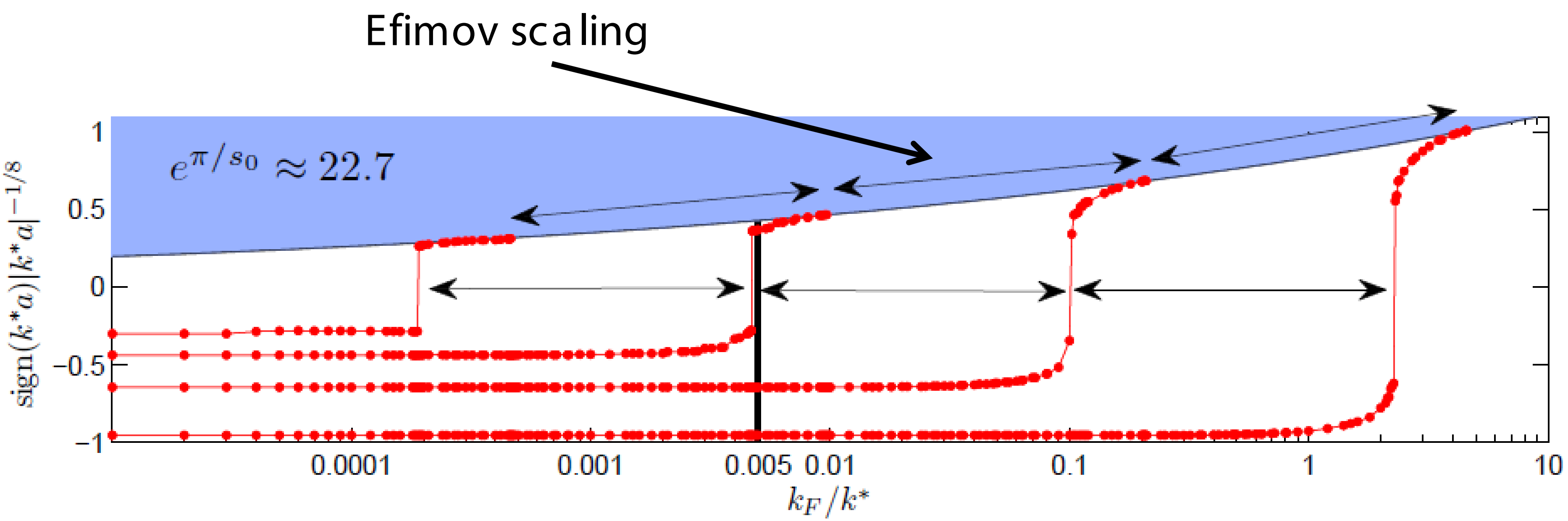
Needs regularization! Use method of Danilov, Zh.Eksp. Teor. Fiz. **40**, 498 (1961). Nice recent discuss by Pricoupenko, Phys. Rev. A **82**, 043633 (2010)



N.G. Nygaard and N.T. Zinner, New J. Phys. **16**, 023026 (2014)

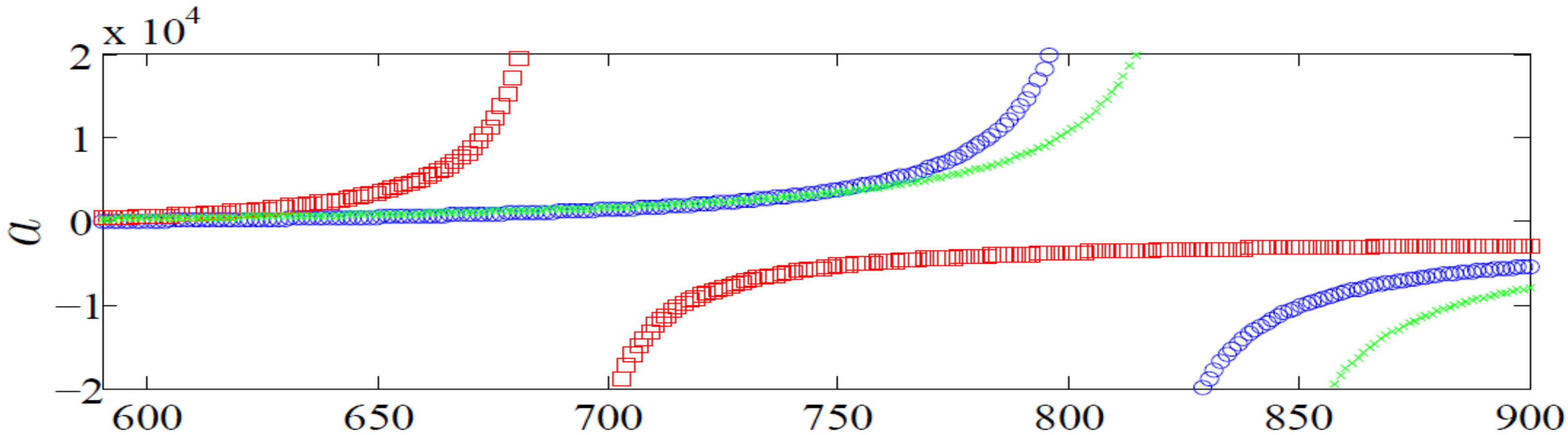
WHAT ABOUT EFIMOV SCALING?

We find many-body Efimov scaling!

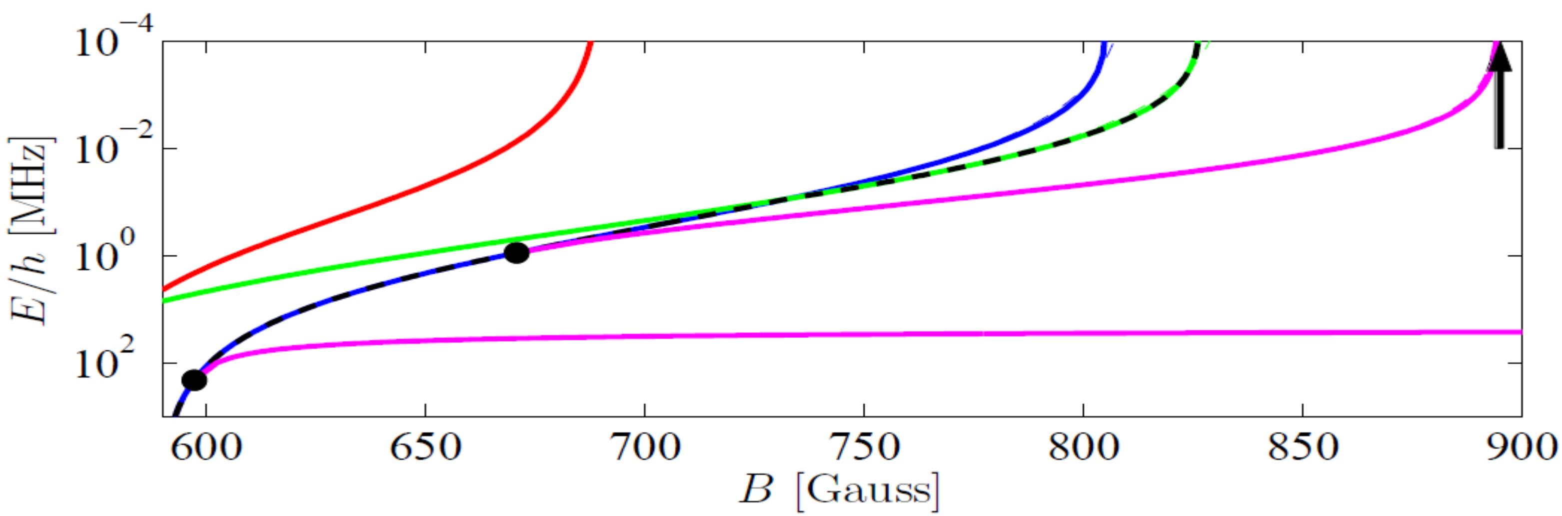


N.G. Nygaard and N.T. Zinner, New J. Phys. **16**, 023026 (2014)

THE ^{6}LI SYSTEM



$k^* = 6.9 \cdot 10^{-3}\ a_0^{-1}$ $k_F = 0.01k^*$ $n \sim 10^{11}\ cm^{-3}$



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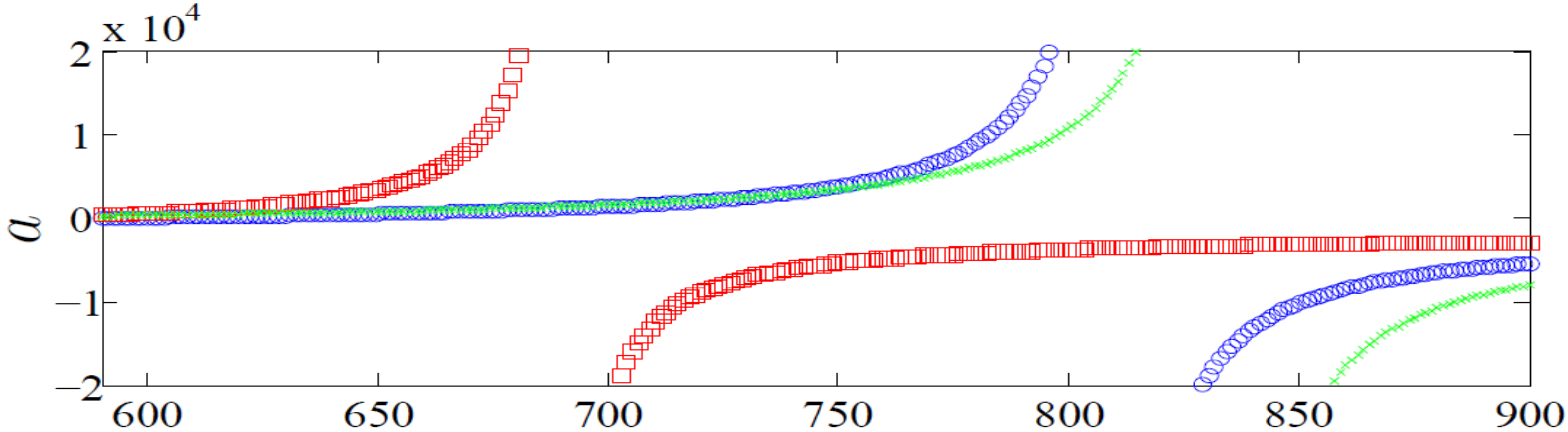
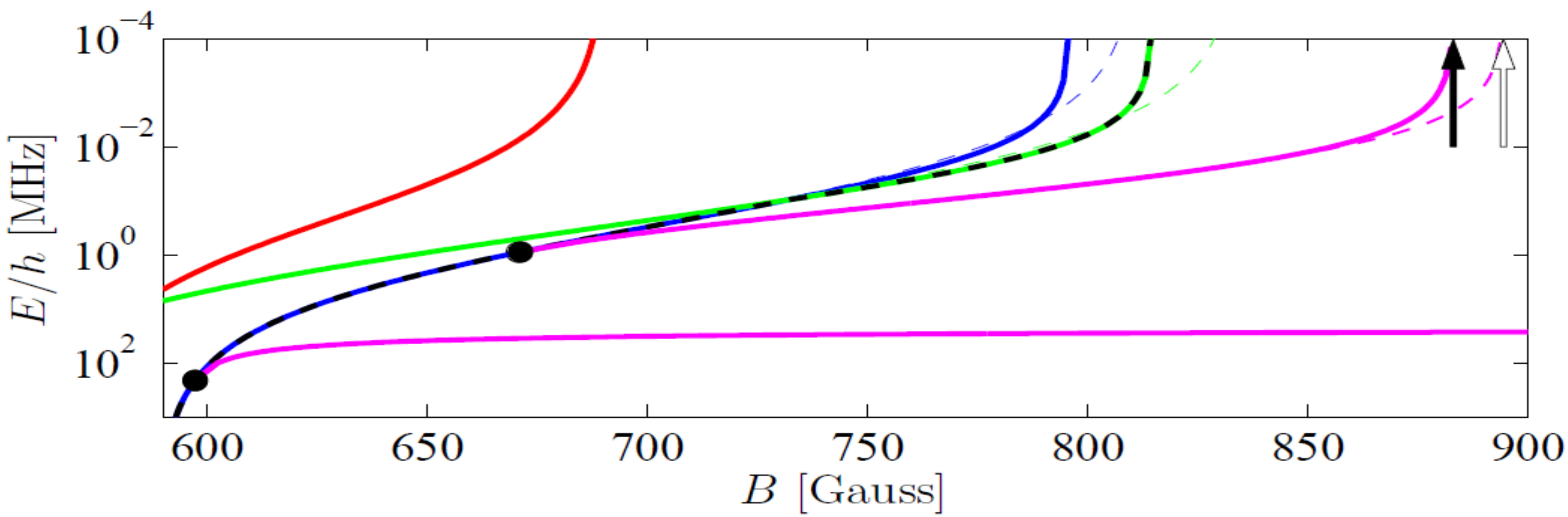
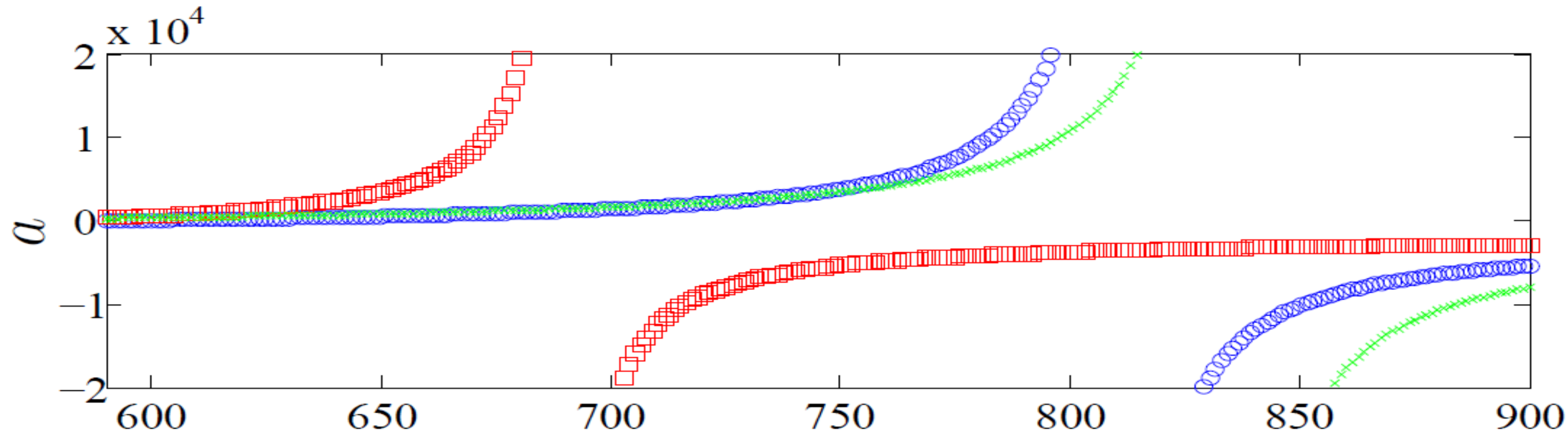


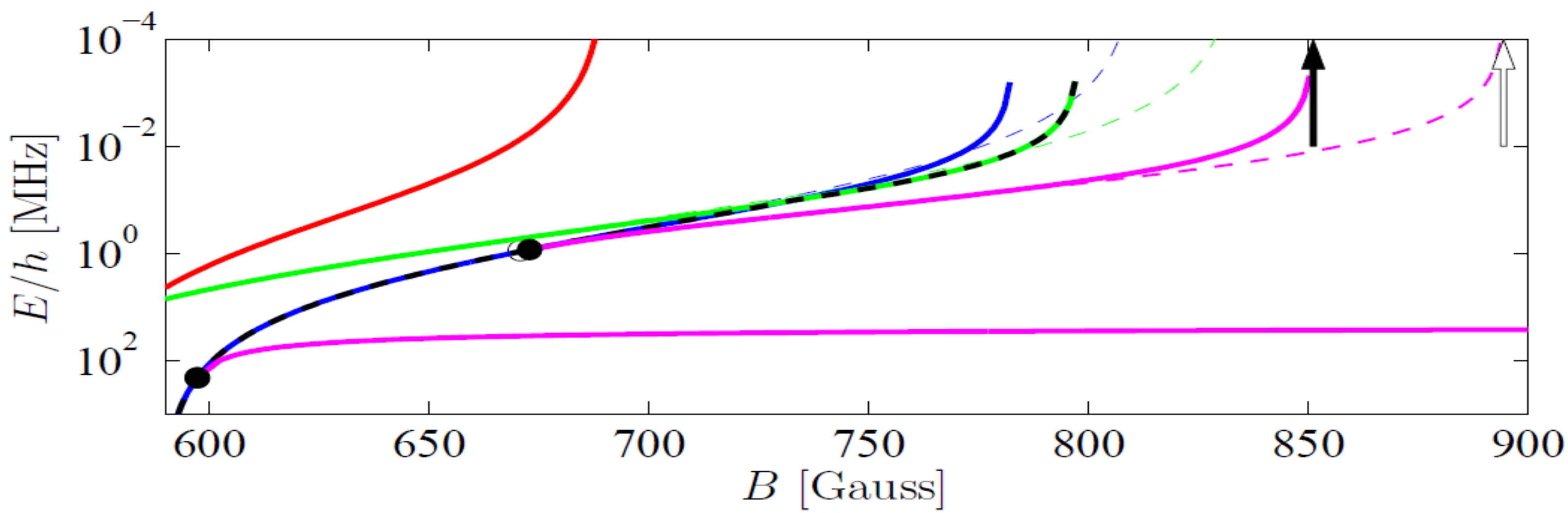
$k^*=6.9*10^{-3}\ a_0^{-1}$ $k_F=0.03k^*$ $n\sim10^{12}\ cm^{-3}$



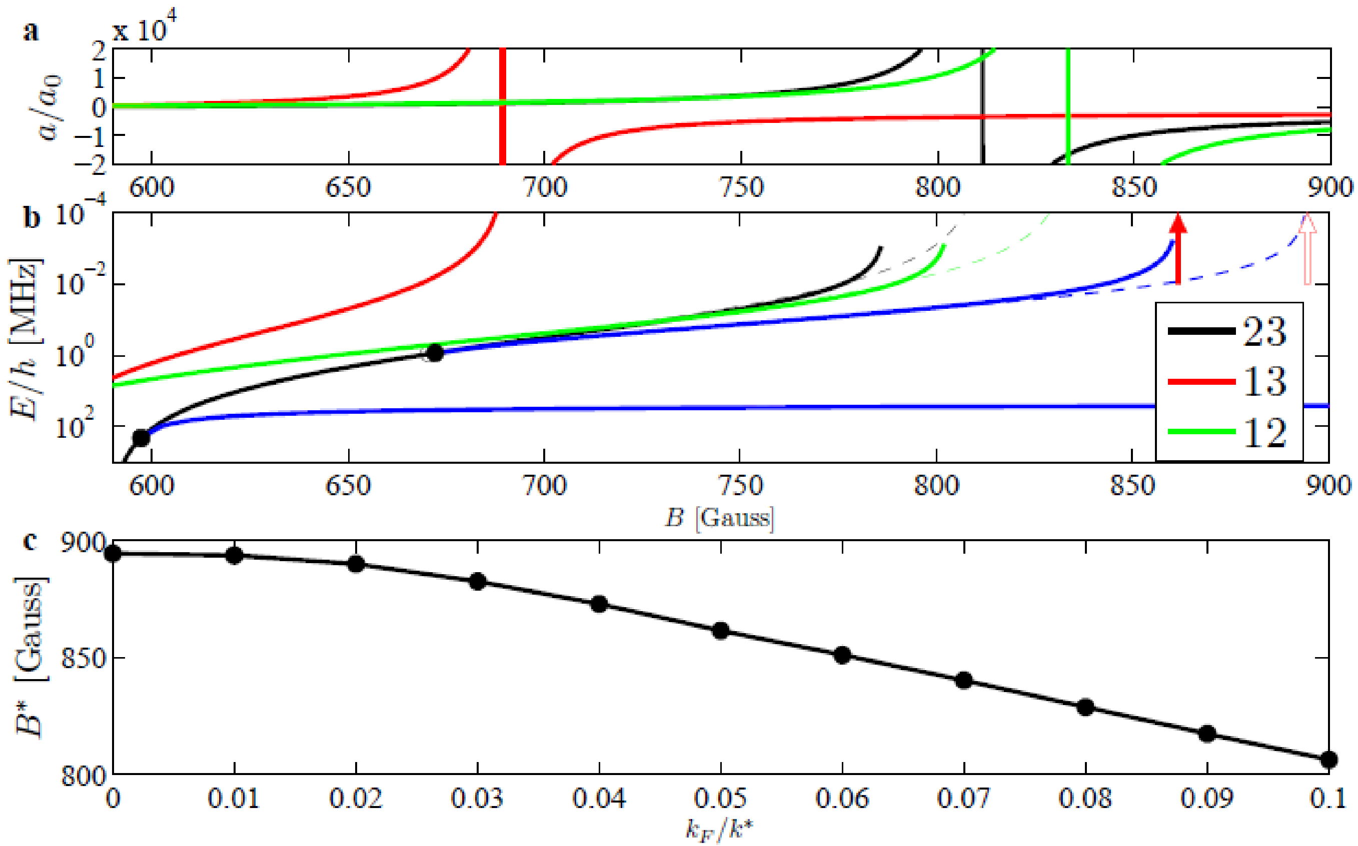
THE ^{6}LI SYSTEM



$k^* = 6.9 \cdot 10^{-3}\ a_0^{-1}$ $k_F = 0.06 k^*$ $n \sim 10^{13}\ cm^{-3}$



A REALISTIC SYSTEM



N.G. Nygaard and N.T. Zinner, New J. Phys. **16**, 023026 (2014)

OBSERVABILITY?

- Densities have been too small or measurements have not been around the second trimer threshold point.
- Trimer moves outside threshold regime D'Incao *et al.* PRL **93**, 123201 (2004).
- Perhaps not a problem Wang and Esry New. J. Phys. **13**, 035025 (2011).
- Dimer regime is harder since lowest Efimov state has large binding energy.

BOUND STATES AND BACKGROUND SUMMARY

Bound states are rarely alone in the world when we probe them

Separation of scales usually comes to the rescue

Cold atomic three-body results are largely consistent with no background effect

HOWEVER: Background density has energy scale that is slightly smaller than binding energy. Effects should be addressable in current experiments!

Important lesson: Cooper pair problem
No bound states in vacuum, but bound states with Fermi sea background!

Need to generalize the Cooper problem to three (or more)-body states!

N.G. Nygaard and N.T. Zinner, New J. Phys. **16**, 023026 (2014)

OUTLOOK ON DRESSED EFIMOV

More Fermi seas will not change the results qualitatively

Niemann and Hammer Phys. Rev. A **86**, 013628 (2012).

Fluctuations are an important outstanding question!

Scattering states and recombination in a Fermi sea

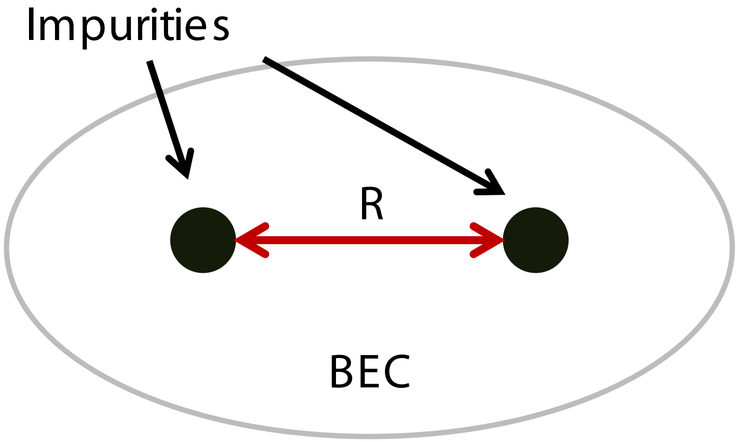
Mixed systems of bosonic and fermionic atoms

Superfluid or condensed states?

Can many-body effects provide a three-body parameter?
Can it be universal?

AARHUS UNIVERSITET

TRIMERS IN CONDENSATES



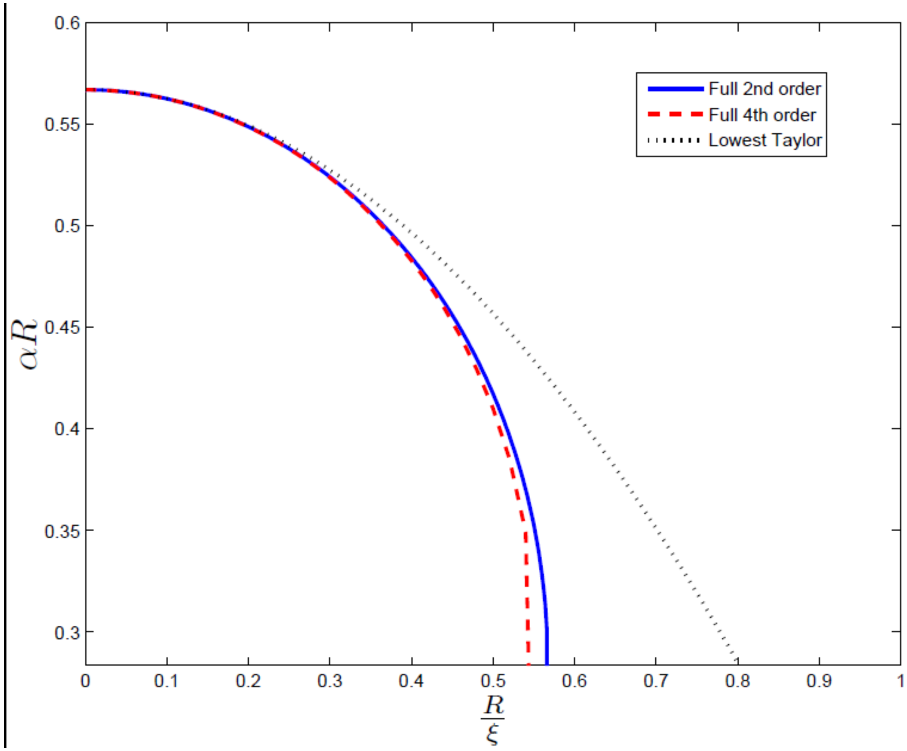
Born-Oppenheimer potential with no condensate

$$V(R) = -\frac{\Omega^2 \hbar^2}{mR^2}\ ,\ \Omega = e^{\Omega}$$

Two impurities in BEC of light bosons – BEC is weakly interacting – ξ is large

Born-Oppenheimer result is strongly modified by presence of condensate

$$N_T \simeq \frac{1}{\pi}\mathrm{Log}\left[\frac{a}{r_0}\right] \to \frac{1}{\pi}\mathrm{Log}\left[\frac{a}{\xi}\right]$$



NTZ, EPL **101** (2013) 60009

OUTLOOK ON LOWER DIMENSION

No Efimov effect in 1D or 2D

Zero-range limit provides 2 bound states in 2D and one bound state in 1D

Realistic cold gases have traps, must be taken into account through oscillator potentials. This complicates matters

Pauli blocking on three-body states in 2D might be similar to 3D, but 1D will be very different

An interpolation scheme would be extremely nice!

M.T. Yamashita *et al.*, arXiv:1404.7002 (2014).

CONDENSED-MATTER APPLICATIONS

› Multi-band superconductors
› Surfaces and wires – low-dimensional bound state problems
› Excitons and polarons
› Trion states – carbon nanotubes
› Surface states on non-trivial insulators

THREE ANGLES OF APPROACH

- Characterize low-energy bound states in different geometries, dimensionalities, and with both short- and long-range interactions.
- Apply many-body effects in either a top-down or a bottom-up fashion.
- Merge findings to improve formalism that accounts for both many- and few-body correlations in a general setting.

IMPLEMENTING MANY-BODY

A top-down scheme

Advantages:
Green's function approach is very general
Can work with both fermions and with bosons
Can accommodate degenerate backgrounds
Can be generalized to finite temperature 'easily'

Disadvantages:
Long-range interactions are difficult to handle
Self-energy term is complicated
Requires truncation at low order

IMPLEMENTING MANY-BODY

Bottom-up approach

Use exact diagonalization, stochastic methods, and other numerical schemes

Advantages:
Includes all correlations within model space
Can work both short- and long-range interactions
Easy access to full N-body wave function

Disadvantages:
Can not be extended to large systems
Truncation implies effective interaction must be used

SYNTHESIZE FORMALISM

- Top-down and bottom-up approaches complement each other.
- Results will help pin down precise varitional wave functions for general systems.
- Ultimate goal is to predict the low-energy behavior of a quantum system based on knowledge of its few-body structure.
- Develop a classification scheme for quantum structures.

NUCLEAR, PARTICLE, AND ASTROPHYSICS

Particle physics

Quark bound states – strongly-interacting systems

Quark matter – dense stars

Possibility of superfluid behavior – polarized superfluids
Bound states in a medium?

Nuclear physics

Nuclear physics methods and inspiration is important

Nuclear matter – neutron matter – neutron stars

An (almost) universal interaction type of problem

Renormalization group nuclear studies are interesting
Can one turn it around to integrate out *low*-momentum degrees of freedom?
That would be great for handling Pauli blocking effects for instance

COLD ATOMS AND SOLID-STATE PHYSICS

Cold atoms are nice, but 'difficult' to interact with for technological purposes

Technology relies mostly on solid-state devices, i.e. electronics

Simulation and quantum manipulation with cold atoms is very powerful

To harness the power we need cold atom-solid-state interfaces!

Atoms on a chip, atoms close to wires and nanotubes
Atoms trapped around optical fibers
Microstructured potentials that can beat optical wavelength limit

Hybrid quantum systems

COLLABORATION

AU
Nicolai Nygaard
Aksel Jensen
Dmitri Fedorov
Peder Sørensen
Artem Volosniev
Jeremy Armstrong
Georg Bruun
Jens Kusk
Jan Arlt
Jakob Sherson

Taiwan
Daw-Wei Wang
Sheng-Jie Huang
Hao Lee

Harvard
Eugene Demler
Bernard Wunsch

Caltech
David Pekker

Oxford
Igor Mekhov

Brazil
Filipe Bellotti
Marcelo Yamashita
Tobias Frederico

Thank you and enjoy the weelend!