Dynamical Systems and Financial Instability

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A brief history of Macroeconomics

- Classics (Smith, Ricardo, Marx): no distinction between micro and macro, Say's law, emphasis on long run.
- Beginning of the 20th century (Wicksell, Fisher): natural rate of interest, quantity theory of money.
- Keynesian revolution (1936): shift to demand, fallacies of composition, role of expectations, and much more!
- Neoclassical synthesis - 1945 to 1970 (Hicks, Samuelson, Solow): Keynesian consensus.
- Rational Expectations Revolution - 1972 (Lucas, Prescott, Sargent): internal consistency, microfoundations.
- Start of Macro Wars: Real Business Cycles versus New Keynesian.
- 1990's: impression of consensus around DSGE models, but with different flavours.

Dynamic Stochastic General Equilibrium

- Seeks to explain the aggregate economy using theories based on strong microeconomic foundations.
- Collective decisions of rational individuals over a range of variables for both present and future.
- All variables are assumed to be simultaneously in equilibrium.
- The only way the economy can be in disequilibrium at any point in time is through decisions based on wrong information.
- Money is neutral in its effect on real variables.
- Largely ignores uncertainty by simply subtracting risk premia from all risky returns and treat them as risk-free.

Really bad economics: hardcore (freshwater) DSGE

- The strand of DSGE economists affiliated with RBC theory made the following predictions after 2008:
  1. Increases government borrowing would lead to higher interest rates on government debt because of "crowding out".
  2. Increases in the money supply would lead to inflation.
  3. Fiscal stimulus has zero effect in an ideal world and negative effect in practice (because of decreased confidence).

Wrong prediction number 1

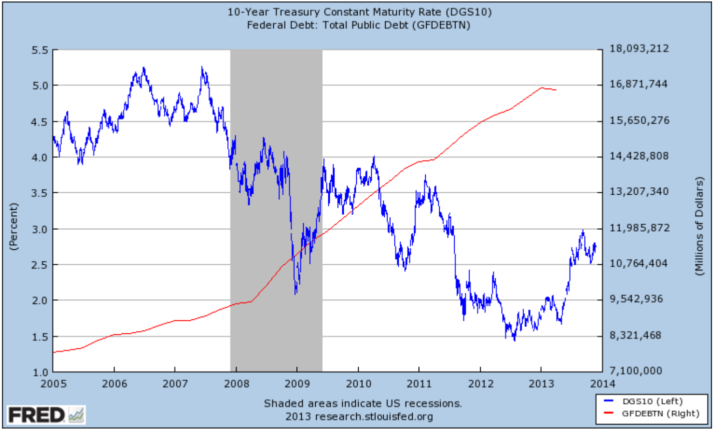


Figure: Government borrowing and interest rates.

Wrong prediction number 2

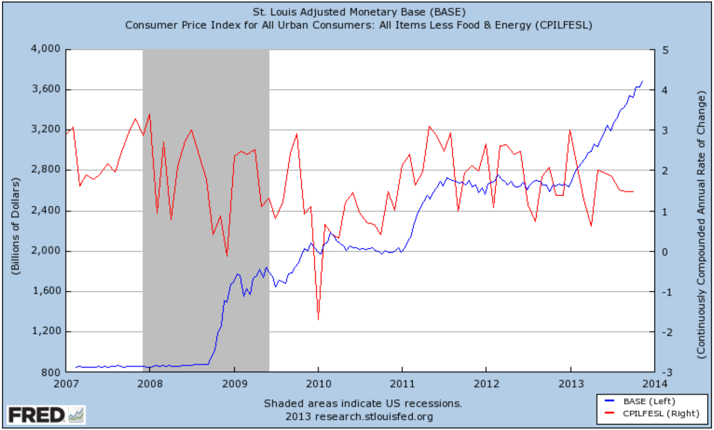


Figure: Monetary base and inflation.

Wrong prediction number 3

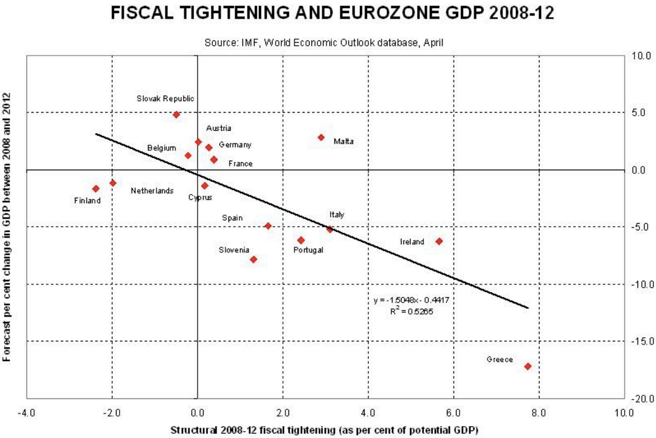


Figure: Fiscal tightening and GDP.

Better (but still bad) economics: soft core (saltwater) DSGE

- The strand of DSGE economists affiliated with New Keynesian theory got all these predictions right.
- They did so by augmented DSGE with 'imperfections' (wage stickiness, asymmetric information, imperfect competition, etc).
- Still DSGE at core - analogous to adding epicycles to Ptolemaic planetary system.
- For example: "Ignoring the foreign component, or looking at the world as a whole, the overall level of debt makes no difference to aggregate net worth – one person's liability is another person's asset." (Paul Krugman and Gauti B. Eggertsson, 2010, pp. 2-3)

Finance in DSGE models

- The financial sector merely serve as intermediaries channeling savings from households to business.
- Banks provide indirect finance by borrowing short and lending long (business loans), thereby solving the problem of liquidity preferences (Diamond and Dybvig (1986) model).
- Financial market provide direct finance through shares, thereby introducing market prices and discipline.
- Financial Frictions (e.g borrowing constraints, market liquidity) create persistence and amplification of real shocks (Bernanke and Gertler (1989), Kiyotaki and Moore (1997) models)
- See Brunnermeier and Sannikov (2013) for a recent contribution to this strand of literature in light of the financial crisis, in particular in the context of macro-prudential regulation.

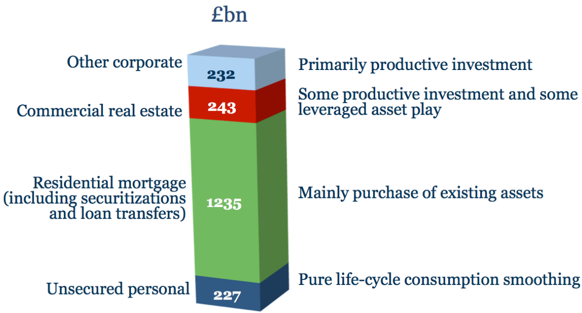
Frictions literature still missing the point

Turner 2013 observes that:

- "Quantitative impacts suggested by the models were far smaller than those empirically observed in real world episodes such as the Great Depression or the 2008 crisis"
- "Most of the literature omits consideration of behaviourally driven 'irrational' cycles in asset prices".
- "the vast majority of the literature ignores the possibilities of credit extension to finance the purchase of already existing assets".
- "the dominant model remains one in which household savers make deposits in banks, which lend money to entrepreneurs/businesses to pursue 'investment projects'. The reality of a world in which only a small proportion (e.g. 15%) of bank credit funds 'new investment projects' has therefore been left largely unexplored."

Turner (2013) slide

Categories of bank debt: UK, 2009

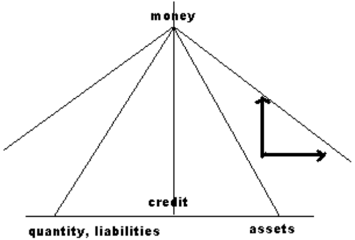


A parallel history of Macroeconomics

- Classical 19th century monetarism (Bagehot, Allan Young): role of banks in trade (Britain) and development (U.S.), central banking.
- Several prominent disciples of Keynes (Kaldor, Robinson, Davidson) immediately rejected the Neoclassical synthesis as "bastardized Keynesianism".
- Flow of Funds accounting - 1952 (Copeland): alternative to both $Y = C + I + G + X - M$ (finals sales) and $MV = PT$ (money transactions) by tracking exchanges of both goods and financial assets.
- Gurley, Shaw, Tobin, Minsky: financial intermediation at centre stage.
- Kindleberger (1978): detailed history of financier crises.
- Stock-flow consistent models (Godley, Lavoie)
- Revival of interest after the 2008 crisis.

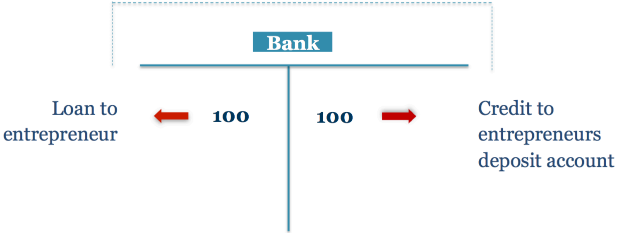
Key insight 1: money is not neutral

- Money is hierarchical: currency is a promise to pay gold (or taxes); deposits are promises to pay currency; securities are promises to pay deposits.
- Financial institutions are market-makers straddling two levels in the hierarchy: central banks, banks, security dealers.
- The hierarchy is dynamic: discipline and elasticity change in time.



Key insight 2: money is endogenous

- Banks create money and purchasing power.
- Reserve requirements are never binding.



Key insight 3: private debt matters

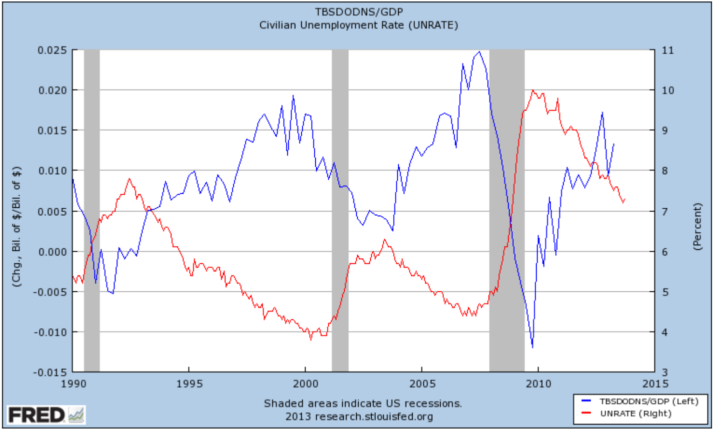


Figure: Change in debt and unemployment.

Key insight 4: finance is not just intermediation

- Market never clear in all states: set of events is larger than what can be contracted.
- The financial sector absorbs the risk of unfulfilled promises.
- The cone of acceptable losses defines the size of the real economy.

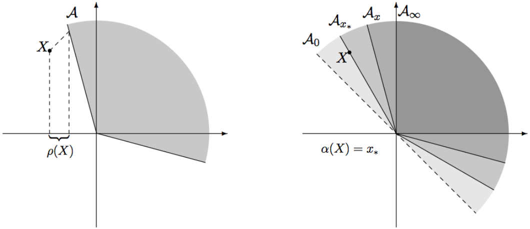


Figure: Cherny and Madan (2009)

Much better economics: SFC models

- Stock-flow consistent models emerged in the last decade as a common language for many heterodox schools of thought in economics.
- Consider both real and monetary factors from the start
- Specify the balance sheet and transactions between sectors
- Accommodate a number of behavioural assumptions in a way that is consistent with the underlying accounting structure.
- Reject silly (and mathematically unsound!) hypotheses such as the RARE individual (representative agent with rational expectations).
- See Godley and Lavoie (2007) for the full framework.

Balance Sheets

Balance Sheet	Households	Firms		Banks	Central Bank	Government	Sum
		current	capital				
Cash	$+H_h$			$+H_b$	$-H$		0
Deposits	$+M_h$		$+M_f$	$-M$			0
Loans			$-L$	$+L$			0
Bills	$+B_h$			$+B_b$	$+B_c$	$-B$	0
Equities	$+p_f E_f + p_b E_b$		$-p_f E_f$	$-p_b E_b$			0
Advances				$-A$	$+A$		0
Capital			$+pK$				pK
Sum (net worth)	V_h	0	V_f	V_b	0	$-B$	pK

Table: Balance sheet in an example of a general SFC model.

Transactions

Transactions	Households	Firms current	Firms capital	Banks	Central Bank	Government	Sum
Consumption	$-pC_h$	$+pC$		$-pC_b$			0
Investment		$+pI$	$-pI$				0
Gov spending		$+pG$				$-pG$	0
Acct memo [GDP]		$[pY]$					
Wages	$+W$	$-W$					0
Taxes	$-T_h$	$-T_f$				$+T$	0
Interest on deposits	$+r_M.M_h$	$+r_M.M_f$		$-r_M.M$			0
Interest on loans		$-r_L.L$		$+r_L.L$			0
Interest on bills	$+r_B.B_h$			$+r_B.B_b$	$+r_B.B_c$	$-r_B.B$	0
Profits	$+\Pi_d + \Pi_b$	$-\Pi$	$+\Pi_u$	$-\Pi_b$	$-\Pi_c$	$+\Pi_c$	0
Sum	S_h	0	$S_f - pI$	S_b	0	S_g	0

Table: Transactions in an example of a general SFC model.

Flow of Funds

Flow of Funds	Households	Firms current	Firms capital	Banks	Central Bank	Government	Sum
Cash	$+\dot{H}_h$			$+\dot{H}_b$	$-\dot{H}$		0
Deposits	$+\dot{M}_h$		$+\dot{M}_f$	$-\dot{M}$			0
Loans			$-\dot{L}$	$+\dot{L}$			0
Bills	$+\dot{B}_h$			$+\dot{B}_b$	$+\dot{B}_c$	$-\dot{B}$	0
Equities	$+p_f\dot{E}_f + p_b\dot{E}_b$		$-p_f\dot{E}_f$	$-p_b\dot{E}_b$			0
Advances				$-\dot{A}$	$+\dot{A}$		0
Capital			$+pI$				pI
Sum	S_h	0	S_f	S_b	0	S_g	pI
Change in Net Worth	$(S_h + \dot{p}_f E_f + \dot{p}_b E_b)$	$(S_f - \dot{p}_f E_f + \dot{p}K - p\delta K)$		$(S_b - \dot{p}_b E_b)$		S_g	$\dot{p}K + p\dot{K}$

Table: Flow of funds in an example of a general SFC model.

General Notation

- Employed labor force: ℓ
- Production function: $Y = f(K, \ell)$
- Labour productivity: $a = \frac{Y}{\ell}$
- Capital-to-output ratio: $\nu = \frac{K}{Y}$
- Employment rate: $\lambda = \frac{\ell}{N}$
- Change in capital: $\dot{K} = I - \delta K$
- Inflation rate: $i = \frac{\dot{p}}{p}$

Goodwin Model (1967) - Assumptions

- Assume that

$$N = N_0 e^{\beta t} \qquad \text{(total labour force)}$$
$$a = a_0 e^{\alpha t} \qquad \text{(productivity per worker)}$$
$$Y = \min\left\{\frac{K}{\nu}, a\ell\right\} \qquad \text{(Leontief production)}$$

- Assume further that

$$Y = \frac{K}{\nu} = a\ell \qquad \text{(full capital utilization)}$$
$$\dot{\mathrm{w}} = \Phi(\lambda, i, i^e)\mathrm{w} \qquad \text{(Phillips curve)}$$
$$pI = pY - \mathrm{w}\ell \qquad \text{(Say's Law)}$$

- NOTE: In the original paper, Goodwin assumed that w above was the real wage rate, so all quantities were normalized by p.

Goodwin Model - SFC matrix

Balance Sheet	Households	Firms		Sum
		current	capital	
Capital			$+pK$	pK
Sum (net worth)	0	0	V_f	pK
Transactions				
Consumption	$-pC$	$+pC$		0
Investment		$+pI$	$-pI$	0
Acct memo [GDP]		$[pY]$		
Wages	$+W$	$-W$		0
Profits		$-\Pi$	$+\Pi_u$	0
Sum	0	0	0	0
Flow of Funds				
Capital			$+pI$	pI
Sum	0	0	Π_u	pI
Change in Net Worth	0	$pI + \dot{p}K - p\delta K$		$\dot{p}K + p\dot{K}$

Table: SFC table for the Goodwin model.

Goodwin Model - Differential equations

- Define

$$\omega = \frac{\mathrm{w}L}{pY} = \frac{\mathrm{w}}{pa} \quad \text{(wage share)}$$

$$\lambda = \frac{L}{N} = \frac{Y}{aN} \quad \text{(employment rate)}$$

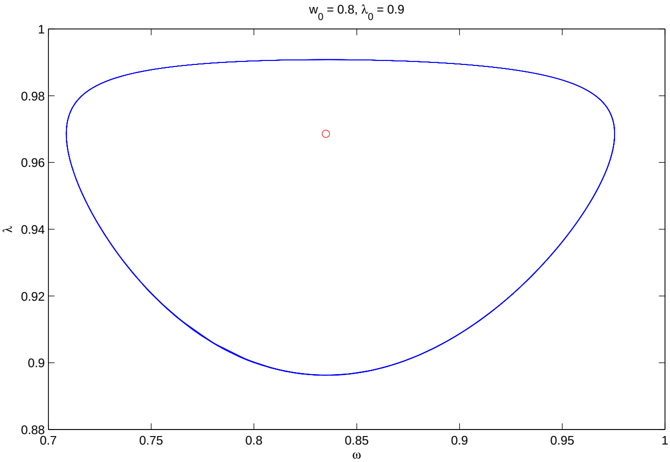
- It then follows that

$$\frac{\dot{\omega}}{\omega} = \frac{\dot{\mathrm{w}}}{\mathrm{w}} - \frac{\dot{p}}{p} - \frac{\dot{a}}{a} = \Phi(\lambda, i, i^e) - i - \alpha$$

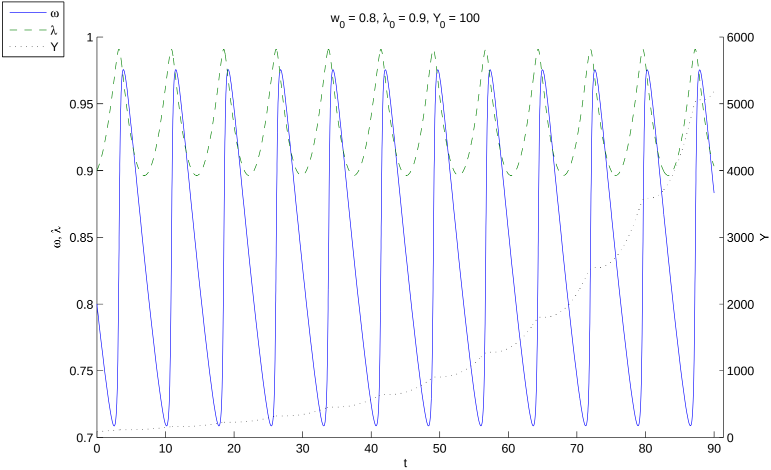
$$\frac{\dot{\lambda}}{\lambda} = \frac{1-\omega}{\nu} - \alpha - \beta - \delta$$

- In the original model, all quantities were real (i.e divided by p), which is equivalent to setting $i = i^e = 0$.

Example 1: Goodwin model



Example 1 (continued): Goodwin model



Goodwin Model - Extensions, structural instability, and empirical tests

- Desai 1972: Inflation leads to a stable equilibrium.
- Ploeg 1985: CES production function leads to stable equilibrium.
- Goodwin 1991: Pro-cyclical productivity growth leads to explosive oscillations.
- Solow 1990: US post-war data shows three sub-cycles with a "bare hint of a single large clockwise sweep" in the (ω, λ) plot.
- Harvie 2000: Data from other OECD confirms the same qualitative features and shows unsatisfactory quantitative estimations.

Testing Goodwin on OECD countries

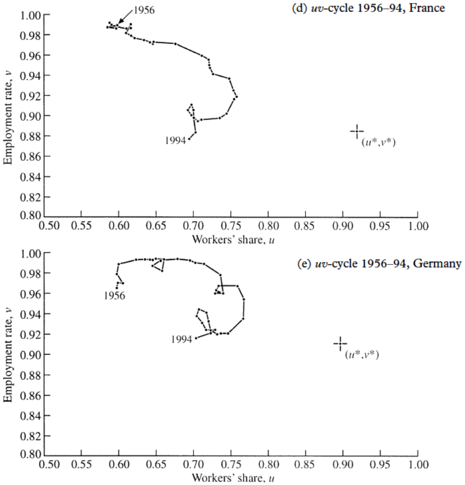


Figure: Harvie (2000)

Correcting Harvie

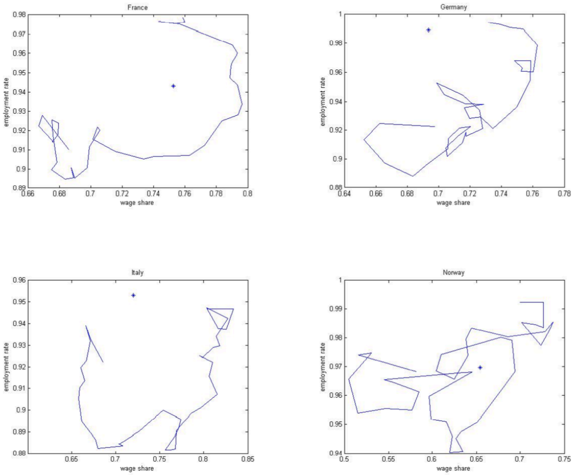


Figure: Grasselli and Maheshwari (2012)

SFC table for Keen (1995) model

Balance Sheet	Households	Firms current	Firms capital	Banks	Sum
Deposits	$+D$			$-D$	0
Loans			$-L$	$+L$	0
Capital			$+pK$		pK
Sum (net worth)	V_h	0	V_f	0	pK
Transactions					
Consumption	$-pC$	$+pC$			0
Investment		$+pI$	$-pI$		0
Acct memo [GDP]		$[pY]$			
Wages	$+W$	$-W$			0
Interest on deposits	$+rD$			$-rD$	0
Interest on loans		$-rL$		$+rL$	0
Profits		$-\Pi$	$+\Pi_u$		0
Sum	S_h	0	$S_f - pI$	0	0
Flow of Funds					
Deposits	$+\dot{D}$			$-\dot{D}$	0
Loans			$-\dot{L}$	$+\dot{L}$	0
Capital			$+pI$		pI
Sum	S_h	0	Π_u	0	pI
Change in Net Worth	S_h	$(S_f + \dot{p}K - p\delta K)$			$\dot{p}K + p\dot{K}$

Table: SFC table for the Keen model.

Keen model - Investment function

- Assume now that new investment is given by

$$\dot{K} = \kappa(1 - \omega - rd)Y - \delta K$$

where $\kappa(\cdot)$ is a nonlinear increasing function of profits $\pi = 1 - \omega - rd$.

- This leads to external financing through debt evolving according to

$$\dot{D} = \kappa(1 - \omega - rd)Y - (1 - \omega - rd)Y$$

Keen model - Differential Equations

Denote the debt ratio in the economy by $d = D/Y$, the model can now be described by the following system

$$
\begin{aligned}
\dot{\omega} &= \omega\left[\Phi(\lambda) - \alpha\right] \\
\dot{\lambda} &= \lambda\left[\frac{\kappa(1-\omega-rd)}{\nu} - \alpha - \beta - \delta\right] \\
\dot{d} &= d\left[r - \frac{\kappa(1-\omega-rd)}{\nu} + \delta\right] + \kappa(1-\omega-rd) - (1-\omega)
\end{aligned}
\tag{1}
$$

Keen model - equilibria

- The system (1) has a good equilibrium at

$$\overline{\omega} = 1 - \overline{\pi} - r\frac{\nu(\alpha+\beta+\delta) - \overline{\pi}}{\alpha+\beta}$$
$$\overline{\lambda} = \Phi^{-1}(\alpha)$$
$$\overline{d} = \frac{\nu(\alpha+\beta+\delta) - \overline{\pi}}{\alpha+\beta}$$

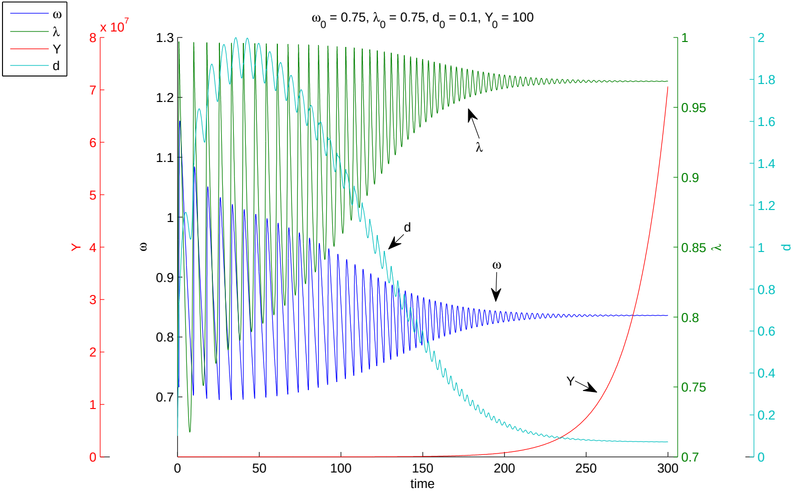
  with

$$\overline{\pi} = \kappa^{-1}(\nu(\alpha+\beta+\delta)),$$

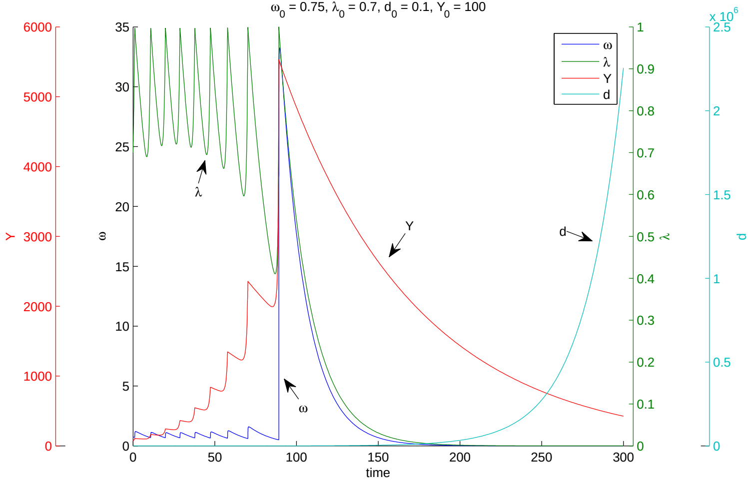
  which is stable for a large range of parameters
- It also has a bad equilibrium at $(0, 0, +\infty)$, which is stable if

$$\frac{\kappa(-\infty)}{\nu} - \delta < r \tag{2}$$

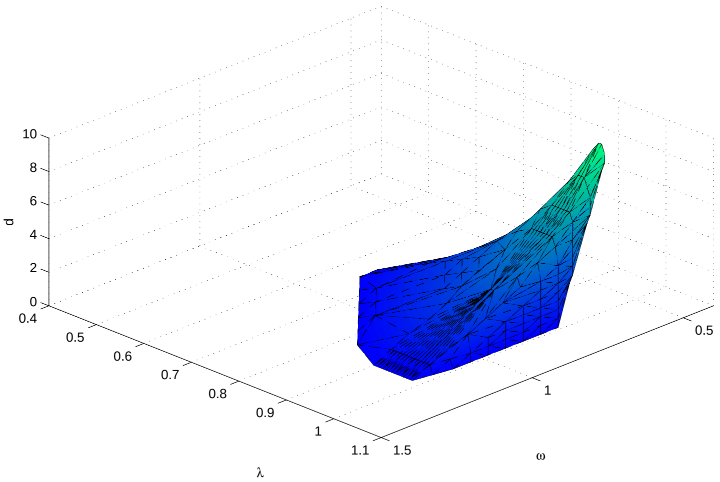
Example 2: convergence to the good equilibrium in a Keen model



Example 3: explosive debt in a Keen model



Basin of convergence for Keen model



Ponzi financing

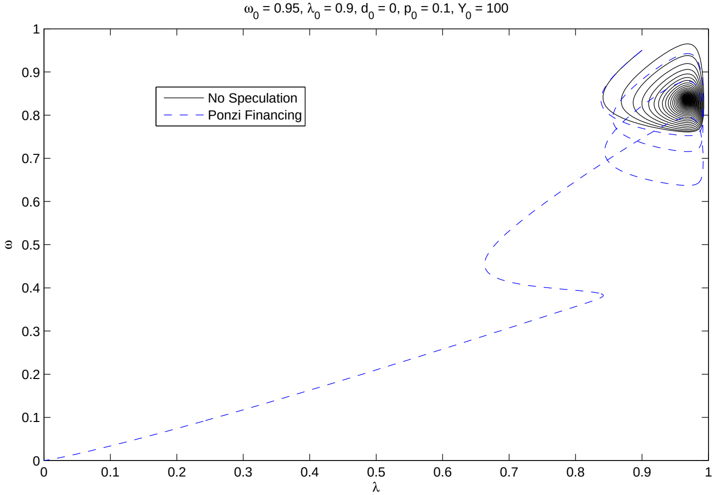
To introduce the destabilizing effect of purely speculative investment, we consider a modified version of the previous model with

$$\dot{D} = \kappa(1 - \omega - rd)Y - (1 - \omega - rd)Y + P$$
$$\dot{P} = \Psi(g(\omega, d)P$$

where $\Psi(\cdot)$ is an increasing function of the growth rate of economic output

$$g = \frac{\kappa(1 - \omega - rd)}{\nu} - \delta.$$

Example 4: effect of Ponzi financing



Stock prices

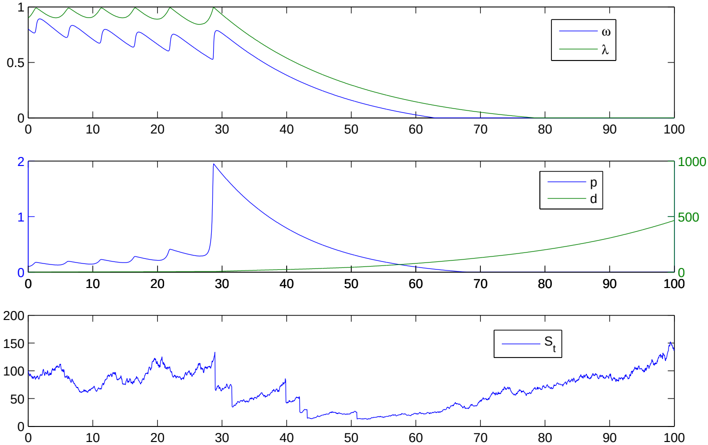
- Consider a stock price process of the form

$$\frac{dS_t}{S_t} = r_b dt + \sigma dW_t + \gamma \mu_t dt - \gamma dN^{(\mu_t)}$$

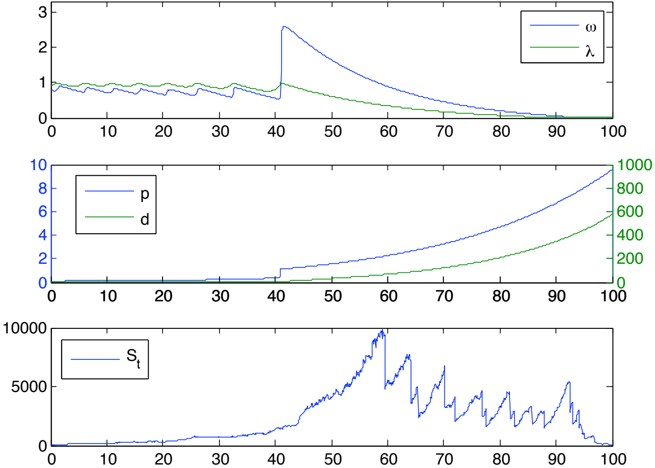
  where N_t is a Cox process with stochastic intensity $\mu_t = M(p(t))$.
- The interest rate for private debt is modelled as $r_t = r_b + r_p(t)$ where

$$r_p(t) = \rho_1 (S_t + \rho_2)^{\rho_3}$$

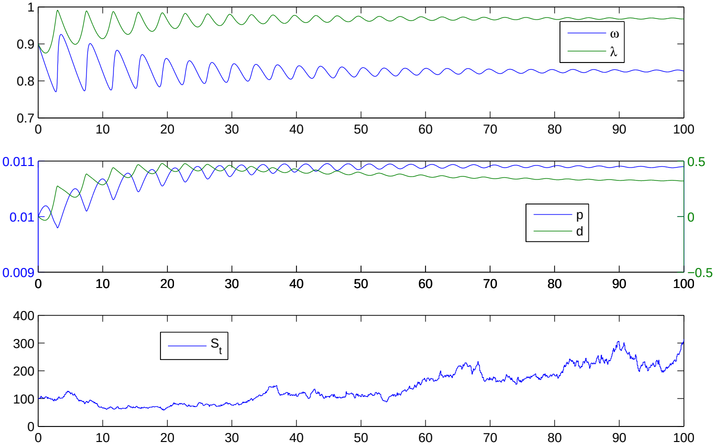
Example 5: stock prices, explosive debt, zero speculation



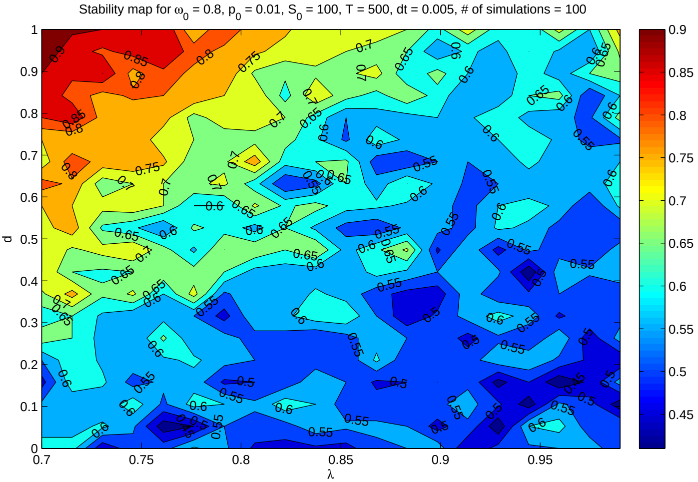
Example 6: stock prices, explosive debt, explosive speculation



Example 7: stock prices, finite debt, finite speculation



Stability map



Introducing a government sector

- Following Keen (and echoing Minsky) we add discretionary government subsidied and taxation into the original system in the form

$$G = G_1 + G_2$$
$$T = T_1 + T_2$$

where

$$\dot{G}_1 = \eta_1(\lambda)Y \qquad \dot{G}_2 = \eta_2(\lambda)G_2$$
$$\dot{T}_1 = \Theta_1(\pi)Y \qquad \dot{T}_2 = \Theta_2(\pi)T_2$$

- Defining $g = G/Y$ and $\tau = T/Y$, the net profit share is now

$$\pi = 1 - \omega - rd + g - \tau,$$

and government debt evolves according to

$$\dot{B} = rB + G - T.$$

Differential equations - reduced system

- Notice that π does not depend on b, so that the last equation can be solved separately.
- Observe further that we can write

$$\dot{\pi} = -\dot{\omega} - r\dot{d} + \dot{g} - \dot{\tau} \tag{3}$$

leading to the five-dimensional system

$$\begin{aligned}
\dot{\omega} &= \omega\left[\Phi(\lambda) - \alpha\right], \\
\dot{\lambda} &= \lambda\left[\gamma(\pi) - \alpha - \beta\right] \\
\dot{g}_2 &= g_2\left[\eta_2(\lambda) - \gamma(\pi)\right] \\
\dot{\tau}_2 &= \tau_2\left[\Theta_2(\pi) - \gamma(\pi)\right] \\
\dot{\pi} &= -\omega(\Phi(\lambda) - \alpha) - r(\kappa(\pi) - \pi) + (1 - \omega - \pi)\gamma(\pi) \\
&\quad + \eta_1(\lambda) + g_2\eta_2(\lambda) - \Theta_2(\pi) - \tau_2\Theta_2(\pi)
\end{aligned} \tag{4}$$

Good equilibrium

- The system (4) has a good equilibrium at

$$
\begin{aligned}
\overline{\omega} &= 1 - \overline{\pi} - r\frac{\nu(\alpha+\beta+\delta) - \overline{\pi}}{\alpha+\beta} + \frac{\eta_1(\overline{\lambda}) - \Theta_1(\overline{\pi})}{\alpha+\beta} \\
\overline{\lambda} &= \Phi^{-1}(\alpha) \\
\overline{\pi} &= \kappa^{-1}(\nu(\alpha+\beta+\delta)) \\
\overline{g}_2 &= \overline{\tau}_2 = 0
\end{aligned}
$$

and this is locally stable for a large range of parameters.

- The other variables then converge exponentially fast to

$$
\overline{d} = \frac{\nu(\alpha+\beta+\delta) - \overline{\pi}}{\alpha+\beta}, \qquad \overline{g}_1 = \frac{\eta_1(\overline{\lambda})}{\alpha+\beta}
$$

$$
\overline{\tau}_1 = \frac{\Theta_1(\overline{\pi})}{\alpha+\beta}
$$

Bad equilibria - destabilizing a stable crisis

- Recall that $\pi = 1 - \omega - rd + g - \tau$.
- The system (4) has bad equilibria of the form

$$(\omega, \lambda, g_2, \tau_2, \pi) = (0, 0, 0, 0, -\infty)$$
$$(\omega, \lambda, g_2, \tau_2, \pi) = (0, 0, \pm\infty, 0, -\infty)$$

- If $g_2(0) > 0$, then any equilibria with $\pi \to -\infty$ is locally unstable provided $\eta_2(0) > r$.
- On the other hand, if $g_2(0) < 0$ (austerity), then these equilibria are all locally stable.

Persistence results

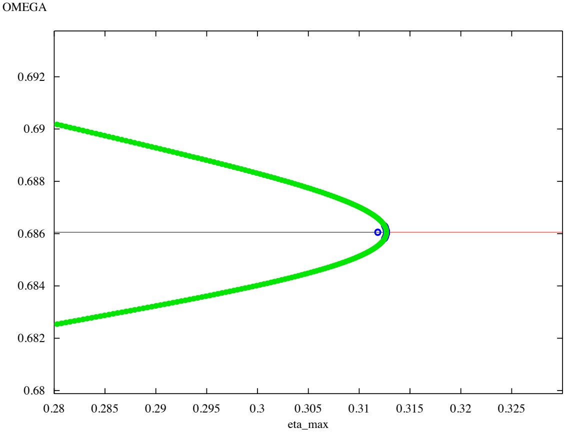
Proposition 1: Assume $g_2(0) > 0$, then the system (4) is e^{π}-UWP if either

1. $\lambda\eta_1(\lambda)$ is bounded below as $\lambda \to 0$, or
2. $\eta_2(0) > r$.

Proposition 2: Assume $g_2(0) > 0$ and $\tau_2(0) = 0$, then the system (4) is λ-UWP if either of the following three conditions is satisfied:

1. $\lambda\eta_1(\lambda)$ is bounded below as $\lambda \to 0$, or
2. $\eta_2(0) > \max\{r, \alpha+\beta\}$, or
3. $r < \eta_2(0) \leq \alpha+\beta$ and $-r(\kappa(x)-x)+(1-x)\gamma(x)+\eta_1(0)-\Theta_1(x) > 0$ for $\gamma(x) \in [\eta_2(0), \alpha+\beta]$.

Hopft bifurcation with respect to government spending.



The Great Moderation in the U.S. - 1984 to 2007

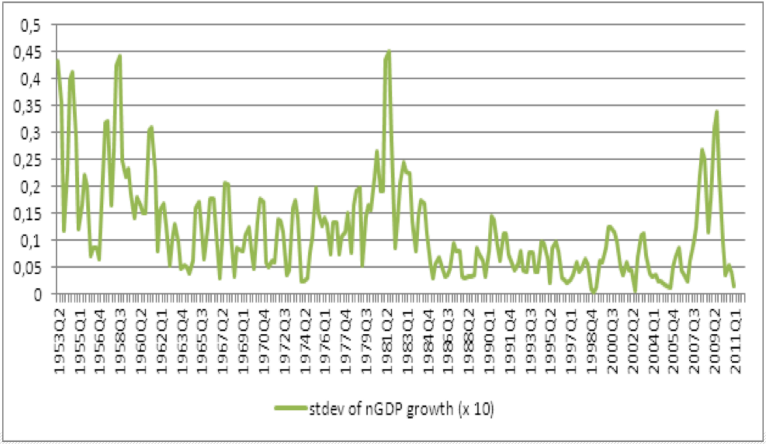


Figure: Grydaki and Bezemer (2013)

Possible explanations

- Real-sector causes: inventory management, labour market changes, responses to oil shocks, external balances , etc.
- Financial-sector causes: credit accelerator models, financial innovation, deregulation, better monetary policy, etc.
- Grydaki and Bezemer (2013): growth of debt in the real sector.

Bank credit-to-GDP ratio in the U.S

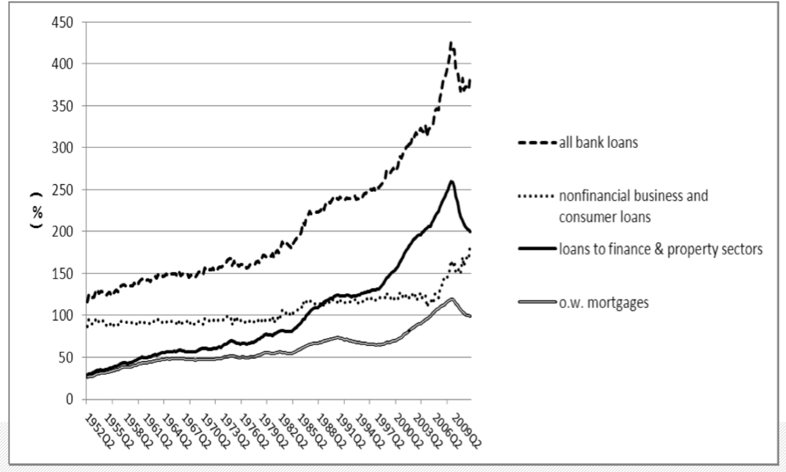


Figure: Grydaki and Bezemer (2013)

Cumulative percentage point growth of excess credit growth, 1952-2008

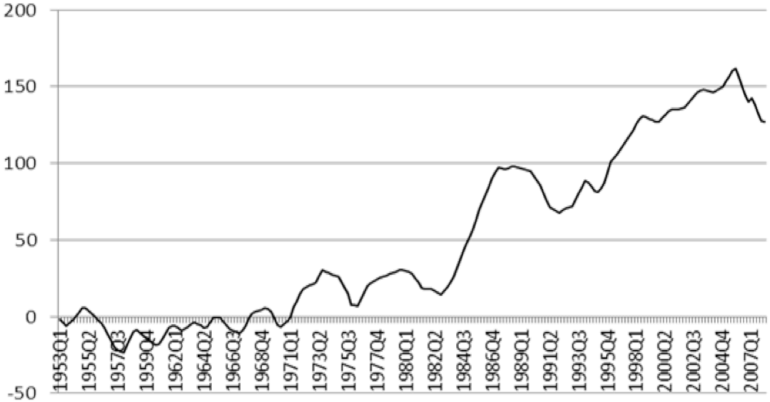


Figure: Grydaki and Bezemer (2013)

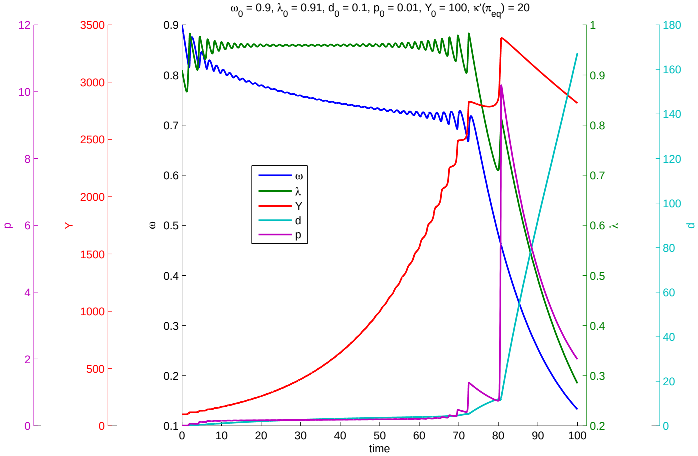
Excess credit growth moderated output volatility during, but not before the Great Moderation

Before the Great Moderation	*During the Great Moderation*
change in interest rate (-) => output volatility	excess credit growth (-) => output volatility
change in interest rate (+) => inflation	output volatility (+) => excess credit growth
excess credit growth (+) => change in interest rate	output volatility (-) => change in interest rate
	excess credit growth (+) => change in interest rate
	inflation (+) => change in interest rate

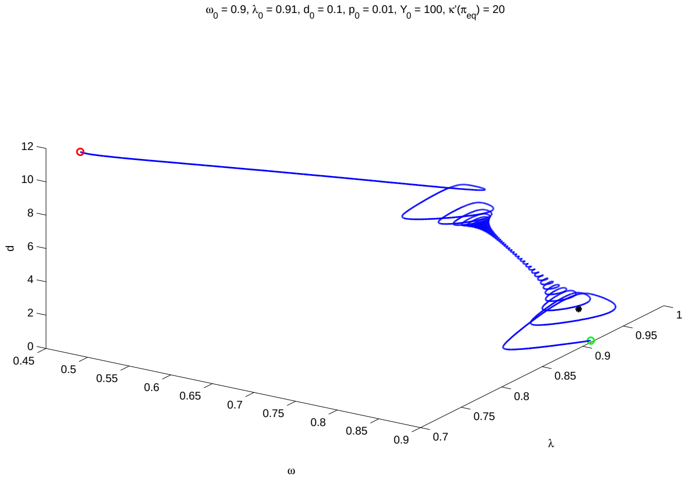
Note: In the table, x (-) => y denotes that a one-standard deviation shock in variable x impacts negatively on the change of variable y. Similarly, x (+) => y indicates a positive impact.

Figure: Grydaki and Bezemer (2013)

Example 8: strongly moderated oscillations



Example 9 (cont): Shilnikov bifurcation



Shortcomings of Goodwin and Keen models

- No independent specification of consumption (and therefore savings) for households:

$$C = W, \quad S_h = 0 \qquad \text{(Goodwin)}$$
$$C = (1-\kappa(\pi))Y, \quad S_h = \dot{D} = \Pi_u - I \qquad \text{(Keen)}$$

- Full capacity utilization.
- Everything that is produced is sold.
- No active market for equities.
- Skott (1989) uses prices as an accommodating variable in the short run.
- Chiarella, Flaschel and Franke (2005) propose a dynamics for inventory and expected sales.
- Grasselli and Nguyen (2013) provide a synthesis, including equities and Tobin's portfolio choices.

Concluding remarks

- Macroeconomics is too important to be left to macroeconomists.
- Since Keynes's death it has developed in two radically different approaches:
  1. The dominant one has the appearance of mathematical rigour (the SMD theorems notwithstanding), but is based on implausible assumptions, has poor fit to data in general, and is disastrously wrong during crises. Finance plays a negligible role
  2. The heterodox approach is grounded in history and institutional understanding, takes empirical work much more seriously, but is generally averse to mathematics. Finance plays a major role.
- It's clear which approach should be embraced by mathematical finance "to boldly go where no man has gone before" $\cdots$

Qatlho!