The Standard Model and Neutrino Mass Models

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Quick Preliminaries

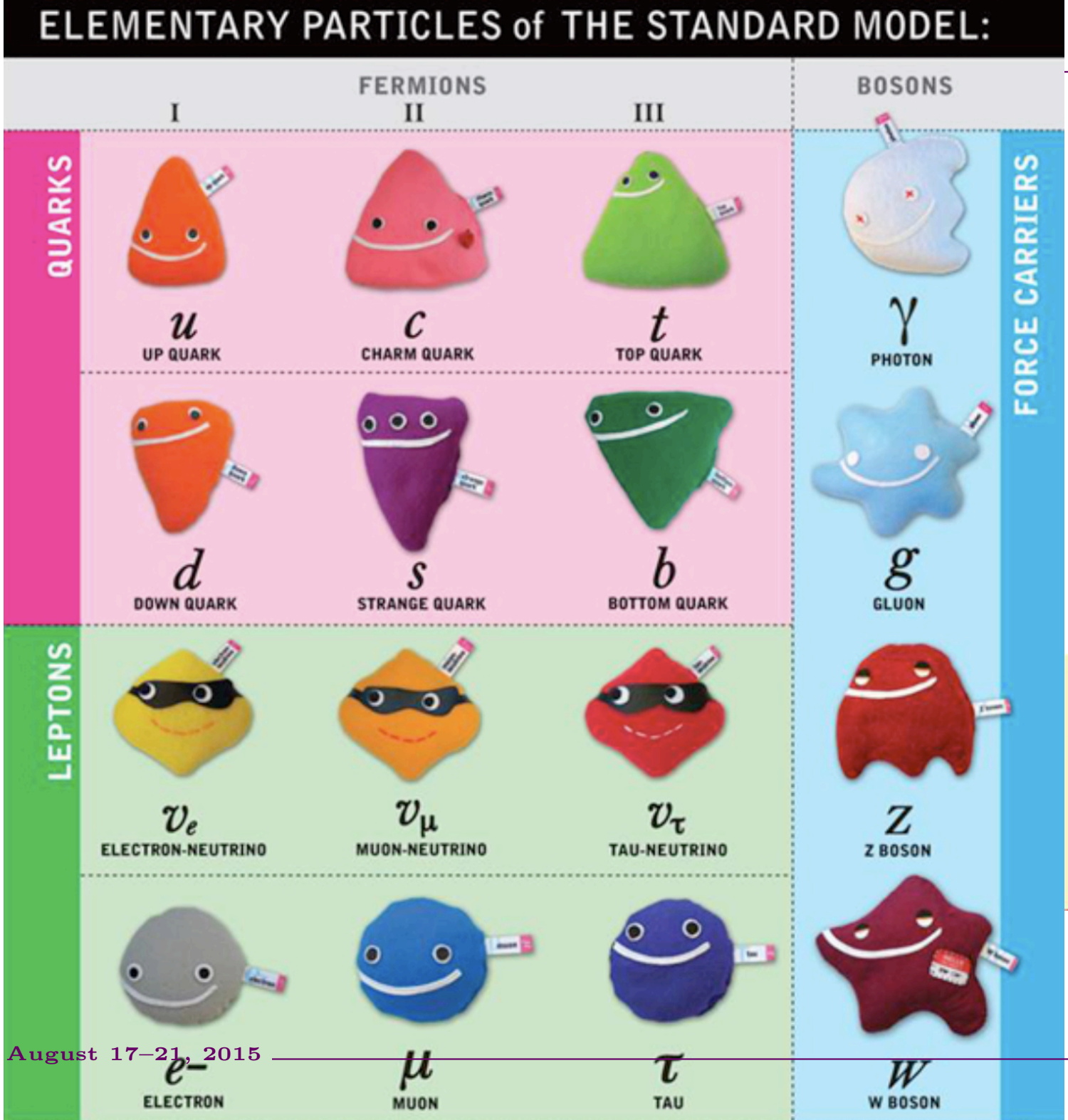
I will assume that everyone has been exposed to Quantum Field Theory and some Particle Physics in the past. These lectures are aimed at helping us understand the consequences of non-zero neutrino masses for particle physics and what might lie beyond the Standard Model.

Some good references include:

- "Introduction to Quantum Field Theory," Peskin and Schroeder;
- "Gauge Theories of the Strong, Weak, and Electromagnetic Interactions," Quigg;
- Dynamics of the Standard Model," Donoghue, Golowich, Holstein;
- S. Willenbrock, TASI 2004 Lectures, arXiv:hep-ph/0410370.

Tentative Outline for the Five Lectures

- SM Overview and QFT Basics;
- The Fundamental Fields, including Weyl, Dirac and Majorana fermions;
- Gauge Symmetries and SM Interactions;
- Spontaneous Symmetry Breaking and the Origin of Mass;
- Evidence for New Physics;
- Neutrino Masses.



21st Century Periodic Table

(Now with Higgs boson!)

http://www.particlezoo.net

- Result of over 60 years of particle physics theoretical and experimental research.
- Theoretical formalism based on the marriage of Quantum Mechanics and Special Relativity – Relativistic Quantum Field Theory.
- Very Powerful – once we specify the model ingredients: field content (matter particles) and the internal symmetries (interactions), the dynamics of the system is uniquely specified by a finite set of free parameters.

[courtesy of Pilar Hernández]

SM Nobel tapestry

Quantum Field Theory in a (Tiny) Nutshell

The objects we care about are quantum fields, real or complex functions of space-time. They are operator-value objects. In the case of free fields, the differential equation that describes the different fields are solvable. These act on a Fock-space, and create and destroy free-particles with well-defined energy and momentum (including a dispersion relation: $E^2 - |\vec{p}|^2 = m^2$).

In the presence of interactions, the differential equations that describe the operators are non-linear and, in general, cannot be solved exactly. We make use of perturbation theory in order to compute useful observables. Mostly, we care about scattering: distinct asymptotic states in $t = \pm\infty$, and the matrix elements that connect them (e.g., $\mathcal{S} = {}_{+\infty}\langle \mu(p_\mu)\bar{\nu}(p_\nu)|\pi(p_\pi)\rangle_{-\infty}$). These asymptotic states are treated as if they were destroyed and created by the free fields.

[When perturbation theory is not effective, we need to get smarter. I won't talk about this at all!]

Elementary particles or interactions represented by (complex) causal quantum fields (operators in Fock space):

$$\phi(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_{\mathbf{p}}}} \left(u_s^a(\mathbf{p}) \hat{a}^s_{\mathbf{p},a} e^{-ipx} + v_s^a(\mathbf{p}) \hat{b}^{s\dagger}_{\mathbf{p},a} e^{ipx} \right) \Big|_{p^0 = E_{\mathbf{p}} \equiv \sqrt{\mathbf{p}^2 + m^2}}$$

a : flavour index

s : spin index

One (anti)particle states:

$$\begin{cases} |\mathbf{p}, s, a\rangle_+ = \hat{a}^{s\dagger}_{\mathbf{p},a} |0\rangle \\ |\mathbf{p}, s, a\rangle_- = \hat{b}^{s\dagger}_{\mathbf{p},a} |0\rangle \end{cases}$$

$u_s^a(\mathbf{p}), v_s^a(\mathbf{p})$ wave functions in spin and internal space

[courtesy of Pilar Hernández]

Lorentz Invariance

We only care about QFTs which are Lorentz Invariant (unlike, say, folks in condensed matter physics). This is because, of course, Nature seems to like Lorentz invariance, at least at "small" distances and in the limit where we can ignore gravitational interactions. We will always ignore gravity, for two reasons. (i) Gravitational interactions are really weak compared to all other interactions for the types of processes we care about, and (ii) we don't know how to write down a mathematically well-defined QFT for the gravitational interactions.

Lorentz invariance dictates the objects (fields) we are allowed to use as ingredients. They are

- Massive fields with spin 0, 1/2, 1, 3/2, 2, etc
- Massless fields with spin 0, 1/2, 1, 3/2, 2, etc

Lorentz Invariance

If we want the theory to be renormalizable, we must stick to the following "types" of fundamental fields:

- Real Scalar. One degree of freedom.
- Complex Scalar. Two degrees of freedom.
- Spin 1/2 Fermion. Two degrees of freedom (Weyl and Majorana fermions) or four degrees of freedom (Dirac fermion). More on this later.
- Vector field. Two degrees of freedom when massless, three when massive. The latter are problematic. More on this later.

This is it! [Composite objects can be more complicated. As you know, the $\Delta^{++}, \Delta^{+}, \Delta^{0}, \Delta^{-}$ baryons have spin-3/2, the f_2 mesons have spin 2, etc. Check out the Particle Data Book(let) if you want to be impressed!]

Weyl, Majorana, and Dirac Fermions

The generators of the Lorentz group are the rotations, J_i, and the boosts, K_i. They satisfy the algebra

$$[J_i, J_j] = i\epsilon_{ijk} J_k$$
$$[K_i, K_j] = -i\epsilon_{ijk} J_k$$
$$[J_i, K_j] = i\epsilon_{ijk} K_k \ .$$

The J_i are Hermitian, and the K_i are anti-Hermitian. The J_i satisfy the algebra of the rotation group, $SU(2)$. The last commutation relation expresses the fact that a boost transforms as a three-vector under rotations.

To disentangle the algebra, define the Hermitian generators

$$\begin{aligned} A_i &= \frac{1}{2}(J_i + iK_i) \\ B_i &= \frac{1}{2}(J_i - iK_i) \ . \end{aligned}$$

It is easy to show (try it!) that the A_i and B_i satisfy the algebra

$$\begin{aligned} [A_i, A_j] &= i\epsilon_{ijk}A_k \\ [B_i, B_j] &= i\epsilon_{ijk}B_k \\ [A_i, B_j] &= 0 \ . \end{aligned}$$

A_i and the B_i: algebra of two independent $SU(2)$.

The Lorentz group, $SO(3,1)$, is locally isomorphic to $SU(2) \times SU(2)$.

Representations of $SU(2)$ are familiar from the representations of the rotation group: each representation is label by "spin," which can have integer or half-integer values.

Representations of the Lorentz group are labeled (a, b), where $a, b = 1/2, 1, 3/2, \ldots$. The simplest representation is $(0, 0)$, which corresponds to a scalar field. The simplest nontrivial representation is $(1/2, 0)$, which corresponds to a Weyl spinor, χ.

A Weyl spinor is a 2-component object that transforms under rotations and boosts as

$$\begin{aligned} \chi &\rightarrow e^{-\frac{i}{2}\sigma\cdot\theta}\chi \\ \chi &\rightarrow e^{-\frac{1}{2}\sigma\cdot\eta}\chi \end{aligned}$$

where η is the rapidity, which is related to the velocity by $\beta = \tanh\eta$. The transformation under rotations shows that a Weyl spinor carries spin 1/2.

Massive Weyl Spinor

$$\mathcal{L} \supset \frac{1}{2} m (\chi^T \epsilon \chi + h.c.)$$

where $\epsilon \equiv i\sigma_2$ is the 2×2 antisymmetric matrix. This is Lorentz invariant.

(Proof: Denote a Lorentz transformation acting on χ by the matrix M, where

$$M = e^{-\frac{i}{2}\sigma\cdot\theta} \text{ or } e^{-\frac{1}{2}\sigma\cdot\eta} .$$

Thus under a Lorentz transformation,

$$\chi^T \epsilon \chi \to \chi^T M^T \epsilon M \chi .$$

Displaying indices,

$$(M^T)_{\alpha\beta}\epsilon_{\beta\gamma}M_{\gamma\delta} = \epsilon_{\beta\gamma}M_{\beta\alpha}M_{\gamma\delta} = \epsilon_{\alpha\delta}\det M = \epsilon_{\alpha\delta}$$

where the last step uses the fact that $\det M = 1$. Thus $M^T \epsilon M = \epsilon$, which completes the proof in This also shows that the Lorentz group is locally isomorphic to $SL(2, \mathcal{C})$, the group of 2×2 complex matrices of unit determinant.)

Although a Majorana mass is less familiar than a Dirac mass, it is actually a more basic quantity, constructed from a single Weyl spinor. In this sense a Majorana mass is the simplest fermion mass term. However, if χ carries an unbroken global or local $U(1)$ charge, a Majorana mass is forbidden, since it would violate this symmetry.

None of the fermions of the standard model (except neutrinos – much more on this later!) can have a Majorana mass, since they carry electric charge. This is why we often don't hear about them.

[More generally, if χ transforms under a complex or pseudoreal representation of an unbroken global or local internal symmetry, a Majorana mass is forbidden. Let

$$\chi \to U\chi$$

where U is a unitary transformation acting on a set of Weyl spinors. The Majorana mass term transforms as

$$\chi^T \epsilon \chi \to \chi^T U^T \epsilon U \chi = \chi^T \epsilon U^T U \chi$$

This is invariant only if $U^T U = 1$, which is true only if the unitary transformation U is real ($U^* = U$).]

Physically, a fermion with a Majorana mass is its own antiparticle. It is referred to as a Majorana fermion. It cannot carry an unbroken global or local $U(1)$ charge (or, more generally, transform under a complex or pseudoreal representation) because a particle and an antiparticle must carry opposite charge.

Exercise - Gluinos are hypothetical Majorana fermions that are the superpartners of gluons. Why can they carry color charge?

Exercise - Consider a Weyl fermion that transforms under the defining representation of an unbroken $SU(2)$ group. Show that the Majorana mass term

$$\mathcal{L} = \frac{1}{2} m (\epsilon_{ab} \chi^{aT} \epsilon \chi^{b} + h.c.)$$

is invariant under the $SU(2)$ symmetry. However, show that this term vanishes.

If a Weyl fermion transforms under a complex or pseudoreal representation of an unbroken global or local symmetry, then we need to introduce a second Weyl fermion that transforms under the complex-conjugate representation in order to construct a mass term. This is a Dirac mass. Let χ, ξ transform under the $(1/2, 0)$ representation of the Lorentz group, and transform under some unbroken global or local symmetry as

$$\begin{aligned} \chi &\to U\chi \\ \xi &\to U^*\xi \,. \end{aligned}$$

Then a Lorentz-invariant mass term may be formed which respects the unbroken symmetry,

$$\mathcal{L} = m(\xi^T \epsilon \chi + h.c.)$$

since

$$\xi^T \epsilon \chi \to \xi^T U^\dagger \epsilon U \chi = \xi^T \epsilon \chi \,.$$

Thus it takes two Weyl spinors to construct a Dirac mass. A fermion with a Dirac mass is called a Dirac fermion.

The Dirac Spinor (The One With Which Everyone is Familiar)

Dirac spinor, which is a four-component object constructed from a pair of $(1/2, 0)$ Weyl spinors χ, ξ via

$$\psi = \begin{pmatrix} \chi \\ \epsilon\xi^* \end{pmatrix} .$$

In terms of a Dirac spinor, a Dirac mass is written in the familiar form

$$\begin{aligned} \mathcal{L} = -m\bar{\psi}\psi &= -m\left(\chi^\dagger, -\xi^T\epsilon\right)\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \chi \\ \epsilon\xi^* \end{pmatrix} \\ &= m(\xi^T\epsilon\chi - \chi^\dagger\epsilon\xi^*). \end{aligned}$$

We are using the so-called Weyl or chiral basis for the gamma matrices,

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix} \quad \gamma_5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \quad ,$$

where each entry in the above matrices is itself a 2×2 matrix. In this basis, the chiral projection operator $(1 \pm \gamma_5)/2$ projects out the Weyl spinors,

$$\psi = \frac{1-\gamma_5}{2}\psi + \frac{1+\gamma_5}{2}\psi = \psi_L + \psi_R$$

where

$$\psi_L = \begin{pmatrix} \chi \\ 0 \end{pmatrix}$$

$$\psi_R = \begin{pmatrix} 0 \\ \epsilon\xi^* \end{pmatrix} .$$

ψ_L is the four-component Dirac-spinor version of the Weyl spinor χ, and similarly for ψ_R and $\epsilon\xi^*$.

Exercise - If ξ transforms under the $(1/2, 0)$ representation of the Lorentz group, show that $\epsilon\xi^*$ transforms under the $(0, 1/2)$ representation. [Hint: recall $M^T \epsilon M = \epsilon$].

A Dirac spinor transforms under the $(1/2, 0) \oplus (0, 1/2)$ representation of the Lorentz group, corresponding to $\psi = \psi_L + \psi_R$.

The Majorana Spinor

While a Dirac spinor is composed of two Weyl spinors, a Majorana spinor is a four-component object composed of a single Weyl spinor,

$$\psi_M = \begin{pmatrix} \chi \\ \epsilon\chi^* \end{pmatrix} .$$

Thus it is simply a four-component version of a Weyl spinor.

Exercise - Show that

$$\mathcal{L} = -\frac{1}{2} m \bar{\psi}_M \psi_M$$

is a Majorana mass term.

The charge-conjugation matrix C in the Weyl representation is

$$C = \begin{pmatrix} -\epsilon & 0 \\ 0 & \epsilon \end{pmatrix} .$$

Given a Dirac spinor ψ, we can form the conjugate spinor via

$$\begin{aligned} \psi^c &\equiv C\gamma^0\psi^* \\ &= \begin{pmatrix} -\epsilon & 0 \\ 0 & \epsilon \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \chi^* \\ \epsilon\xi \end{pmatrix} \\ &= \begin{pmatrix} \xi \\ \epsilon\chi^* \end{pmatrix} . \end{aligned}$$

Thus

$$\psi_L = \begin{pmatrix} \chi \\ 0 \end{pmatrix}$$

$$(\psi^c)_L = \begin{pmatrix} \xi \\ 0 \end{pmatrix} = (\psi_R)^c .$$

Exercise - Show that

$$\mathcal{L} = -m((\psi^c)_L^T C\psi_L + h.c.)$$

is a Dirac mass.

Exercise - Show that

$$\mathcal{L} = -\frac{1}{2}m(\psi_L^T C\psi_L + h.c.)$$

is a Majorana mass. Thus one can write a Majorana mass in terms of a Dirac spinor.

Exercise - Show that

$$\psi_M^c = \psi_M \ .$$

This is called the Majorana condition.

One last point: We can write a Dirac mass in terms of Majorana spinors

Consider a Dirac mass written in terms of a Dirac spinor,

$$\mathcal{L} = -m\bar{\psi}\psi = -\frac{1}{2}m(\bar{\psi}\psi + \bar{\psi}^c\psi^c)$$

where I'll let you verify the last equality. Now define the Majorana spinors

$$\begin{aligned} \psi_M^1 &\equiv \frac{1}{\sqrt{2}}(\psi + \psi^c) \\ \psi_M^2 &\equiv \frac{1}{\sqrt{2}}(\psi - \psi^c)\ . \end{aligned}$$

and

$$\mathcal{L} = -\frac{1}{2}m(\bar{\psi}_M^1\psi_M^1 + \bar{\psi}_M^2\psi_M^2)\ .$$

Thus a Dirac fermion is equivalent to two degenerate Majorana fermions. However

$$\begin{aligned} (\psi_M^1)^c &= \frac{1}{\sqrt{2}}(\psi^c + \psi) = \psi_M^1 \\ (\psi_M^2)^c &= \frac{1}{\sqrt{2}}(\psi^c - \psi) = -\psi_M^2\ . \end{aligned}$$

The two Majorana spinors have the opposite sign under charge conjugation.

spinor	Majorana mass	Dirac mass
Weyl	$\frac{1}{2}m(\chi^T\epsilon\chi + h.c.)$	$m(\xi^T\epsilon\chi + h.c.)$
Majorana	$-\frac{1}{2}m\bar{\psi}_M\psi_M$	$-\frac{1}{2}m(\bar{\psi}^1_M\psi^1_M + \bar{\psi}^2_M\psi^2_M)$
Dirac	$-\frac{1}{2}m(\psi^T_L C\psi_L + h.c.)$	$-m((\psi^c)^T_L C\psi_L + h.c.)$
Dirac	$-\frac{1}{2}m(\overline{(\psi^c)}_R\psi_L + h.c.)$	$-m\bar{\psi}\psi$

Table 1: A Majorana mass and a Dirac mass may be constructed from Weyl, Majorana, or Dirac spinors.

[from S. Willenbrock, arXiv:hep-ph/0410370.]

Very quickly: Kinetic Energy terms.

$$\chi^T \left(\bar{\sigma}^\mu\right) \chi, \quad \bar{\sigma}^\mu = \left(1, -\vec{\sigma}\right),$$

transforms like a four-vector, similar to $\bar{\psi}\gamma^\mu\psi$. In fact, (check this!)

$$\bar{\psi}\gamma^\mu\partial_\mu\psi = \chi^T \left(\bar{\sigma}^\mu\partial_\mu\right) \chi + \xi^T \left(\bar{\sigma}^\mu\partial_\mu\right) \xi$$

These don't "mix" χ and ξ. As far as they are concerned, these two objects are completely unrelated. The same is true of the four-component ψ_R and ψ_L. As far as the kinetic energy terms are concerned, they are different, unrelated objects.

Gauge Invariance and the Gauge Interactions

All fundamental interactions – strong, weak, electromagnetic (remember we pretend gravity does not exist) – except for the so-called Yukawa interactions, are mediated by vector fields.

We need to be careful when it comes to considering the vector fields. It turns out that we can only write renormalizable QFTs with vector fields if we also introduce gauge invariance.

Gauge invariance is not a choice.

Gauge Symmetry: U(1)

Maxwell eqs. in terms of gauge potentials are invariant under

$$A_\mu(x) \to A_\mu(x) + \partial_\mu \alpha(x)$$

Eqs. of motion for any charged field will also remain invariant if

$$\Psi(x) \to e^{iq\alpha(x)}\Psi(x)$$

e.m. gauge invariance <-> U(1) local gauge transformation

$$D_\mu \Psi \equiv (\partial_\mu - iqA_\mu)\Psi \to e^{iq\alpha} D_\mu \Psi$$

U(1) invariants:

$$\bar{\Psi}\gamma_\mu D_\mu \Psi \qquad \bar{\Psi}\Psi \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

[courtesy of Pilar Hernández]

Gauge Symmetry: U(1)

$$\mathcal{L}_{QED} = -\frac{1}{4}F_{\mu\nu}^2 + \bar{\Psi}(i\not{D} - m)\Psi$$

$$D_\mu = \partial_\mu - iqA_\mu$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

A mass term $-\frac{m_A^2}{2}A_\mu^2$ breaks symmetry: photon massless

[courtesy of Pilar Hernández]

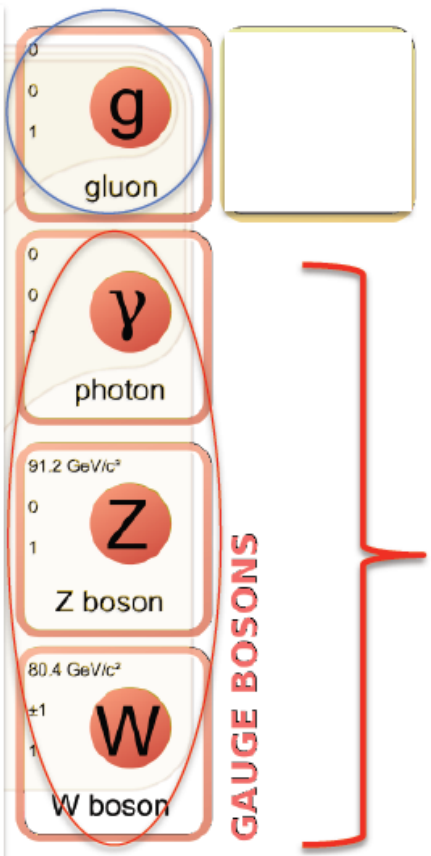
[courtesy of Pilar Hernandez]

SM is a gauge theory

$$SU(3) \times SU(2) \times U(1)_Y$$

$SU(3)$

$SU(2) \times U(1)_Y$



$G_\mu^{a=1,..,8}$

$W_{\mu\nu}^{a=1,2,3}, B_\mu$

[courtesy of Pilar Hernández]

				$SU(3)$	$SU(2)$	$U(1)_Y$
$Q^i =$	$\begin{pmatrix} u \\ d \end{pmatrix}$	$\begin{pmatrix} c \\ s \end{pmatrix}$	$\begin{pmatrix} t \\ b \end{pmatrix}$	3	2	$\frac{1}{6}$
$(u^c)^i =$	u^c	c^c	t^c	$\bar{3}$	1	$-\frac{2}{3}$
$(d^c)^i =$	d^c	s^c	b^c	$\bar{3}$	1	$\frac{1}{3}$
$L^i =$	$\begin{pmatrix} \nu_e \\ e \end{pmatrix}$	$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}$	$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$	1	2	$-\frac{1}{2}$
$(e^c)^i =$	e^c	μ^c	τ^c	1	1	1

Table 2: All fields can be interpreted as two-component Weyl fermions of the left-handed type.

$$\mathcal{L} \supset i\bar{Q}^i_L \not{D} Q^i_L + i\overline{(u^c)}^i_L \not{D} (u^c)^i_L + i\overline{(d^c)}^i_L \not{D} (d^c)^i_L + i\bar{L}^i_L \not{D} L^i_L + i\overline{(e^c)}^i_L \not{D} (e^c)^i_L \ .$$

or

$$\mathcal{L} \supset i\bar{Q}^i_L \not{D} Q^i_L + i\bar{u}^i_R \not{D} u^i_R + i\bar{d}^i_R \not{D} d^i_R + i\bar{L}^i_L \not{D} L^i_L + i\bar{e}^i_R \not{D} e^i_R \ .$$

All SM gauge interactions are included in the $\not{D}$ terms. At this point, this is a theory of chiral, charged massless fermions which interact with massless spin-1 particles.

All fermion fields are completely independent. They are interpreted as follows:

- Q destroys left-handed quark doublets and creates right-handed anti-quark-doublets.
- u^c destroys left-handed anti-up-quarks and creates right-handed up-quarks.
- L destroys left-handed lepton doublets and creates right-handed anti-lepton-doublets.
- etc

What is wrong with this picture

- Aren't the W-boson and the Z-boson massive? Where are the longitudinal gauge bosons (extra degrees of freedom of the spin-1 massive fields)?
- Where is the photon? Where are the electric charges of all the different fermion fields?
- Aren't the fermions massive?

The answer is the introduction of new field(s) and spontaneous gauge symmetry breaking, as we will discuss momentarily.

A couple of asides

- Global Symmetries: In the absence of fermion masses, $\mathcal{L}$ has a good deal of (accidental) global symmetry, $U(3)^5$

$$\begin{aligned} Q^i &\rightarrow U_Q^{ij} Q^j, \\ (u^c)^i &\rightarrow U_u^{ij} (u^c)^j, \\ (d^c)^i &\rightarrow U_d^{ij} (d^c)^j, \\ L^i &\rightarrow U_L^{ij} L^j, \\ (e^c)^i &\rightarrow U_e^{ij} (e^c)^j \ . \end{aligned}$$

- We have what is called a chiral gauge theory. This means that, unlike electromagnetism, there are no Weyl fermions with opposite charges. These theories are susceptible to anomalies!

Anomalies in a Tiny Nutshell

There are circumstances where Lagrangian possesses a classical symmetry that is broken at the quantum mechanical level.

This is not a problem for global symmetries. Indeed, anomalies help explain certain low-energy phenomena ($\pi^0 \to \gamma\gamma$).

This is a disaster for gauge symmetries. If these are anomalous, gauge invariance is not around to ensure the theory is mathematically consistent.

The "poster child" for potentially anomalous theories are chiral gauge theories. A.k.a. the SM. It requires anomaly cancellations. This means one adds up the contributions to the anomaly of all different fields and hopes that they add up to zero. While this is mysterious, it is stable.

Constraints include

$$\sum_{i=Q,u,d,L,e} Y_i = 0, \qquad \sum_{i=Q,u,d,L,e} Y_i^3 = 0, \qquad \sum_{i=Q,u,d} Y_i^3 = 0, \sum_{i=Q,L} Y_i^3 = 0.$$

If you have never seen this before, check it!

SSB of Gauge symmetry

The effective field theory of superconductivity: a complex scalar field with charge q coupled to the U(1) gauge field

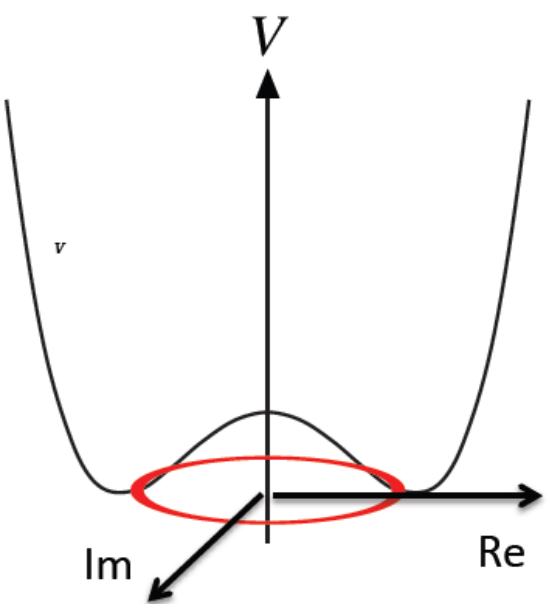
$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\phi)^\dagger D_\mu\phi - V(\phi) \qquad D_\mu = \partial_\mu - iqA_\mu$$

$$V(\phi) = -\mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2$$

$$\lambda > 0$$

Lowest energy configuration (vacuum):

$$\phi^\dagger\phi = \frac{\mu^2}{2\lambda} \equiv \frac{v^2}{2} \qquad \phi = \frac{v}{\sqrt{2}}e^{i\theta}$$



[courtesy of Pilar Hernández]

SSB of Gauge symmetry

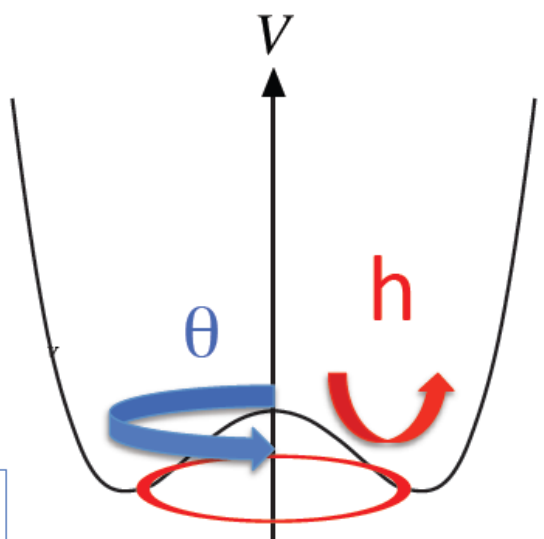
Perturbing around the true vacuum $\phi = \frac{v + h(x)}{\sqrt{2}} e^{i\theta(x)}$

$$\mathcal{L}(\phi, A_\mu) = -\tfrac{1}{4} F_{\mu\nu} F_{\mu\nu}$$

$$+ \tfrac{1}{2}\partial_\mu h \partial_\mu h + \tfrac{v^2}{2}\partial_\mu\theta\partial_\mu\theta - qv^2\partial_\mu\theta A_\mu + \tfrac{q^2v^2}{2} A_\mu A_\mu$$

$$- \mu^2 h^2 - \lambda v h^3 - \tfrac{\lambda}{4} h^4$$

θ is a Goldstone Boson



GB:field with only derivative couplings

[courtesy of Pilar Hernández]

The ABEGH^{2}KN Mechanism

Anderson-Brout-Englert-Guralnik-Hagen-Higgs-Kibble-Nambu

$$\mathcal{L}^{(2)} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{q^2v^2}{2}A_\mu A_\mu + \frac{1}{2}(\partial_\mu h\,\partial_\mu h - \mu^2 h^2) + \frac{v^2}{2}\partial_\mu\theta\,\partial_\mu\theta - qv^2\partial_\mu\theta A_\mu$$

Gauge transformation: $A'_\mu = A_\mu - \frac{1}{q}\partial_\mu\theta$

$$= -\frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} + \frac{q^2v^2}{2}A'_\mu A'_\mu + \frac{1}{2}(\partial_\mu h\,\partial_\mu h - \mu^2 h^2)$$

Goldstone mode -> massive gauge field (ie. longitudinal polarization)

Goldstone mode "is eaten" by the gauge field to get massive: unitary gauge

Radial mode -> massive neutral scalar field

[courtesy of Pilar Hernández]

SM BEH mechanism

A complex doublet with quantum numbers Y=+1/2 and no color

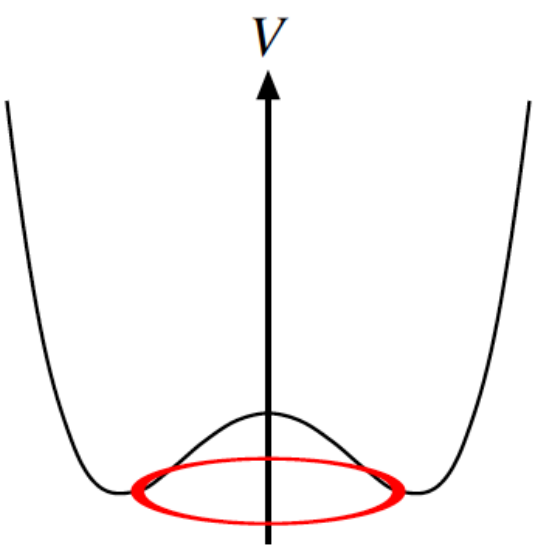
$$\mathcal{L}_\phi = D_\mu\phi^\dagger D_\mu\phi - V(\phi) \qquad D_\mu\phi = (\partial_\mu - igW_\mu^a\frac{\sigma^a}{2} - i\frac{g'}{2}B_\mu)\phi$$

$$\phi = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_0 + i\phi_3 \end{pmatrix} \qquad \phi \to e^{iT^a\alpha^a}\phi \quad T^a = \left(\frac{\sigma^0}{2}, \frac{\vec{\sigma}}{2}\right)$$

A potential with a minimum at

$$\langle\phi^\dagger\phi\rangle = \frac{v^2}{2} = \frac{\mu^2}{2\lambda}$$

$$\langle\phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$$



[courtesy of Pilar Hernández]

SM BEH mechanism

Of the full symmetry group: $\phi \to e^{iT^a\alpha^a}\phi \quad T^a = \left(\frac{\sigma^0}{2}, \frac{\vec{\sigma}}{2}\right)$

A U(1) subgroup remains unbroken

$$(T^0 + T^3)\langle\phi\rangle = 0$$

$$SU(2) \times U(1) \to U(1)_{em}$$

v

three broken generators: three massive gauge fields (W^{+-}, Z^0) and one massless photon

[courtesy of Pilar Hernández]

Gauge boson masses

$$\phi = e^{i\alpha^a T^a} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

In unitary gauge:

$$D_\mu \phi^\dagger D_\mu \phi = \frac{1}{2}\partial_\mu h \, \partial_\mu h + \frac{1}{2}(0 \quad v+h) \left(g W_\mu^a \frac{\sigma^a}{2} + \frac{1}{2} g' B_\mu \right)^2 \begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

$$= \frac{1}{2}\frac{v^2}{4} \left\{ g^2 \left((W_\mu^1)^2 + (W_\mu^2)^2 \right) + (g' B_\mu - g W_\mu^3)^2 \right\} = m_W^2 W_\mu^+ W_\mu^- + \frac{1}{2} m_Z^2 Z_\mu Z_\mu + O(h)$$

Charged weak: $W_\mu^\pm = \frac{1}{\sqrt{2}} \left(W_\mu^1 \pm i W_\mu^2 \right)$ $\qquad m_W \equiv g \frac{v}{2}$

Neutral weak $Z_\mu = \frac{1}{\sqrt{g^2 + g'^2}} \left(g W_\mu^3 - g' B_\mu \right)$ $\qquad m_Z \equiv \sqrt{g^2 + g'^2} \frac{v}{2}$

Electromagnetic $A_\mu = \frac{1}{\sqrt{g^2 + g'^2}} \left(g' W_\mu^3 + g B_\mu \right)$

[courtesy of Pilar Hernández]

Gauge Boson masses

Weak mixing angle

$$Z_\mu = \frac{1}{\sqrt{g^2 + g'^2}} \left(g W_\mu^3 - g' B_\mu \right)$$

$$A_\mu = \frac{1}{\sqrt{g^2 + g'^2}} \left(g' W_\mu^3 + g B_\mu \right)$$

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}$$

$$\cos\theta_W \equiv \frac{g}{\sqrt{g^2 + g'^2}} \qquad \sin\theta_W = \frac{g'}{\sqrt{g^2 + g'^2}}$$

$$m_W = m_Z \cos\theta_W$$

[courtesy of Pilar Hernández]

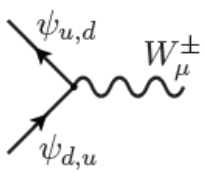
Neutral currents

All fermions are doublets $D_\mu \Psi = (\partial_\mu - i g W_\mu^a \frac{\sigma^a}{2} - i Y_\Psi g' B_\mu)\Psi$

$$D_\mu \Psi = \left(\partial_\mu - i\frac{g}{\sqrt{2}}(W_\mu^+ T^+ + W_\mu^- T^-) - i\frac{g}{\cos\theta_W}(T^3 - \sin^2\theta_W Q_\Psi) Z_\mu - ieQ_\Psi A_\mu \right) \Psi$$

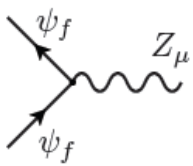
$$Q_\Psi \equiv T^3 + Y_\Psi \quad e \equiv \frac{g g'}{\sqrt{g^2 + g'^2}} = g \sin\theta_W$$

Off-diagonal in isospin!

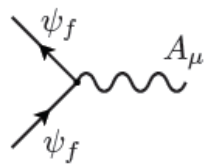


$$i\frac{g}{\sqrt{2}}\gamma_\mu \frac{1-\gamma_5}{2}$$

Diagonal in isospin!



$$i\frac{g}{\cos\theta_W}\gamma_\mu \left(g_V^f - g_A^f \gamma_5\right)$$



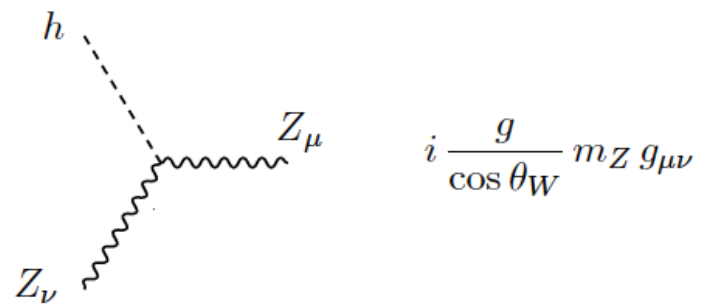
$$-ieQ_f\gamma_\mu$$

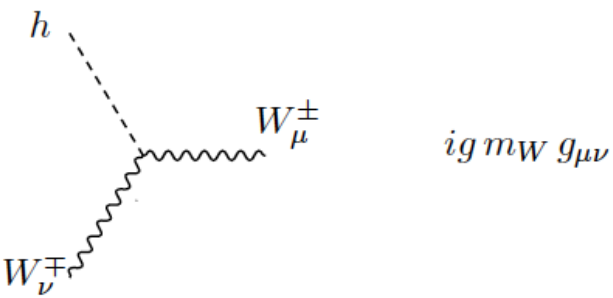
$$g_V^f = \frac{1}{2}T_f^3 - Q_f \sin^2\theta_W, \quad g_A^f = \frac{1}{2}T_f^3 \,.$$

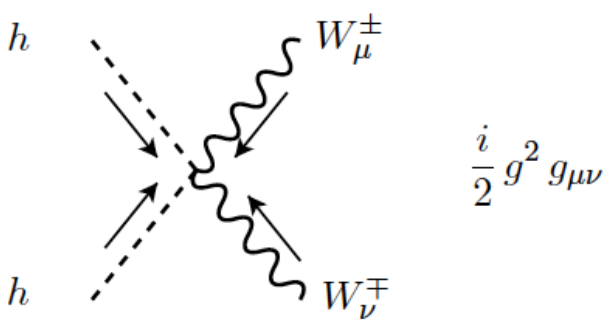
[courtesy of Pilar Hernández]

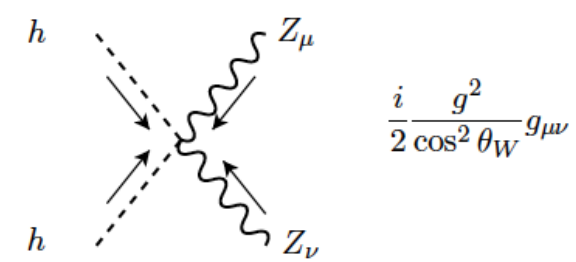
Higgs-Gauge couplings

$$(1 + h/v)^2 (m_W^2 W_\mu^+ W_\mu^- + \frac{1}{2} m_Z^2 Z_\mu Z_\mu)$$









[courtesy of Pilar Hernández]

Fermion Masses

Given the $SU(2)$ and $U(1)$ charges of the Higgs field and the fermions, Yukawa interactions are allowed:

$$-\mathcal{L} \supset Y_u Q u^c \phi + Y_d Q d^c \tilde{\phi} + Y_e L e^c \tilde{\phi} + h.c.,$$

where $\tilde{\phi} \equiv i\sigma_2 \phi^*$. Using

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad \langle \tilde{\phi} \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v \\ 0 \end{pmatrix},$$

$$-\mathcal{L} \supset (m_u)_{ij} u^i (u^c)^j + (m_d)_{ij} d^i (d^c)^j + (m_e)_{ij} e^i (e^c)^j + h.c.,$$

where $m_f \equiv Y_f v/\sqrt{2}$.

$u + u^c$, $d + d^c$, $e + e^c$ merge into Dirac fermions [note that all these fields have equal-but-opposite electric charges]. Neutrinos are left unpaired! $\rightarrow$ neutrinos are massless. Robust prediction, stable under quantum corrections.

Global Symmetries: $U(3)^5$ explicitly broken down to $U(1)_B \times U(1)_e \times U(1)_\mu \times U(1)_\tau$. I will come back to this.

How many physically relevant parameters remain after the field redefinitions are performed? [Quark Sector]

The number of parameters contained in the complex matrices Y_u, Y_d is $2 \times 3 \times 3 \times 2 = 36$.

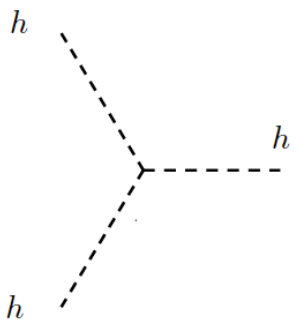
The unitary symmetries U_Q, U_u, U_d are a subset of the quark field redefinitions. There are $3 \times 3 \times 3$ degrees of freedom in these symmetries (a unitary $N \times N$ matrix has N^2 free parameters), so the total number of parameters that remain in the full Lagrangian after field redefinitions is

$$2 \times 3 \times 3 \times 2 - (3 \times 3 \times 3 - 1) = 10,$$

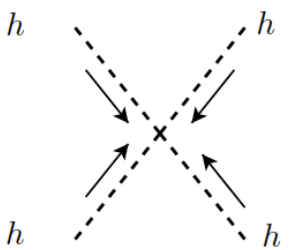
subtracting baryon number from the subset of field redefinitions that are symmetries of the matter Lagrangian. $U(1)_B$ is a symmetry of the Yukawa Lagrangian, and hence cannot be used to diagonalize the mass matrices. The ten remaining parameters correspond to the six quark masses and the four parameters of the Cabibbo-Kobayashi-Maskawa (CKM) matrix (three mixing angles and one CP-violating phase).

Higgs self-couplings

From the Higgs potential:



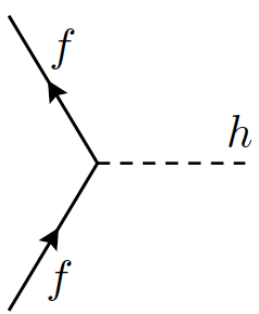
$$-\frac{3}{2}\,i\,g\frac{m_h^2}{m_W^2}$$



$$-\frac{3}{4}\,i\,g^2\frac{m_h^2}{m_W^2}$$

Higgs-fermion couplings

$$(1+h/v)\left(-m_d\bar{d}_L d_R - m_u\bar{u}_L u_R - m_l\bar{l}_L l_R\right) + h.c.$$



$$-i\,\frac{g}{2}\,\frac{m_f}{m_W}$$

[courtesy of Pilar Hernández]

Quark mixing: Cabbibo-Kobayashi-Maskawa

The Yukawa couplings are generic matrices in flavour space:

basis where CC and NC diagonal ≠ mass eigenbasis

$$\mathcal{L}_{SM} \supset -(\bar{d}_L, \bar{s}_L, \bar{b}_L) \underbrace{m_d}_{3\times3} \begin{pmatrix} d_R \\ s_R \\ b_R \end{pmatrix} - (\bar{u}_L, \bar{c}_L, \bar{t}_L) \underbrace{m_u}_{3\times3} \begin{pmatrix} u_R \\ c_R \\ t_R \end{pmatrix} - (\bar{e}_L, \bar{\mu}_L, \bar{\tau}_L) \underbrace{m_u}_{3\times3} \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix}$$

$$m_i = U_{Li}^{\dagger} \mathrm{Diag}(m_i) V_{Ri}$$

$$u'_L = U_{Lu} u_L, \;\; d'_L = U_{Ld} d_L, \;\; l'_L = U_{Ll} l_L, \;\; u'_R = V_{Ru} u_R, \;\; d'_R = V_{Rd} d_R, \;\; l'_R = V_{Rl} l_R$$

$$\mathcal{L}_{SM}^{CC} \supset -\frac{g}{\sqrt{2}} \bar{u}'_{Li} \underbrace{(U_{Lu} U_{Ld}^{\dagger})_{ij}}_{CKM} \gamma_\mu W_\mu^+ d'_{Lj} + h.c.$$

[courtesy of Pilar Hernández]

Quark mixing: Cabbibo-Kobayashi-Maskawa

The Yukawa couplings are generic matrices in flavour space:

basis where CC and NC diagonal ≠ mass eigenbasis

$$\mathcal{L}_{SM} \supset -(\bar{d}_L, \bar{s}_L, \bar{b}_L) \underbrace{m_d}_{3\times3} \begin{pmatrix} d_R \\ s_R \\ b_R \end{pmatrix} - (\bar{u}_L, \bar{c}_L, \bar{t}_L) \underbrace{m_u}_{3\times3} \begin{pmatrix} u_R \\ c_R \\ t_R \end{pmatrix} - (\bar{e}_L, \bar{\mu}_L, \bar{\tau}_L) \underbrace{m_u}_{3\times3} \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix}$$

$$m_i = U_{Li}^{\dagger} \mathrm{Diag}(m_i) V_{Ri}$$

$$u'_L = U_{Lu} u_L, \;\; d'_L = U_{Ld} d_L, \;\; l'_L = U_{Ll} l_L, \;\; u'_R = V_{Ru} u_R, \;\; d'_R = V_{Rd} d_R, \;\; l'_R = V_{Rl} l_R$$

$$\mathcal{L}_{SM}^{NC} \supset -\frac{g}{\cos\theta_W} \bar{d}'_{Li} \left(-\frac{1}{2} + \frac{1}{3}\sin^2\theta_W \right) \underbrace{(U_{Ld} U_{Ld}^{\dagger})_{ij}}_{\delta_{ij}} \gamma_\mu Z_\mu d'_{Lj}$$

$$-\frac{g}{\cos\theta_W} \bar{u}'_{Li} \left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W \right) \underbrace{(U_{Lu} U_{Lu}^{\dagger})_{ij}}_{\delta_{ij}} \gamma_\mu Z_\mu u'_{Lj}$$

[courtesy of Pilar Hernández]

Quark mixing: Cabbibo-Kobayashi-Maskawa

Neutral currents diagonal also in the mass eigenbasis: only quarks in the same family can exchange a Z boson

It was quite of a challenge to come up with this when only 1.5 quark family was known: u, d, s -> prediction of the charm !

Charged currents not diagonal: CKM 3x3 unitary matrix

$$|V|_{\rm CKM} = \begin{pmatrix} 0.97427 \pm 0.00015 & 0.22534 \pm 0.0065 & (3.51 \pm 0.15) \times 10^{-3} \\ 0.2252 \pm 0.00065 & 0.97344 \pm 0.00016 & (41.2^{+1.1}_{-5}) \times 10^{-3} \\ (8.67^{+0.29}_{-0.31}) \times 10^{-3} & (40.4^{+1.1}_{-0.5}) \times 10^{-3} & 0.999146^{+0.000021}_{-0.000046} \end{pmatrix}$$

PDG

[courtesy of Pilar Hernández]

$$\mathcal{L} \supset \bar{e}_L U W^\mu \gamma_\mu \nu_L - \bar{e}_L(M_e) e_R - \overline{\nu_L^c}(M_\nu)\nu_L + H.c.$$

Write $U = E^{-i\xi/2} U' E^{i\alpha/2}$, where $E^{i\beta/2} \equiv \mathrm{diag}(e^{i\beta_1/2}, e^{i\beta_2/2}, e^{i\beta_3/2})$, $\beta = \alpha, \xi$

$$\mathcal{L} \supset \bar{e}_L U' W^\mu \gamma_\mu \nu_L - \bar{e}_L E^{i\xi/2}(M_e) e_R - \overline{\nu_L^c}(M_\nu) E^{-i\alpha} \nu_L + H.c.$$

ξ phases can be "absorbed" by e_R,

α phases cannot go away!

on the other hand

Dirac Case:

$$\mathcal{L} \supset \bar{e}_L U W^\mu \gamma_\mu \nu_L - \bar{e}_L(M_e) e_R - \bar{\nu}_R(M_\nu)\nu_L + H.c.$$

$$\mathcal{L} \supset \bar{e}_L U' W^\mu \gamma_\mu \nu_L - \bar{e}_L E^{i\xi/2}(M_e) e_R - \bar{\nu}_R(M_\nu) E^{-i\alpha/2} \nu_L + H.c.$$

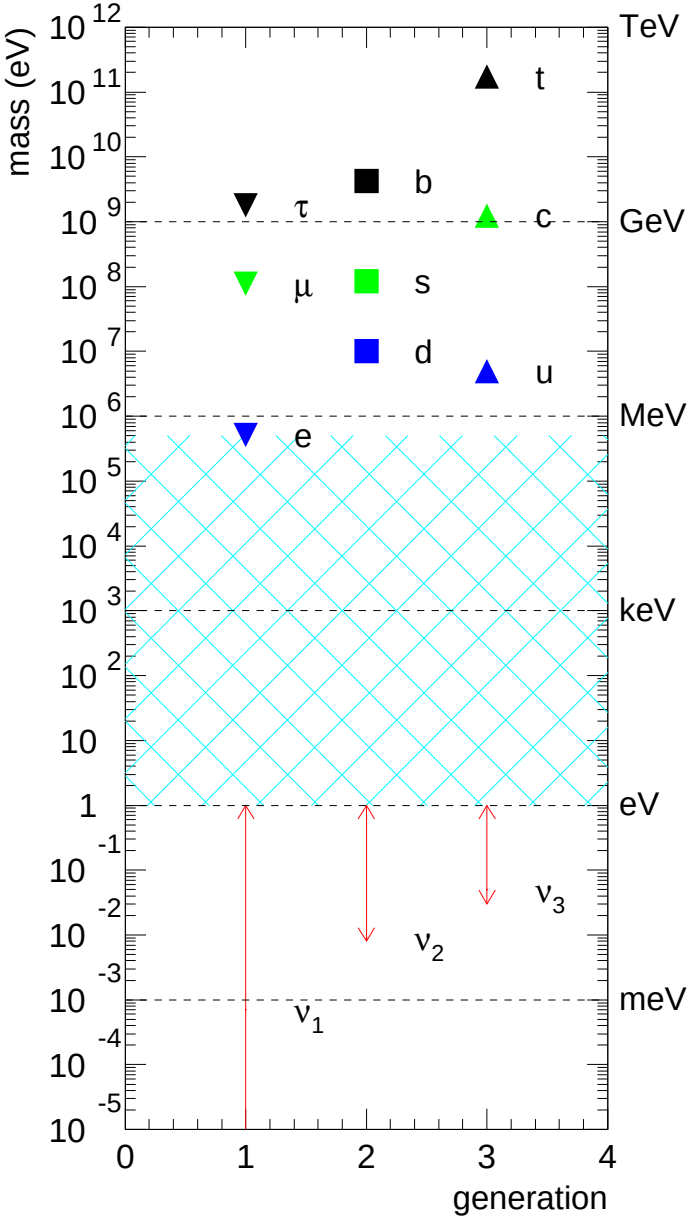
ξ phases can be "absorbed" by e_R, α phases can be "absorbed" by ν_R,

$$V_{MNS} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{e\tau 2} & U_{\tau 3} \end{pmatrix}' \begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & e^{i\alpha_3/2} \end{pmatrix}.$$

It is easy to see that the Majorana phases never show up in neutrino oscillations ($A \propto U_{\alpha i} U^*_{\beta_i}$).

Furthermore, they only manifest themselves in phenomena that vanish in the limit $m_i \to 0$ – after all they are only physical if we "know" that lepton number is broken.

$$A(\alpha_i) \propto m_i/E \quad \to \quad \text{tiny!}$$



NEUTRINOS HAVE MASS

albeit very tiny ones...

SO WHAT?

Only* "Palpable" Evidence of Physics Beyond the Standard Model

The SM we have been discussing predicts that neutrinos are strictly massless. Hence, massive neutrinos imply that the the SM is incomplete and needs to be replaced/modified.

Furthermore, the SM has to be replaced by something qualitatively different.

---

* I will discuss the rest of the evidence for new physics later.

Standard Model So Far, Summary in One Slide, No Equations

The SM is a quantum field theory with the following defining characteristics:

- Gauge Group $(SU(3)_c \times SU(2)_{\rm L} \times U(1)_{\rm Y})$;
- Particle Content (fermions: Q, u^c, d^c, L, e^c, scalars: ϕ).

Once this is specified, the SM is unambiguously determined:

- Most General Renormalizable Lagrangian;
- Measure All Free Parameters, and You Are Done! (after several decades of hard experimental work...)

If you follow these rules, neutrinos have no mass. Something has to give.

What is the New Standard Model? [νSM]

The short answer is – WE DON'T KNOW. Not enough available info!

Equivalently, there are several completely different ways of addressing neutrino masses. The key issue is to understand what else the νSM candidates can do. [are they falsifiable?, are they "simple"?, do they address other outstanding problems in physics?, etc]

We need more experimental input, and it looks like it may be coming in the near/intermediate future!

Neutrino Masses, EWSB, and a New Mass Scale of Nature

The LHC has revealed that the minimum SM prescription for electroweak symmetry breaking — the one Higgs double model — is at least approximately correct. What does that have to do with neutrinos?

The tiny neutrino masses point to three different possibilities.

1. Neutrinos talk to the Higgs boson very, very **weakly** (Dirac neutrinos);
2. Neutrinos talk to a **different Higgs** boson – there is a new source of electroweak symmetry breaking! (Majorana neutrinos);
3. Neutrino masses are small because there is **another source of mass** out there — a new energy scale indirectly responsible for the tiny neutrino masses, a la the seesaw mechanism (Majorana neutrinos).

Searches for $0\nu\beta\beta$ help tell (1) from (2) and (3), the LHC, charged-lepton flavor violation, *et al* may provide more information.

The νSM – Everyone's Favorite Scenario

SM as an effective field theory – non-renormalizable operators

$$\mathcal{L}_{\nu\text{SM}} \supset -\lambda_{ij}\frac{L^i H L^j H}{2M} + \mathcal{O}\left(\frac{1}{M^2}\right) + H.c.$$

There is only one dimension five operator [Weinberg, 1979]. If $M \gg 1$ TeV, it leads to only one observable consequence...

$$\text{after EWSB } \mathcal{L}_{\nu\text{SM}} \supset \frac{m_{ij}}{2}\nu^i\nu^j; \quad m_{ij} = \lambda_{ij}\frac{v^2}{M}.$$

- Neutrino masses are small: $M \gg v \to m_\nu \ll m_f$ ($f = e, \mu, u, d$, etc)
- Neutrinos are Majorana fermions – Lepton number is violated!
- νSM effective theory – not valid for energies above at most M.
- What is M? First naive guess is that M is the Planck scale – does not work. Data require $M \sim 10^{14}$ GeV (anything to do with the GUT scale?)

What else is this "good for"? Depends on the ultraviolet completion!

Note that this VERY similar to the "discovery" weak interactions. Imagine the following scenario:

$$U(1)_{E\&M} \; + \; e(q=-1), \; \mu(q=-1), \; \nu_e(q=0), \; \nu_\mu(q=0).$$

The most general renormalizable Lagrangian explains *all* QED phenomena once all couplings are known (α, m_f).

New physics: the muon decays! $\mu^- \to e^- \bar{\nu}_e \nu_\mu$. This can be interpreted as evidence of effective four fermion theory (nonrenormalizable operators):

$$-\frac{4G_F}{\sqrt{2}} \sum_\gamma g_\gamma \left(\bar{e}\Gamma^\gamma \nu\right)\left(\bar{\nu}\Gamma_\gamma \mu\right), \qquad \Gamma_\gamma = 1, \gamma_5, \gamma_\mu, \ldots$$

Prediction: will discover new physics at an energy scale **below** $\sqrt{1/G_F} \simeq 250$ GeV. We know how this turned out $\Rightarrow$ $W^\pm, Z^0$ discovered slightly below 100 GeV!

Full disclosure:

All higher dimensional operators are completely negligible for $M \sim 10^{14}$ GeV, **except** those that mediate proton decay, like:

$$\frac{\lambda_B}{M^2} QQQL$$

The fact that the proton does not decay forces M/λ_B to be much larger than the energy scale required to explain neutrino masses.

Why is that? We don't know...

Example: the Seesaw Mechanism

A simple[a], renormalizable Lagrangian that allows for neutrino masses is

$$\mathcal{L}_\nu = \mathcal{L}_{\rm old} - \lambda_{\alpha i} L^\alpha H N^i - \sum_{i=1}^{3} \frac{M_i}{2} N^i N^i + H.c.,$$

where N_i ($i = 1, 2, 3$, for concreteness) are SM gauge singlet fermions. $\mathcal{L}_\nu$ is the most general, renormalizable Lagrangian consistent with the SM gauge group and particle content, plus the addition of the N_i fields.

After electroweak symmetry breaking, $\mathcal{L}_\nu$ describes, besides all other SM degrees of freedom, six Majorana fermions: **six neutrinos.**

[a] Only requires the introduction of three fermionic degrees of freedom, no new interactions or symmetries.

To be determined from data: λ and M.

The data can be summarized as follows: there is evidence for three neutrinos, mostly "active" (linear combinations of ν_e, ν_μ, and ν_τ). At least two of them are massive and, if there are other neutrinos, they have to be "sterile."

This provides very little information concerning the magnitude of M_i (assume $M_1 \sim M_2 \sim M_3$)

Theoretically, there is prejudice in favor of very large M: $M \gg v$. Popular examples include $M \sim M_{\rm GUT}$ (GUT scale), or $M \sim 1$ TeV (EWSB scale).

Furthermore, $\lambda \sim 1$ translates into $M \sim 10^{14}$ GeV, while thermal leptogenesis requires the lightest M_i to be around 10^{10} GeV.

we can impose very, very few experimental constraints on M

What We Know About M:

- $M = 0$: the six neutrinos "fuse" into three Dirac states. Neutrino mass matrix given by $\mu_{\alpha i} \equiv \lambda_{\alpha i} v$.

  The symmetry of $\mathcal{L}_\nu$ is enhanced: $U(1)_{B-L}$ is an exact global symmetry of the Lagrangian if all M_i vanish. Small M_i values are 'tHooft natural.

- $M \gg \mu$: the six neutrinos split up into three mostly active, light ones, and three, mostly sterile, heavy ones. The light neutrino mass matrix is given by $m_{\alpha\beta} = \sum_i \mu_{\alpha i} M_i^{-1} \mu_{\beta i}$ $\qquad [m = 1/\Lambda \;\Rightarrow\; \Lambda = M/\mu^2]$.

  This the **seesaw mechanism.** Neutrinos are Majorana fermions. Lepton number is not a good symmetry of $\mathcal{L}_\nu$, even though L-violating effects are hard to come by.

- $M \sim \mu$: six states have similar masses. Active–sterile mixing is very large. This scenario is (generically) ruled out by active neutrino data (atmospheric, solar, KamLAND, K2K, etc).

[ASIDE: Why are Neutrino Masses Small in the $M \neq 0$ Case?]

If $\mu \ll M$, below the mass scale M,

$$\mathcal{L}_5 = \frac{LHLH}{\Lambda}.$$

Neutrino masses are small if $\Lambda \gg \langle H \rangle$. Data require $\Lambda \sim 10^{14}$ GeV.

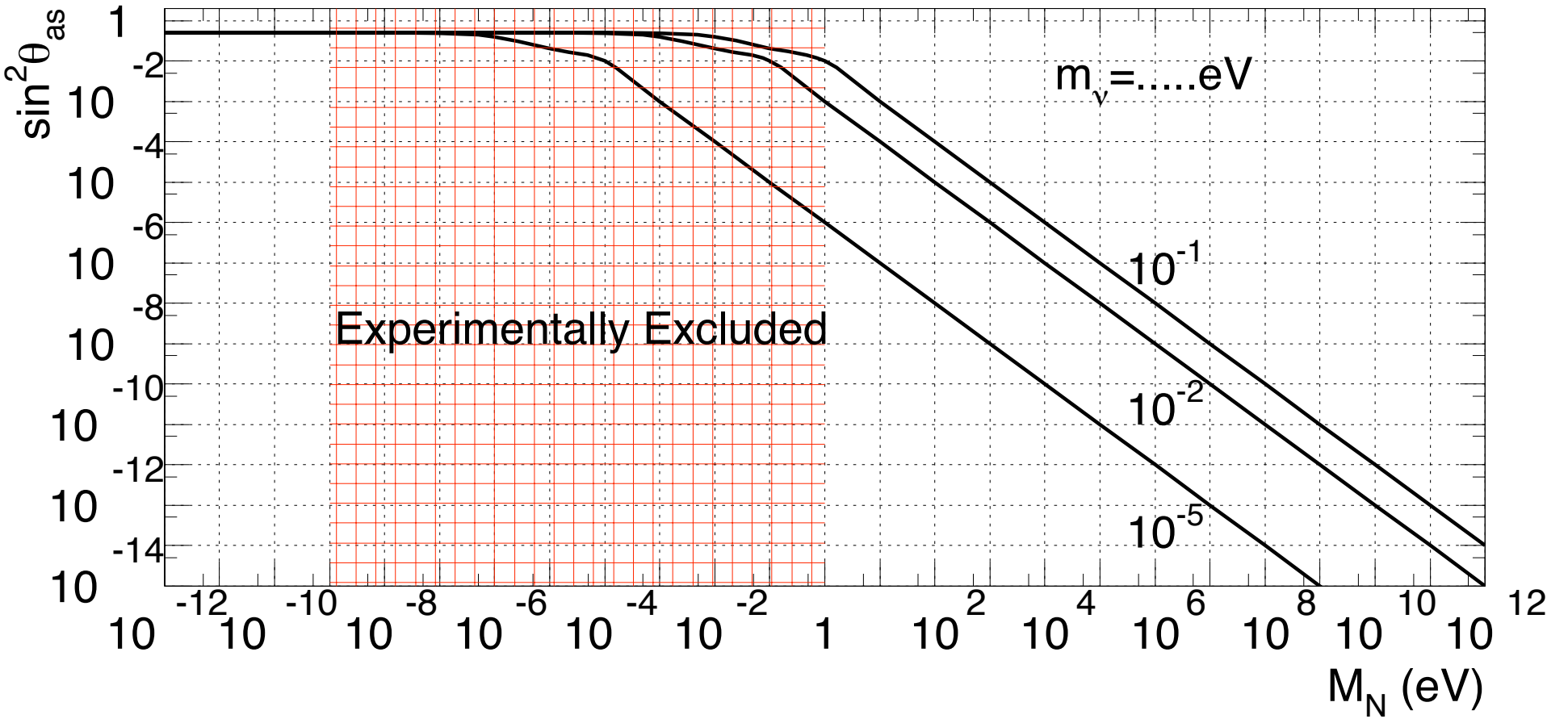
In the case of the seesaw,

$$\Lambda \sim \frac{M}{\lambda^2},$$

so neutrino masses are small if either

- they are generated by physics at a very high energy scale $M \gg v$ (high-energy seesaw); or
- they arise out of a very weak coupling between the SM and a new, hidden sector (low-energy seesaw); or
- cancellations among different contributions render neutrino masses accidentally small ("fine-tuning").

Constraining the Seesaw Lagrangian



[AdG, Huang, Jenkins, arXiv:0906.1611]

High-Energy Seesaw: Brief Comments

- This is everyone's favorite scenario.
- Upper bound for M (e.g. Maltoni, Niczyporuk, Willenbrock, hep-ph/0006358):

$$M < 7.6 \times 10^{15}\ \text{GeV} \times \left(\frac{0.1\ \text{eV}}{m_\nu}\right).$$

- Hierarchy problem hint (e.g., Casas, Espinosa, Hidalgo, hep-ph/0410298):

$$M < 10^7\ \text{GeV}.$$

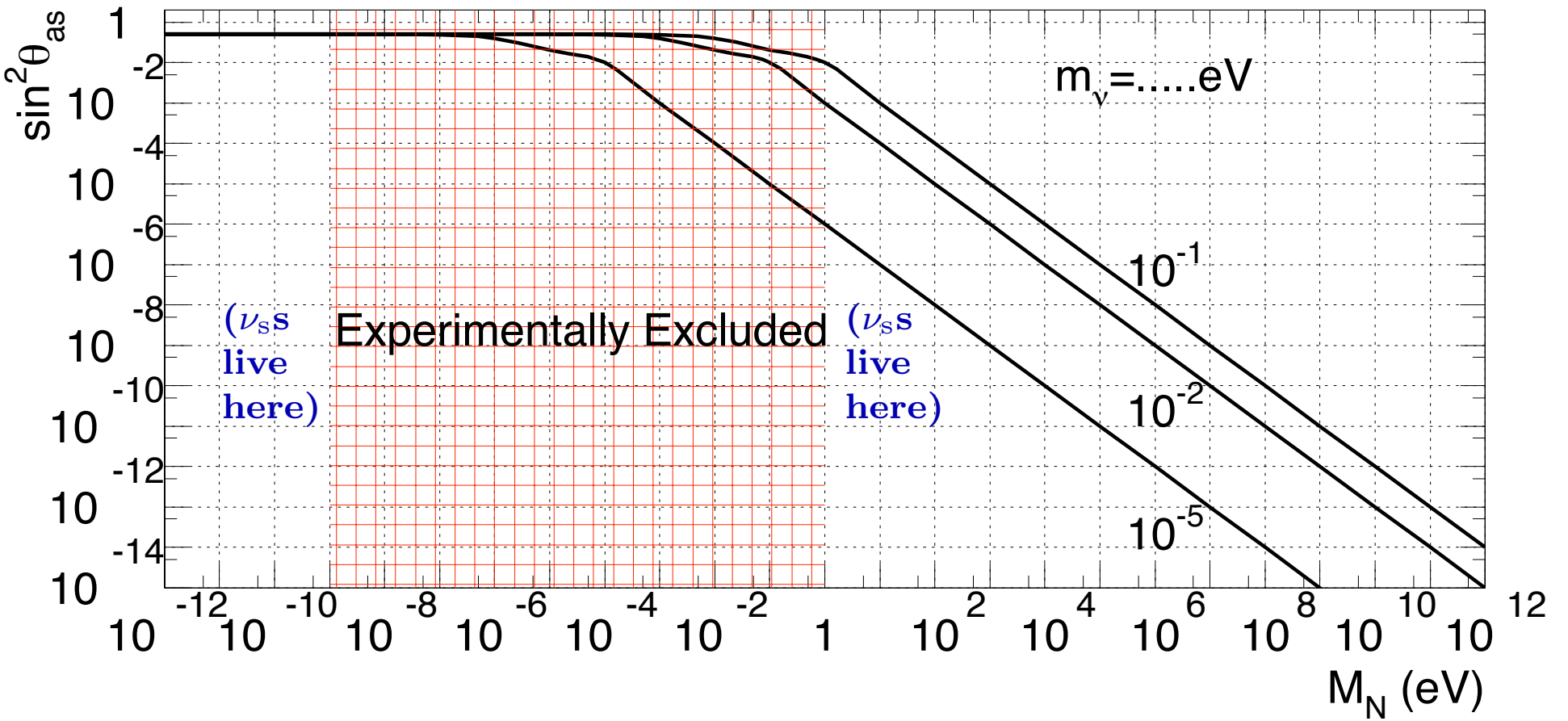
- Physics "too" heavy! No observable consequence other than leptogenesis. From thermal leptogenesis $M > 10^9$ GeV. Will we ever convince ourselves that this is correct? (e.g., Buckley, Murayama, hep-ph/0606088)

Low-Energy Seesaw [AdG PRD72,033005)]

The other end of the M spectrum ($M < 100$ GeV). What do we get?

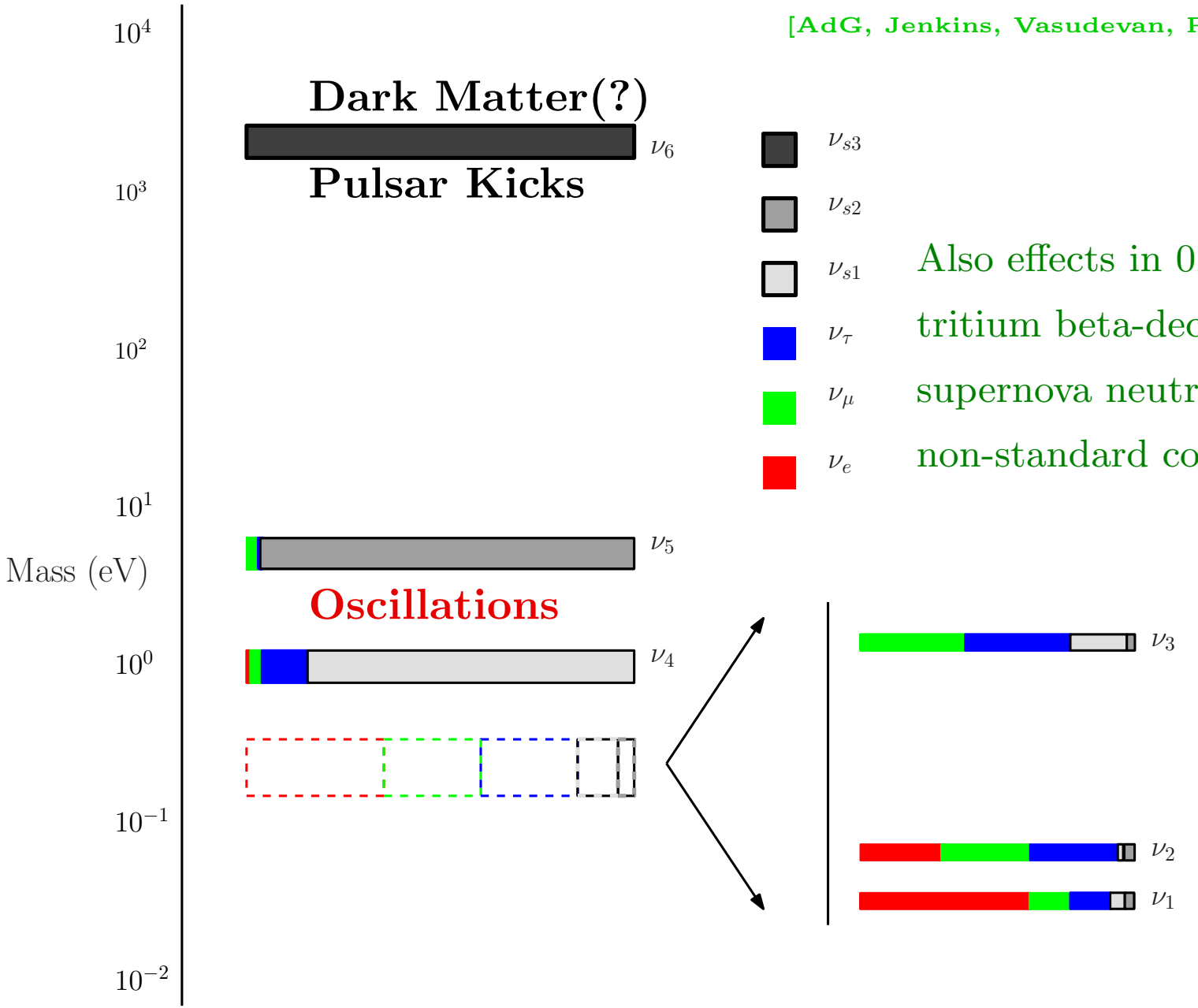
- Neutrino masses are small because the Yukawa couplings are very small $\lambda \in [10^{-6}, 10^{-11}]$;
- No standard thermal leptogenesis – right-handed neutrinos way too light? [For a possible alternative see Canetti, Shaposhnikov, arXiv: 1006.0133 and reference therein.]
- No obvious connection with other energy scales (EWSB, GUTs, etc);
- Right-handed neutrinos are propagating degrees of freedom. They look like sterile neutrinos $\Rightarrow$ sterile neutrinos associated with the fact that the active neutrinos have mass;
- sterile–active mixing can be predicted – hypothesis is falsifiable!
- Small values of M are natural (in the 'tHooft sense). In fact, theoretically, no value of M should be discriminated against!

Constraining the Seesaw Lagrangian



[AdG, Huang, Jenkins, arXiv:0906.1611]

[AdG, Jenkins, Vasudevan, PRD75, 013003 (2007)]



Also effects in $0\nu\beta\beta$,

tritium beta-decay,

supernova neutrino oscillations,

non-standard cosmology.

Other Versions of the Seesaw – Tree-Level

There are three ways of generating nonzero neutrino Majorana masses – the Weinberg Operator – at the tree-level with the introduction of new degrees of freedom:

- Right-handed neutrinos: $SU(2)$ gauge-singlet fermions with hypercharge 0 (N);
- New complex scalar field, a Higgs $SU(2)$ Triplet with hypercharge +1 (T);
- Vector-like fermions, triplets of $SU(2)$ with hypercharge 0 (Σ).

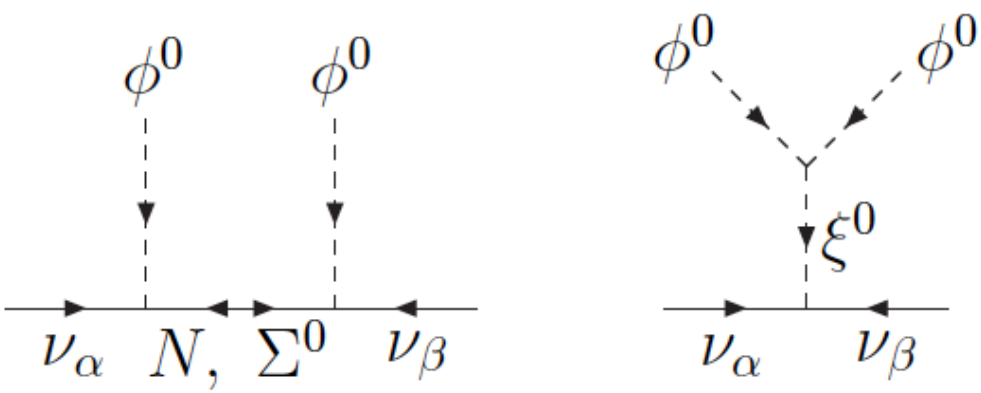
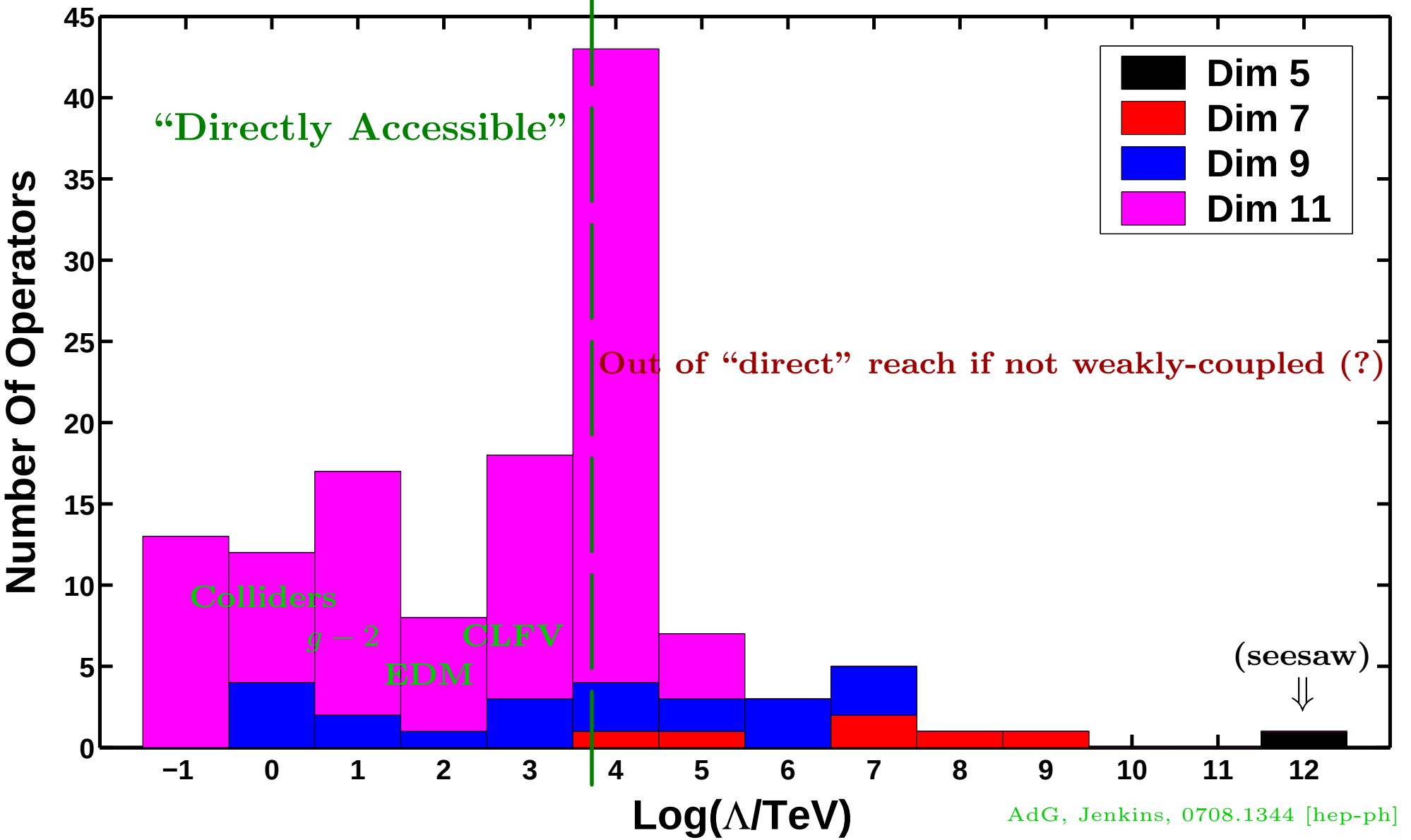


FIG. 1: Three tree-level realizations of seesaw Majorana neutrino mass.

This is Just the Tip of the Model-Iceberg!



AdG, Jenkins, 0708.1344 [hep-ph]

AdG, Jenkins, 0708.1344 [hep-ph]

Effective Operator Approach

(there are 129 of them if you discount different Lorentz structures!)

classified by Babu and Leung in NPB**619**,667(2001)

4_a	$L^iL^j\overline{Q}_i\bar{u}^cH^k\epsilon_{jk}$	$\frac{y_u}{16\pi^2}\frac{v^2}{\Lambda}$	4×10^9	$\beta\beta0\nu$
4_b	$L^iL^j\overline{Q}_k\bar{u}^cH^k\epsilon_{ij}$	$\frac{y_ug^2}{(16\pi^2)^2}\frac{v^2}{\Lambda}$	6×10^6	$\beta\beta0\nu$
5	$L^iL^jQ^kd^cH^lH^m\overline{H}_i\epsilon_{jl}\epsilon_{km}$	$\frac{y_d}{(16\pi^2)^2}\frac{v^2}{\Lambda}$	6×10^5	$\beta\beta0\nu$
6	$L^iL^j\overline{Q}_k\bar{u}^cH^lH^k\overline{H}_i\epsilon_{jl}$	$\frac{y_u}{(16\pi^2)^2}\frac{v^2}{\Lambda}$	2×10^7	$\beta\beta0\nu$
7	$L^iQ^j\bar{e}^c\overline{Q}_kH^kH^lH^m\epsilon_{il}\epsilon_{jm}$	$y_{\ell_\beta}\frac{g^2}{(16\pi^2)^2}\frac{v^2}{\Lambda}\left(\frac{1}{16\pi^2}+\frac{v^2}{\Lambda^2}\right)$	4×10^2	mix
8	$L^i\bar{e}^c\bar{u}^cd^cH^j\epsilon_{ij}$	$y_{\ell_\beta}\frac{y_dy_u}{(16\pi^2)^2}\frac{v^2}{\Lambda}$	6×10^3	mix
9	$L^iL^jL^ke^cL^le^c\epsilon_{ij}\epsilon_{kl}$	$\frac{y_\ell^2}{(16\pi^2)^2}\frac{v^2}{\Lambda}$	3×10^3	$\beta\beta0\nu$
10	$L^iL^jL^ke^cQ^ld^c\epsilon_{ij}\epsilon_{kl}$	$\frac{y_\ell y_d}{(16\pi^2)^2}\frac{v^2}{\Lambda}$	6×10^3	$\beta\beta0\nu$
11_a	$L^iL^jQ^kd^cQ^ld^c\epsilon_{ij}\epsilon_{kl}$	$\frac{y_d^2g^2}{(16\pi^2)^3}\frac{v^2}{\Lambda}$	30	$\beta\beta0\nu$
11_b	$L^iL^jQ^kd^cQ^ld^c\epsilon_{ik}\epsilon_{jl}$	$\frac{y_d^2}{(16\pi^2)^2}\frac{v^2}{\Lambda}$	2×10^4	$\beta\beta0\nu$
12_a	$L^iL^j\overline{Q}_i\bar{u}^c\overline{Q}_j\bar{u}^c$	$\frac{y_u^2}{(16\pi^2)^2}\frac{v^2}{\Lambda}$	2×10^7	$\beta\beta0\nu$
12_b	$L^iL^j\overline{Q}_k\bar{u}^c\overline{Q}_l\bar{u}^c\epsilon_{ij}\epsilon^{kl}$	$\frac{y_u^2g^2}{(16\pi^2)^3}\frac{v^2}{\Lambda}$	4×10^4	$\beta\beta0\nu$
13	$L^iL^j\overline{Q}_i\bar{u}^cL^le^c\epsilon_{jl}$	$\frac{y_\ell y_u}{(16\pi^2)^2}\frac{v^2}{\Lambda}$	2×10^5	$\beta\beta0\nu$
14_a	$L^iL^j\overline{Q}_k\bar{u}^cQ^kd^c\epsilon_{ij}$	$\frac{y_dy_ug^2}{(16\pi^2)^3}\frac{v^2}{\Lambda}$	1×10^3	$\beta\beta0\nu$
14_b	$L^iL^j\overline{Q}_i\bar{u}^cQ^ld^c\epsilon_{jl}$	$\frac{y_dy_u}{(16\pi^2)^2}\frac{v^2}{\Lambda}$	6×10^5	$\beta\beta0\nu$
15	$L^iL^jL^kd^c\overline{L}_i\bar{u}^c\epsilon_{jk}$	$\frac{y_dy_ug^2}{(16\pi^2)^3}\frac{v^2}{\Lambda}$	1×10^3	$\beta\beta0\nu$
16	$L^iL^je^cd^c\bar{e}^c\bar{u}^c\epsilon_{ij}$	$\frac{y_dy_ug^4}{(16\pi^2)^4}\frac{v^2}{\Lambda}$	2	$\beta\beta0\nu$, LHC
17	$L^iL^jd^cd^c\bar{d}^c\bar{u}^c\epsilon_{ij}$	$\frac{y_dy_ug^4}{(16\pi^2)^4}\frac{v^2}{\Lambda}$	2	$\beta\beta0\nu$, LHC
18	$L^iL^jd^cu^c\bar{u}^c\bar{u}^c\epsilon_{ij}$	$\frac{y_dy_ug^4}{(16\pi^2)^4}\frac{v^2}{\Lambda}$	2	$\beta\beta0\nu$, LHC
19	$L^iQ^jd^cd^c\bar{e}^c\bar{u}^c\epsilon_{ij}$	$y_{\ell_\beta}\frac{y_d^2y_u}{(16\pi^2)^3}\frac{v^2}{\Lambda}$	1	$\beta\beta0\nu$, HElnv, LHC, mix
20	$L^id^c\overline{Q}_i\bar{u}^c\bar{e}^c\bar{u}^c$	$y_{\ell_\beta}\frac{y_dy_u^2}{(16\pi^2)^3}\frac{v^2}{\Lambda}$	40	$\beta\beta0\nu$, mix
21_a	$L^iL^jL^ke^cQ^lu^cH^mH^n\epsilon_{ij}\epsilon_{km}\epsilon_{ln}$	$\frac{y_\ell y_u}{(16\pi^2)^2}\frac{v^2}{\Lambda}\left(\frac{1}{16\pi^2}+\frac{v^2}{\Lambda^2}\right)$	2×10^3	$\beta\beta0\nu$
21_b	$L^iL^jL^ke^cQ^lu^cH^mH^n\epsilon_{il}\epsilon_{jm}\epsilon_{kn}$	$\frac{y_\ell y_u}{(16\pi^2)^2}\frac{v^2}{\Lambda}\left(\frac{1}{16\pi^2}+\frac{v^2}{\Lambda^2}\right)$	2×10^3	$\beta\beta0\nu$
22	$L^iL^jL^ke^c\overline{L}_k\bar{e}^cH^lH^m\epsilon_{il}\epsilon_{jm}$	$\frac{g^2}{(16\pi^2)^3}\frac{v^2}{\Lambda}$	4×10^4	$\beta\beta0\nu$
23	$L^iL^jL^ke^c\overline{Q}_k\bar{d}^cH^lH^m\epsilon_{il}\epsilon_{jm}$	$\frac{y_\ell y_d}{(16\pi^2)^2}\frac{v^2}{\Lambda}\left(\frac{1}{16\pi^2}+\frac{v^2}{\Lambda^2}\right)$	40	$\beta\beta0\nu$
24_a	$L^iL^jQ^kd^cQ^ld^cH^m\overline{H}_i\epsilon_{jk}\epsilon_{lm}$	$\frac{y_d^2}{(16\pi^2)^3}\frac{v^2}{\Lambda}$	1×10^2	$\beta\beta0\nu$
24_b	$L^iL^jQ^kd^cQ^ld^cH^m\overline{H}_i\epsilon_{jm}\epsilon_{kl}$	$\frac{y_d^2}{(16\pi^2)^3}\frac{v^2}{\Lambda}$	1×10^2	$\beta\beta0\nu$
25	$L^iL^jQ^kd^cQ^lu^cH^mH^n\epsilon_{im}\epsilon_{jn}\epsilon_{kl}$	$\frac{y_dy_u}{(16\pi^2)^2}\frac{v^2}{\Lambda}\left(\frac{1}{16\pi^2}+\frac{v^2}{\Lambda^2}\right)$	4×10^3	$\beta\beta0\nu$

B. Zee/Wolfenstein Model

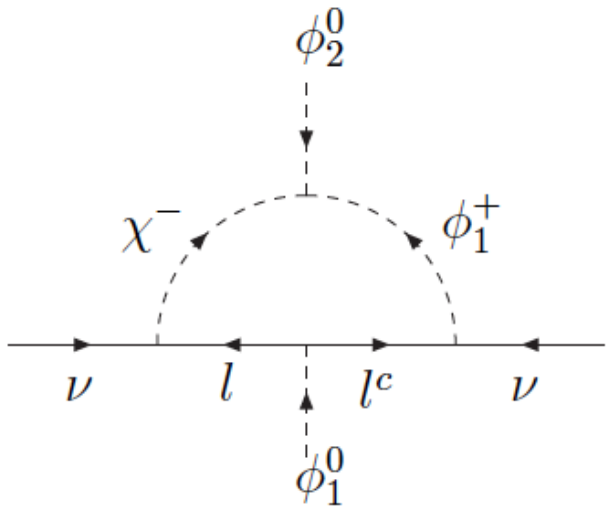


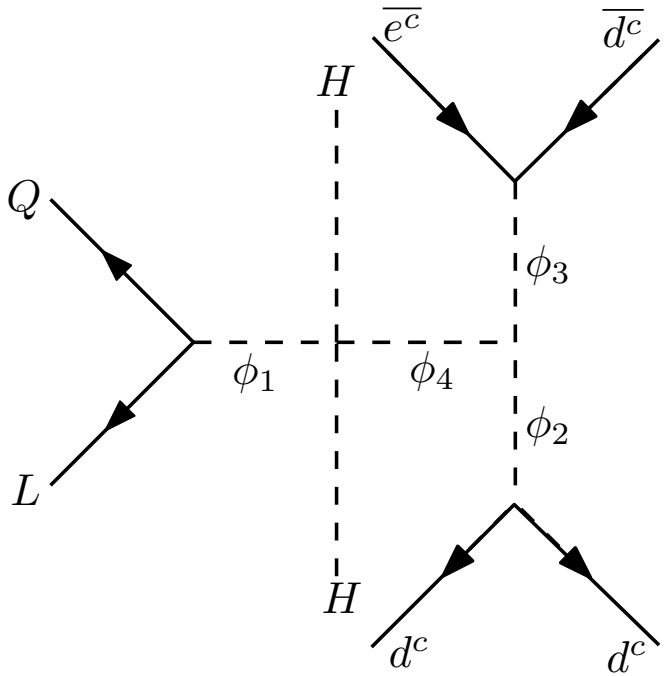
FIG. 3: One-loop radiative neutrino mass.

$$\mathcal{L} \supset \frac{y_l}{v} L l^c \phi_1 + h_{ij}(L^i L^j)\chi + \kappa(\phi_1\phi_2)\chi + M_\chi|\chi^2|.$$

$h_{ij} = h_{ij}$, and two scalar doublets. SM Higgs boson, $\phi_1 \neq$ the "sterile" Higgs, ϕ_2.

$$\mathcal{M}_\mu = \begin{pmatrix} 0 & f_{\mu e}(m_\mu^2 - m_e^2) & f_{\tau e}(m_\tau^2 - m_e^2) \\ f_{\mu e}(m_\mu^2 - m_e^2) & 0 & f_{\tau\mu}(m_\tau^2 - m_\mu^2) \\ f_{\tau e}(m_\tau^2 - m_e^2) & f_{\tau\mu}(m_\tau^2 - m_\mu^2) & 0 \end{pmatrix}$$

[arXiv:0708.1344 [hep-ph]]



Order-One Coupled, Weak Scale Physics

Can Also Explain Naturally Small

Majorana Neutrino Masses:

Multi-loop neutrino masses from lepton number violating new physics.

$$-\mathcal{L}_{\nu\text{SM}} \supset \sum_{i=1}^{4} M_i \phi_i \bar{\phi}_i + i y_1 Q L \phi_1 + y_2 d^c d^c \phi_2 + y_3 e^c d^c \phi_3 + \lambda_{14} \bar{\phi}_1 \phi_4 H H + \lambda_{234} M \phi_2 \bar{\phi}_3 \phi_4 + h.c.$$

$m_\nu \propto (y_1 y_2 y_3 \lambda_{234}) \lambda_{14} / (16\pi)^4 \;\rightarrow$ neutrino masses at 4 loops, requires $M_i \sim 100$ GeV!

WARNING: For illustrative purposes only. Details still to be worked out. Scenario most likely ruled out by charged-lepton flavor-violation, LEP, Tevatron, and HERA.

Comments on Dirac Neutrinos

- If the neutrinos are Dirac fermions, the Majorana masses for the N fields are zero. This can be imposed by requiring a symmetry, for example, $U(1)_{B-L}$. If this is the case, the status of $B-L$: is "upgraded" from an accidental symmetry to a new fundamental symmetry. It implies, for example, that there are no new particles or physics processes that can violate lepton-number, at any scale!

- Why would the neutrino Yukawa couplings be so small? One possibility is that they are forbidden by some new, hidden symmetry. For example, if there is a new (e.g. $U(1)$) gauge interaction that couples only to N, and the symmetry is spontaneous broken by the vev of some new scalar field $\phi_{\rm new}$, then the neutrino Yukawa operator, LHN is forbidden, but the higher dimensional operator

$$\frac{1}{\Lambda}(LH)(N\phi_{\rm new})$$

is allowed. If this is the case, the neutrino Dirac masses are proportional to $m_D \propto (v_{\rm new}/M)v$ – potentially parametrically suppressed, exactly like the Weinberg operator!

Before I move on

Some Reference for Effective Field Theories:

- David B. Kaplan, nucl-th/0510023, nucl-th/9506035;
- S. Weinberg, "Baryon and Lepton Nonconserving Processes," Phys. Rev. Lett. 43, 1566 (1979), and "Phenomenological Lagrangians," Physica A96, 327 (1979);
- F. Wilczek and A. Zee, "Operator Analysis of Nucleon Decay," Phys. Rev. Lett. 43, 1571 (1979).
- W. Buchmuller and D. Wyler, "Effective Lagrangian Analysis of New Interactions and Flavor Conservation," Nucl. Phys. B268, 621 (1986).

Understanding Fermion Mixing

The other puzzling phenomenon uncovered by the neutrino data is the fact that Neutrino Mixing is Strange. What does this mean?

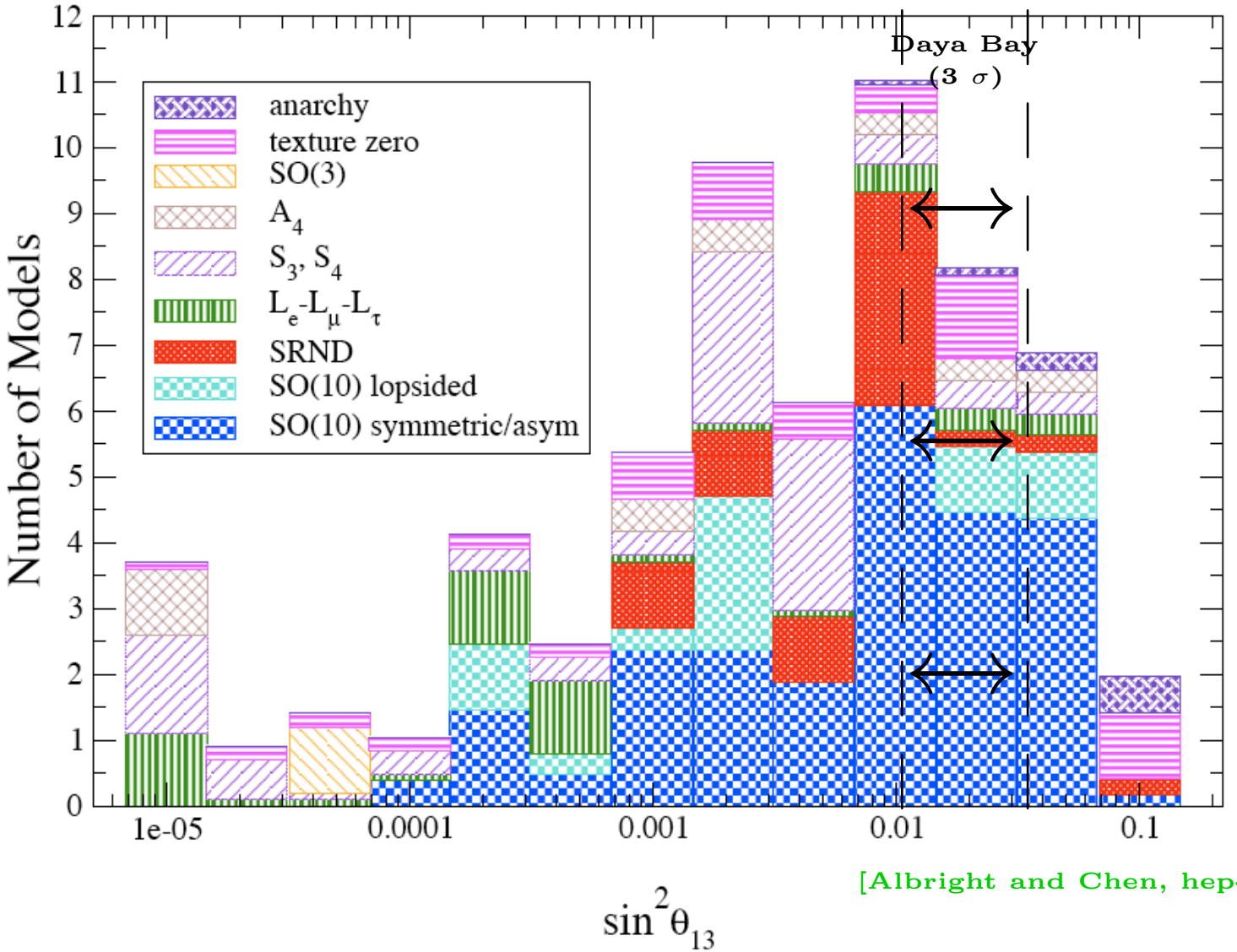
It means that lepton mixing is very different from quark mixing:

$$V_{MNS} \sim \begin{pmatrix} 0.8 & 0.5 & \mathbf{0.2} \\ 0.4 & 0.6 & 0.7 \\ 0.4 & 0.6 & 0.7 \end{pmatrix} \qquad V_{CKM} \sim \begin{pmatrix} 1 & 0.2 & 0.001 \\ 0.2 & 1 & 0.01 \\ 0.001 & 0.01 & 1 \end{pmatrix}$$

$[|(V_{MNS})_{e3}| < 0.2]$

WHY?

They certainly look VERY different, but which one would you label as "strange"?



[Albright and Chen, hep-ph/0608137]

"Left-Over" Predictions: δ, mass-hierarchy, $\cos 2\theta_{23}$. More important: CORRELATIONS!

Beyond the Standard Model: Evidence, both "Direct" and "Indirect."

There are many open **theoretical** questions surrounding the Standard Model of Particle Physics (SM).

- How come the Higgs boson mass is so light? (Naturalness, or the Hierarchy Problem)
- How do we reconcile quantum field theory with general relativity? (Quantum Gravity)
- Are the parameters in the SM related? Do the gauge couplings unify? (Grand Unification)
- ...

I will talk about a couple of these later, time permitting.

First, I want to talk about the concrete **experimental** evidence we have for phenomena that the SM fails to explain. These are very few – four depending on how you count ...

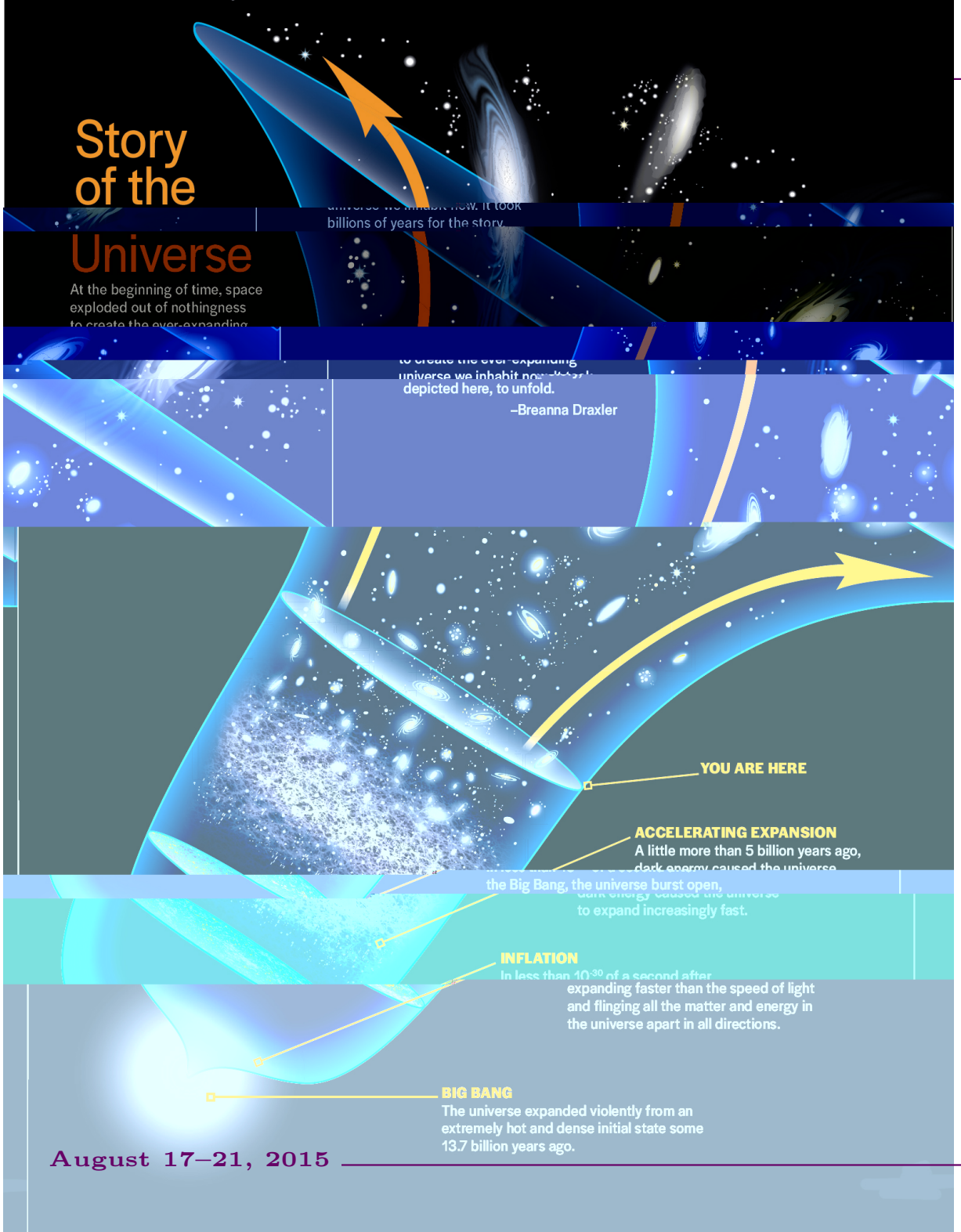
Evidence for Physics Beyond the Standard Model

1. The expansion rate of the universe seems to accelerate, both early on (inflation) and right now (dark energy).
2. Dark matter seems to exist.
3. Why is there so much baryonic matter in the universe?
4. Neutrino masses are not zero. (✓)

1. and 2. are consequences of astrophysical/cosmological observations. It is fair to ask whether we are sure they have anything to do with particle physics.

3. is also related to our understanding of the early history of the universe and requires some more explaining.

4. is the only palpable evidence for new physics.



Measurements of the expansion rate of the universe reveal exponential growth early and now.

We don't know what causes this.

Whatever it is, it is not in the SM.

Is there a cosmological constant?

What causes inflation to begin and end?

Is this a particle physics question?

Evidence for Dark Matter

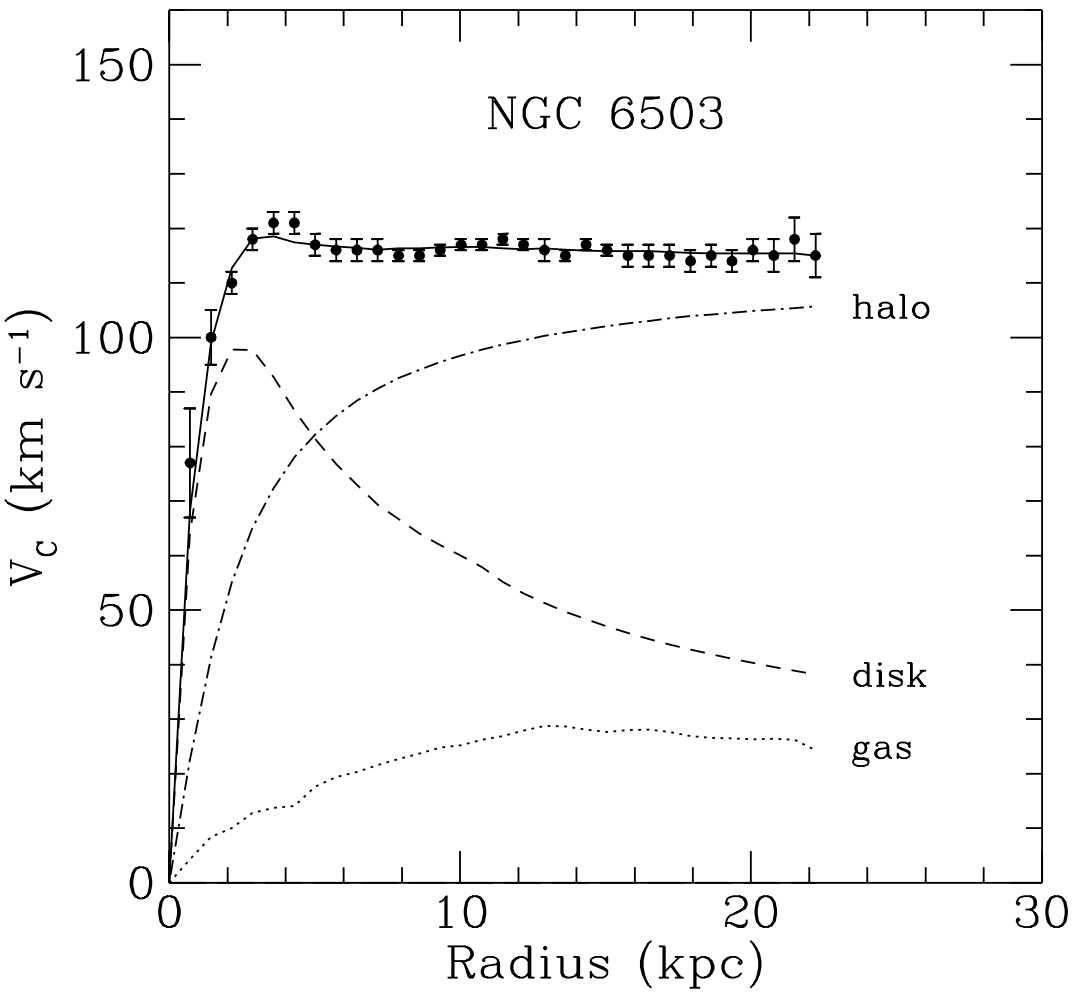
We have been able to measure gravitaty at work at all kinds of distance scales. At the galactic level and above, we consistently run into trouble: there seems to be more matter than originally expected. Here, I'll mention just a few examples.

The first evidence [Zwicky, 1933] for "dark matter" came from studies of the velocity dispersions of galaxies in the Coma cluster (Galaxy Cluster) – mass-to-light ratio is around 200.

A little more recently (APJ 1996), survey of 16 clusters find

$$\rightarrow \ \Omega_M = 0.24 \pm 0.05 \pm 0.09$$

Rotation Curves of Galaxies

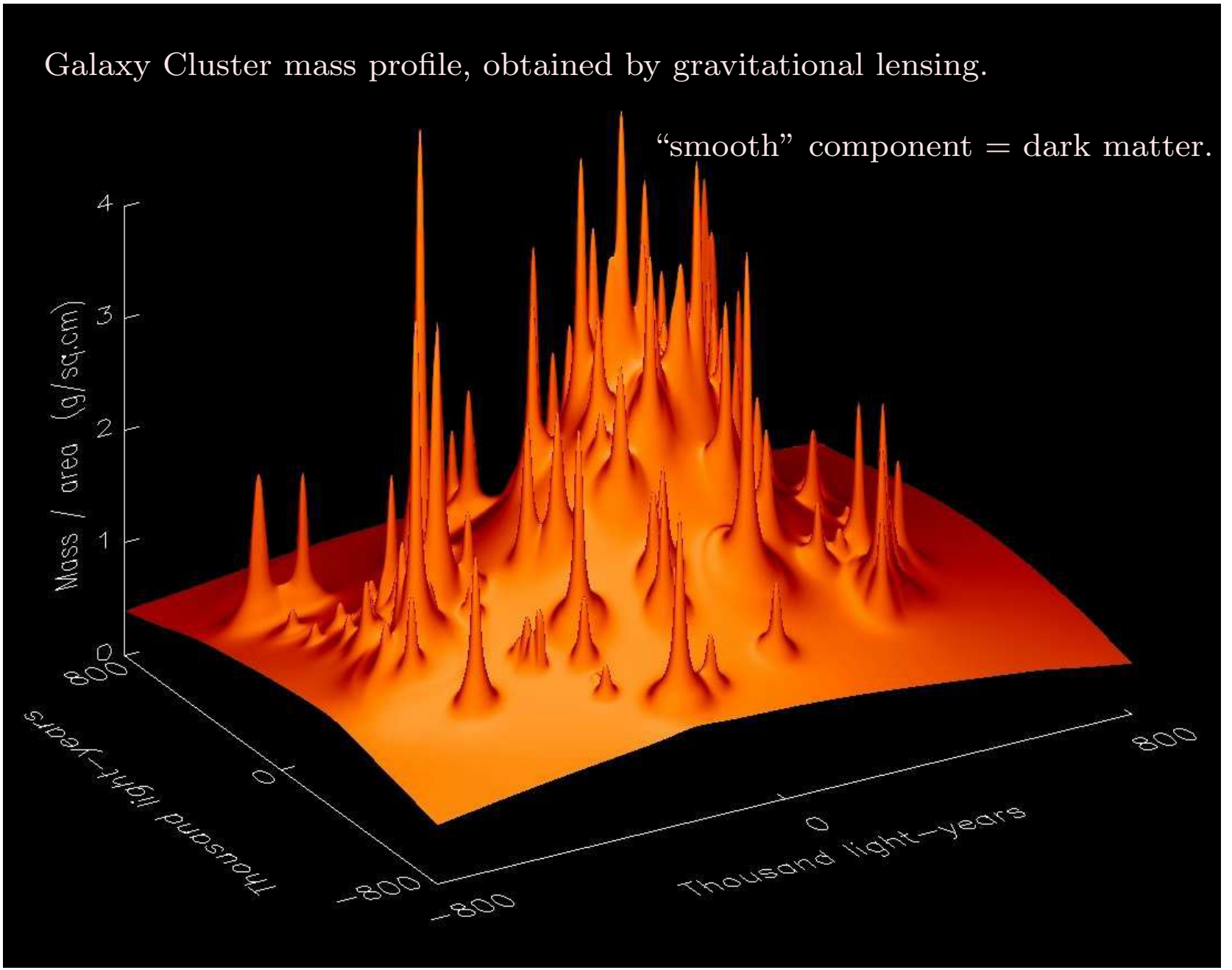


There is more matter within galaxies than can be seen. In particular, the invisible radius of the galaxy is much larger than the visible one.

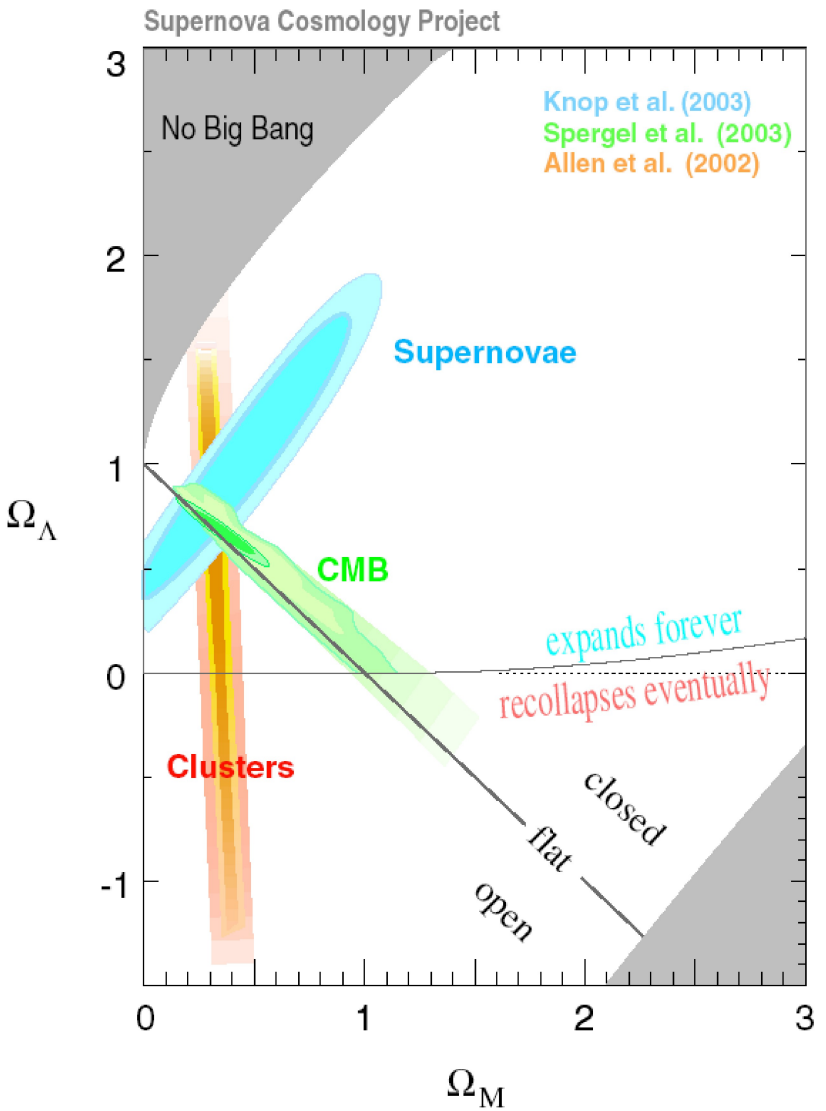
Remember: $v(r) = \sqrt{\frac{GM(r)}{r}}$,

where $M(r) = 4\pi \int_0^r r^2 \mathrm{d}r \rho(r)$.

Galaxy Cluster mass profile, obtained by gravitational lensing.



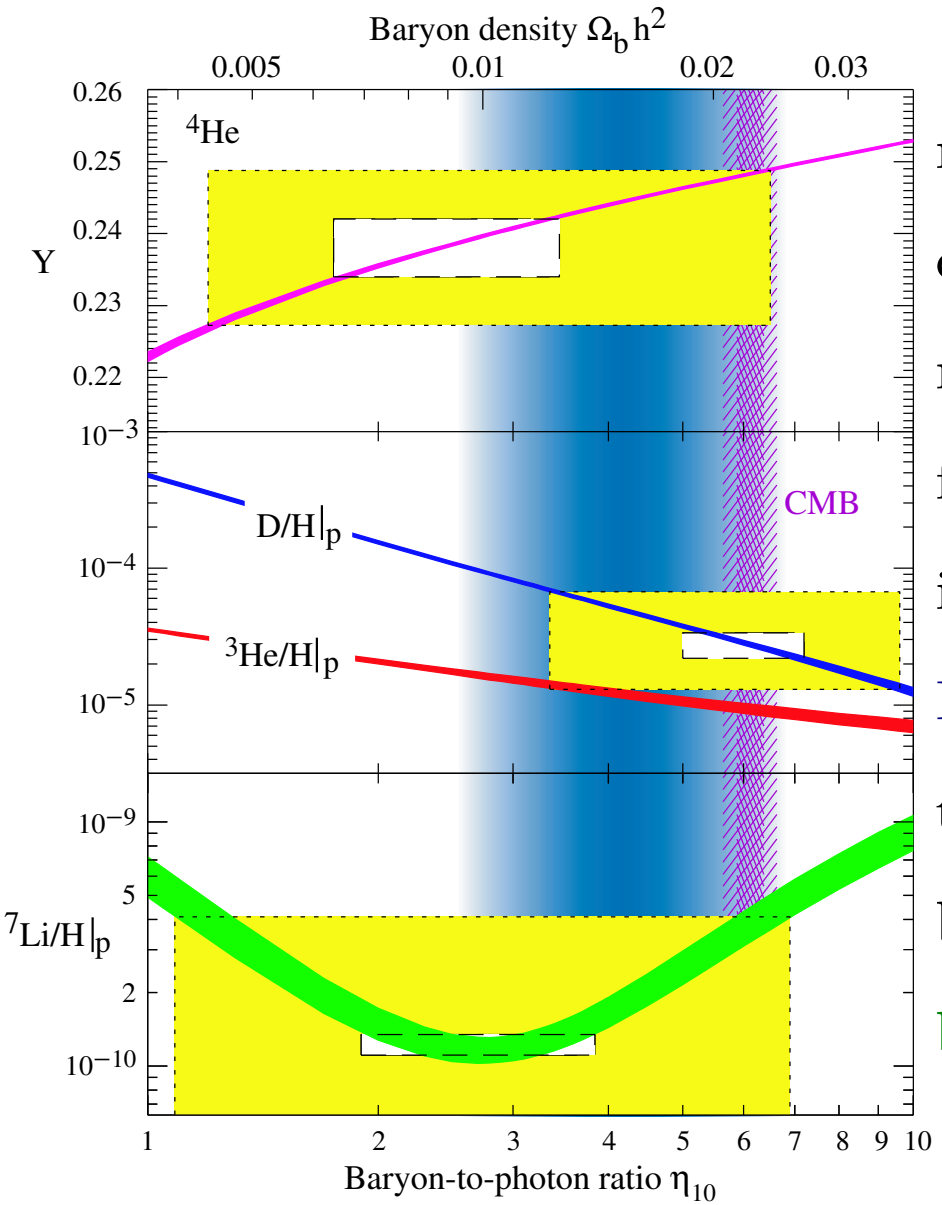
Evidence From Cosmological Probes



"Concordance" ΛCDM Model

$$\Omega_M = 0.14170 \pm 0.00097$$
$$\Omega_c h^2 = 0.1188 \pm 0.0010$$
$$\Omega_b h^2 = 0.02230 \pm 0.00014$$
$$h = 0.6774 \pm 0.0046$$

[Planck Coll., arXiv:1502.01589]



not only is there more matter than we can see, but it is not ordinary (baryonic) matter. There is overwhelming evidence for that, from direct searches for heavy invisible objects, to

Big Bang Nucleosynthesis (← figure),

to analysis of the cosmic microwave background.

baryons = things made up of protons, neutrons and electrons.

And there is the Bullet Cluster!

We'd like to believe that DM is a new particle, about which we still know very little.

What Can We Learn...? – Cosmology

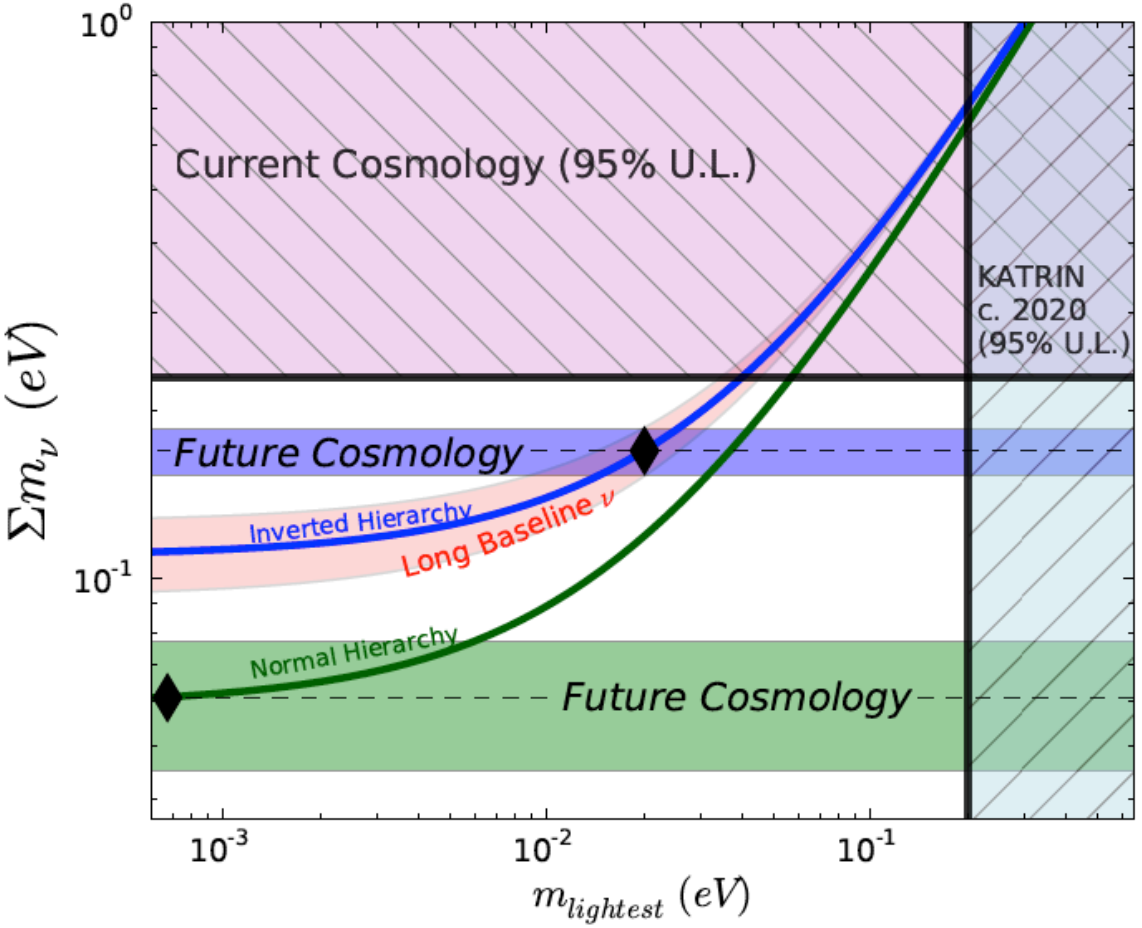
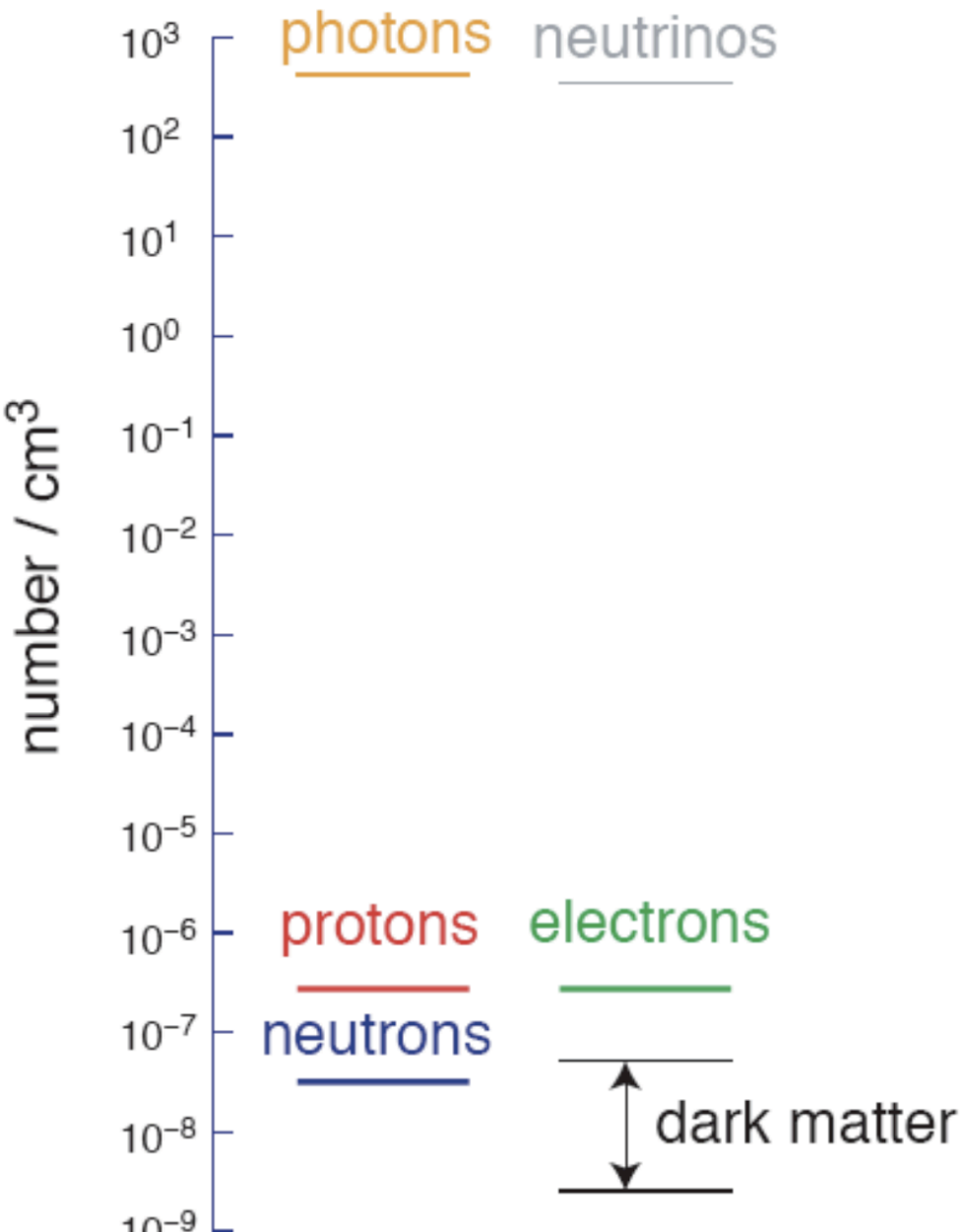
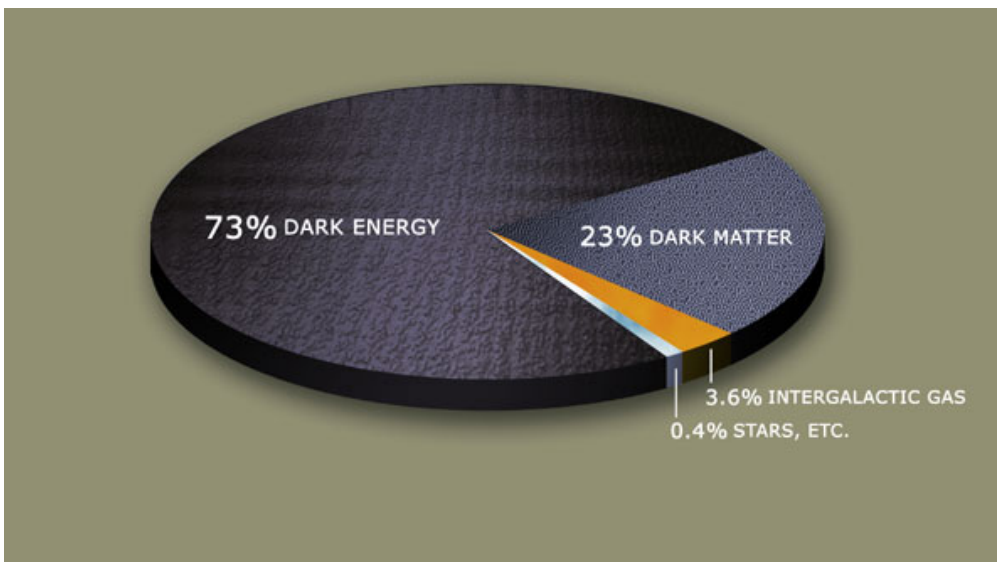


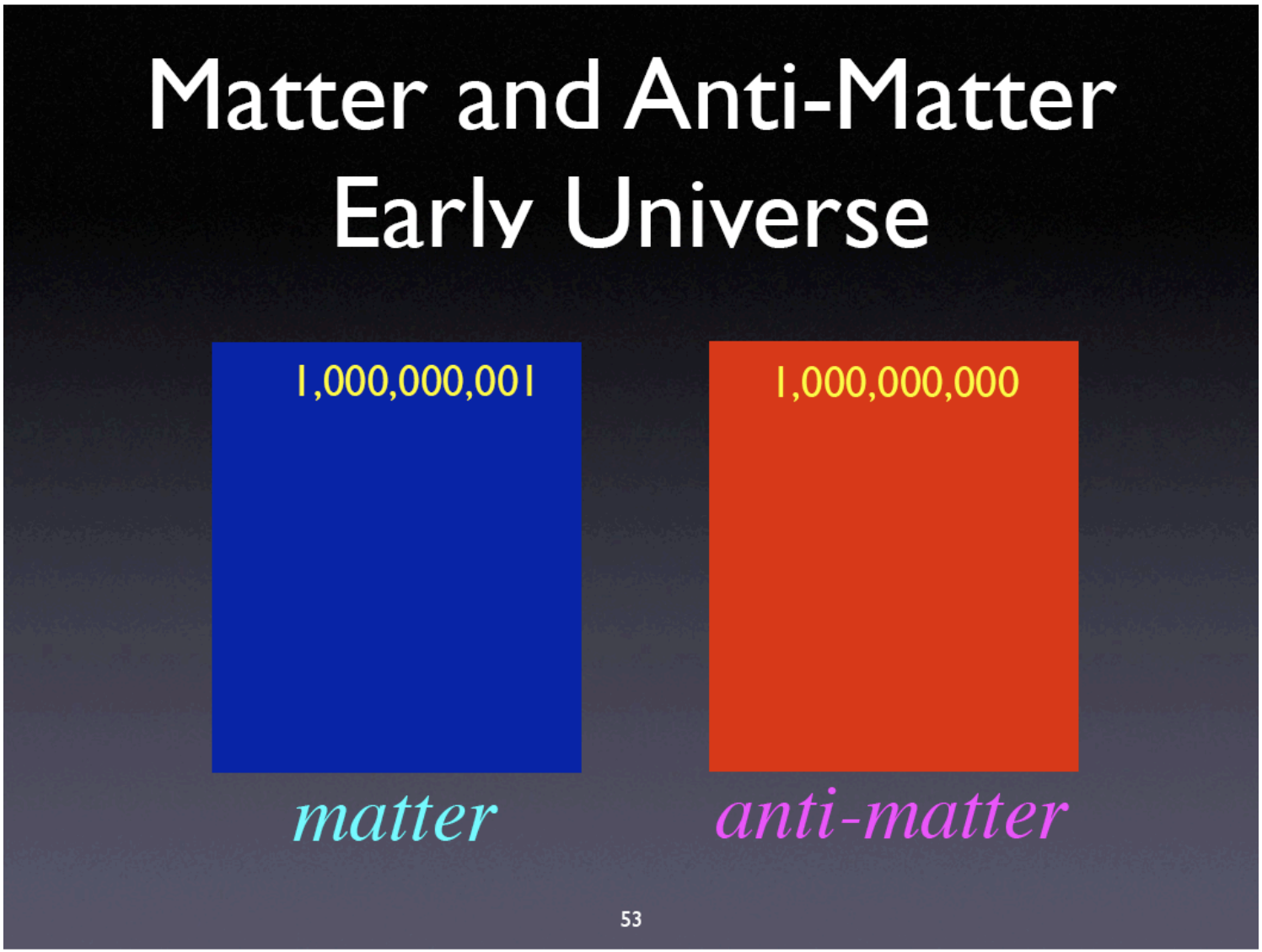
Figure 7. *Current constraints and forecast sensitivity of cosmology to the sum of neutrino masses. In the case of an "inverted hierarchy," with an example case marked as a diamond in the upper curve, future combined cosmological constraints would have a very high-significance detection, with 1-σ error shown as a blue band. In the case of a normal neutrino mass hierarchy with an example case marked as diamond on the lower curve, future cosmology would still detect the lowest* $\sum m_\nu$ *at greater than 3-σ.*

[K. Abazajian *et al.* arXiv:1309.5386]

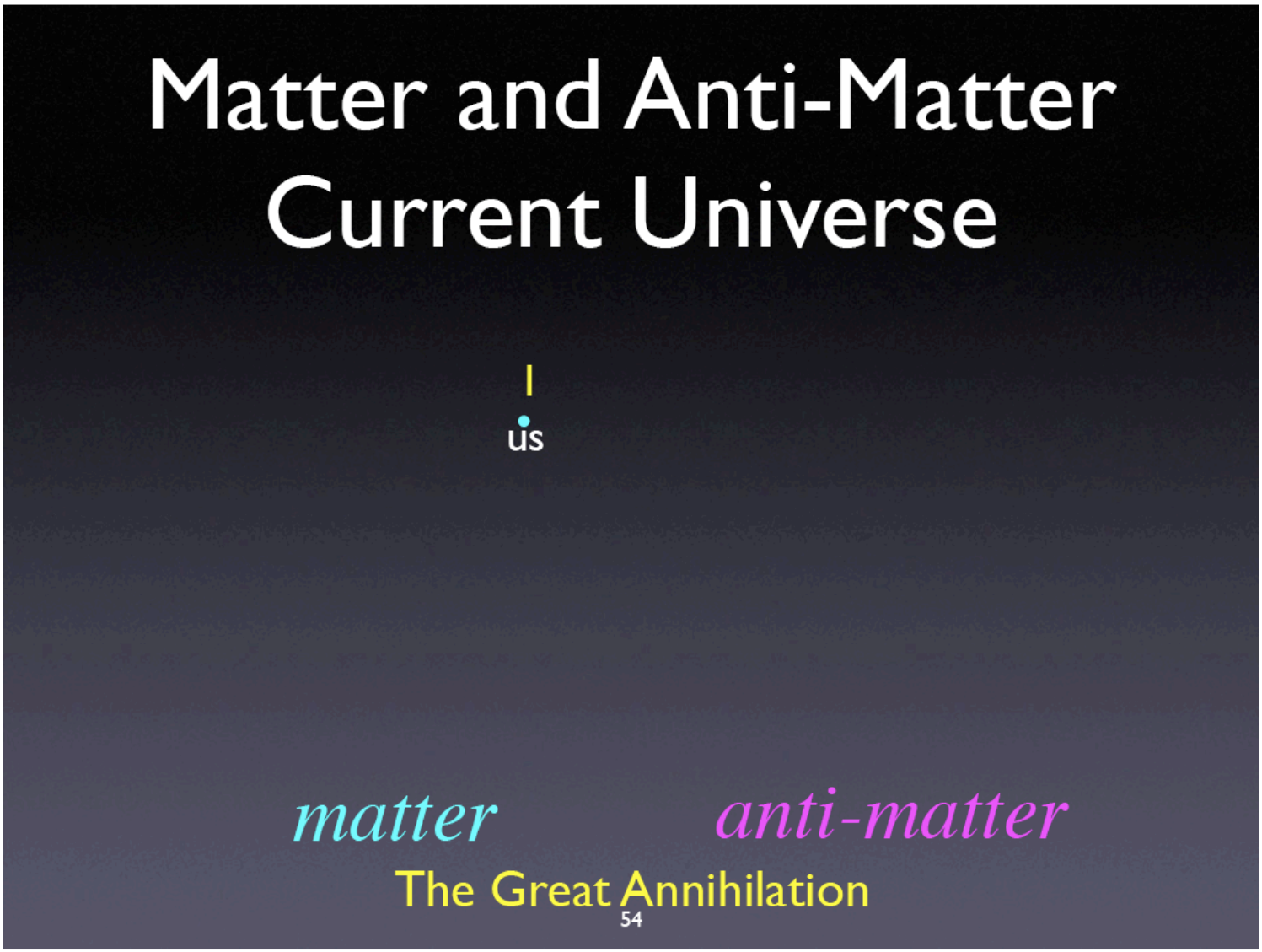
The Particle Universe







[from Hitoshi Murayama]



[from Hitoshi Murayama]

Baryogenesis

One of the most basic questions we are allowed to ask (with any real hope of getting an answer) is whether the observed baryon asymmetry of the Universe can be obtained from a baryon–antibaryon symmetric initial condition plus well understood dynamics. [Baryogenesis]

This isn't just for aesthetic reasons. If the early Universe undergoes a period of inflation, baryogenesis is required, as inflation would wipe out any pre-existing baryon asymmetry.

Ingredients for Baryogenesis

Scenarios: leptogenesis, EW baryogenesis, Afflek-Dine, asymmetric DM, cold baryogenesis, post-sphaleron baryogenesis…

Testable

	Standard Model	*BSM*
• *B violation (sphalerons)*	✔	✔
• *C & CP violation*	✖	✔
• *Out-of-equilibrium or CPT violation*	✖	✔

[from Michael Ramsey-Musolf]

Bottom Line:

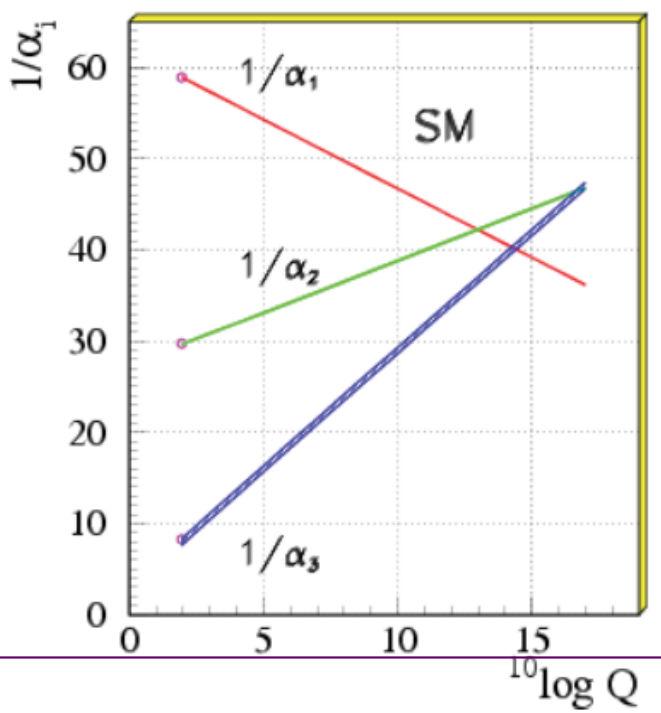
- There are new particles out there. What are they? What are their masses? Can we see them in the laboratory? There is a very popular scenario – electroweak baryogenesis – where there are new particles around the weak scale.
- There are new sources of CP-invariance violation. Can we see it? Where?

SM gauge group: interactions

$$SU(3)\times SU(2)\times U(1)_Y$$

$G_\mu^{a=1,..,8}$ $W_\mu^{a=1,2,3}$ B_μ

Gauge unification ?



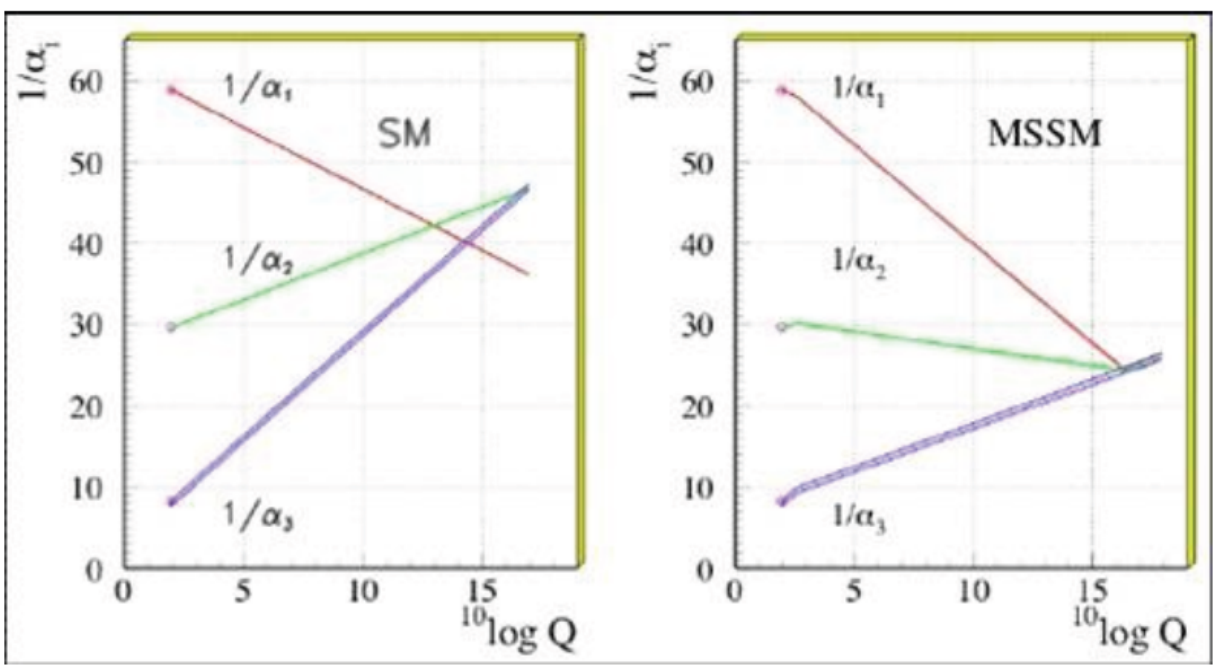
$$\alpha_i = \frac{g_i^2}{4\pi}$$

[courtesy of Pilar Hernández]

The motivation for SUSY

No quadratic corrections to the Higgs mass: cancellation between particles and SUSY partners.

Gauge unification



[courtesy of Pilar Hernández]

Is the Higgs Boson Mass Too Light? – A Controversial Topic

[based on arXiv:1402.2658, with D. Hernández and Tim M.P. Tait]

Other references most closely related to what I will present (and there are several more):

- A. Casas, J.R. Espinoza, and I. Hidalgo, hep-ph/0410298 and hep-ph/0607279
- M. Farina, D. Pappadopulo, A. Strumia, arXiv:1303.7244

All of the results presented are obtained at the order-of-magnitude level, using naive perturbation theory and dimensional analysis. Much more quantitative results can be found in arXiv:1303.7244.

We only worry about one hierarchy problem at a time (this will become more clear. Landau poles will also be ignored.

THE HIERARCHY PROBLEM

Higgs potential: $V(H) = -\mu^2|H|^2 + \lambda|H|^4$

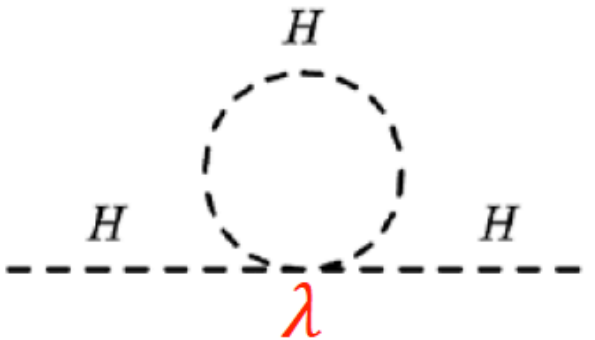
Classically: μ is equivalent to the mass of the Higgs

Quantum mechanics introduces corrections to this simple relation

$$\mu^2 \to \mu^2 + \delta\mu^2$$

THE HIERARCHY PROBLEM

Quantum corrections correspond to loop Feynman diagrams



$$\delta\mu^2 \propto \lambda\Lambda^2$$

unknown mass scale (regulator)

- This correction is infinite when $\Lambda \to \infty$
- It depends on the regulation procedure
- Renormalization gets rid of these

However, this may not be the right way to look at this. Quadratic divergences are not observable – they can be renormalized away. **They don't even show up if one uses dimensional regularization!** Some level of interpretation, often implied, is required in order to state that the presence of quadratic divergences implies that the weak scale is unstable.

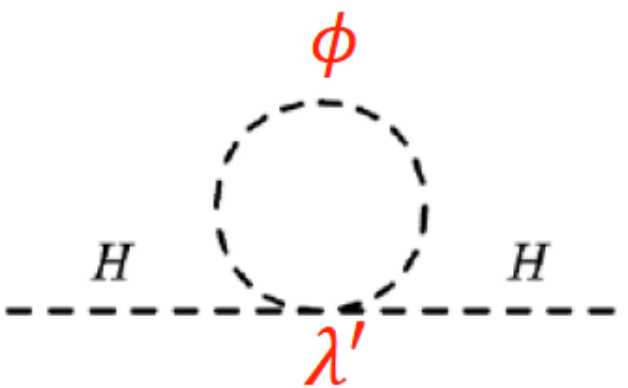
By itself, the Standard Model is a **one-mass-scale theory**. The weak scale, at the tree-level or the one-loop level, is the weak scale. And you can't predict it, it has to be measured.

Life is quite different if, on top of the weak scale, there is **another mass scale** $M_{\rm new}$. Is this case, there are finite corrections to the Higgs mass-squared parameter. These may de-stabilize the weak scale. We use this as the definition of the hierarchy problem:

Can the Weak Scale Co-Exist with Another Mass Scale?

The key point is that the answer **depends on the new physics** and **how it talks to the Higgs boson**. Estimating it from Standard Model parameters may be dangerous.

THE HIERARCHY PROBLEM (Finite corrections)



Known mass scale

$$\delta\mu^2 \propto \lambda' m_\phi^2$$

- **Finite for $\Lambda \to \infty$. Hierarchy Problem when $m_\phi \gg \mu$.**
- **It is independent of the regulator**

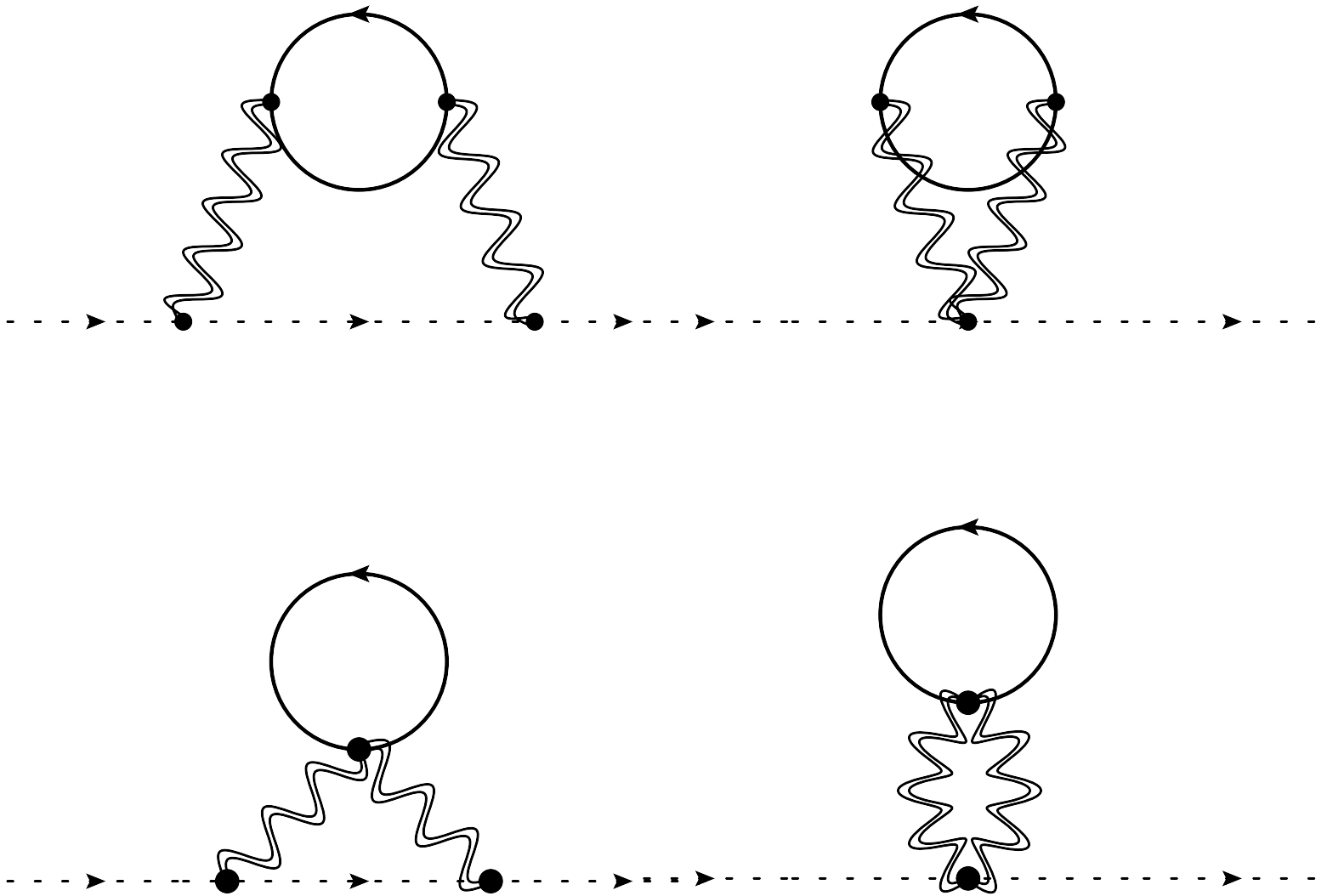
Case Study: New Fermion Ψ, uncoupled to the Standard Model

Complaint: Gravity exists, and the weak scale is way lower than the Planck scale. There is no way these two mass scales can co-exist.

Answer: I don't know how to compute quantum gravity corrections to the Higgs mass-squared. I do know that, perturbatively and at low-energies (below the Planck scale), corrections are tiny since the coupling goes like $1/M_{Pl}^2$:

$$\delta\mu^2 \sim \mu^2 \left(\frac{\mu^2}{M_{Pl}^2}\right)^N$$

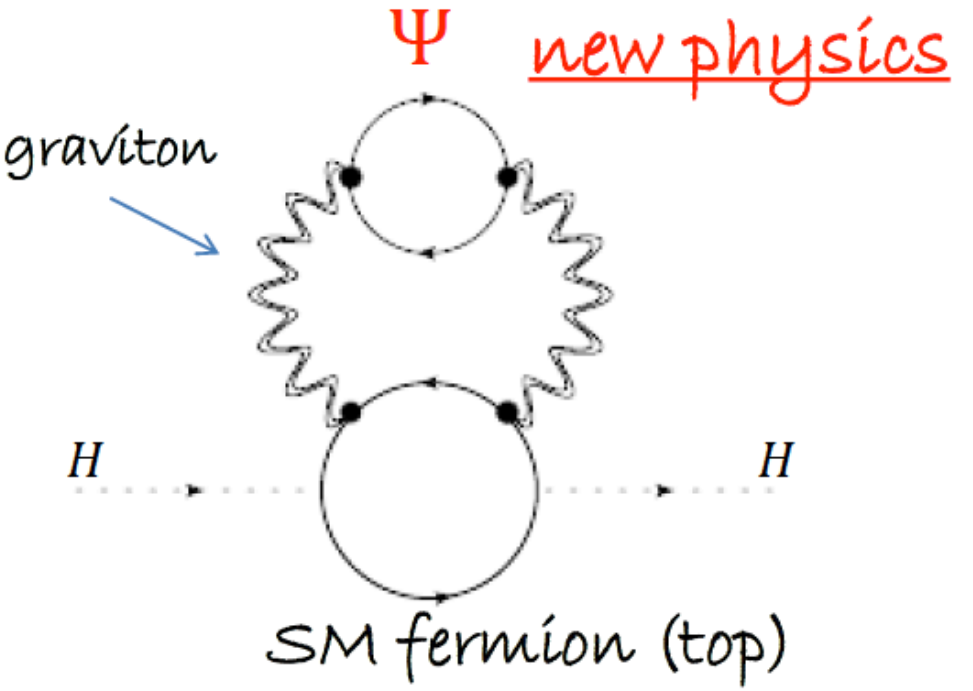
Toy-model: Add a new vector-like fermion with mass M_Ψ that does not couple to the Standard Model at all, except through gravity.



$$\delta\mu^2 \sim \frac{1}{(16\pi^2)^2}\frac{M_\Psi^4}{M_{Pl}^4} \times \mu^2.$$

Uncoupled:

(except for gravity)



$$\delta\mu^2 \sim \frac{\lambda_t^2}{(16\pi^2)^3}\frac{M_\Psi^4}{M_{Pl}^4} \times M_\Psi^2$$

$$\delta\mu^2 \lesssim 100^2\ \mathrm{GeV}^2 \Rightarrow M_\Psi \lesssim 10^{14}\ \mathrm{GeV}$$

Other Case Studies with a new Fermion

Fermion charged under $SU(2) \times U(1)$:

$$\delta\mu^2 = \left(\frac{g^2}{16\pi^2}\right)^2 \times F\left(\frac{M_{W,Z}^2}{M_\Psi^2}\right) \times M_\Psi^2,$$

(two loops). Get in trouble for M_Ψ above tens of TeV.

Yukawa coupled ($y_{\rm new}(\psi H)\Psi$):

$$\delta\mu^2 \sim C\,\frac{y_{\rm new}^2}{16\pi^2} \times M_\Psi^2.$$

(one loop). Get in trouble for M_Ψ above . . . ? Depends on the Yukawa coupling!

Hints for New Mass Scales

We know there is new physics beyond the standard model!

1. Nonzero neutrino masses. New mass scale → maybe. Not necessarily very high.
2. Dark matter. New mass scale → most likely. Not necessarily very high.
3. Gauge coupling unification. New mass scale → certainly. Most likely very high.
4. Flavor symmetries. New mass scale → certainly. Probably pretty high.
5. Inflation. New mass scale → most likely. Not necessarily very high (?).
6. Baryogenesis. New mass scale → probably. Not necessarily very high.
7. Dark Energy. New mass scale → ???

"Thats all Folks!"