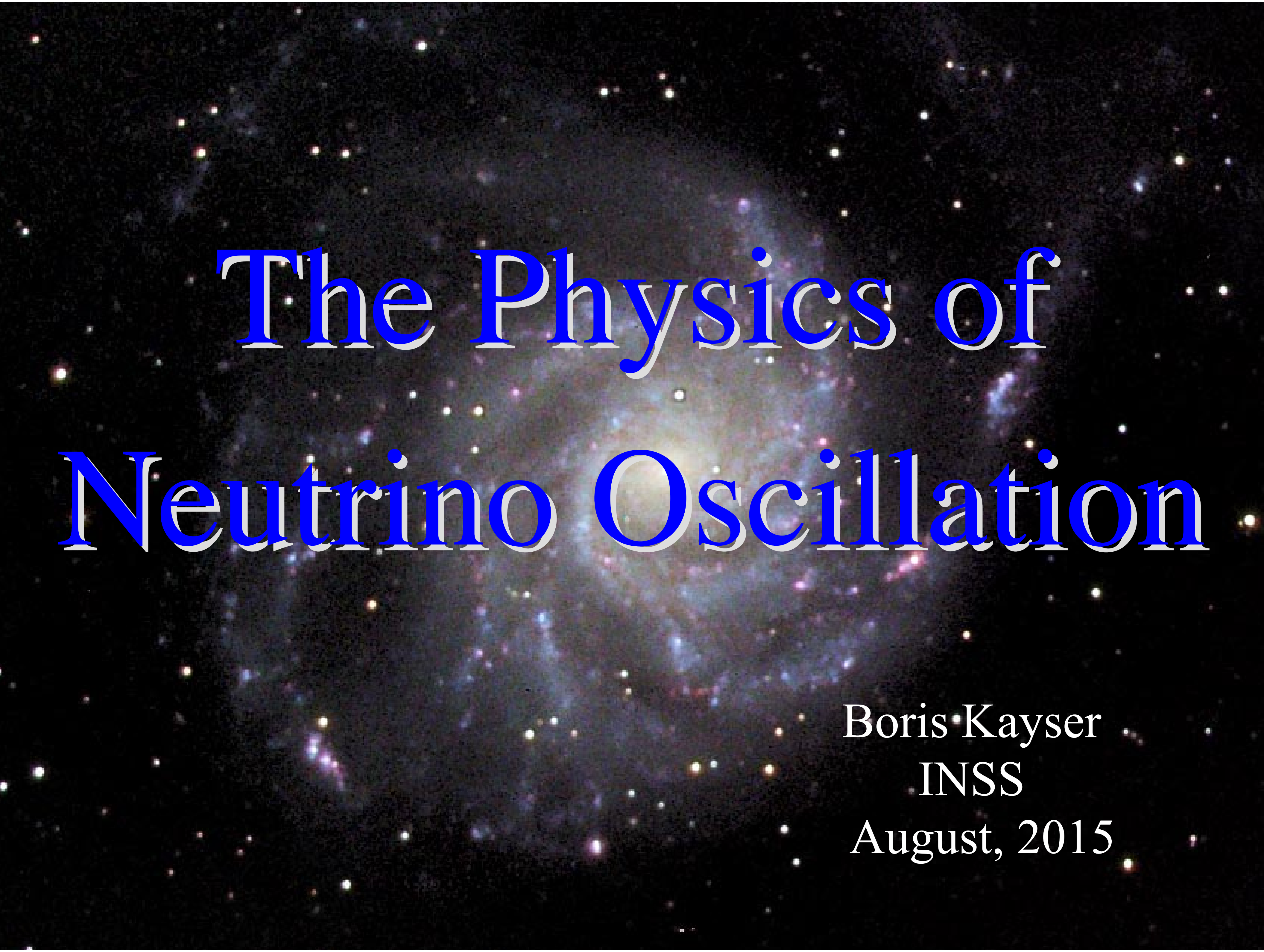

The Physics of Neutrino Oscillation

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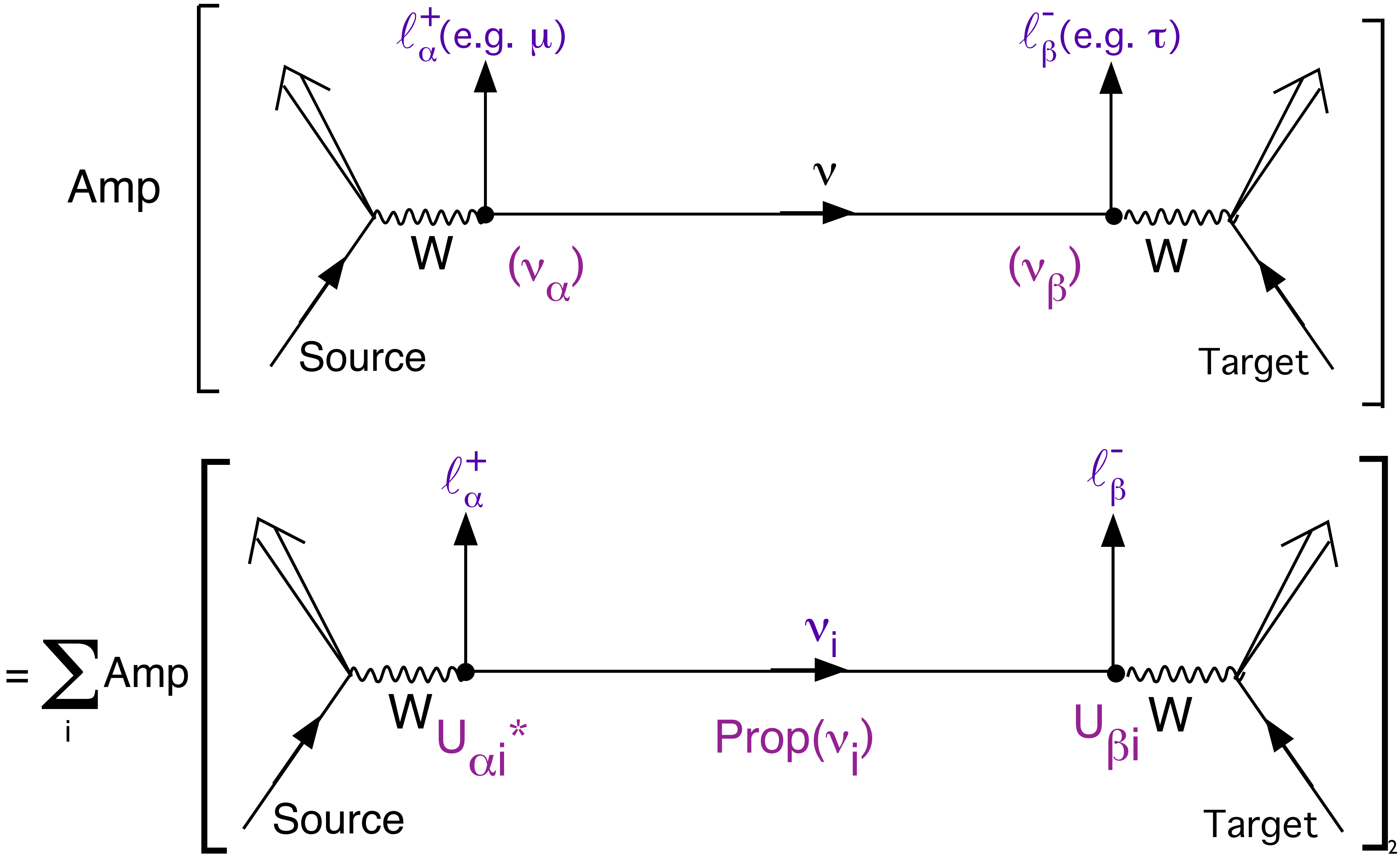
August, 2015

Neutrino Flavor Change (Oscillation) in Vacuum

(Approach of
B.K. & Stodolsky)



Amp $[\nu_\alpha \rightarrow \nu_\beta] = \Sigma U_{\alpha i}{}^* \, \mathrm{Prop}(\nu_i) U_{\beta i}$

What is Propagator $(\nu_i) \equiv \mathrm{Prop}(\nu_i)$?

In the ν_i rest frame, where the proper time is τ_i,

$$i\frac{\partial}{\partial\tau_i}|\nu_i(\tau_i)>= m_i|\nu_i(\tau_i)> \quad .$$

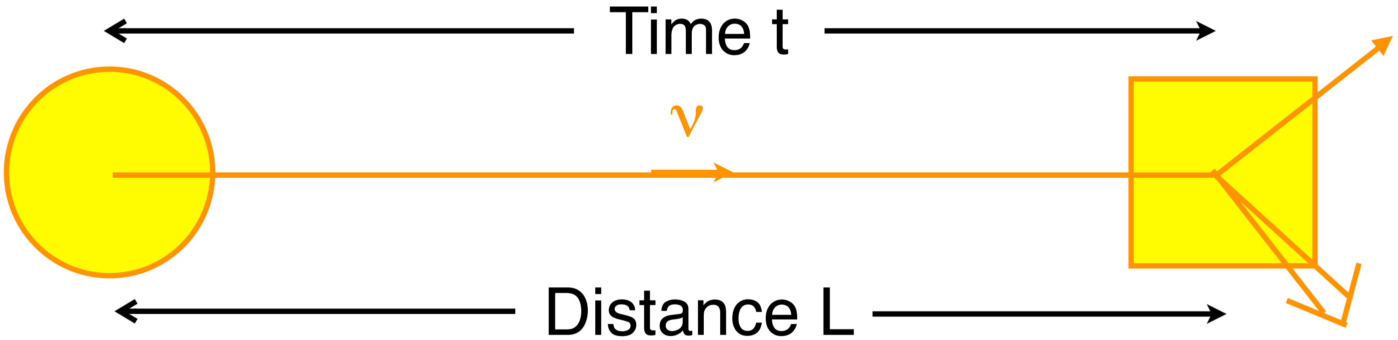
Thus,

$$|\nu_i(\tau_i)>= e^{-im_i\tau_i}|\nu_i(0)> \quad .$$

Then, the amplitude for propagation for time τ_i is —

$$\mathrm{Prop}(\nu_i) \equiv< \nu_i(0)|\nu_i(\tau_i)>= e^{-im_i\tau_i} \quad .$$

In the laboratory frame —



The experimenter chooses L and t.

They are common to all components of the beam.

For each ν_i, by Lorentz invariance,

$$(E_i, p_i) \times (t, L) = m_i \tau_i = E_i t - p_i L \ .$$

Neutrino sources are ~ constant in time.

Averaged over time, the

$$e^{-iE_1t} - e^{-iE_2t}$$ interference

is —

$$< e^{-i(E_1-E_2)t} >_t \;=\; 0$$

unless $E_2 = E_1$.

Only neutrino mass eigenstates with a common energy E are coherent.

(Stodolsky)

For each mass eigenstate ,

$$p_i = \sqrt{E^2 - m_i^2} \cong E - \frac{m_i^2}{2E} \quad .$$

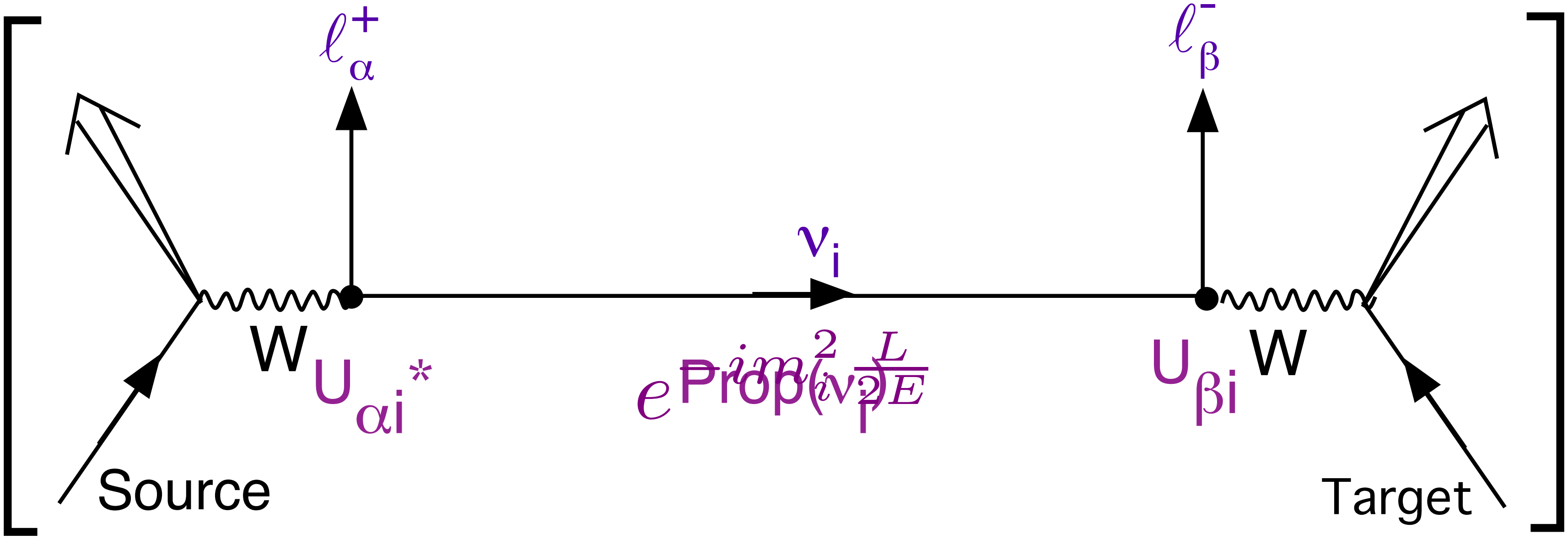
Then the phase in the ν_i propagator $exp[-im_i\tau_i]$ is

$$m_i\tau_i = E_i t - p_i L \cong Et - (E - m_i^2/2E)L$$

$$= \underbrace{E(t - L)}_{\text{Irrelevant overall phase}} + m_i^2 L/2E \quad .$$

Amp $[\nu_\alpha \rightarrow \nu_\beta]$

$= \sum_i \mathrm{Amp}$



$$= \sum_i U^*_{\alpha i} e^{-im_i^2 \frac{L}{2E}} U_{\beta i}$$

Probability for Neutrino Oscillation in Vacuum

$$P(\nu_\alpha \to \nu_\beta) = |\mathrm{Amp}(\nu_\alpha \to \nu_\beta)|^2 =$$

$$= \delta_{\alpha\beta} - 4 \sum_{i>j} \Re(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}) \sin^2(\Delta m^2_{ij} \frac{L}{4E})$$

$$+2 \sum_{i>j} \Im(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}) \sin(\Delta m^2_{ij} \frac{L}{2E})$$

$$\text{where } \Delta m^2_{ij} \equiv m^2_i - m^2_j$$

For Antineutrinos –

We assume the world is CPT invariant.

Our formalism assumes this.

C: Particle → Antiparticle

P: Helicity → Reversed helicity

$$\overline{\nu}(\text{right-handed}) = \mathrm{CP}[\nu(\text{left-handed})]$$

$$P(\overline{\nu_\alpha} \to \overline{\nu_\beta}) \overset{CPT}{=} P(\nu_\beta \to \nu_\alpha) = P(\nu_\alpha \to \nu_\beta; U \to U^*)$$

Thus,

$$P(\overset{(-)}{\nu_\alpha} \to \overset{(-)}{\nu_\beta}) =$$

$$= \delta_{\alpha\beta} - 4 \sum_{i>j} \Re(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}) \sin^2(\Delta m^2_{ij} \frac{L}{4E})$$

$$\underset{(-)}{+} 2 \sum_{i>j} \Im(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}) \sin(\Delta m^2_{ij} \frac{L}{2E})$$

A complex U would lead to the CP violation
$P(\overline{\nu_\alpha} \to \overline{\nu_\beta}) \neq P(\nu_\alpha \to \nu_\beta)$.

Must we assume all mass eigenstates have the same *E*?

No, we can take entanglement into account, and use energy conservation.

The oscillation probabilities are still the same.

B.K., arXiv:1206.4325

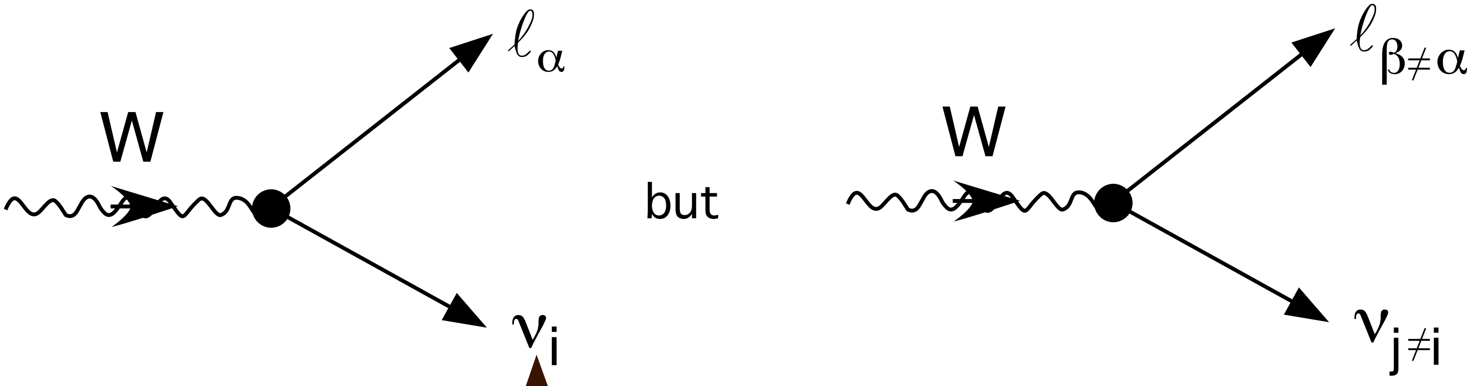
— Comments —

1. If all $m_i = 0$, so that all $\Delta m_{ij}^2 = 0$,

$$P\,(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\beta) = \delta_{\alpha\beta}$$

Flavor *change* ⇒ ν Mass

2. If there is no mixing,



always same ν_i

$$\Rightarrow U_{\alpha i}U_{\beta\neq\alpha,i} = 0, \text{ so that } P\,(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\beta) = \delta_{\alpha\beta}.$$

Flavor *change* ⇒ Mixing

3. One can detect ($\nu_\alpha \rightarrow \nu_\beta$) in two ways:

See $\nu_{\beta\neq\alpha}$ in a ν_α beam (Appearance)

See some of known ν_α flux disappear (Disappearance)

4. Including $\hbar$ and c

$$\Delta m^2 \frac{L}{4E} = 1.27\Delta m^2(\mathrm{eV}^2)\frac{L(\mathrm{km})}{E(\mathrm{GeV})}$$

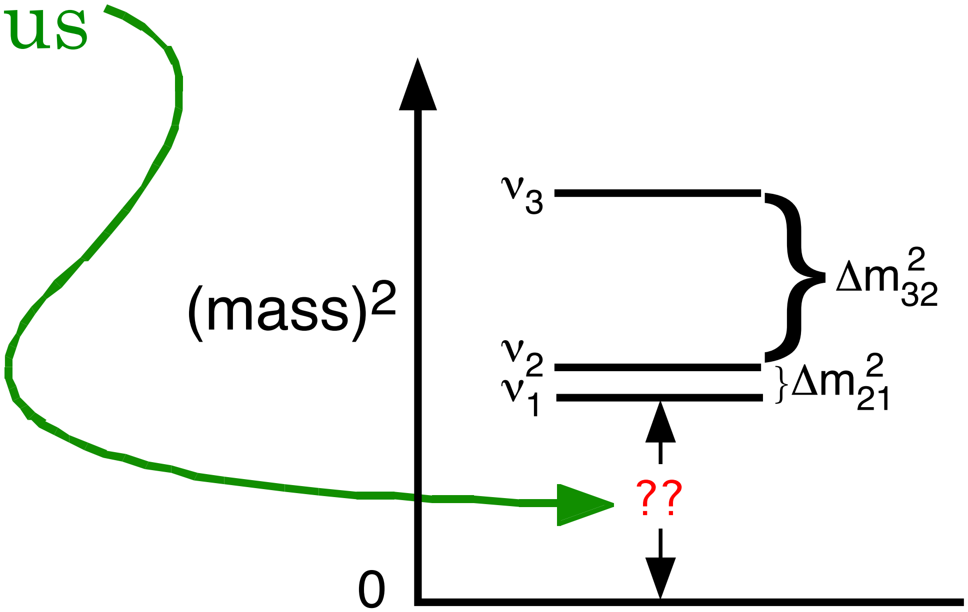
$\sin^2[1.27\Delta m^2(\mathrm{eV})^2 \frac{L(\mathrm{km})}{E(\mathrm{GeV})}]$ becomes appreciable when its argument reaches $\mathcal{O}(1)$.

An experiment with given L/E is sensitive to

$$\Delta m^2(\mathrm{eV}^2) \gtrsim \frac{E(\mathrm{GeV})}{L(\mathrm{km})} \quad .$$

5. Flavor change in vacuum oscillates with L/E. Hence the name "neutrino oscillation". {The L/E is from the proper time τ.}

6. P ($\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta$) depends only on squared-mass splittings. Oscillation experiments cannot tell us



7. Neutrino flavor change does not change the total flux in a beam.

It just redistributes it among the flavors.

$$\sum_{\text{All }\beta} P(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\beta) = 1$$

But some of the flavors β≠α could be sterile.

Then some of the *active* flux disappears:

$$\phi_{\nu_e} + \phi_{\nu_\mu} + \phi_{\nu_\tau} < \phi_{\text{Original}}$$

Important Special Cases

Three Flavors

For $\beta \neq \alpha$,

$$e^{-im_1^2 \frac{L}{2E}} \mathrm{Amp}^*(\nu_\alpha \rightarrow \nu_\beta) = \sum_i U_{\alpha i} U^*_{\beta i} e^{im_i^2 \frac{L}{2E}} e^{-im_1^2 \frac{L}{2E}}$$

$$= U_{\alpha 3} U^*_{\beta 3} e^{2i\Delta_{31}} + U_{\alpha 2} U^*_{\beta 2} e^{2i\Delta_{21}} \underbrace{- (U_{\alpha 3} U^*_{\beta 3} + U_{\alpha 2} U^*_{\beta 2})}_{\text{Unitarity}}$$

$$= 2i[U_{\alpha 3} U^*_{\beta 3} e^{i\Delta_{31}} \sin \Delta_{31} + U_{\alpha 2} U^*_{\beta 2} e^{i\Delta_{21}} \sin \Delta_{21}]$$

$$\text{where } \Delta_{ij} \equiv \Delta m^2_{ij} \frac{L}{4E} \equiv (m_i^2 - m_j^2) \frac{L}{4E} \quad .$$

$$P(\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta) = \left| e^{-im_1^2 \frac{L}{2E}} \, \mathrm{Amp}^* (\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta) \right|^2$$

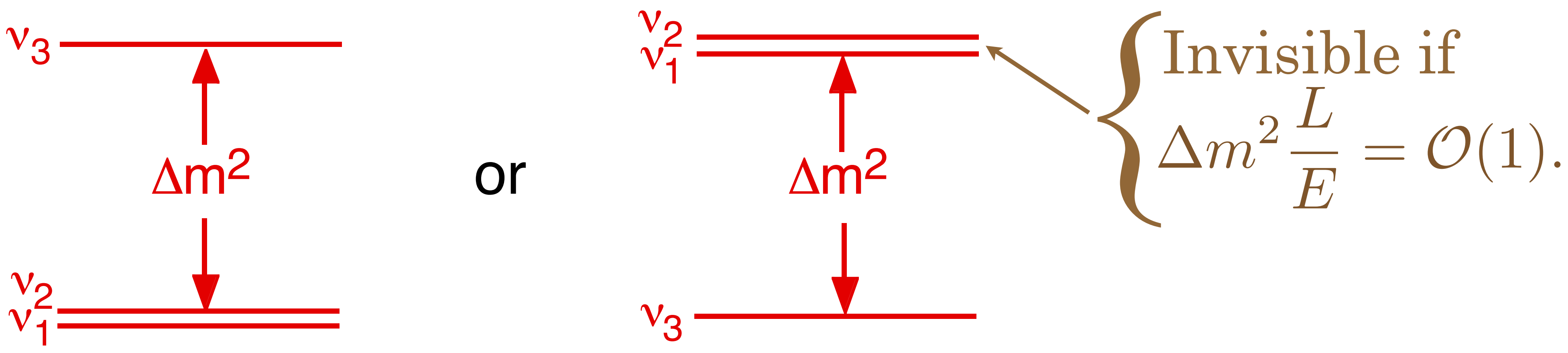
$$= 4[|U_{\alpha 3} U_{\beta 3}|^2 \sin^2 \Delta_{31} + |U_{\alpha 2} U_{\beta 2}|^2 \sin^2 \Delta_{21}$$

$$+2|U_{\alpha 3} U_{\beta 3} U_{\alpha 2} U_{\beta 2}| \sin \Delta_{31} \sin \Delta_{21} \cos(\Delta_{32} \underset{(-)}{+} \delta_{32})] \quad .$$

$$\text{Here } \delta_{32} \equiv \arg(U_{\alpha 3} U^*_{\beta 3} U^*_{\alpha 2} U_{\beta 2}) \,, \;\; \text{a CP} - \text{violating phase.}$$

Two waves of different frequencies,
and their ~~CP~~ interference.

When the Spectrum Is—



Invisible if $\Delta m^2 \frac{L}{E} = \mathcal{O}(1)$.

For $\beta \neq \alpha$,

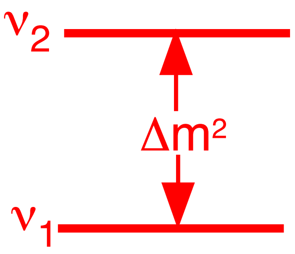
$$P(\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta) \cong 4|U_{\alpha 3}U_{\beta 3}|^2 \sin^2(\Delta m^2 \tfrac{L}{4E}) \quad .$$

For no flavor change,

$$P(\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\alpha) \cong 1 - 4|U_{\alpha 3}|^2(1 - |U_{\alpha 3}|^2) \sin^2(\Delta m^2 \tfrac{L}{4E}) \quad .$$

Experiments with $\Delta m^2 \frac{L}{E} = \mathcal{O}(1)$ can determine the flavor content of ν_3.

When There are Only Two Flavors and Two Mass Eigenstates



Majorana
$\cancel{CP}$ phase

$$U = \begin{bmatrix} U_{\alpha 1} & U_{\alpha 2} \\ U_{\beta 1} & U_{\beta 2} \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} e^{i\xi} & 0 \\ 0 & 1 \end{bmatrix}$$

Mixing angle

For $\beta \neq \alpha$, $P(\overset{(-)}{\nu}_\alpha \leftrightarrow \overset{(-)}{\nu}_\beta) = \sin^2 2\theta \sin^2(\Delta m^2 \frac{L}{4E})$.

For no flavor change, $P(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\alpha) = 1 - \sin^2 2\theta \sin^2(\Delta m^2 \frac{L}{4E})$.