Ação para partícula com massa M e carga e num background eletromagnética:

$$S = \int d\tau \left[M\sqrt{\dot{x}^2} + \frac{e}{c}\, \dot{x}^m A_m(x) \right]$$ $\dot{x}_m = \frac{\partial}{\partial\tau} x_m$

$m = 0$ a 3

Eq. de mov: $M \frac{\ddot{x}_m}{\sqrt{\dot{x}^2}} + \frac{e}{c} \dot{A}_m = \frac{e}{c} \dot{x}^n \partial_m A_n$

$\frac{\partial}{\partial\tau}\left(\frac{\partial L}{\partial \dot{x}}\right) = \frac{\partial L}{\partial x}$

$$\Rightarrow M \frac{\ddot{x}_m}{\sqrt{\dot{x}^2}} = \frac{e}{c} \dot{x}^n F_{mn}$$ $(F_{mn} = \partial_{[m} A$

Pode usar inv. de reparam. para escolher $\dot{x}^2 = c^2$.

Tambem pode começar com $S = \int d\tau \left[\frac{M}{2} \dot{x}^2 + \frac{e}{c} \dot{x}^m A_m(x) \right]$

Eq. de mov: $M \ddot{x}_m = \frac{e}{c} \dot{x}^n F_{mn}$

No limite não-relativ: $x_0 = c\tau$, $\dot{x}_0 = c$, $\dot{x}_h = v_h$

$|\vec{v}| << c$

(letras $\leq h$ vai de 1 a 3)

Eq. de mov: $M\vec{a} = e\vec{E} + \frac{e}{c} \vec{v} \times \vec{B}$

Eq. de mov. para partícula com espín $\vec{S}$:

$$M\vec{a} = e\vec{E} + \frac{e}{c}\vec{v} \times \vec{B} + \frac{ge}{2Mc} \vec{\nabla}(\vec{B} \cdot \vec{S})$$

$$\dot{\vec{S}} = \frac{ge}{2Mc} \vec{S} \times \vec{B}$$ g = razão giromagnético

Existe uma ação que produz estas eq. de mov?

Sim, quando $g = 2$ e espin $= \frac{1}{2}$ (i.e. $|\vec{S}|^2 = \frac{3}{4}\hbar^2$).

Ação tem $D=1$ supersimetria (Brink et al, Phys. Lett. B64 (1976) 435).

$\mathcal{L}\left(x^m_{(\tau)}, \psi^m_{(\tau)}\right)$ — variavel anti-comutante

$$S = \int d\tau \left[\frac{M}{2}\left(\dot{x}^2 + i\dot{\psi}^m \psi_m\right) + \frac{e}{c}\left(\dot{x}^m A_m + \frac{i}{2}\psi^m \psi^n F_{mn}\right)\right]$$

(Uma ação tambem existe com inv. de reparam., mas é mais complicada.
Veja referência.

Ação tem simetria de translação: $x^m \to x^m - i\Lambda \dot{x}^m$, $\psi^m \to \psi^m - i\Lambda\dot{\psi}^m$
(P)

Mas tambem tem supersimetria: $x^m \to x^m + i\alpha\psi^m$, $\psi^m \to \psi^m + \alpha\dot{x}^m$
(Q)
$\uparrow$ parametro anti-comutante

$$\delta_Q\left(\dot{x}^2 + i\dot{\psi}^m\psi_m\right) = 2i\alpha\dot{x}^m\dot{\psi}_m + i\alpha\ddot{x}^m\psi_m + i\dot{\psi}^m\alpha\dot{x}^m$$

$$= i\alpha\frac{\partial}{\partial\tau}\left(\dot{x}^m\psi_m\right)$$

$$\delta_Q\left(\dot{x}^m A_m + \frac{i}{2}\psi^m\psi^n F_{mn}\right) = i\alpha\dot{\psi}^m A_m + \dot{x}^m\left(i\alpha\psi^n\partial_n A_m\right)$$

$$+ \frac{i}{2}\left(\alpha\dot{x}^m\psi^n + \psi^m\alpha\dot{x}^n\right)F_{mn} + \frac{i}{2}\psi^m\psi^n\left(i\alpha\psi^p\partial_p F_{mn}\right)$$

$$= i\alpha\frac{\partial}{\partial\tau}\left(\psi^m A_m\right) + (i-i)\alpha\psi^m\dot{x}^n F_{mn}$$

Então $\delta_Q S = i\alpha\int d\tau\,\frac{\partial}{\partial\tau}\left[\frac{M}{2}\dot{x}^m\psi_m + \frac{e}{c}\psi^m A_m\right] = 0$

(suponha que não tem termos
de superfície)

$$\left(\delta_{Q_1}\delta_{Q_2} - \delta_{Q_2}\delta_{Q_1}\right)x^m = \delta_{Q_1}\left(i\alpha_2\psi^m\right) - \delta_{Q_2}\left(i\alpha_1\psi^m\right) = -2i\alpha_1\alpha_2\dot{x}^m = \delta_P x^m$$

$$\left(\delta_{Q_1}\delta_{Q_2} - \delta_{Q_2}\delta_{Q_1}\right)\psi^m = \delta_{Q_1}\left(\alpha_2\dot{x}^m\right) - \delta_{Q_2}\left(\alpha_1\dot{x}^m\right) = -2i\alpha_1\alpha_2\dot{\psi}^m = \delta_P\psi^m$$

onde $\Lambda = 2\alpha_1\alpha_2$

$\{Q, Q\} = 2P$ "algebra de supersimetria em $D=1$"

Onde está o espín? Define momentum angular M_{jk} (sem fontes)

Inv. Lorentz $\Rightarrow \delta \mathcal{S} = 0$ quando $\delta x^m = \Lambda^{mn} x_n$, $\delta \psi^m = \Lambda^{mn} \psi_n$

$$\Rightarrow \partial^m \mathcal{M}_{mn} = 0 \text{ onde } \mathcal{M}_{mn} = \frac{\partial L}{\partial \dot{x}^p} \frac{\delta x^p}{\delta \Lambda^{mn}} + \frac{\delta L}{\delta \dot{\psi}^p} \frac{\delta \psi^p}{\delta \Lambda^{mn}}$$ "corrente de Noether"

$$\frac{\delta x^p}{\delta \Lambda^{mn}} = x_{[m} \delta^p_{n]} , \quad \frac{\delta \psi^p}{\delta \Lambda^{mn}} = \psi_{[m} \delta^p_{n]}$$

$$\Rightarrow \mathcal{M}_{mn} = M \Big(\underbrace{\dot{x}_{[n} x_{m]}}_{\text{cont. orbital}} + i \underbrace{\psi_n \psi_m}_{\text{cont. de espín}} \Big)$$

$$\Rightarrow \vec{S} = \frac{i}{2} M \vec{\psi} \times \vec{\psi} , \text{ i.e. } \vec{S} = i M \left(\psi^2 \psi^3 , \psi^3 \psi^1 , \psi^1 \psi^2 \right)$$

Quantização do $\mathcal{S} \Rightarrow [x^m , \dot{x}_n] = \frac{i \hbar}{M} \delta^m_n$; $\{ \psi^m , \psi_n \} = - \frac{\hbar}{M} \delta^m_n$

$$\Rightarrow [S_f , S_g] = i \epsilon_{fgh} S_h , \quad |\vec{S}|^2 = \tfrac{3}{4} \hbar^2 .$$

Eq. de mov.: $$M \ddot{x}_m + \frac{e}{c} \dot{A}_m = \frac{e}{c} \left(\dot{x}^n \partial_m A_n + \frac{i}{2} \psi^n \psi^p \partial_m F_{np} \right)$$

$$\frac{\partial}{\partial \tau} \left(\frac{\partial L}{\partial \dot{x}} \right) = \frac{\partial L}{\partial x} \qquad \Rightarrow M \ddot{x}_m = \frac{e}{c} \left(\dot{x}^n F_{mn} + \frac{i}{2} \psi^n \psi^p \partial_m F_{np} \right)$$

$$\frac{\partial}{\partial \tau} \left(\frac{\partial L}{\partial \dot{\psi}} \right) = \frac{\partial L}{\partial \psi} \qquad \frac{i}{2} M \dot{\psi}_m = \frac{i e}{c} F_{mn} \psi^n - \frac{i}{2} M \dot{\psi}_m$$

(sempre da esquerda)

$$\Rightarrow M \dot{\psi}_m = \tfrac{e}{c} F_{mn} \psi^n$$

Limite não-relativístico ($|\vec{v}| << c$) $\Rightarrow \dot{x}_m = (c, \vec{v})$, $\psi_m = (0, \vec{\psi})$

($\dot{x}^m \psi_m$ é constante

Então, $$M \vec{a} = e \vec{E} + \frac{e}{c} \vec{v} \times \vec{B} + \frac{e}{Mc} \vec{\nabla} (\vec{B} \cdot \vec{S})$$

$$\dot{\vec{S}} = i M \dot{\vec{\psi}} \times \vec{\psi} = \frac{i e}{c} (\vec{\psi} \times \vec{B}) \times \vec{\psi} = \frac{i e}{2c} (\vec{\psi} \times \vec{\psi}) \times \vec{B}$$

$$\Rightarrow \dot{\vec{S}} = \frac{e}{Mc} \vec{S} \times \vec{B} \qquad (\Rightarrow g = 2)$$

Supersimetria manifesta:

Define $\mathbb{X}^m(\tau, \kappa) = X^m(\tau) + i\kappa \Psi^m(\tau)$

$\uparrow$ parâmetro anti-comutante

$$P: \tau \to \tau - i\Lambda \, , \; \kappa \to \kappa \quad \left(X^m \to X^m - i\Lambda \dot{X}^m \, , \; \Psi^m \to \Psi^m - i\Lambda \dot{\Psi}^m \right)$$

$$Q: \tau \to \tau - i\alpha\kappa \, , \; \kappa \to \kappa + \alpha \quad \left(X^m \to X^m + i\alpha \Psi^m \, , \; \Psi^m \to \Psi^m + \alpha \dot{X}^m \right.$$

$$P = -i\frac{\partial}{\partial\tau} \, , \quad Q = \frac{\partial}{\partial\kappa} - i\kappa\frac{\partial}{\partial\tau} \quad \Rightarrow \quad \{Q, Q\} = 2P$$

$$\delta_Q f(\mathbb{X}) = \left(\frac{\partial}{\partial\kappa} - i\kappa\frac{\partial}{\partial\tau}\right) f(\mathbb{X}) = \left(\frac{\partial}{\partial\kappa}\mathbb{X}^m - i\kappa\frac{\partial}{\partial\tau}\mathbb{X}^m\right)\partial_m f(\mathbb{X})$$

$$= i\left(\Psi^m - \dot{X}^m\kappa\right)\partial_m f(\mathbb{X})$$

Integração Berezin: $\int d\kappa \, (a + \kappa b) = b$

$$\Rightarrow \int d\kappa \, f(\mathbb{X}) = \left[\frac{\partial}{\partial\kappa} f(\mathbb{X})\right]\Big|_{\kappa=0}$$

$\int d\tau \int d\kappa \, f(\mathbb{X})$ tem supersimetria porque

$$\int d\tau \int d\kappa \, \delta_Q f = \int d\tau \int d\kappa \left[\left(\frac{\partial}{\partial\kappa} - i\kappa\frac{\partial}{\partial\tau}\right) f(\mathbb{X})\right] = \int d\tau \frac{\partial}{\partial\kappa}\left[\left(\frac{\partial}{\partial\kappa} - i\kappa\frac{\partial}{\partial\tau}\right) f(\mathbb{X})\right]\Big|_{\kappa}$$

$$= \int d\tau \left[\left(\frac{\partial}{\partial\kappa}\right)^2 f(\mathbb{X}) - i\frac{\partial}{\partial\tau} f(\mathbb{X})\right]\Big|_{\kappa=0} = 0$$

É útil definir a derivada fermiônica: $D = \frac{\partial}{\partial\kappa} + i\kappa\frac{\partial}{\partial\tau}$

Note que $\{D, Q\} = \left\{\frac{\partial}{\partial\kappa} + i\kappa\frac{\partial}{\partial\tau}, \frac{\partial}{\partial\kappa} - i\kappa\frac{\partial}{\partial\tau}\right\} = 0$

Então $\int d\tau \int d\kappa \, f(\mathbb{X}) \, D g(\mathbb{X})$ também é supersimétric

$$S = -i\int d\tau \int d\kappa \left[\frac{M}{2} D\mathbb{X}^m \dot{\mathbb{X}}_m + \frac{e}{c} D\mathbb{X}^m A_m(\mathbb{X})\right]$$

$$= \int d\tau \left[\frac{M}{2}\left(\dot{x}^2 + i\dot{\psi}^m\psi_m\right) + \frac{e}{c}\left(\dot{x}^m A_m + \frac{i}{2}\psi^m\psi^n F_{mn}\right)\right]$$

$$D\mathbb{X}^m = \left(\frac{\partial}{\partial\kappa} + i\kappa\frac{\partial}{\partial\tau}\right)\mathbb{X}^m = i\psi^m + i\kappa\dot{x}^m,$$

$$\dot{\mathbb{X}}^m = \dot{x}^m + i\kappa\dot{\psi}, \quad A_m(\mathbb{X}) = A_m(x) + i\kappa\psi^n\partial_n A_m(x)$$

Então $-\frac{iM}{2}\int d\tau\int d\kappa\, D\mathbb{X}^m\dot{\mathbb{X}}_m = -\frac{iM}{2}\int d\tau\int d\kappa\,(i\psi^m + i\kappa\dot{x}^m)(\dot{x}_m + i\kappa\dot{\psi}_m$

$$= -\frac{iM}{2}\int d\tau\int d\kappa\,[i\psi^m\dot{x}_m + i\kappa\dot{x}^2 - \kappa\dot{\psi}_m\psi^m$$

$$= \frac{M}{2}\int d\tau\,(\dot{x}^2 + i\dot{\psi}_m\psi^m)$$

Exercícios para 21/3 (a semana que vem):

1) Mostre que $\{D, Q\} = 0$

2) Mostre que $\int d\tau\int d\kappa\, f(\mathbb{X})\, Dg(\mathbb{X})$ é supersimétrica

3) Mostre que $-\frac{ie}{c}\int d\tau\int d\kappa\, D\mathbb{X}^m A_m(\mathbb{X}) = \frac{e}{c}\int d\tau\,(\dot{x}^m A_m + \frac{i}{2}\psi^n\psi^m F_m$

4) As eq. de movimento são

$$\frac{\partial}{\partial\tau}\left(\frac{\partial\hat{\mathcal{L}}}{\partial\dot{\mathbb{X}}^m}\right) + D\left(\frac{\partial\hat{\mathcal{L}}}{\partial D\mathbb{X}^m}\right) = \frac{\partial\hat{\mathcal{L}}}{\partial\mathbb{X}^m} \quad \text{onde } S = \int d\tau\int d\kappa\,\hat{\mathcal{L}}$$

Mostre que esta eq. de mov. $\Rightarrow M D\dot{\mathbb{X}}_m = \frac{e}{c} D\mathbb{X}^n F_{mn}$

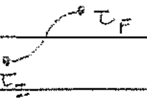
Mostre (expandindo em κ) que esta eq. de mov. implica

$$M\ddot{x}_m = \frac{e}{c}\left(\dot{x}^n F_{mn} + \frac{i}{2}\psi^n\psi^p\partial_m F_{np}\right) \text{ e } M\dot{\psi}_m = \frac{e}{c}F_{mn}\psi^n$$

Partícula: $S = M \int_{\tau_I}^{\tau_F} d\tau \sqrt{\dot{x}^m \dot{x}_m}$ $m = 0$ a $D-1$

linha de universo é unidimensiona

$= M \cdot$ comprimento de caminho

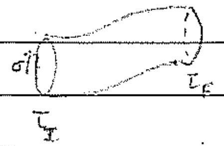


$$P_m = \frac{\partial \mathcal{L}}{\partial \dot{x}^m} = \frac{M \dot{x}_m}{\sqrt{\dot{x}^2}} \Rightarrow P_m P^m = M^2 \quad (\Rightarrow \text{massa} = M)$$

$\dot{x}^2 = 1$: $\ddot{x}^m = 0 \Rightarrow x^m(\tau) = x_0^m + \frac{P^m}{M}\tau$, $[x^m, P_n] = i\hbar \delta_n^m$

$(c=1)$

Corda:



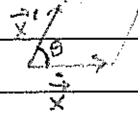
$\sigma \in (0, 2\pi]$ $X^m(\tau, \sigma)$

$m = 0$ a $D-1$

superfície de universo é duo-dimensiona

$$S = \overset{\text{tensão}}{T} \int d\tau d\sigma \sqrt{\dot{x}^2 x'^2 - (\dot{x} \cdot x')^2}$$

$\dot{x}^m = \frac{\partial x^m}{\partial \tau}$, $x'^m = \frac{\partial x^m}{\partial \sigma}$



tem área $= |\vec{\dot{x}}| \, |\vec{x}'| \sin\theta$

$= \sqrt{\dot{x}^2 x'^2 (1 - \cos^2\theta)}$

$= \sqrt{\dot{x}^2 x'^2 - (\dot{x} \cdot x')^2}$

$$P_m = \frac{\partial \mathcal{L}}{\partial \dot{x}^m} = \frac{\dot{x}^m (x')^2 - x'^m (\dot{x})^2}{\sqrt{\dot{x}^2 x'^2 - (\dot{x} \cdot x')^2}} \Rightarrow P^2 = T^2 (x')^2$$

$$\left.\begin{array}{l} \dot{x}^2 + x'^2 = 0 \\ \dot{x} \cdot x' = 0 \end{array}\right\} S = \frac{T}{2} \int d\tau d\sigma \left(\dot{x}^2 - x'^2\right) = \frac{T}{2} \int d\tau d\sigma \, \partial_+ x^m \partial_- x_m$$

$$\boxed{\partial_\pm = \frac{\partial}{\partial \tau} \mp \frac{\partial}{\partial \sigma}}$$

Eq de mov: $\partial_+ \partial_- x^m = 0 \Rightarrow x^m(\tau, \sigma) = x_0^m + \frac{P_0^m}{T}\tau + \sum_{N \neq 0} a_N^m e^{iN(\tau+\sigma)}$

$x^m(\tau, \sigma) = x^m(\tau, \sigma + 2\pi)$

$+ \sum_{N \neq 0} \tilde{a}_N^m e^{iN(\tau - \sigma)}$

$$[x^m(\sigma), P_n(\sigma')] = i\hbar \delta_n^m \delta(\sigma - \sigma') \Rightarrow [a_N^m, a_R^n] = [\tilde{a}_N^m, \tilde{a}_R^n] = \frac{1}{R} \eta^{mn} \delta_{N+R}$$

$$P^2 = T^2 (x')^2 \Rightarrow M^2 = p_0^2 = T^2 \sum_{N \neq 0} \left(N^2 a_N \cdot a_{-N} + N^2 \tilde{a}_N \cdot \tilde{a}_{-N}\right)$$

$$M^2 = T^2 \sum_{N > 0} \left(2N^2 a_N \cdot a_{-N} + 2N^2 \tilde{a}_N \cdot \tilde{a}_{-N} + 2N(D-2)\right) , \quad \overbrace{\sum_{N>0} N = -}^{\text{reg. ze}}$$

$D = 26 \Rightarrow |0\rangle$ $P^2 = -4T^2$ "táquion", $a_1^m \tilde{a}_1^n |0\rangle$ $P^2 = 0$ "graviton", ...

[illegible]

$$\mathcal{S} = \frac{T}{2}\int d\tau\, d\sigma \left(\partial_+ X^m \partial_- X_m - i\Psi_-^m \partial_+ \Psi_{-m} - i\Psi_+^m \partial_- \Psi_{+m} \right)$$

$$X^m(\tau,\sigma)\,,\ \underbrace{\Psi_+^m(\tau,\sigma)\,,\ \Psi_-^m(\tau,\sigma)}_{\text{anti-comutante}}$$

SUSY: $\delta X^m = i\alpha^+ \Psi_+^m + i\alpha^- \Psi_-^m\,,\quad \delta\Psi_+^m = \alpha^+ \partial_+ X^m\,,\quad \delta\Psi_-^m = \alpha^- \partial_- X$

$$\delta\mathcal{S} = \frac{T}{2}\int d\tau\, d\sigma \Big(-2\left(i\alpha^+\Psi_+^m + i\alpha^-\Psi_-^m\right)\partial_+\partial_- X_m - 2i\alpha^-(\partial_- X^m)\partial_+\Psi_{-m}$$

$$- 2i\alpha^+(\partial_+ X^m)\partial_-\Psi_{+m}\Big)$$

$$= \frac{T}{2}\int d\tau\, d\sigma \left(-2i\alpha^+ \partial_-\left(\Psi_+^m \partial_+ X_m\right) - 2i\alpha^- \partial_+\left(\Psi_-^m \partial_- X_m\right)\right)$$

$$= 0$$

Tem 2 SUSY's $\begin{cases} \alpha^+ Q_+ : \ \delta X^m = i\alpha^+\Psi_+^m\,, & \delta\Psi_+^m = \alpha^+\partial_+ X^m\,, \quad \delta\Psi_-^m = 0 \\ \alpha^- Q_- : \ \delta X^m = i\alpha^-\Psi_-^m\,, & \delta\Psi_+^m = 0\,, \quad \delta\Psi_-^m = \alpha^-\partial_- X \end{cases}$

Eq. de mov: $\partial_+\partial_- X^m = 0 \Rightarrow X^m(\tau,\sigma) = x_0^m + \frac{p_0^m}{T} + \sum_{N\neq 0}\left(a_N^m e^{iN(\tau+\sigma)} + \tilde{a}_N^m \cdot e^{iN(\tau-\sigma}\right.$

$\partial_-\Psi_+^m = 0 \Rightarrow \Psi_+^m(\tau,\sigma) = \sum_N b_N^m e^{iN(\tau+\sigma)}$ $\qquad \{b_N^m, b_R^n\} = \eta^{mn}\delta_{N}$

$\partial_+\Psi_-^m = 0 \Rightarrow \Psi_-^m(\tau,\sigma) = \sum_N \tilde{b}_N^m e^{iN(\tau-\sigma)}$ $\qquad \{\tilde{b}_N^m, \tilde{b}_R^n\} = \eta^{mn}\delta$

$$P^2 = T^2\left(X'^2 - i\Psi_+^m \partial_+ \Psi_{+m} - i\Psi_-^m \partial \Psi_{-m}\right)$$

$$\Rightarrow M^2 = p_0^2 = T^2 \sum_N \left(N^2 a_N \cdot a_{-N} + N^2 \tilde{a}_N \cdot \tilde{a}_{-N} + N b_N \cdot b_{-N} + N \tilde{b}_N \cdot \tilde{b}_{-N}\right)$$

$$= T^2 \sum_{N>0} \Big(2N^2 a_N \cdot a_{-N} + 2N^2 \tilde{a}_N \cdot \tilde{a}_{-N} + 2N b_N \cdot b_{-N} + 2N \tilde{b}_N \cdot \tilde{b}_{-N}$$

$$+ (D-2)\left(N^2[a_{-N}, a_N] + N^2[\tilde{a}_{-N}, a_N] - N\{b_{-N}, b_N\} - N\{\tilde{b}_{-N}, \tilde{b}_N\}\right)\Big)$$

$$\Rightarrow M^2 = 2T^2 \sum_{N>0}\left(N^2 a_N \cdot a_{-N} + N^2 \tilde{a}_N \cdot \tilde{a}_{-N} + N b_N \cdot b_{-N} + N \tilde{b}_N \cdot \tilde{b}_{-N}\right)$$

Não tem taquions (ainda tem grávitons sem massa).

Algebra de SUSY:

$\alpha^+ Q_+ + \alpha^- Q_-$: $\delta X^m = i\alpha^+\Psi_+^m + i\alpha^-\Psi_-^m$, $\delta\Psi_+^m = \alpha^+\partial_+ X^m$, $\delta\Psi_-^m = \alpha^-\partial_- X^m$

$\Lambda^+ P_+ + \Lambda^- P_-$: $\delta X^m = -i\Lambda^+\partial_+ X^m - i\Lambda^-\partial_- X^m$, $\delta\Psi_+^m = (-i\Lambda^+\partial_+ - i\Lambda^-\partial_-)\Psi_+^m$

($\Lambda^\pm$ é imaginaria) $\qquad \delta\Psi_-^m = (-i\Lambda^+\partial_+ - i\Lambda^-\partial_-)\Psi_-^m$

$$(\delta_1^Q\delta_2^Q - \delta_2^Q\delta_1^Q)X^m = \delta_1^Q(i\alpha_2^+\Psi_+^m + i\alpha_2^-\Psi_-^m) - \delta_2^Q(i\alpha_1^+\Psi_+^m + i\alpha_1^-\Psi_-^m)$$

$$= -2i\alpha_1^+\alpha_2^+\partial_+ X^m - 2i\,\alpha_1^-\alpha_2^-\,\partial_- X^m$$

$$= \delta^P X^m \quad \text{onde} \quad \Lambda^+ = 2\alpha_1^+\alpha_2^+ \ \text{e} \ \Lambda^- = 2\alpha_1^-\alpha_2^- .$$

$$\{Q_+, Q_+\} = 2P_+ \ , \quad \{Q_-, Q_-\} = 2P_- \ ; \quad \{Q_+, Q_-\} = 0$$

$$(\delta_1^Q\delta_2^Q - \delta_2^Q\delta_1^Q)\Psi_+^m = \delta_1^Q(\alpha_2^+\partial_+ X^m) - \delta_2^Q(\alpha_1^+\partial_+ X^m)$$

$$= \alpha_2^+(i\alpha_1^+\partial_+\Psi_+^m + i\alpha_1^-\partial_+\Psi_-^m) - \alpha_1^+(i\alpha_2^+\partial_+\Psi_+^m + i\alpha_2^-\partial_+\Psi$$

$$= -2i(\alpha_1^+\alpha_2^+\partial_+ + \alpha_1^-\alpha_2^-\partial_-)\Psi_+^m + \text{termos que são zero usand[o] eq. de mov.}$$

$$= \delta^P\Psi_+^m \quad \text{na camada de massa}$$

Então, a álgebra de SUSY somente fecha na camade de masse (i.e. usando eq. de movimento).

Modifica S com campos "auxiliores" (não-propegando)

$$(**) \quad S = \frac{T}{2}\int d\tau\, d\sigma\, (\partial_+ X^m\partial_- X_m - i\Psi_+^m\partial_-\Psi_{+m} - i\Psi_-^m\partial_+\Psi_{-m} + \underset{\substack{\uparrow \\ \text{comutante}}}{F^m}F_m)$$

Eq. de mov para F^m é $F^m = 0$.

$\alpha^+ Q_+ + \alpha^- Q_-$: $\delta X^m = i\alpha^+\Psi_+^m + i\alpha^-\Psi_-^m$, $\delta\Psi_+^m = \alpha^+\partial_+ X^m + \alpha^- F^m$

$\delta\Psi_-^m = \alpha^-\partial_- X^m \quad \alpha^+ F^m \quad \delta F^m$

$$(\delta_1 \delta_2 - \delta_2 \delta_1)\psi_+ = \delta_1(\alpha_2^+ \partial_+ x^m + \alpha_2^- F^m) - \delta_2(\alpha_1^+ \partial_+ x^m + \alpha_1^- F^m)$$

$$= \alpha_2^+ (i\alpha_1^+ \partial_+ \psi_+^m + i\alpha_1^- \partial_+ \psi_-^m) + i\alpha_2^- (\alpha_1^- \partial_- \psi_+^m - \alpha_1^+ \partial_+ \psi_-$$

$$- \alpha_1^+ (i\alpha_2^+ \partial_+ \psi_+^m + i\alpha_2^- \partial_+ \psi_-^m) - i\alpha_1^- (\alpha_2^- \partial_- \psi_+^m - \alpha_2^+ \partial_+ \psi_-^m$$

$$= -2i(\alpha_1^+ \alpha_2^+ \partial_+ + \alpha_1^- \alpha_2^- \partial_-)\psi_+^m$$

A álgebra agora fecha fora de camada da massa

(i.e. sem usar eq. de mov.)

Super-variável: $\mathbb{X}^m(\tau, \sigma; \kappa^+, \kappa^-) = x^m(\tau,\sigma) + i\kappa^+ \psi_+^m(\tau,\sigma)$

$\tau, \sigma; \underbrace{\kappa^+, \kappa^-}_{\text{anti-comutante}}$ $\qquad + i\kappa^- \psi_-^m(\tau,\sigma) + i\kappa^+\kappa^- F^m(\tau,$

$$\boxed{\tau^\pm = \frac{1}{2}(\tau \pm \sigma)}$$

$\alpha^+ Q_+ : \tau^+ \to \tau^+ - i\alpha^+ \kappa^+ , \quad \tau^- \to \tau^- , \quad \kappa^+ \to \kappa^+ + \alpha^+ , \quad \kappa^- \to \kappa^-$

$\alpha^- Q_- : \tau^+ \to \tau^+ , \quad \tau^- \to \tau^- - i\alpha^- \kappa^- , \quad \kappa^+ \to \kappa^+ , \quad \kappa^- \to \kappa^- + \alpha^-$

$\Lambda^+ P_+ : \tau^+ \to \tau^+ - i\Lambda^+ , \quad \tau^- \to \tau^- , \quad \kappa^+ \to \kappa^+ , \quad \kappa^- \to \kappa^-$

$\Lambda^- P_- : \tau^+ \to \tau^+ , \quad \tau^- \to \tau^- - i\Lambda^- , \quad \kappa^+ \to \kappa^+ , \quad \kappa^- \to \kappa^-$

(*) $Q_+ = \frac{\partial}{\partial \kappa^+} - i\kappa^+ \partial_+ , \quad Q_- = \frac{\partial}{\partial \kappa^-} - i\kappa^- \partial_- , \quad P_+ = -i\partial_+ , \quad P_- = -i\partial_-$

$\delta^Q \mathbb{X}^m = (\alpha^+ Q_+ + \alpha^- Q_-)\mathbb{X}^m , \quad \delta^Q f(\mathbb{X}) = \left[(\alpha^+ Q_+ + \alpha^- Q_-)\mathbb{X}^m\right] \partial_m f($

Integração de Berezin: $\int d\kappa^+ d\kappa^- (A + \kappa^+ B + \kappa^- C + \kappa^- \kappa^+ D) = D$

$$\int d\kappa^+ d\kappa^- f(\mathbb{X}) = \left[\frac{\partial}{\partial \kappa^+} \frac{\partial}{\partial \kappa^-} f(\mathbb{X})\right]\Big|_{\kappa^+ = \kappa^- = 0}$$

$\int d\tau d\sigma \int d\kappa^+ d\kappa^- f(\mathbb{X})$ é supersimétrico porque

$\int d\tau d\sigma \int d\kappa^+ d\kappa^- \delta^Q f(\mathbb{X}) = \int d\tau d\sigma \int d\kappa^+ d\kappa^- (\alpha^+ Q_+ + \alpha^- Q_-) f(\mathbb{X}) =$

(prox. pág

$$= \int d\tau d\sigma \int dK^+ dK^- \Big(\underbrace{\alpha^+ \frac{\partial}{\partial K^+} + \alpha^- \frac{\partial}{\partial K^-}}_{\text{zero porque } \left(\frac{\partial}{\partial K^+}\right)^2 f = \left(\frac{\partial}{\partial K^-}\right)^2 f = 0} + \underbrace{\alpha^+ K^+ \partial_+ + \alpha^- K^- \partial_-}_{\text{zero porque } \int d\tau d\sigma\, \partial_\pm g = 0} \Big) f(\mathbb{X})$$

$$= 0$$

Defina derivadas que ant-comutam com $Q_\pm$:

$$D_+ = \frac{\partial}{\partial K^+} + iK^+ \partial_+ \quad , \quad D_- = \frac{\partial}{\partial K^-} + iK^- \partial_-$$

Então $\int d\tau d\sigma \int dK^+ dK^- \, f(\mathbb{X})\, D_+ g(\mathbb{X})$ é supersimétrica

porque $\delta(f D_+ g) = [(\alpha^+ Q_+ + \alpha^- Q_-) f] D_+ g + f D_+ (\alpha^+ Q_+ + \alpha^- Q_-) g = (\alpha^+ Q_+ + \alpha^- Q_-)(f D_+ g)$

Exercícios para 4 de abril:

1) Mostre que $(\delta_1^Q \delta_2^Q - \delta_2^Q \delta_1^Q) F^m = -2i(\alpha_1^+ \alpha_2^+ \partial_+ + \alpha_1^- \alpha_2^- \partial_-) F$

2) Mostre que $\delta^Q \mathbb{X}^m = (\alpha^+ Q_+ + \alpha^- Q_-) \mathbb{X}^m$ implica

as transf. de x^m, ψ_+^m, ψ_-^m, F^m da aula.

3) Mostre que as operadores de $(*)$ satisfazem

$\{Q_\alpha , Q_\beta\} = 2\sigma^a_{\alpha\beta} P_a$ onde $\sigma^0_{\alpha\beta} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $\sigma^1_{\alpha\beta} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$Q_1 = Q_+$, $Q_2 = Q_-$, $P_0 = \frac{1}{2}(P_+ + P_-)$, $P_1 = \frac{1}{2}(P_+ - P_-)$

4) Mostre que $\int d\tau d\sigma \left(\psi_+^m \partial_- \psi_{+m} + \psi_-^m \partial_+ \psi_{-m} \right)$

$= \int d\tau d\sigma \; \psi_\alpha \bar{\sigma}_a^{\alpha\beta} \partial^a \psi_\beta$ onde $\bar{\sigma}^{a\,\beta\alpha} = \varepsilon^{\alpha\gamma} \varepsilon^{\beta\delta} \sigma^a_{\gamma\delta}$

$\psi_1 = \psi_+$, $\psi_2 = \psi_-$

5) Mostre que $\frac{T}{2} \int d\tau d\sigma \int dK^+ dK^- \; D_+ \mathbb{X}^m D_- \mathbb{X}_m$ é

igual à ação da $(**)$.

Ex. II.2: Mostre que $\int d\tau \int d\kappa\, f(X)\, Dg(X)$ é supersimétrica:

$$X^m \to X^m + \epsilon Q X^m \quad \text{onde} \quad QX^m = \left(\frac{\partial}{\partial\kappa} - i\kappa\frac{\partial}{\partial\tau}\right)X^m = i(\psi^m - \kappa \dot{x}^m)$$

$$\Rightarrow f(X)Dg(X) \to f(X+\epsilon QX)\, D[g(X+\epsilon QX)]$$

$$= f(X)Dg(X) + \epsilon(QX^m)(\partial_m f(X))Dg(X) + f(X)D[(\epsilon QX^m)\partial_m g(X)]$$

$(\epsilon^2=0$

$$= f(X)Dg(X) + \epsilon[Qf(X)]Dg(X) + f(X)D[\epsilon Qg(X)]$$

$$= f(X)Dg(X) + \epsilon[(Qf(X))Dg(X) \mp f(X)(DQg(X))]$$

$\left[f \text{ é} \begin{cases}\text{bôson} \\ \text{férmi}\end{cases}\right.$

$$= f(X)Dg(X) + \epsilon[(Qf(X))Dg(X) \pm f(X)(QDg(X))]$$

$$= f(X)Dg(X) + \epsilon Q[f(X)Dg(X)]$$

$$\Rightarrow \int d\tau d\kappa\, f(X)Dg(X) \to \int d\tau d\kappa \left[f(X)Dg(X) + \epsilon \cancelto{0}{\left(\frac{\partial}{\partial\kappa} - i\kappa\frac{\partial}{\partial\tau}\right)}[f(X)Dg(X)\right.$$

Ação de Klein-Gordon para scalares (e.g. bôson de Higgs)

$$S = -\int d^4x \left[\partial_m\varphi\, \partial^m\varphi + M^2\varphi^2 + \underset{\text{energia negativa}}{a\varphi^3} + g^2\varphi^4 + \underset{\text{não-renormaliz}}{\cancel{\ldots}}\right]$$

Eq. de mov: $\Box\varphi = M^2\varphi + 2g^2\varphi^3$ $\quad \left(\langle\varphi\rangle = 0 \text{ ou } \langle\varphi\rangle = \sqrt{\frac{-M^2}{2g^2}}\right)$

Propagador: $\frac{1}{\Box - M^2} = \frac{-1}{k^2 - M^2}$, Vértice: $g^2\ X$ (supõe que a=

Correções a massa:

$$M^2 = M_0^2 + g_0^2 \int_0^\Lambda d^4k \frac{-1}{k^2 + M_0^2} + g_0^4 \int_0^\Lambda d^4k_1 \int_0^\Lambda d^4k_2 \frac{-1}{(k_1^2 + M_0^2)(k_2^2 + M_0^2)((k_1+k_2)^2 + M$$

$$= M_0^2 + g_0^2\left(c_1\Lambda^2 + c_2 \ln\left(\frac{\Lambda^2}{M_0^2}\right) + c_3\right) + g_0^4\left(d_1\Lambda^2 + d_2 \ln\left(\frac{\Lambda^2}{M_0^2}\right) + d_3\right) + .$$

Como você deixa c_3 pequeno quando $\Lambda \to M_P$?

Problema de "hierarquia" vai ser resolvido por SUSY.

III : D=4 SUSY , Scalares

; Mostre que $\int d\tau d\kappa\, f(\mathbb{X})\, Dg(\mathbb{X})$ é supersimétrico:

$X^m + \epsilon Q\mathbb{X}^m$ onde $Q\mathbb{X}^m = \left(\frac{\partial}{\partial\kappa} - i\kappa\frac{\partial}{\partial\tau}\right)\mathbb{X}^m = i(\psi^m - \kappa\dot{x}^m)$

$)Dg(\mathbb{X}) \to f(\mathbb{X}+\epsilon Q\mathbb{X})\, D[g(\mathbb{X}+\epsilon Q\mathbb{X})]$

$)Dg(\mathbb{X}) + \epsilon(Q\mathbb{X}^m)\partial_m f(\mathbb{X}))Dg(\mathbb{X}) + f(\mathbb{X})\, D[(\epsilon Q\mathbb{X}^m)\partial_m g(\mathbb{X})]$ $(\epsilon^2=0)$

$)Dg(\mathbb{X}) + \epsilon[Qf(\mathbb{X})]\, Dg(\mathbb{X}) + f(\mathbb{X})\, D[\epsilon Qg(\mathbb{X})]$

$(\mathbb{X})Dg(\mathbb{X}) + \epsilon[(Qf(\mathbb{X}))\, Dg(\mathbb{X}) \mp f(\mathbb{X})(DQg(\mathbb{X}))]$ $\boxed{f\ \acute{e}\begin{cases}\text{bóson}\\ \text{férmion}\end{cases}}$

$(\mathbb{X})Dg(\mathbb{X}) + \epsilon[(Qf(\mathbb{X}))\, Dg(\mathbb{X}) \pm f(\mathbb{X})(QDg(\mathbb{X}))]$

$f(\mathbb{X})Dg(\mathbb{X}) + \epsilon Q[f(\mathbb{X})\, Dg(\mathbb{X})]$

$\int d\tau d\kappa\, f(\mathbb{X})Dg(\mathbb{X}) \to \int d\tau d\kappa\left[f(\mathbb{X})Dg(\mathbb{X}) + \epsilon\left(\frac{\partial}{\partial\kappa} - i\kappa\frac{\partial}{\partial\tau}\right)[f(\mathbb{X})Dg(\mathbb{X})]\right]$ $\overset{0}{\nearrow}$

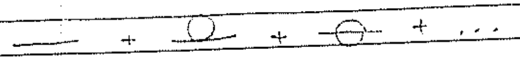
de Klein-Gordon para scalares (e.g. bóson de Higgs)

$d^4x\left[\partial_m\varphi\,\partial^m\varphi + M^2\varphi^2 + \underset{\text{energia negativa}}{a\varphi^3} + g^2\varphi^4 + \underset{\text{não-renormalizável}}{\cancel{\ldots}}\right]$

de mov: $\Box\varphi = M^2\varphi + 2g^2\varphi^3$ $(\langle\varphi\rangle = 0$ ou $\langle\varphi\rangle = \sqrt{\frac{-M^2}{2g^2}})$

opagador: $\frac{1}{\Box - M^2} = \frac{-1}{k^2 - M^2}$, Vértice: g^2 X (supõe que a=0)

correções a massa:



$+ \ldots$

$^2 = M_0^2 + g_0^2\int_0^\Lambda d^4k\,\frac{1}{k^2+M_0^2} + g_0^4\int_0^\Lambda d^4k_1\int_0^\Lambda d^4k_2\,\frac{-1}{(k_1^2+M_0^2)(k_2^2+M_0^2)((k_1+k_2)^2+M_0^2)} + \ldots$

$= M_0^2 + g_0^2\left(c_1\Lambda^2 + c_2\ln\left(\frac{\Lambda^2}{M_0^2}\right) + c_3\right) + g_0^4\left(d_1\Lambda^2 + d_2\ln\left(\frac{\Lambda^2}{M_0^2}\right) + d_3\right) + \ldots$

omo você deixa c_3 pequeno quando $\Lambda \to M_p$?

SUSY

$\mathcal{S}_F = -i\int d^4x\left(\bar\psi_{\dot\alpha}(\bar\sigma^n)^{\dot\alpha\beta}\partial_n\psi_\beta - \frac{iM}{2}(\psi^\alpha\psi_\alpha + \bar\psi_{\dot\alpha}\bar\psi^{\dot\alpha})\right)$

Eq. de mov: $(\bar\sigma^n)^{\dot\alpha\beta}\partial_n\psi_\beta = iM\bar\psi^{\dot\alpha}$ $\Rightarrow$ Tem 2 graus físico

$(\sigma^n)_{\beta\dot\alpha}\partial_n\bar\psi^{\dot\alpha} = iM\psi_\alpha$ $\left(\bar\psi^{\dot\alpha} = -\frac{i}{M}(\bar\sigma^n)^{\dot\alpha\beta}\right.$ $\left.\Box\psi^\alpha = M^2\psi^\alpha\right.$ ①

SUSY ⇒ # graus físicos bosônicos = # graus físicos fermiônicos

⇒ Tem que ter ao menos dois scalares reais (ou um com

$\mathcal{S}_B = -\int d^4x\left[\partial_m\varphi\,\partial^m\bar\varphi + M^2\varphi\bar\varphi\right]$ $\varphi = \varphi_1 + i\varphi_2$

Eqs. de mov: $\Box\varphi = M^2\varphi$, $\Box\bar\varphi = M^2\bar\varphi$ $\bar\varphi = \varphi_1 - i\varphi_2$

$\mathcal{S} = \mathcal{S}_B + \mathcal{S}_F$ é invariante sobre a transf:

$\delta_Q\varphi = \sqrt{2}\,\xi^\alpha\psi_\alpha$, $\delta_Q\bar\varphi = \sqrt{2}\,\bar\xi_{\dot\alpha}\bar\psi^{\dot\alpha}$

$\delta_Q\psi_\alpha = i\sqrt{2}\,\sigma^m_{\alpha\dot\alpha}\bar\xi^{\dot\alpha}\partial_m\varphi - \sqrt{2}M\bar\varphi\xi_\alpha$, $\delta_Q\bar\psi^{\dot\alpha} = i\sqrt{2}(\bar\sigma^m)^{\dot\alpha\beta}\xi_\beta\partial_m\bar\varphi - \sqrt{2}M$

$\delta_Q\mathcal{S}_B = -\sqrt{2}\int d^4x\left[\xi^\alpha\psi_\alpha(-\Box+M^2)\bar\varphi + \bar\xi_{\dot\alpha}\bar\psi^{\dot\alpha}(-\Box+M^2)\varphi\right]$

$\delta_Q\mathcal{S}_F = -\sqrt{2}\int d^4x\left[\partial_m\bar\varphi\,\xi^\beta\sigma^m_{\beta\dot\alpha}(\bar\sigma^n)^{\dot\alpha\gamma}\partial_n\psi_\gamma - M^2\bar\varphi\,\xi^\alpha\psi_\alpha\right.$

$\left.+ \text{conj. complexo}\right]$ ②

$= -\sqrt{2}\int d^4x\left[\xi^\beta\psi_\gamma\left(-\sigma^m_{\beta\dot\alpha}(\bar\sigma^n)^{\dot\alpha\gamma}\right)\partial_m\partial_n\varphi - \xi^\alpha\psi_\alpha M^2\bar\varphi\right.$ + conj. comp

$= -\sqrt{2}\int d^4x\left[\xi^\alpha\psi_\alpha(\Box - M^2)\bar\varphi + \text{conj. complexo}\right]$

$\Rightarrow \delta_Q\mathcal{S} = \delta_Q\mathcal{S}_B + \delta_Q\mathcal{S}_F = 0$

III ; D=4 SUSY , Scalares

Mostre que $\int d\tau dk\, f(X)\, Dg(X)$ é supersimétrico:

$^{m} + \epsilon QX^m$ onde $QX^m = \left(\frac{\partial}{\partial k} - ik\frac{\partial}{\partial \tau}\right)X^m = i(\psi^m - k\dot{x}^m)$

$Dg(X) \to f(X + \epsilon QX)\, D[g(X + \epsilon QX)]$

$Dg(X) + \epsilon(QX^m)\partial_m f(X))Dg(X) + f(X)\, D[(\epsilon QX^m)\partial_m g(X)]$ $(\epsilon^2 = 0)$

$Dg(X) + \epsilon[Qf(X)]Dg(X) + f(X)\, D[\epsilon Qg(X)]$

$X)Dg(X) + \epsilon[(Qf(X))Dg(X) \mp f(X)(DQg(X))]$ $\left[f \in \begin{cases}\text{bóson}\\ \text{férmion}\end{cases}\right]$

$(X)Dg(X) + \epsilon[(Qf(X))Dg(X) \pm f(X)(QDg(X))]$

$f(X)Dg(X) + \epsilon Q[f(X)Dg(X)]$

$\int d\tau dk\, f(X)Dg(X) \to \int d\tau dk\left[f(X)Dg(X) + \epsilon\overset{\nearrow 0}{\left(\frac{\partial}{\partial k} - ik\frac{\partial}{\partial \tau}\right)}[f(X)Dg(X)]\right]$

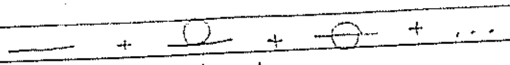
de Klein-Gordon para scalares (e.g. bóson de Higgs)

$d^4x\left[\partial_m\varphi\,\partial^m\varphi + M^2\varphi^2 + a\varphi^3 + g^2\varphi^4 + \cancel{\dots}\right]$

energia negativa   não-renormalizável

de mov: $\Box\varphi = M^2\varphi + 2g^2\varphi^3$ $\left(\langle\varphi\rangle = 0 \text{ ou } \langle\varphi\rangle = \sqrt{\frac{-M^2}{2g^2}}\right)$

opagador: $\frac{1}{\Box - M^2} = \frac{-1}{k^2 - M^2}$, Vértice: g^2 X (supõe que a=0)

orreções a massa: 

$^2 = M_0^2 + g_0^2\int_0^\Lambda d^4k\,\frac{1}{k^2 + M_0^2} + g_0^4\int_0^\Lambda d^4k_1\int_0^\Lambda d^4k_2\,\frac{-1}{(k_1^2 + M_0^2)(k_2^2 + M_0^2)((k_1 + k_2)^2 + M_0^2)} + \dots$

$= M_0^2 + g_0^2\left(c_1\Lambda^2 + c_2\ln\left(\frac{\Lambda^2}{M_0^2}\right) + c_3\right) + g_0^4\left(d_1\Lambda^2 + d_2\ln\left(\frac{\Lambda^2}{M_0^2}\right) + d_3\right) + \dots$

omo você deixa c_3 pequeno quando $\Lambda \to M_p$?

1:1 par SUSY

$\left(\delta_Q^1\delta_Q^2 - \delta_Q^2\delta_Q^1\right)\varphi = \delta_Q^1\left(\sqrt{2}\,\xi_2^\alpha\psi_\alpha\right) - \delta_Q^2\left(\sqrt{2}\,\xi_1^\alpha\psi_\alpha\right)$

$= 2i\left[(\xi_2\sigma^m\bar{\xi}_1)\partial_m\varphi + iM(\xi_2\xi_1) - (\xi_1\sigma^m\bar{\xi}_2)\partial_m\varphi - iM(\xi_1\xi_2\right.$

$= -2i\left[(\xi_1\sigma^m\bar{\xi}_2) + (\bar{\xi}_1\bar{\sigma}^m\xi_2)\right]\partial_m\varphi$

$= 2\,\delta_P\varphi$ onde $\delta_P = -i\lambda^m\partial_m\varphi$ e $\lambda^m = \xi_1\sigma^m\bar{\xi}_2 + \bar{\xi}_1\bar{\sigma}^m\xi_2$

$\left[\xi_1 Q_\alpha + \bar{\xi}_{1\dot\alpha}\bar{Q}^{\dot\alpha}, \xi_2 Q_\alpha + \bar{\xi}_{2\dot\alpha}\bar{Q}^{\dot\alpha}\right]\varphi = 2\left[(\xi_1\sigma^m\bar{\xi}_2 + \bar{\xi}_1\bar{\sigma}^m\xi_2)P_m\right]\varphi$

②

$\{Q_\alpha, Q_\beta\} = 0$, $\{\bar{Q}_{\dot\alpha}, \bar{Q}_{\dot\beta}\} = 0$, $\{Q_\alpha, \bar{Q}_{\dot\beta}\} = 2\sigma^m_{\alpha\dot\beta}P_m$

$\left(\delta_Q^1\delta_Q^2 - \delta_Q^2\delta_Q^1\right)\psi_\alpha = \delta_Q^1\left(i\sqrt{2}(\sigma^m\bar{\xi}_2)_\alpha\partial_m\varphi - \sqrt{2}M\bar{\varphi}\xi_{2\alpha}\right) - 1 \leftrightarrow 2$

$= 2i(\sigma^m\bar{\xi}_2)_\alpha\partial_m(\xi_1^\beta\psi_\beta) - 2M(\bar{\xi}_1\bar{\psi})\xi_{2\alpha} - 1 \leftrightarrow 2$

④

$= -2i(\sigma^m\bar{\xi}_2)_\beta\,\xi_{1\alpha}\partial_m\psi^\beta - 2i\epsilon_{\beta\alpha}(\sigma^m\bar{\xi}_2)_\gamma\,\xi_{1\delta}\,\epsilon^{\gamma\delta}\partial_m\psi^\beta$

$- 2M(\xi_1\bar{\psi})\xi_{2\alpha} - 1 \leftrightarrow$

$= 2i\left((\xi_1\sigma^m\bar{\xi}_2) - (\xi_2\sigma^m\bar{\xi}_1)\right)\epsilon_{\beta\alpha}\partial_m\psi^\beta$

$+ 2i\,\xi_{1\alpha}\left(\bar{\xi}_2\bar{\sigma}^m\partial_m\psi - iM\bar{\xi}_2\bar{\psi}\right) - 2i\,\xi_{2\alpha}\left(\bar{\xi}_1\bar{\sigma}^m\partial_m\psi - iM\bar{\xi}_1\bar{\psi}\right)$

$= -2i\left(\xi_1\sigma^m\bar{\xi}_2 + \bar{\xi}_1\bar{\sigma}^m\xi_2\right)\partial_m\psi_\alpha$ + termos que somem na camada de massa

Para fazer a supersimetria manifesta, tem que incluir campos auxiliares. SUSY manifesta implica # graus bosônicos = # graus fermiônicos

$2 + 2 = 4$

$\varphi, \bar{\varphi}$ $\psi_\alpha, \bar{\psi}_{\dot\alpha}$

⇒ Tem que incluir dois campos bosônicos auxiliares

III : D=4 SUSY , Scalares

Mostre que $\int d\tau\, d\kappa\, f(\mathbb{X})\, Dg(\mathbb{X})$ é supersimétrica:

$X^m + \epsilon Q\mathbb{X}^m$ onde $Q\mathbb{X}^m = \left(\frac{\partial}{\partial\kappa} - i\kappa\frac{\partial}{\partial\tau}\right)\mathbb{X}^m = i(\psi^m - \kappa\dot{x}^m)$

$) Dg(\mathbb{X}) \to f(\mathbb{X}+\epsilon Q\mathbb{X})\, D[g(\mathbb{X}+\epsilon Q\mathbb{X})]$

$) Dg(\mathbb{X}) + \epsilon(Q\mathbb{X}^m)(\partial_m f(\mathbb{X}))Dg(\mathbb{X}) + f(\mathbb{X})\, D[(\epsilon Q\mathbb{X}^m)\partial_m g(\mathbb{X})]$ $(\epsilon^2=0)$

$) Dg(\mathbb{X}) + \epsilon[Qf(\mathbb{X})]Dg(\mathbb{X}) + f(\mathbb{X})\, D[\epsilon Qg(\mathbb{X})]$

$(\mathbb{X})Dg(\mathbb{X}) + \epsilon[(Qf(\mathbb{X}))Dg(\mathbb{X}) \mp f(\mathbb{X})(DQg(\mathbb{X}))]$ $\boxed{f \text{ é } \begin{cases}\text{bôson}\\ \text{férmion}\end{cases}}$

$f(\mathbb{X})Dg(\mathbb{X}) + \epsilon[(Qf(\mathbb{X}))Dg(\mathbb{X}) \pm f(\mathbb{X})(QDg(\mathbb{X}))]$

$f(\mathbb{X})Dg(\mathbb{X}) + \epsilon Q[f(\mathbb{X})Dg(\mathbb{X})]$

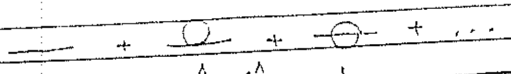
$\int d\tau d\kappa\, f(\mathbb{X})Dg(\mathbb{X}) \to \int d\tau d\kappa\left[f(\mathbb{X})Dg(\mathbb{X}) + \overset{0}{\epsilon\left(\frac{\partial}{\partial\kappa} - i\kappa\frac{\partial}{\partial\tau}\right)[f(\mathbb{X})Dg(\mathbb{X})]}\right]$

de Klein-Gordon para scalares (e.g. bôson de Higgs)

$\int d^4x\left[\partial_m\varphi\,\partial^m\varphi + \underset{\text{energia negativa}}{M^2\varphi^2 + a\varphi^3} + g^2\varphi^4 + \underset{\text{não-renormalizável}}{\cancel{\ldots}}\right]$

de mov: $\Box\varphi = M^2\varphi + 2g^2\varphi^3$ $\left(\langle\varphi\rangle = 0 \text{ ou } \langle\varphi\rangle = \sqrt{\frac{-M^2}{2g^2}}\right)$

Propagador: $\frac{1}{\Box - M^2} = \frac{-1}{k^2 - M^2}$, Vértice: g^2 X (supõe que a=0)

Correções a massa:  + ...

$M^2 = M_0^2 + g_0^2\int_0^\Lambda d^4k\,\frac{1}{k^2+M_0^2} + g_0^4\int_0^\Lambda d^4k_1\int_0^\Lambda d^4k_2\,\frac{-1}{(k_1^2+M_0^2)(k_2^2+M_0^2)((k_1+k_2)^2+M_0^2)} + \ldots$

$= M_0^2 + g_0^2\left(c_1\Lambda^2 + c_2\ln\left(\frac{\Lambda^2}{M_0^2}\right) + c_3\right) + g_0^4\left(d_1\Lambda^2 + d_2\ln\left(\frac{\Lambda^2}{M_0^2}\right) + d_3\right) + \ldots$

Como você deixa c_3 pequeno quando $\Lambda \to M_P$?

[illegible] por SUSY

$\mathcal{L} = [illegible]\left[-\partial_m\bar\varphi\,\partial^m\varphi + i\bar\psi_{\dot\alpha}(\bar\sigma^m)^{\dot\alpha\beta}\partial_m\psi_\beta + \bar F F\right.$

$\left.+ M\left(\varphi F + \bar\varphi\bar F - \frac{1}{2}\psi^\alpha\psi_\alpha - \frac{1}{2}\bar\psi_{\dot\alpha}\bar\psi^{\dot\alpha}\right)\right]$

Eq. de mov.: $\Box\varphi = -M\bar F$, $\Box\bar\varphi = -MF$, $\bar F = -M\varphi$, $F = -M$

$\bar\sigma^m\partial_m\psi = iM\bar\psi$, $\sigma^m\partial_m\bar\psi = iM\psi$

$\Rightarrow \Box\varphi = M^2\varphi$, $\Box\bar\varphi = M^2\bar\varphi$, F e $\bar F$ são auxiliares

Define $\delta_\xi\varphi = \sqrt2\,\xi\psi$, $\delta_\xi\bar\varphi = \sqrt2\,\bar\xi\bar\psi$

(**) $\delta_\xi\psi = i\sqrt2(\sigma^m\bar\xi)\partial_m\varphi + \sqrt2\,\xi F$, $\delta_\xi\bar\psi = i\sqrt2(\bar\sigma^m\xi)\partial_m\bar\varphi + \sqrt2\,\bar\xi$

$\delta_\xi F = i\sqrt2\,\bar\xi\bar\sigma^m\partial_m\psi$, $\delta_\xi\bar F = i\sqrt2\,\xi\sigma^m\partial_m\bar\psi$

$(\delta_{\xi_1}\delta_{\xi_2} - \delta_{\xi_2}\delta_{\xi_1})\psi_\alpha = \delta_{\xi_1}\left(i\sqrt2(\sigma^m\bar\xi_2)_\alpha\partial_m\varphi + \sqrt2\,\xi_{2\alpha}F\right) - 2\leftrightarrow1$

$= 2i(\sigma^m\bar\xi_2)_\alpha\partial_m(\xi_1\psi) + 2i\xi_{2\alpha}(\bar\xi_1\bar\sigma^m\partial_m\psi) - 2\leftrightarrow1$

$= -2i(\sigma^m\bar\xi_2)_\beta\,\xi_{1\alpha}\partial\psi^\beta + 2i\epsilon_{\beta\alpha}(\xi_1\sigma^m\bar\xi_2)\partial_m\psi^\beta + 2i\xi_{2\alpha}(\bar\xi_1\bar\sigma^m\partial_m\psi) - 2\leftrightarrow$

$= -2i(\xi_1\sigma^m\bar\xi_2 + \bar\xi_1\sigma^m\xi_2)\partial_m\psi_\alpha = \delta_P\psi_\alpha$

$\Rightarrow$ álgebra de SUSY fecha fora da camada de massa

Introduz supercampos para fazer a SUSY manifesta:

$\Phi(y^m, \theta^\alpha) = \varphi(y) + \sqrt2\,\theta^\alpha\psi_\alpha(y) + \theta^\alpha\theta_\alpha F(y)$

$\bar\Phi(\bar y^m, \bar\theta^{\dot\alpha}) = \varphi(\bar y) + \sqrt2\,\theta_\alpha\bar\psi^{\dot\alpha}(\bar y) + \theta_\alpha\bar\theta^{\dot\alpha}\bar F(y)$

onde $y = x^m + i\theta\sigma^m\theta$, $\bar y = x^m - i\theta\sigma^m\theta$

$\xi^\alpha Q_\alpha + \bar\xi^{\dot\alpha}\bar Q_{\dot\alpha}$: $\delta x^m = -i(\xi\sigma^m\bar\theta + \bar\xi\sigma^m\theta)$, $\delta\theta^\alpha = \xi^\alpha$, $\delta\bar\theta^{\dot\alpha} = \bar\xi$

$\Rightarrow \delta y^m = -2i\bar\xi\sigma^m\theta$, $\delta\bar y^m = -2i\xi\sigma^m\bar\theta$

1) Mostra que as eqs. de mov. para a ação de Dirac

$$\Rightarrow \Box \Psi^\alpha = M^2 \Psi^\alpha$$

2) Mostre que os termos proporcional a M cancelam em $\delta_Q \delta_F$.

3) Mostre que $\left[\xi_1 Q + \bar\xi_1 Q , \xi_2 Q + \bar\xi_2 \bar Q \right] = 2 \left(\xi_1 \sigma^m \bar\xi_2 + \bar\xi_1 \bar\sigma^m \xi_2 \right)$

$$\Rightarrow \{Q_\alpha, Q_\beta\} = \{\bar Q_{\dot\alpha}, \bar Q_{\dot\beta}\} = 0 \quad \text{e} \quad \{Q_\alpha, \bar Q_{\dot\beta}\} = 2 \sigma^m_{\alpha\dot\beta} P_m$$

4) Mostre que $(\sigma^m \bar\xi_2)_\alpha \partial_m (\xi_1 \Psi)$

$$= -(\sigma^m \bar\xi_2)_\beta \xi_{1\alpha} \partial_m \Psi^\beta - (\xi_1 \sigma^m \bar\xi_2) \partial_m \Psi_\alpha$$

5) Mostre que a ação (*) é supersimétrica sobre as transformações de (**).

6) Em termos de x^m, θ^α, $\bar\theta^{\dot\alpha}$, qual é a definição dos operadores Q_α e $\bar Q_{\dot\alpha}$ que produz as transf. de (**)

se $\delta_Q \bar\Phi(y, \theta) = (\xi Q + \bar\xi \bar Q) \bar\Phi(y, \theta)$

e $\delta \bar\Phi_Q(\bar y, \bar\theta) = (\xi Q + \bar\xi \bar Q) \bar\Phi(\bar y, \bar\theta)$?