

Exercises Lecture 2, String Phenomenology

1. Find the massless spectrum of the $E_8 \times E_8$ heterotic string in the fermionic formulation.
2. Find the massless spectrum of the $E_8 \times E_8$ heterotic string in the bosonic formulation.

The even self-dual lattice Γ_{16} is the product of two 8-dimensional lattices of the form

$$\Gamma_8 = \left\{ (n_1, \dots, n_8), (n_1 + \frac{1}{2}, \dots, n_8 + \frac{1}{2}) \parallel n_i \in \mathbb{Z}, \sum_{i=1}^8 n_i = \text{even} \right\}.$$

3. Find the coordinates of the fixed points in the T^6/\mathbb{Z}_3 orbifold.
4. Find the mode expansion for twisted bosonic coordinates that satisfy boundary conditions

$$Z^i(\tau, \sigma + 2\pi) = e^{2i\pi v_i} Z^i(\tau, \sigma) + W^i,$$

where W^i is a complexified lattice vector.

5. Show, by using ζ function regularization or by introducing a cutoff, that

$$\frac{1}{2} \sum_{n=0}^{\infty} (n+v) = -\frac{1}{24} + \frac{1}{4}v(1-v).$$

Here $0 < v < 1$.