III. Heterotic model building

heterosis

SAVE POPULARITY

noun | het·er·o·sis | \ˌhe-tə-ˈrō-səs\

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Definition of HETEROSIS

: the marked vigor or capacity for growth often exhibited by crossbred animals or plants —called also *hybrid vigor*

— **het·er·ot·ic** | \-ˈrä-tik\ | *adjective*

The heterotic string

world-sheet degrees of freedom (fermionic formulation $SO(32)$)

right R

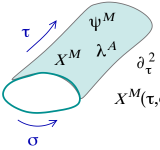
X_R^M , ψ^M 2d fermions

$M = 0, \cdots, 9$, $M = 2, \cdots, 9$ light-cone

left L

X_L^M , λ^A 2d fermions

$A = 1, \cdots, 32$



$$\partial_\tau^2 X^M - \partial_\sigma^2 X^M = 0$$

$$X^M(\tau,\sigma) = X_L^M(\tau+\sigma) + X_R^M(\tau-\sigma)$$

$\psi^M(\tau, \sigma + 2\pi) = \mp\psi^M(\tau, \sigma)$; $\lambda^A(\tau, \sigma + 2\pi) = \mp\lambda^A(\tau, \sigma)$; $-$ Neveu-Schwarz(NS), $+$ Ramond

massless states $|R\rangle \otimes |L\rangle$

NS $\otimes$ NS

e.g. $b^M_{-\frac{1}{2}}\,|0\rangle \;\otimes\; \lambda^A_{-\frac{1}{2}}\,\lambda^B_{-\frac{1}{2}}\,|0\rangle$

$$M_R^2 = \tilde{N}_X + \tilde{N}_\psi - \tfrac{1}{2}$$
$$M_L^2 = N_X + N_\lambda - 1$$

496 gauge vectors of $SO(32)$ in 10d (gauginos in Ramond $\otimes$ NS)

Full massless spectrum

$R \otimes L$ in light cone

$$[(\mathbf{8}_v \oplus \mathbf{8}_s, \mathbf{1})]_R \otimes [(\mathbf{8}_v, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{496})]_L$$

$$= (\mathbf{1} \oplus \mathbf{35} \oplus \mathbf{28} + \mathbf{8}_c + \mathbf{56}_s, \mathbf{1}) \ \oplus \ (\mathbf{8}_v \oplus \mathbf{8}_s, \mathbf{496})$$

massless fields

$$\{\varphi, G_{MN}, B_{MN}, \Psi, \Psi_M\} \ \oplus \ \left\{A_a^M, \chi_a\right\} \quad a = 1, \ldots, \dim G_{\rm het}$$

$10d$, $\mathcal{N}=1$ supergravity $\oplus$ super Yang-Mills $G_{\rm het}$

$G_{\rm het} = E_8 \times E_8, SO(32)$

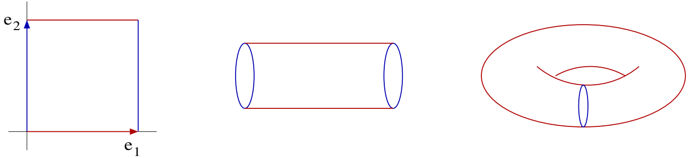
Gauge and gravitational anomalies are cancelled

by the Green-Schwarz mechanism

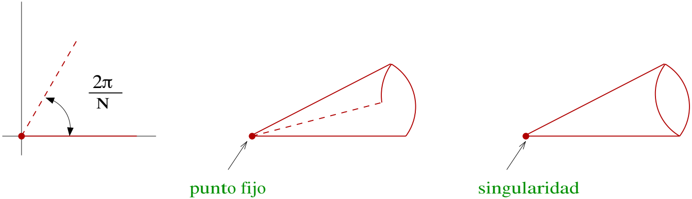
Orbifolds

$\mathcal{O} = \mathcal{M}/\Gamma$; Γ = discrete group of isometries of $\mathcal{M}$

$\mathbf{T}^2 = \mathbf{R}^2 / \Lambda$



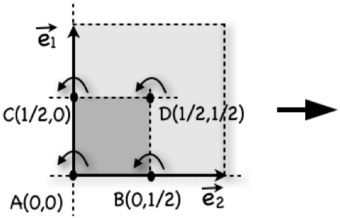
cono $\mathbf{R}^2 / \mathbf{Z_N}$

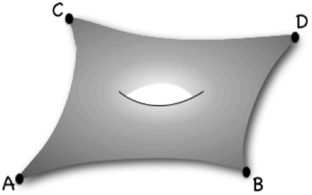


$T^2/\mathbb{Z}_2$ orbifold

$T^2 = \mathbb{R}^2/\Lambda, \quad \Lambda = SO(4)$ root lattice

$\mathbb{Z}_2 = \{\mathbf{1}, \theta\}, \quad \theta =$ rotation by π



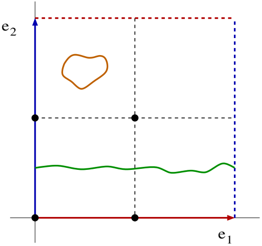


• : fixed points $\longrightarrow$ singularities

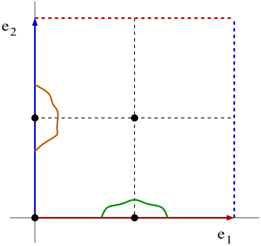
Closed strings on orbifolds $\mathcal{M}/\Gamma$

Dixon, Harvey, Vafa, Witten

* untwisted sector | twisted sector



$$\overrightarrow{X}(\tau,\sigma+2\pi)=\overrightarrow{X}(\tau,\sigma)+n_i\overrightarrow{e_i}$$



$$\overrightarrow{X}(\tau,\sigma+2\pi)=g\overrightarrow{X}(\tau,\sigma)+n_i\overrightarrow{e_i}$$

$g\in\Gamma$ admits fixed points

* Orbifold projection: physical states are invariant under Γ

Both conditions are required by modular invariance

Toroidal orbifolds $\mathrm{T}^6/\mathbb{Z}_N$

$$\mathrm{T}^6 = \mathbb{R}^6/\Lambda, \quad \mathbb{Z}_N = \{\mathbf{1}, \theta, \ldots, \theta^{N-1}\},\ \theta \in SO(6)$$

crystallographic action: $W \in \Lambda,\ \theta W \in \Lambda$

$\theta^N = \mathbf{1} \Rightarrow \theta$ has eigenvalues $e^{\pm 2\pi i v_i},\ v_i = \dfrac{k_i}{N},\ k_i \in \mathbb{Z},\ \ i = 1, 2, 3$

complex internal coordinates $\quad Z^i = \frac{1}{\sqrt{2}}\left(X^{2i+2} + iX^{2i+3}\right), \quad \theta Z^i = e^{2\pi i v_i} Z^i$

$\theta = \exp 2\pi i(v_1 J_{12} + v_2 J_{34} + v_3 J_{56}),\ \ J_{2i-1,2i} : SO(6)$ Cartan generator

action on spinor representation

$$\theta|\pm\tfrac{1}{2}, \pm\tfrac{1}{2}, \pm\tfrac{1}{2}\rangle = e^{i\pi(\pm v_1 \pm v_2 \pm v_3)}|\pm\tfrac{1}{2}, \pm\tfrac{1}{2}, \pm\tfrac{1}{2}\rangle$$

supersymmetry $\ \Rightarrow\ \pm v_1 \pm v_2 \pm v_3 =$ even

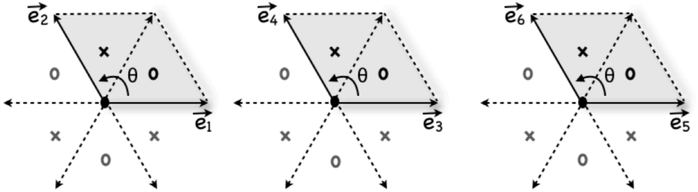
$\theta^N = \mathbf{1}$ acting on fermions $\Rightarrow\ N(v_1 + v_2 + v_3) =$ even

Example: $T^6/\mathbb{Z}_3$ orbifold

$T^6 = \mathbb{R}^6/\Lambda, \quad \Lambda =$ product of three $SU(3)$ root lattices

$\mathbb{Z}_3 = \{\mathbf{1}, \theta, \theta^2\}, \quad \theta =$ rotation by $\frac{2\pi}{3}$ in each sub-lattice

$(v_1, v_2, v_3) = (\frac{1}{3}, \frac{1}{3}, -\frac{2}{3})$



$\bullet, \circ, \times$: fixed points altogether $3 \times 3 \times 3 = 27$ fixed points

Action on $2d$ fermions

right movers

θ-twisted sector

$$\Psi^i = \frac{1}{\sqrt{2}}\left(\psi^{2i+2} + i\psi^{2i+3}\right), \qquad \Psi^i(\tau, \sigma + 2\pi) = \mp e^{2\pi i v_i}\Psi^i(\tau, \sigma + 2\pi)$$

left movers   give gauge degrees of freedom

$$\lambda^A_\pm = \frac{1}{\sqrt{2}}\left(\lambda^{2A-1} \pm i\lambda^{2A}\right), \qquad A = 1, \ldots, 16$$

for $E_8 \times E_8$ divide in two groups: $\lambda^A_\pm,\ \lambda'^A_\pm,\ A = 1, \ldots, 8$

Modular invariance requires that λ^A transforms under θ. The action can be realized by rotation γ with eigenvalues $e^{\pm 2\pi i V_A},\ V_A = \frac{K_A}{N},\ K_A \in \mathbb{Z}$.

$\gamma^N = \mathbf{1} \Rightarrow\ N(V_1 + \ldots + V_{16}) = \text{even}$

Gauge shift vector: $V = (V_1, \ldots, V_8) \times (V'_1, \ldots, V'_8),\ \ E_8 \times E_8$

Modular invariance (level-matching) further requires $N(V^2 - v^2) = \text{even}$

Standard Embedding: $V = (v_1, v_2, v_3, 0, \ldots, 0) \times (0, \ldots, 0)$

Compactification on Calabi-Yau (CY) manifolds

Recall fields in $10d$

$\{\varphi, G_{MN}, B_{MN}, \Psi, \Psi_M\} \;\oplus\; \{A_a^M, \chi_a\}$, $a = 1, \ldots, \dim G_{\rm het}$ $\quad G_{\rm het} = E_8 \times E_8, SO(32)$

Compactification $\mathcal{M}_{10} = \mathcal{M}_4 \times K_6$

Supersymmetry in $4d \;\Rightarrow\; D_m\eta = 0 \;\Rightarrow R_{mn} = 0$

K_6 Ricci-flat $\Rightarrow$ holonomy $SU(3)$ $\quad$ K_6 is CY

Furthermore, K_6 is Kähler (complex with special property of the metric)

$x^m \longrightarrow z^i, \bar{z}^{\bar{i}}$ $\qquad$ k-forms: $\omega_{m_1 \ldots m_k}$ $\qquad$ (p, q)-forms: $\omega_{i_1 \ldots i_p \bar{j}_1 \ldots \bar{j}_q}$

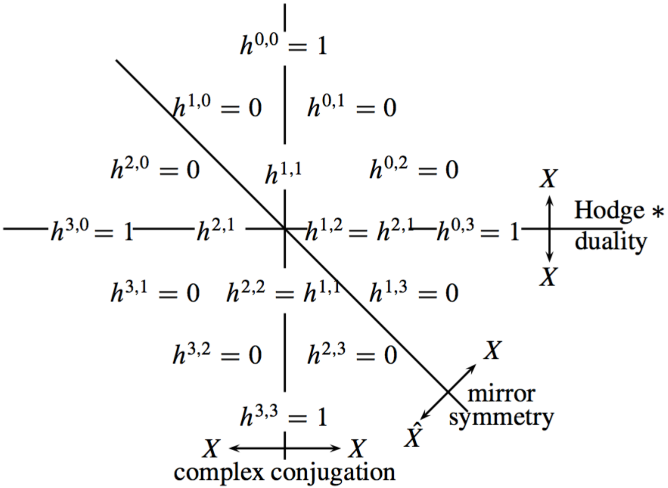
Betti numbers $b_k = \#$ closed (mod exact) k-forms $= \#$ harmonic k-forms

Hodge numbers

$h^{p,q} = \#$ closed (mod exact) (p, q)-forms $= \#$ harmonic (p, q)-forms

$$b_k = \sum_{p=0}^{k} h^{p,k-p}$$

Hodge diamond of a CY X



$\chi = 2(h^{1,1} - h^{1,2})$ Euler characteristic of X

Hodge plot

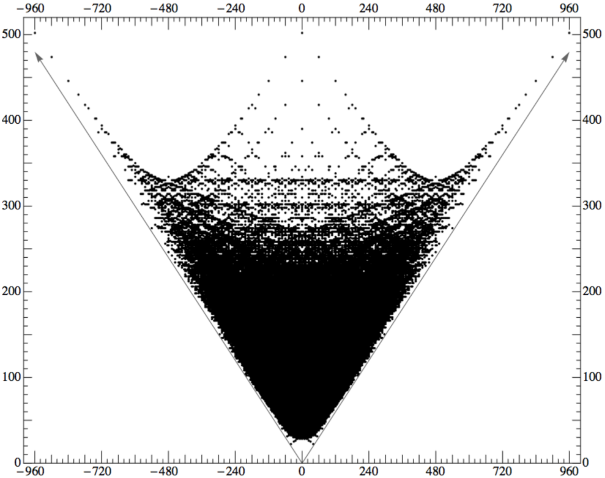


Figure 1: *The Hodge plot for the list or reflexive 4-polytopes. The Euler number* $\chi = 2\left(h^{1,1} - h^{1,2}\right)$ *is plotted against the height* $y = h^{1,1} + h^{1,2}$*. The oblique axes correspond to* $h^{1,1} = 0$ *and* $h^{1,2} = 0$.

Taken from arXiv:1207.4792, based on the Kreuzer-Skarke list