CFTC Centro de Física Teórica e Computacional

Ciências ULisboa

# *Synchronization transitions in the Kuramoto model*

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*School on Complex Networks and Applications to Neuroscience*
02/October/2015

*http://www.namaraujo.net*

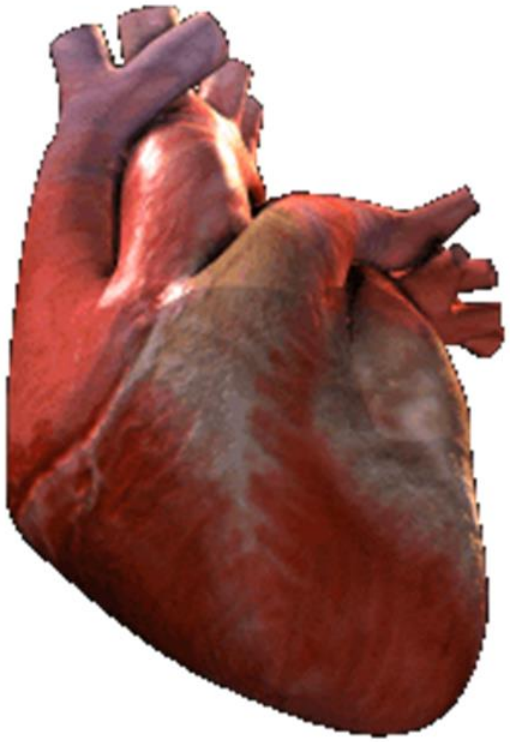


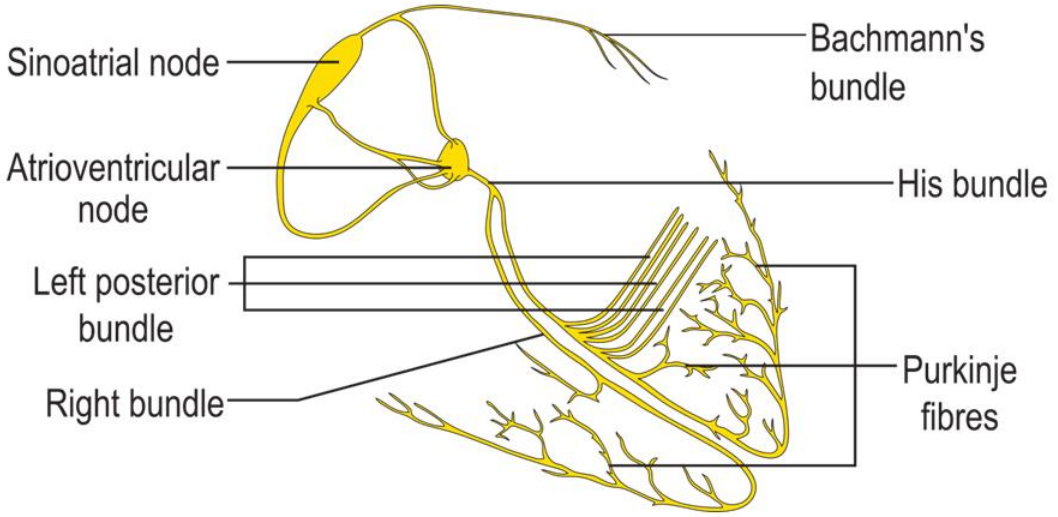
INVESTIGADOR FCT

# The good
# synchronous fireflies

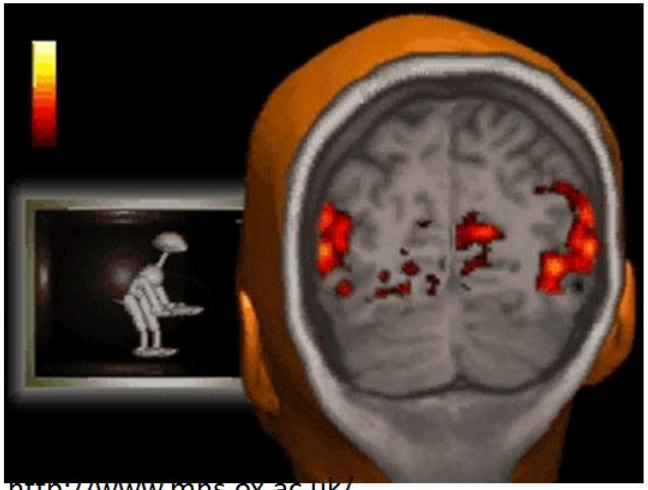
http://www.nps.gov/grsm/naturescience/fireflies.htm
http://www.reed.edu/biology/professors/srenn/pages/teaching/web_2008/mhlo_site/index.html

# The good heartbeat

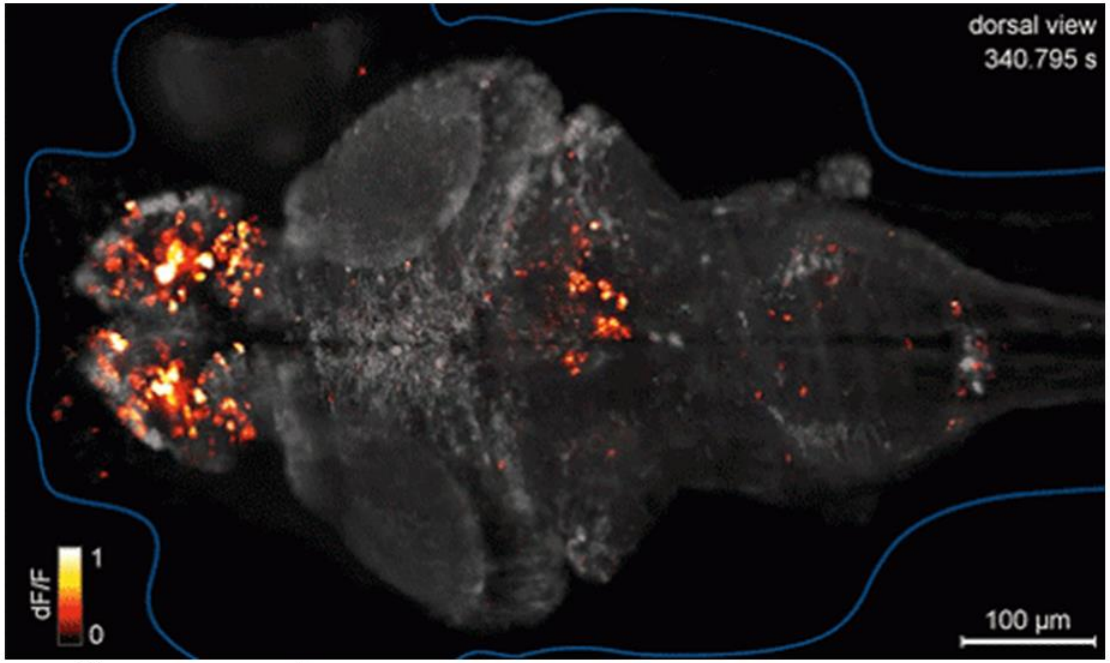




# The good brain



http://www.mhs.ox.ac.uk/

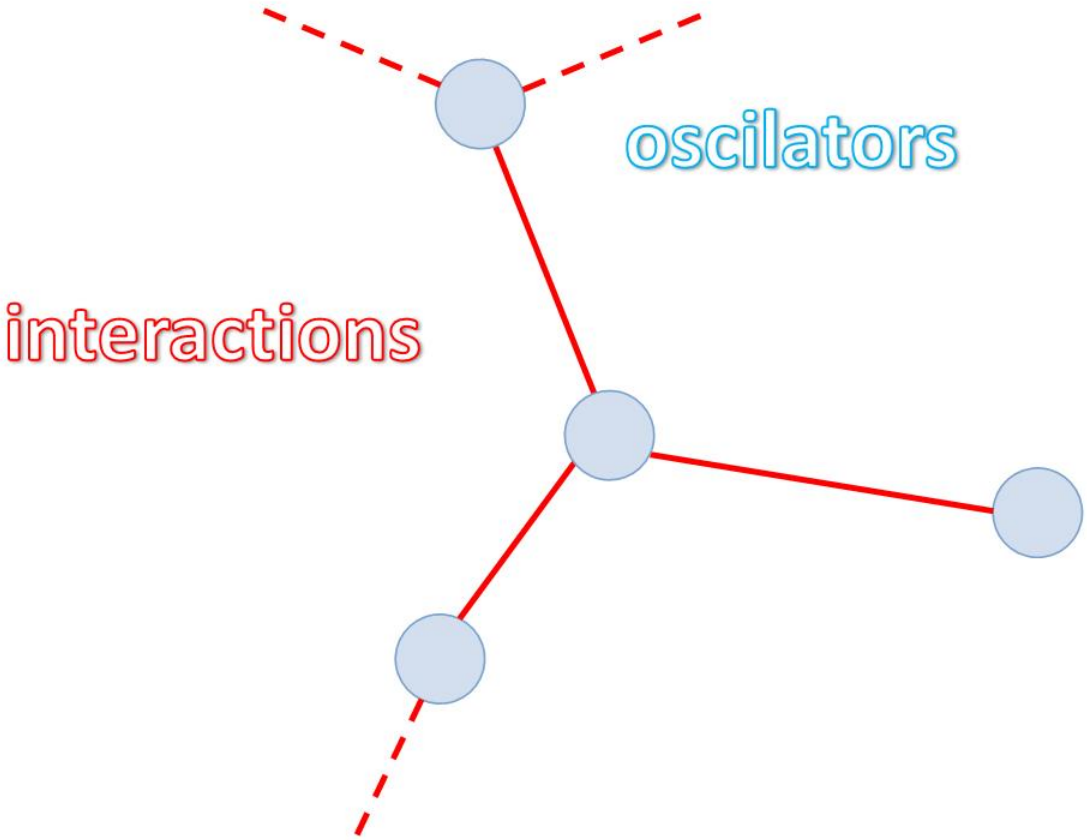



http://cdn.gifbay.com/

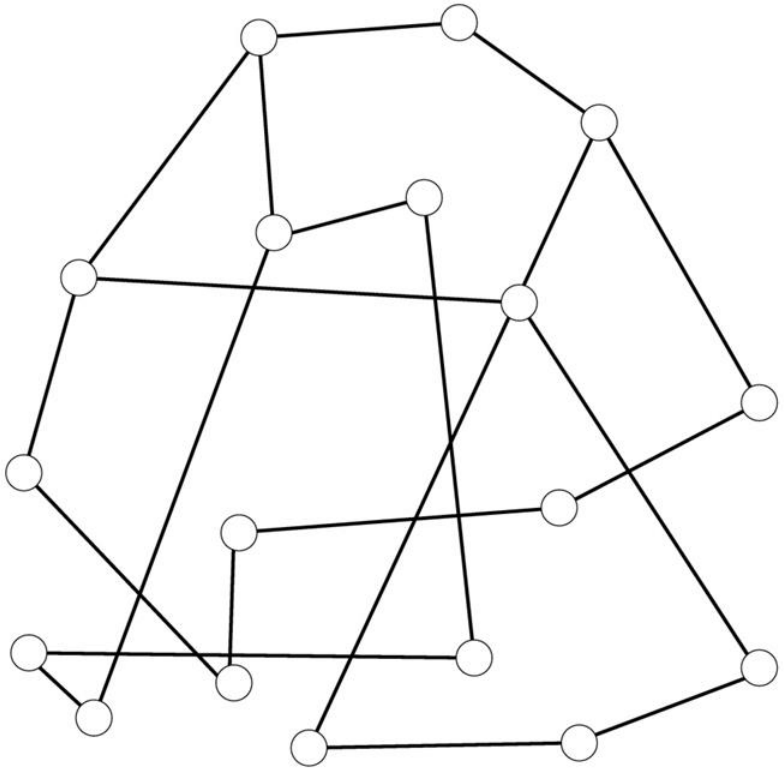
# The good clapping


http://www.guardian.co.uk/music/gallery/2011/jun/24/glastonbury-festival-2011-in-pictures

# The topology (network)
## nodes and links



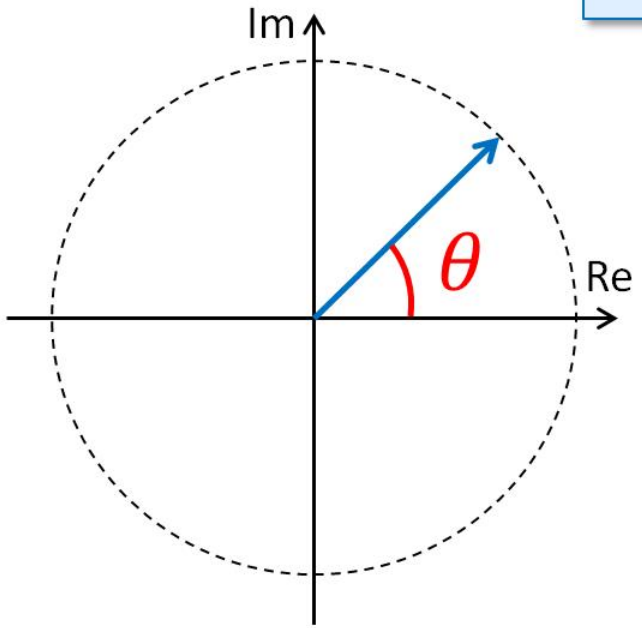
# The topology (network)
## random graph: Erdős-Rényi (ER)

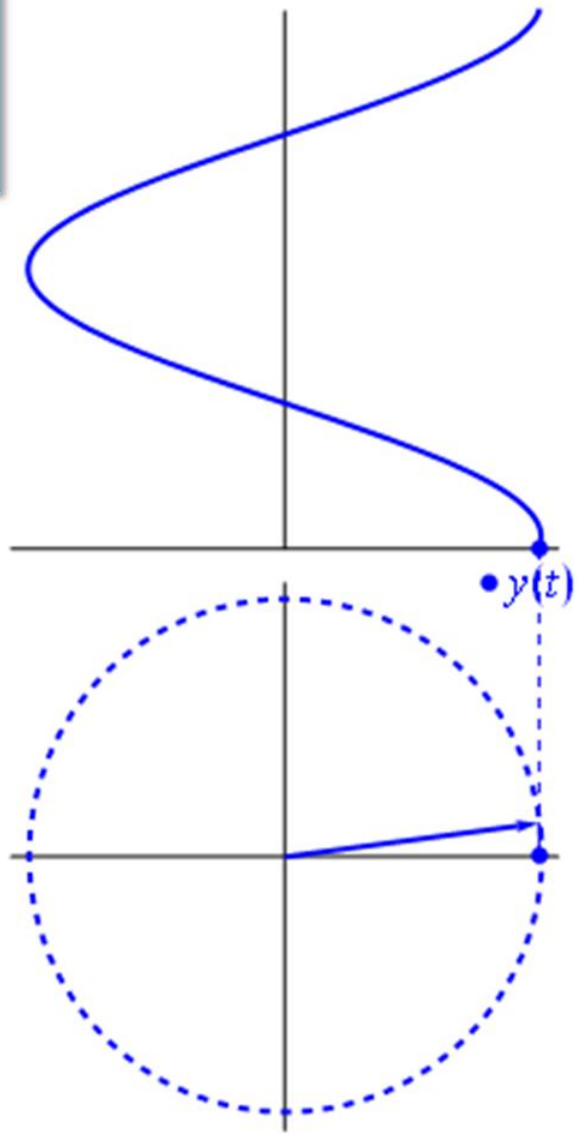


- Nodes are connected with probability p;
- Average degree <k>=(N-1)p.

# Kuramoto model
## phasors

$$Ae^{i\theta}$$




$$Ae^{i\theta} = A\{\cos\theta + i\sin\theta\}$$

$$\theta = \omega t + \theta_0$$

*Image source*: http://en.wikipeida.org/wiki/Phasor

# Kuramoto model

$$\dot{\theta}_i = \omega_i + \lambda \sum_{j \in N(i)} \sin(\theta_j - \theta_i)$$

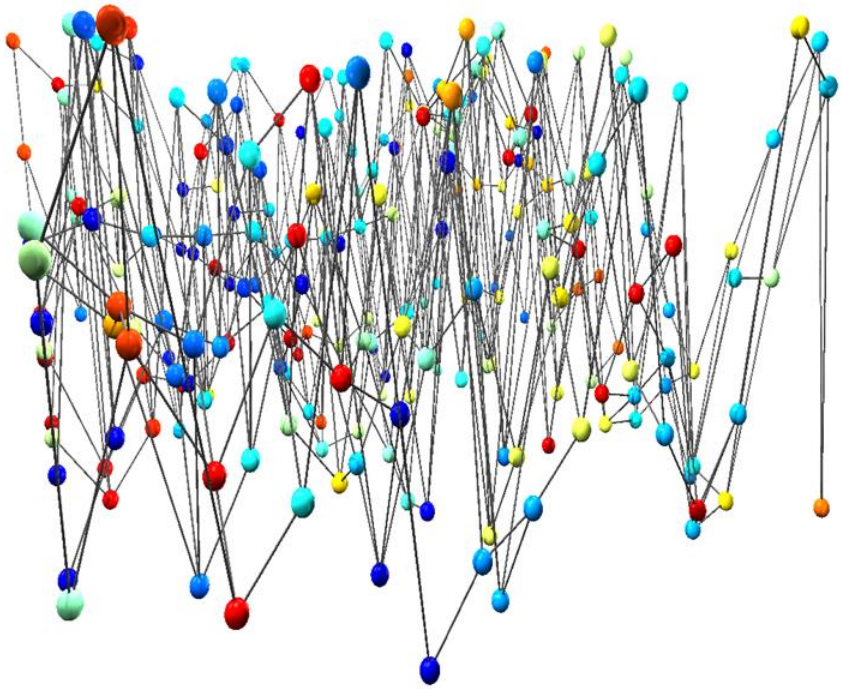
Natural frequency

Coupling strength

Neighbors

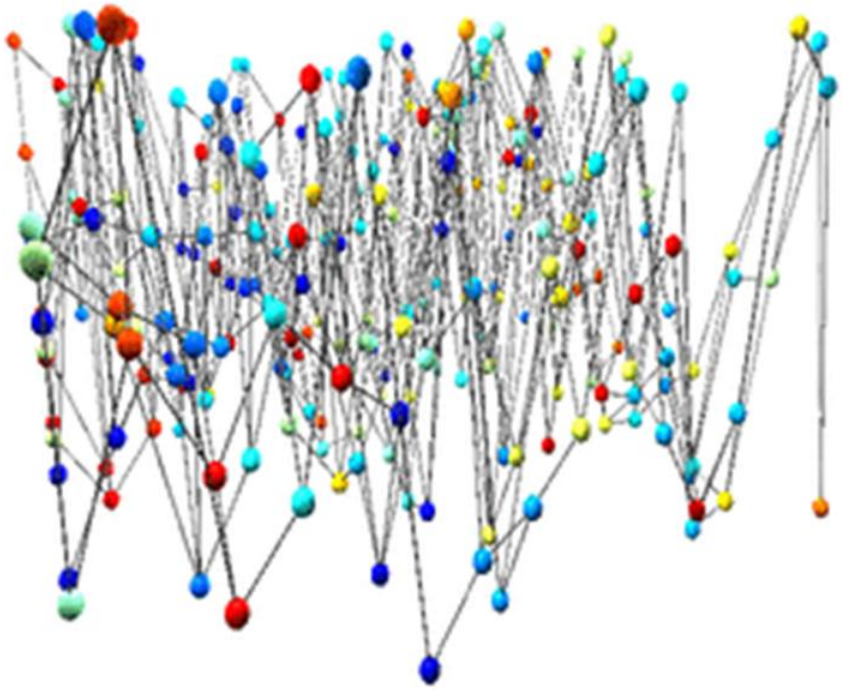
Pikovsky, Rosenblum, and Kurths. Cambridge University Press, Cambridge, (2003).

# Kuramoto model



t=0.00 r=0.01

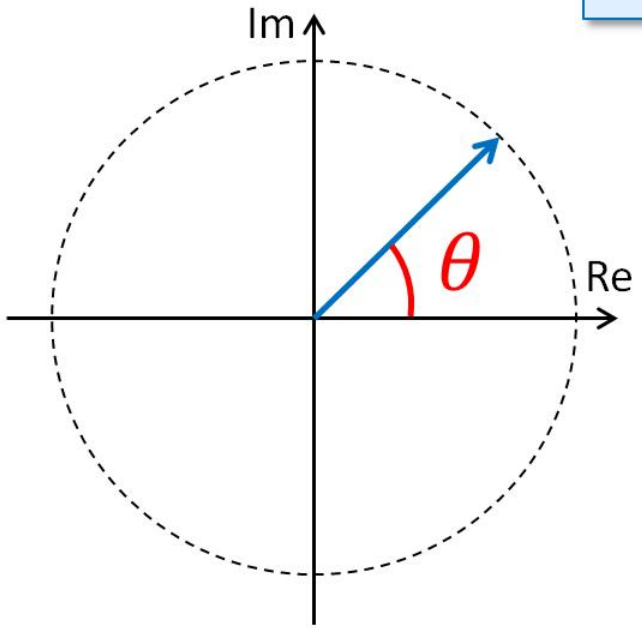
# Kuramoto model



t=0.00 r=0.01

# Kuramoto model
## phasors

$$Ae^{i\theta}$$



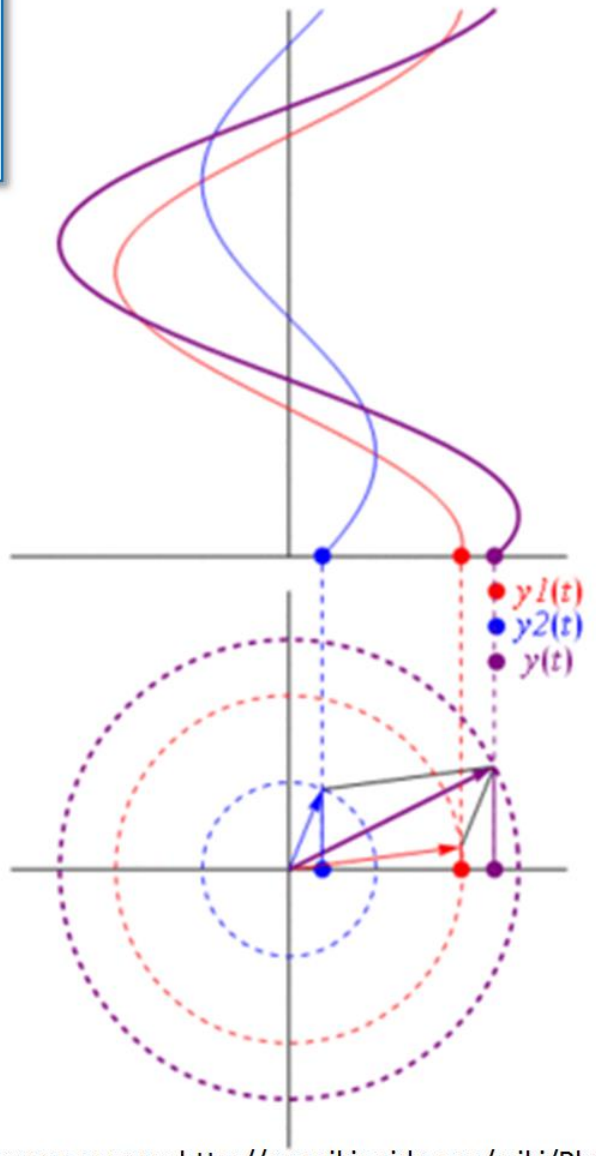




*Image source*: http://en.wikipeida.org/wiki/Phasor

$$Ae^{i\theta} = A\{\cos\theta + i\sin\theta\}$$

$$\theta = \omega t + \theta_0$$

# Kuramoto model
## order parameter

$$r\, e^{i\Psi} = \frac{1}{N}\sum_{j} e^{i\theta_j}$$

$r = 0$

Not Synchronized

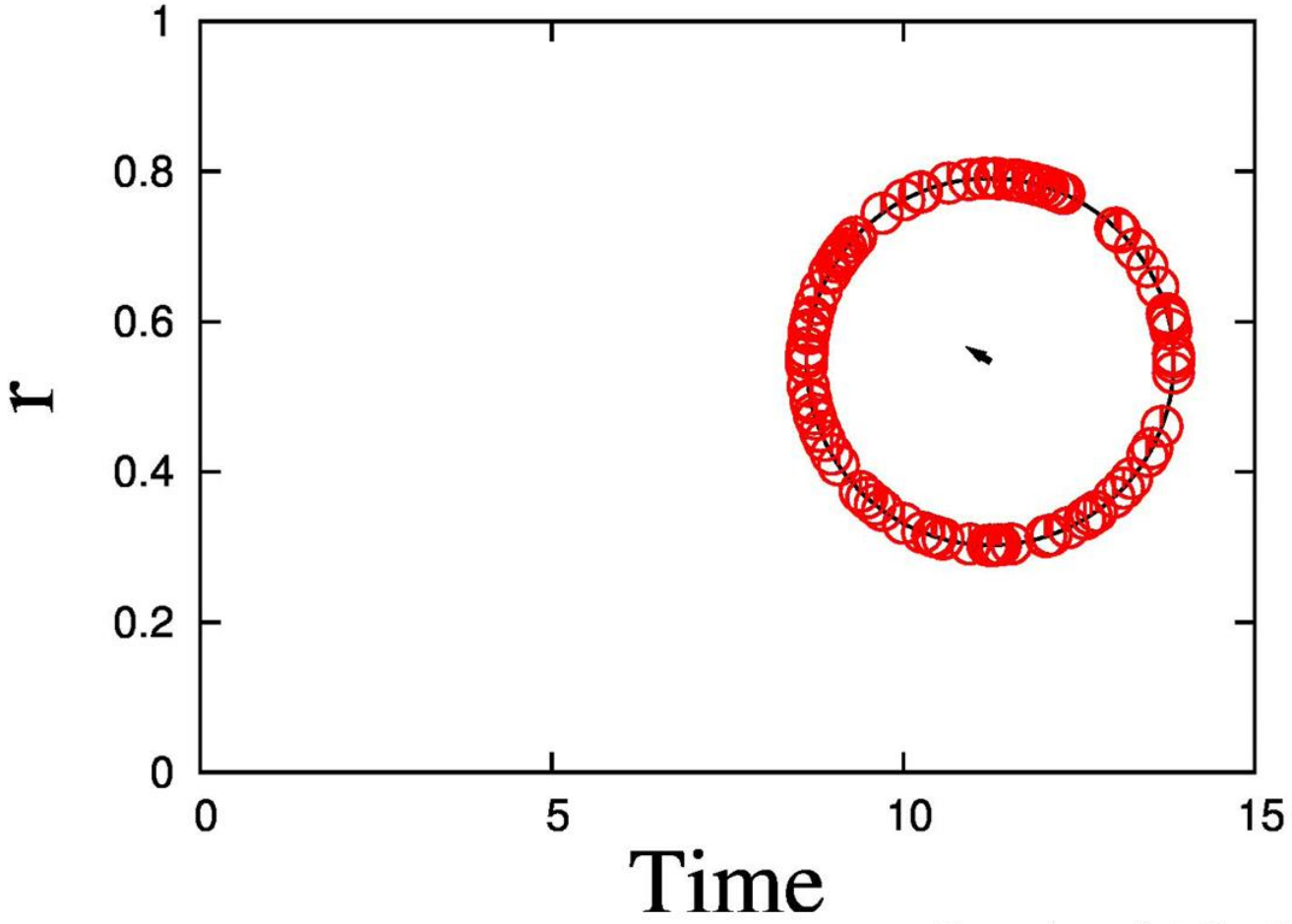
$r = 1$

Synchronized

# Kuramoto model
## no coupling

$$\dot{\theta}_i = \omega_i + \lambda \sum_{j \in N(i)} \sin(\theta_j - \theta_i)$$

$$r\ e^{i\Psi} = \frac{1}{N} \sum_j e^{i\theta_j}$$



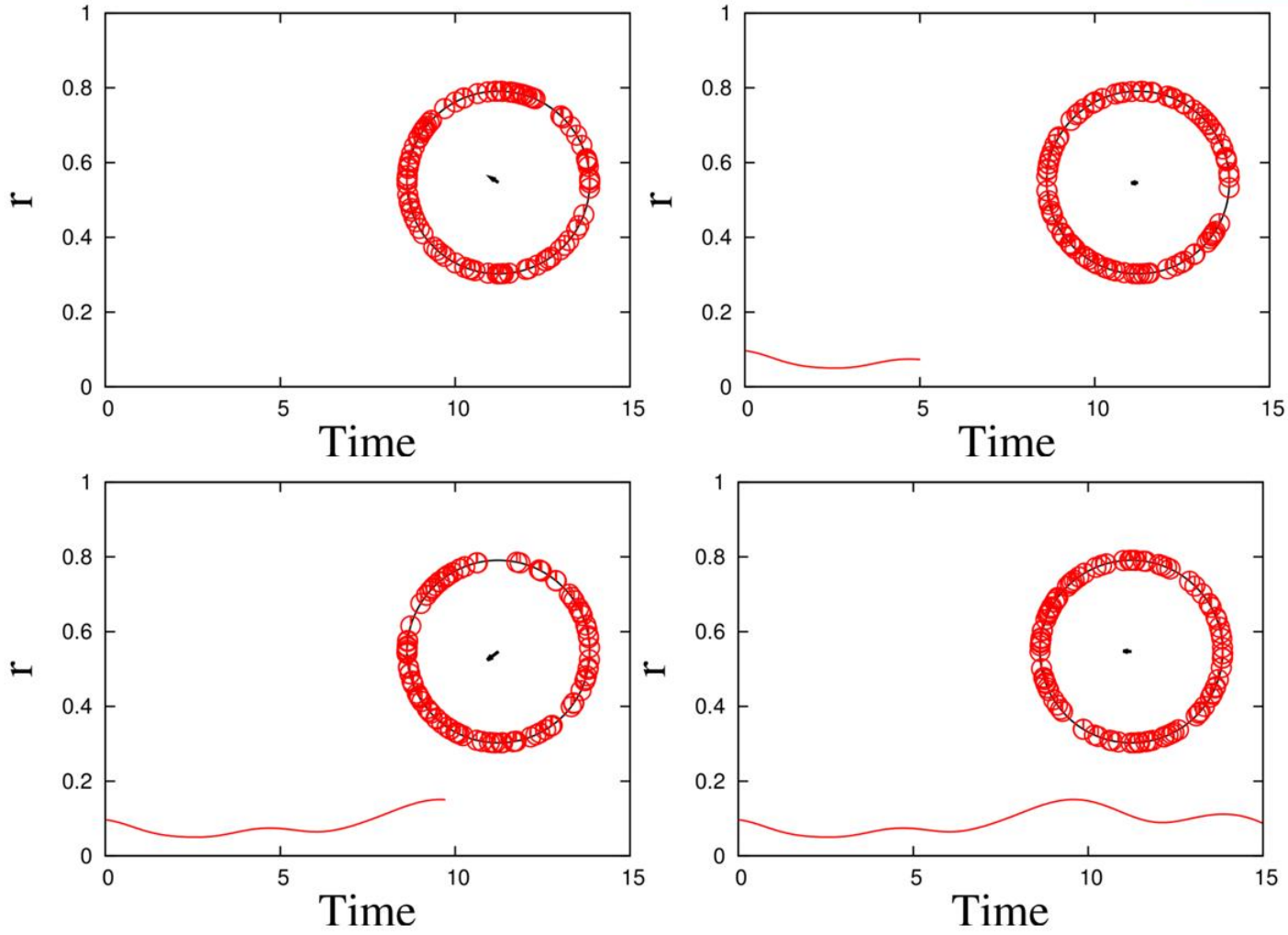

**100 Nodes**
**ER (k=4)**

Gaussian distribution of frequencies

# Kuramoto model
## no coupling

$$\dot{\theta}_i = \omega_i + \lambda \sum_{j \in N(i)} \sin(\theta_j - \theta_i)$$

$$r\ e^{i\Psi} = \frac{1}{N} \sum_j e^{i\theta_j}$$

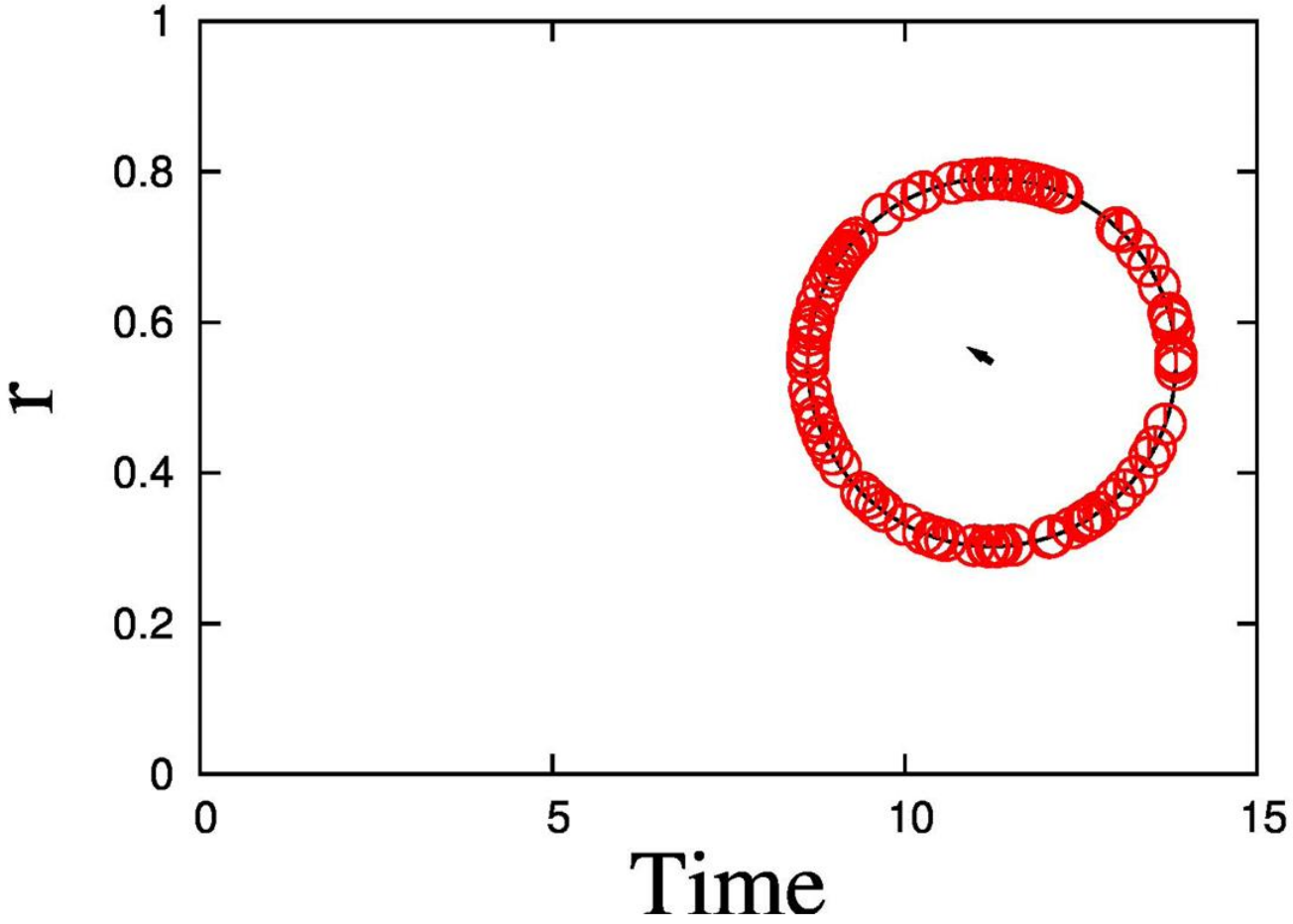



**100 Nodes**
**ER (k=4)**

# Kuramoto model
## strong coupling

$$\dot{\theta}_i = \omega_i + \lambda \sum_{j \in N(i)} \sin(\theta_j - \theta_i)$$

$$r\ e^{i\Psi} = \frac{1}{N} \sum_j e^{i\theta_j}$$



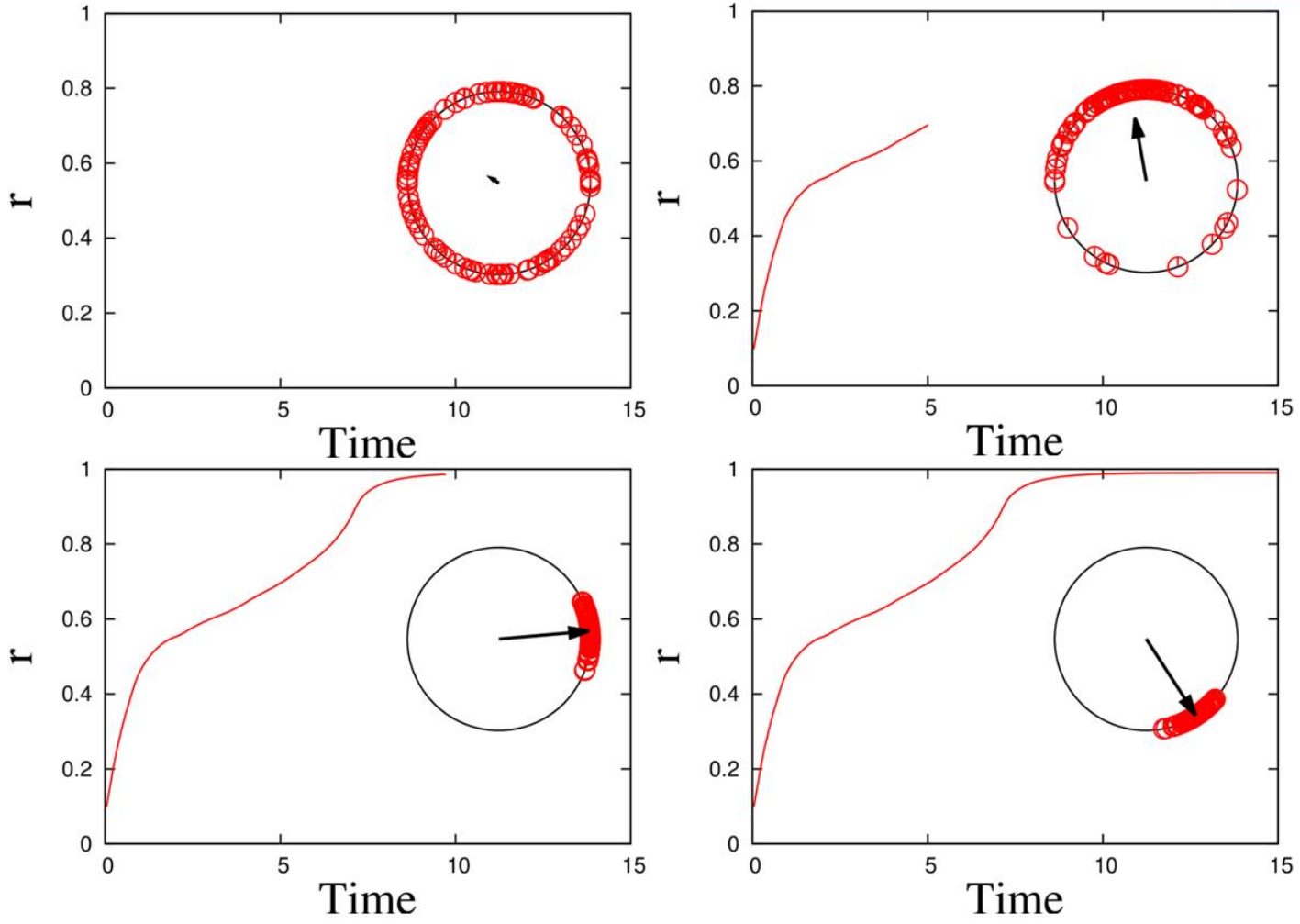

**100 Nodes**
**ER (k=4)**

Gaussian distribution of frequencies

# Kuramoto model
## strong coupling

$$\dot{\theta}_i = \omega_i + \lambda \sum_{j \in N(i)} \sin(\theta_j - \theta_i)$$

$$r\, e^{i\Psi} = \frac{1}{N} \sum_j e^{i\theta_j}$$

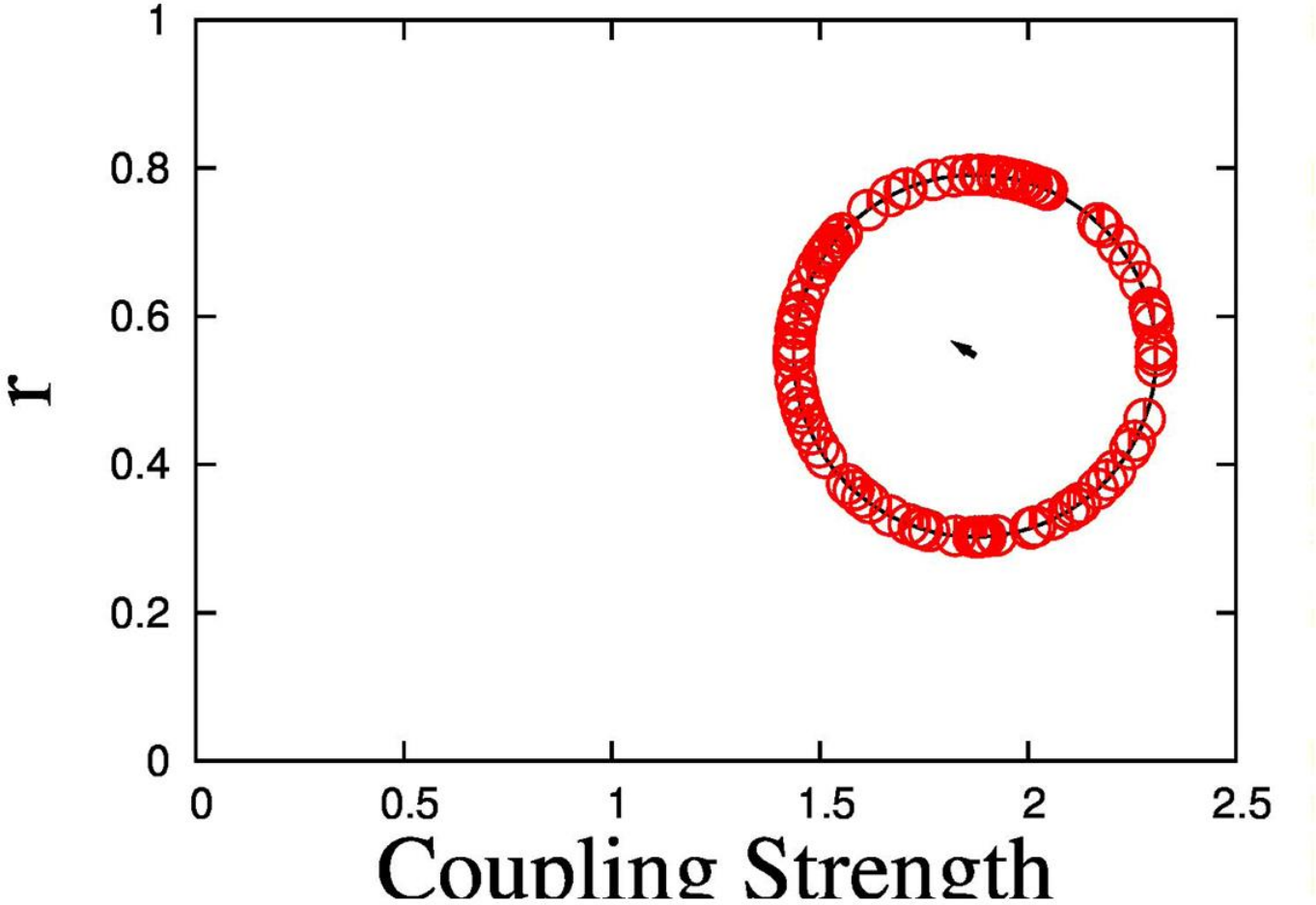



**100 Nodes**
**ER (k=4)**

# Kuramoto model
# increasing coupling

$$\dot{\theta}_i = \omega_i + \lambda \sum_{j \in N(i)} \sin(\theta_j - \theta_i)$$

$$r\ e^{i\Psi} = \frac{1}{N} \sum_j e^{i\theta_j}$$



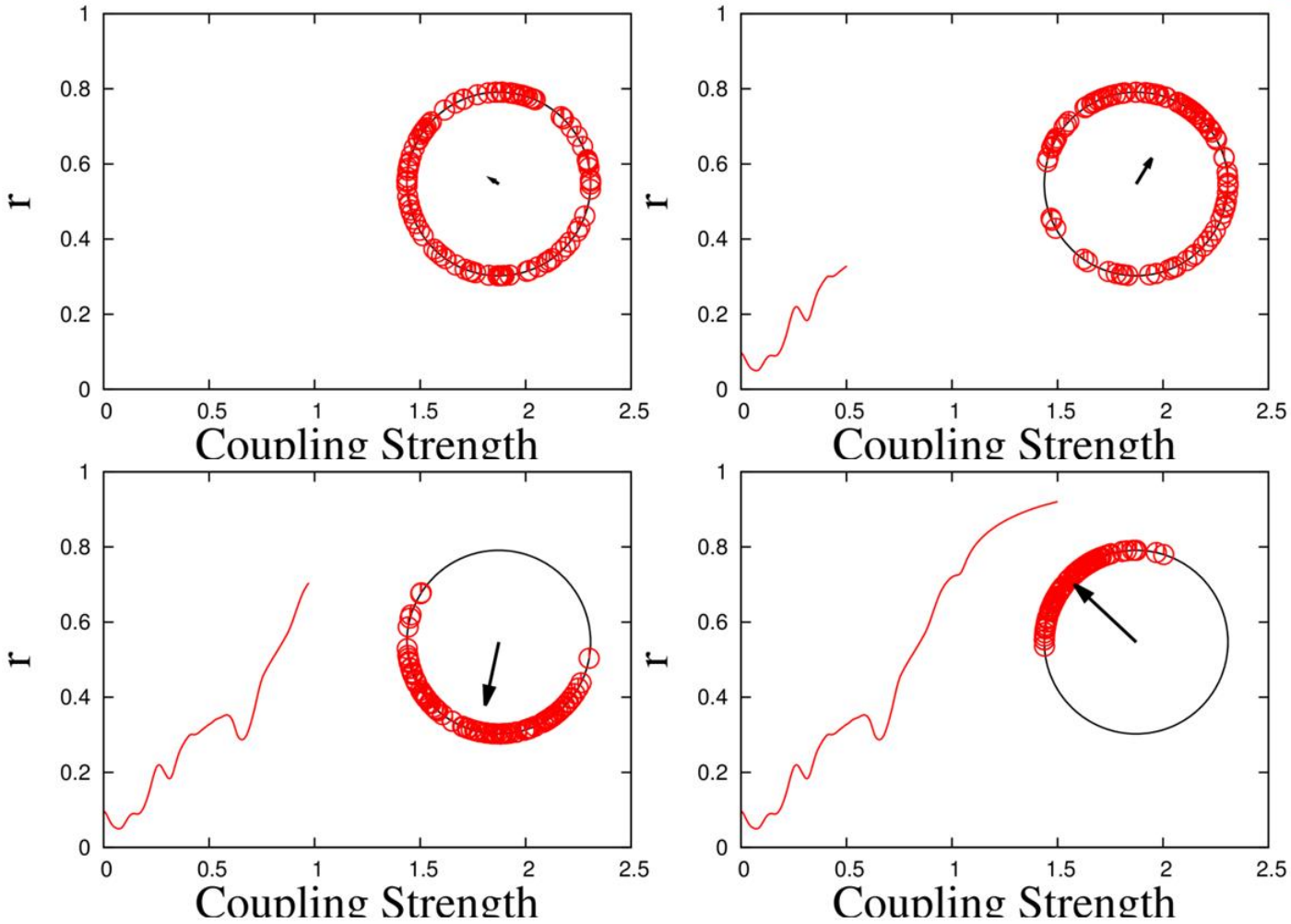

**100 Nodes**
**ER (k=4)**

Gaussian distribution of frequencies

# Kuramoto model
# increasing coupling

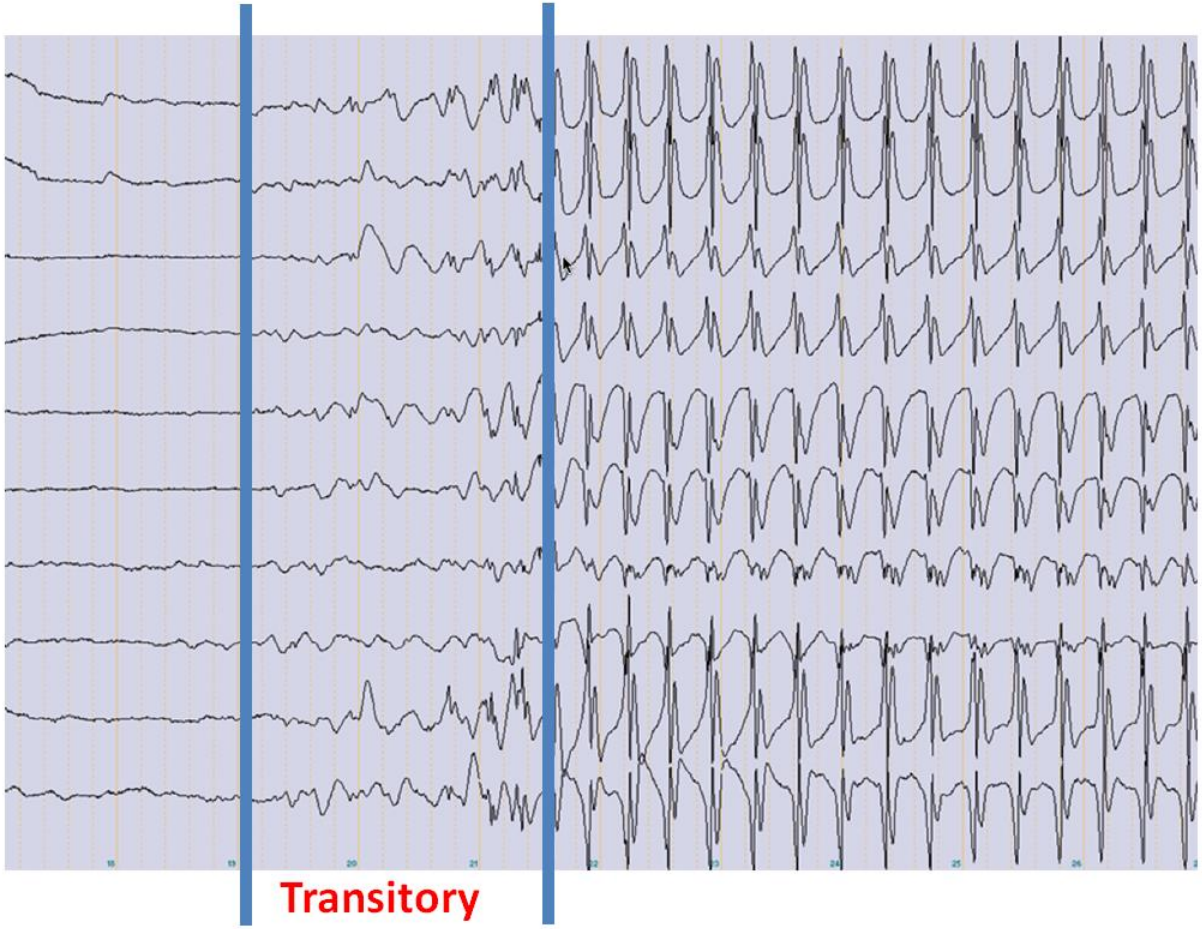
$$\dot{\theta}_i = \omega_i + \lambda \sum_{j \in N(i)} \sin(\theta_j - \theta_i)$$

$$r\ e^{i\Psi} = \frac{1}{N} \sum_j e^{i\theta_j}$$



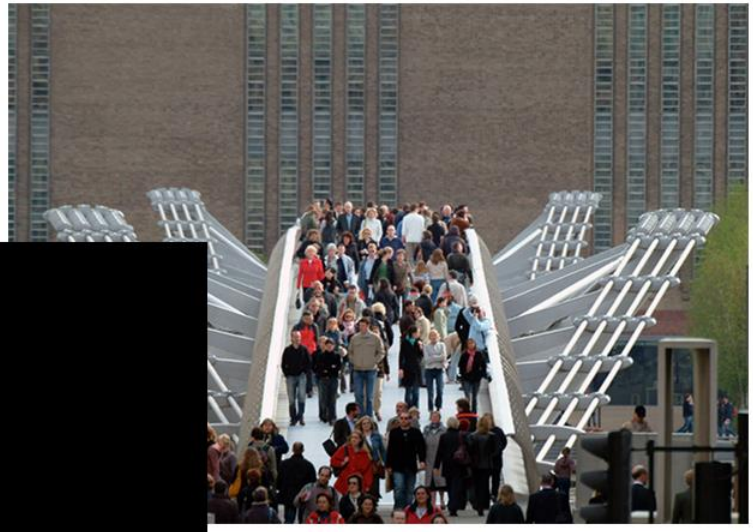

**100 Nodes**
**ER (k=4)**

# The bad epilepsy




http://www.uni-kiel.de/aktuell/pm/2009/2009-003-epilepsie-gen-e.shtml

# The bad
## Millenium bridge



# The bad Brazillian protests


17/06/2013 20h27 - Atualizado em 17/06/2013 21h23

## Manifestantes atravessam Terceira Ponte em protesto de Vitória

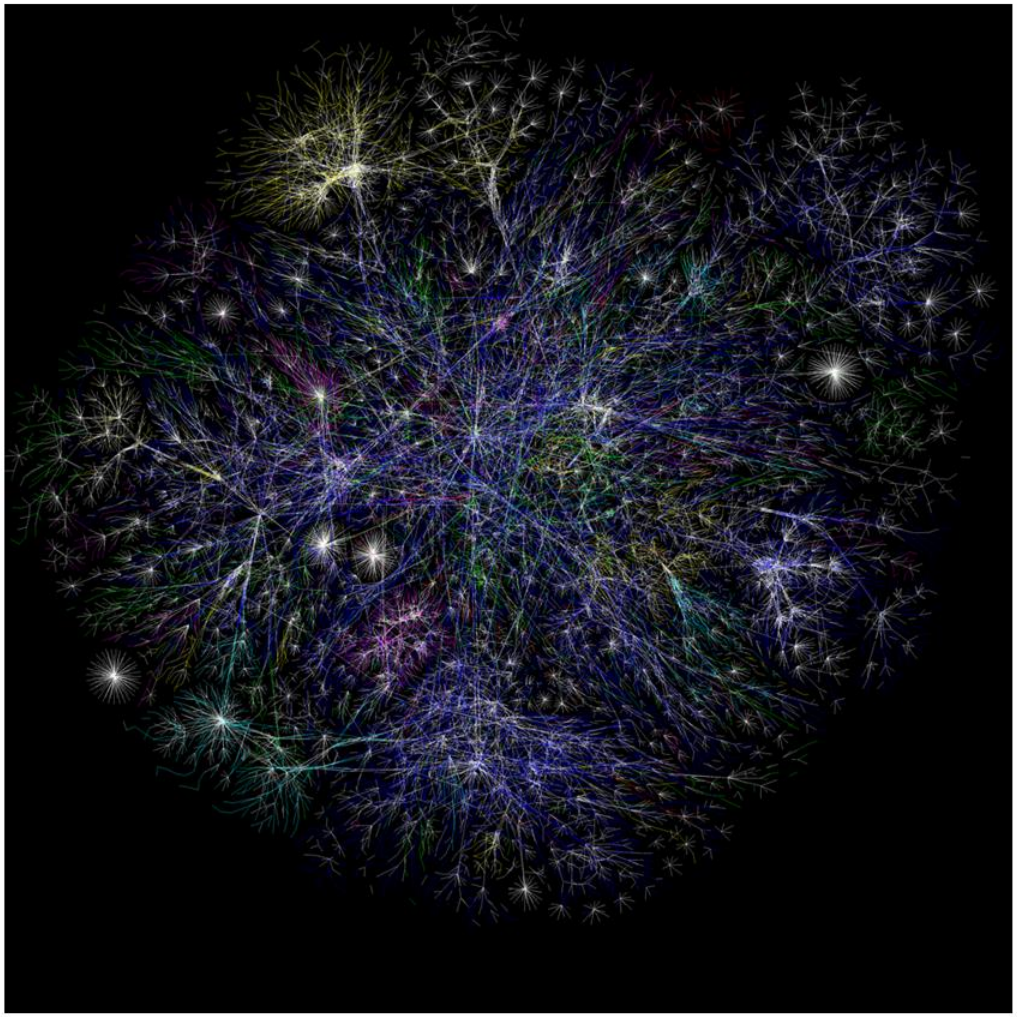
Organização do protesto previa uma passeata até a praça do pedágio. Vão central da via tem 70 metros de altura e é proibida passagem a pé.

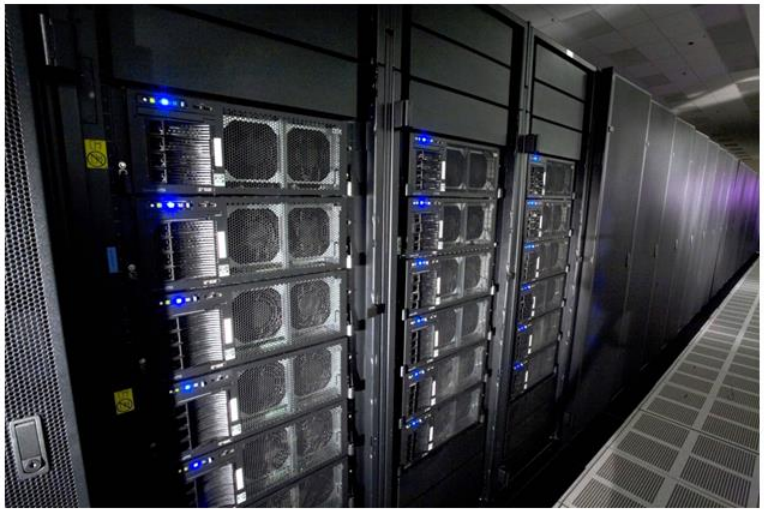
**Leandro Nossa e Juliana Borges**
Do G1 ES

# The bad
## political opponents

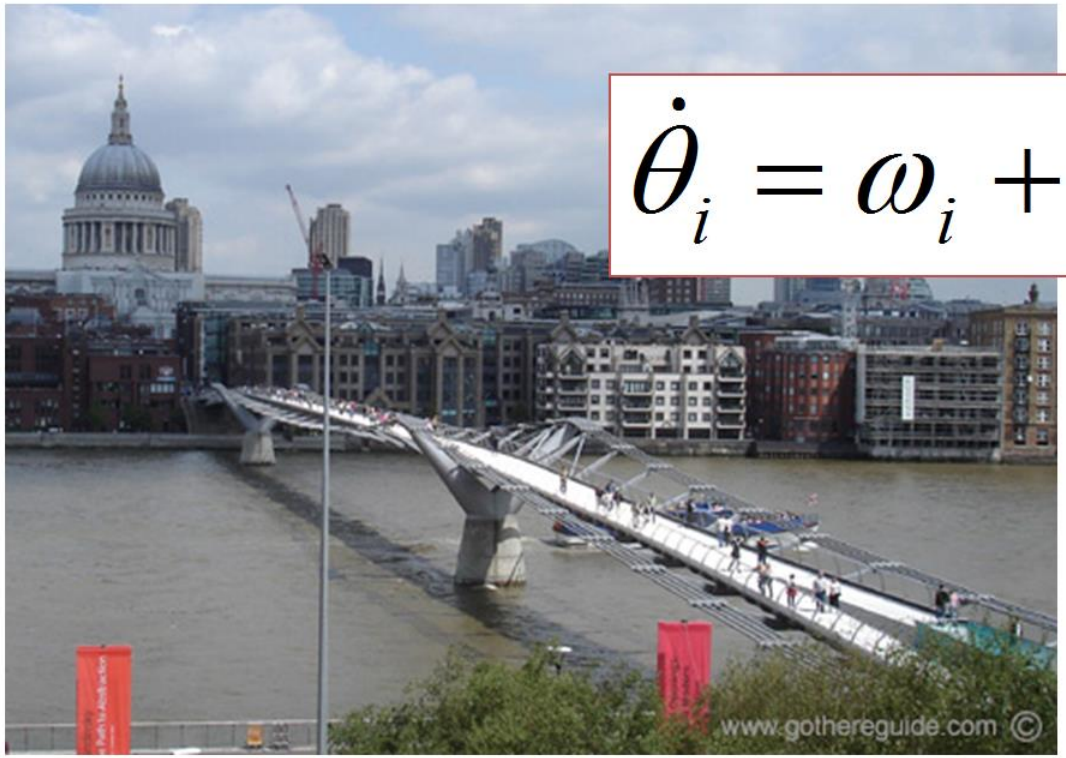
# The bad

## Internet routers: TCP global synchronization





# Contrarians
## global strategy




$$\dot{\theta}_i = \omega_i + \lambda \cos(\Psi - \theta_i)$$

**Average Phase of all oscillators**

V. H. Louzada, N. A. M. Araújo, J. S. Andrade Jr., H. J. Herrmann. *Scientific Reports* **2**, 658 (2012).

# Contrarians
## local strategy

$$\dot{\theta}_i = \omega_i + \lambda \sum_{j \in N(i)} \cos(\theta_j - \theta_i)$$

Sum over the neighbors

V. H. Louzada, N. A. M. Araújo, J. S. Andrade Jr., H. J. Herrmann. *Scientific Reports* **2**, 658 (2012).

# Contrarians
## two strategies

***Global*** *strategy*

$$\dot{\theta}_i = \omega_i + \lambda \cos(\Psi - \theta_i)$$

***Local*** *strategy*

$$\dot{\theta}_i = \omega_i + \lambda \sum_{j \in N(i)} \cos(\theta_j - \theta_i)$$

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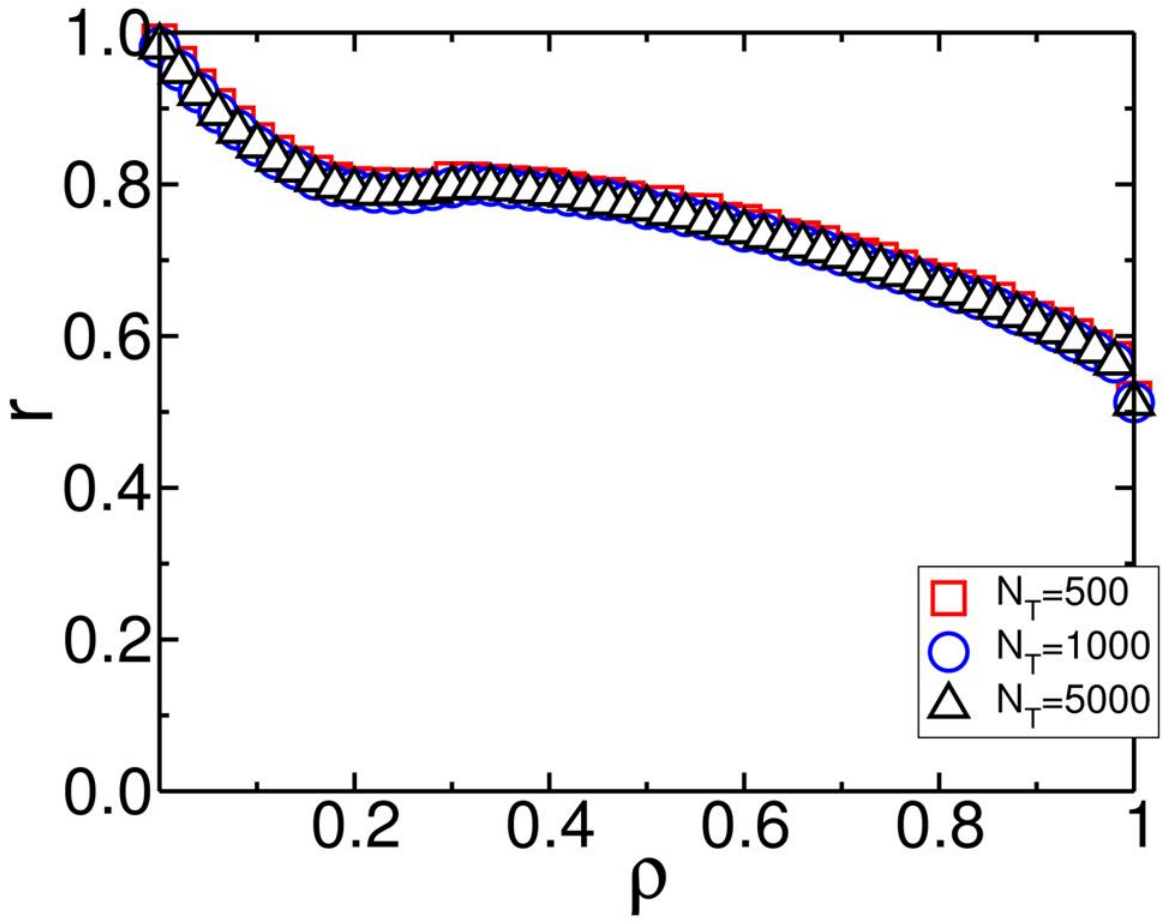
# Contrarians
## global strategy

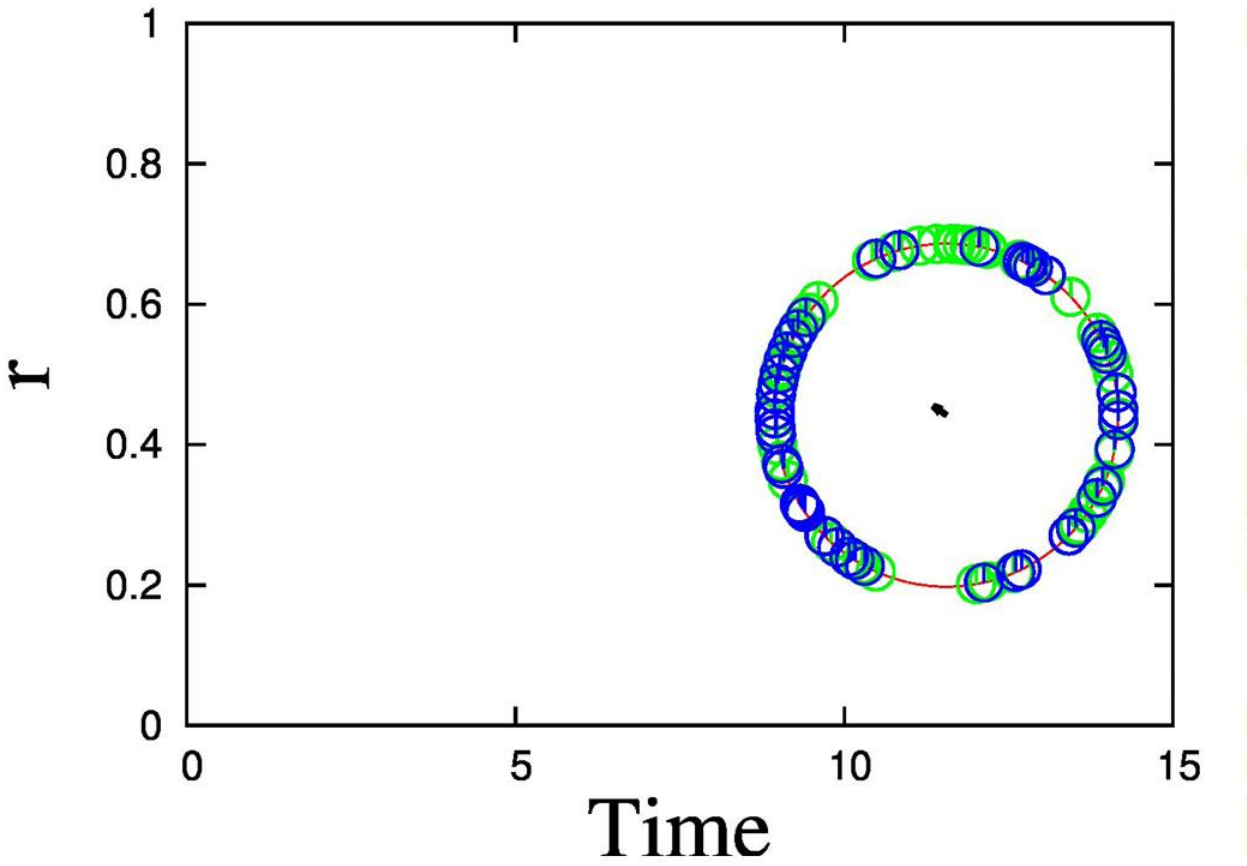
$$\dot{\theta}_i = \omega_i + \lambda \cos(\Psi - \theta_i)$$




V. H. Louzada, N. A. M. Araújo, J. S. Andrade Jr., H. J. Herrmann. *Scientific Reports* **2**, 658 (2012).

# Contrarians
## global strategy

$$\dot{\theta}_i = \omega_i + \lambda \cos(\Psi - \theta_i)$$

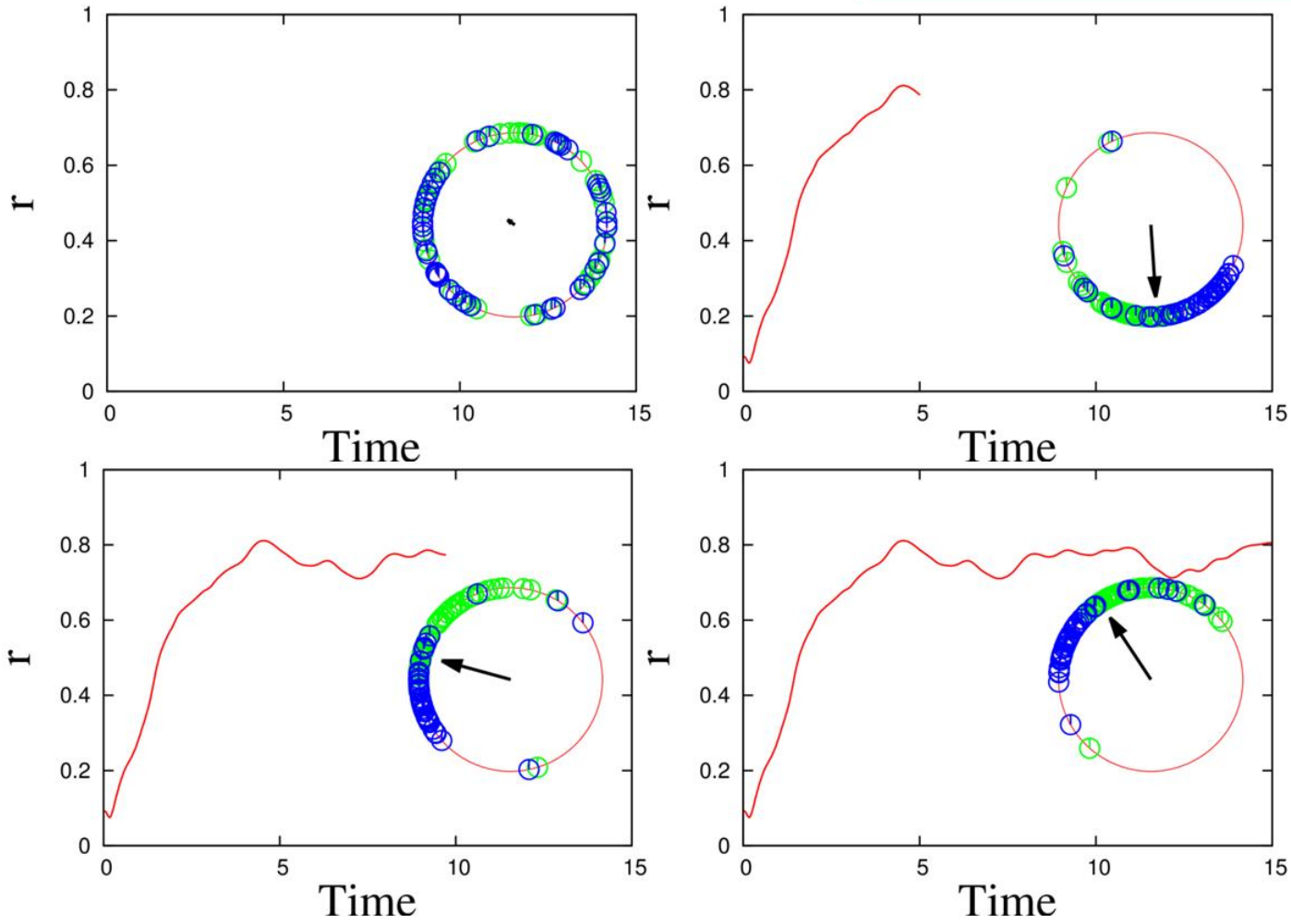



**100 Nodes**
**ER (k=4)**
**50% contrarians**

V. H. Louzada, N. A. M. Araújo, J. S. Andrade Jr., H. J. Herrmann. *Scientific Reports* **2**, 658 (2012).

# Contrarians
## global strategy

$$\dot{\theta}_i = \omega_i + \lambda \cos(\Psi - \theta_i)$$



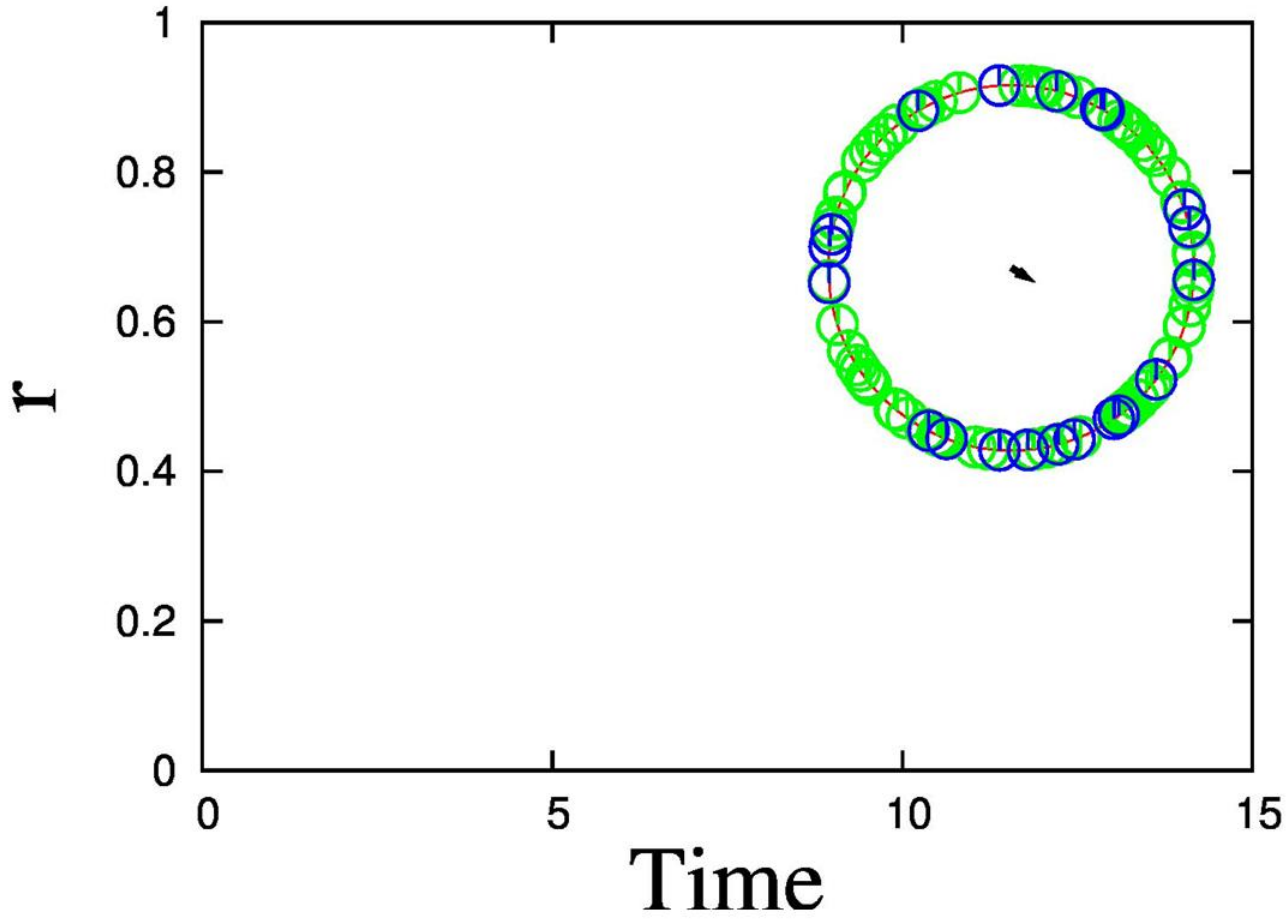

**100 Nodes**
**ER (k=4)**
**50% contrarians**

V. H. Louzada, N. A. M. Araújo, J. S. Andrade Jr., H. J. Herrmann. *Scientific Reports* **2**, 658 (2012).

# Contrarians
# local strategy

$$\dot{\theta}_i = \omega_i + \lambda \sum_{j \in N(i)} \cos(\theta_j - \theta_i)$$

$$r\ e^{i\Psi} = \frac{1}{N} \sum_j e^{i\theta_j}$$



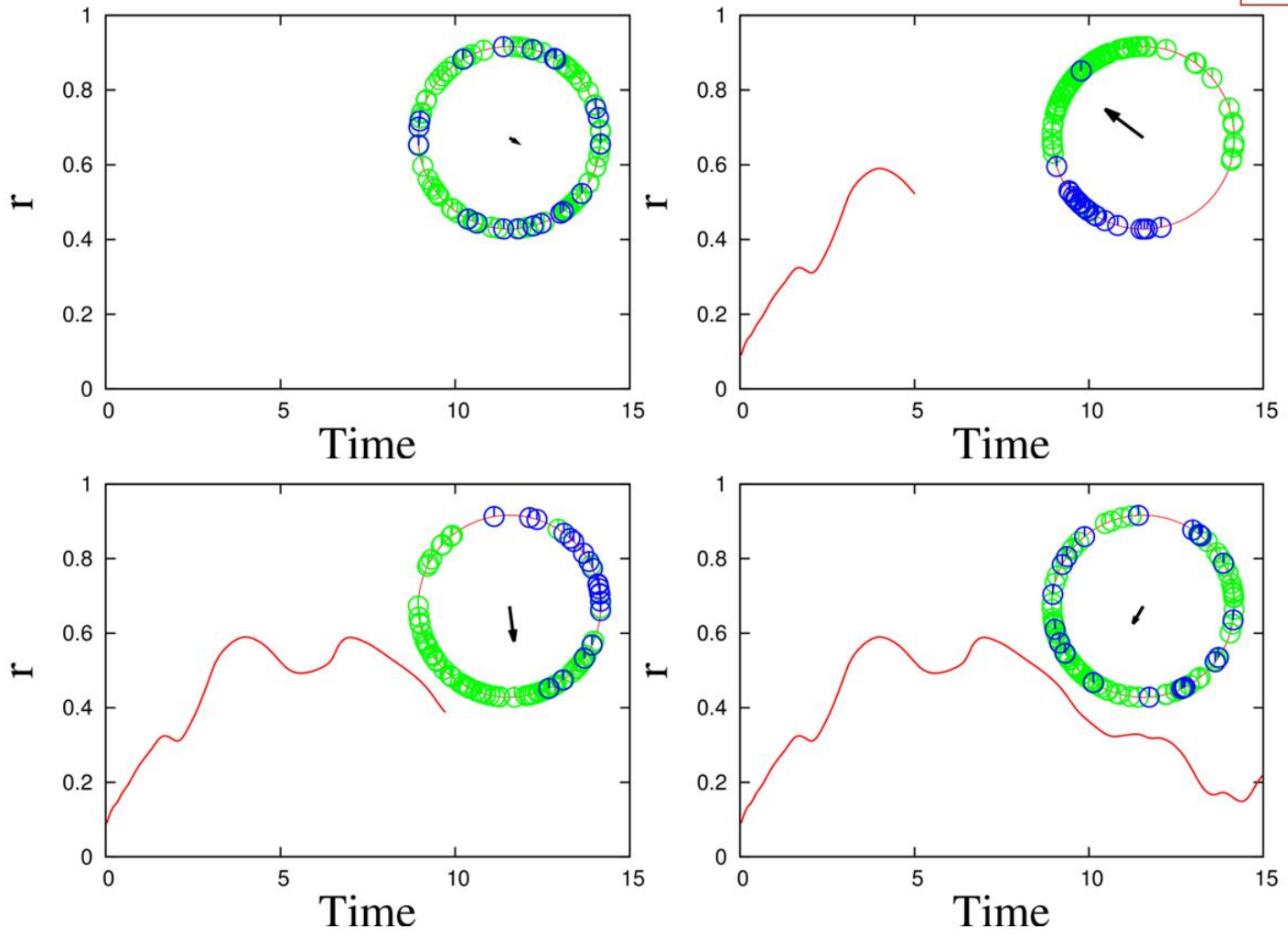

**100 Nodes**
**ER (k=4)**
**20% contrarians**

V. H. Louzada, N. A. M. Araújo, J. S. Andrade Jr., H. J. Herrmann. *Scientific Reports* **2**, 658 (2012).

# Contrarians
## local strategy

$$\dot{\theta}_i = \omega_i + \lambda \sum_{j \in N(i)} \cos(\theta_j - \theta_i)$$

$$r\ e^{i\Psi} = \frac{1}{N} \sum_j e^{i\theta_j}$$



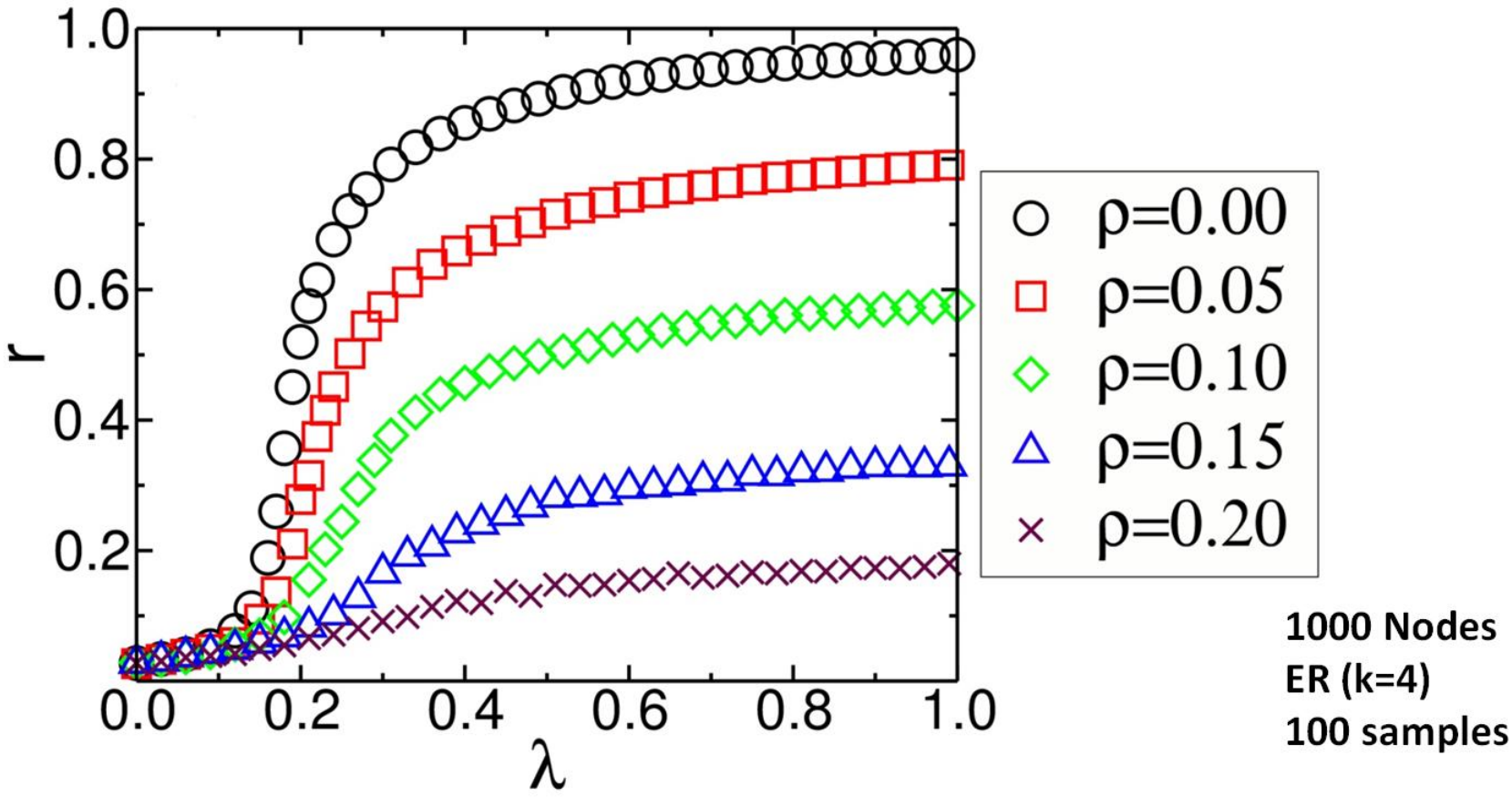

**100 Nodes**
**ER (k=4)**
**20% contrarians**

V. H. Louzada, N. A. M. Araújo, J. S. Andrade Jr., H. J. Herrmann. *Scientific Reports* **2**, 658 (2012).

# Contrarians
# local strategy

$$\dot{\theta}_i = \omega_i + \lambda \sum_{j \in N(i)} \cos(\theta_j - \theta_i)$$

$$r\, e^{i\Psi} = \frac{1}{N} \sum_j e^{i\theta_j}$$




**1000 Nodes**
**ER (k=4)**
**100 samples**

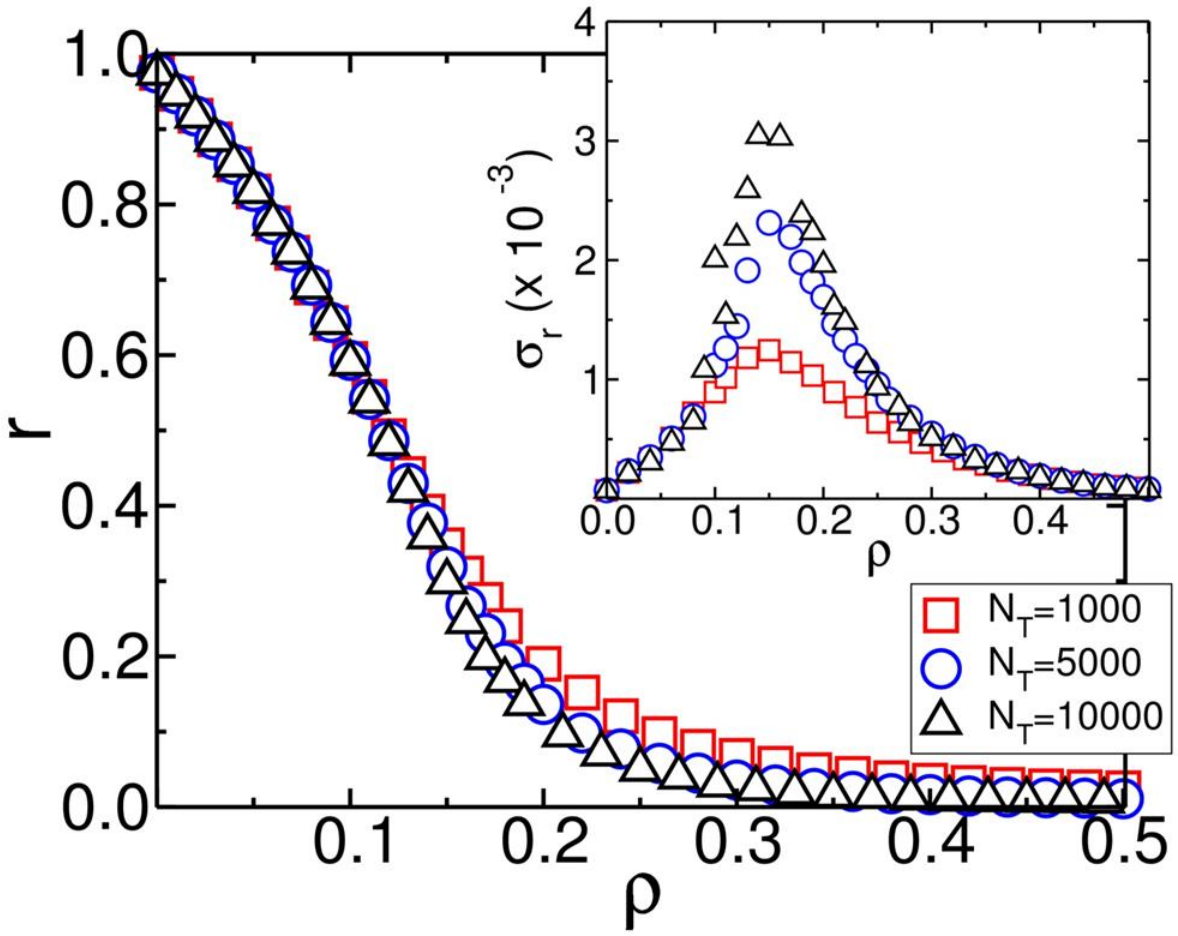
V. H. Louzada, N. A. M. Araújo, J. S. Andrade Jr., H. J. Herrmann. *Scientific Reports* **2**, 658 (2012).

## Contrarians

## local strategy: density of contrarians

$$\dot{\theta}_i = \omega_i + \lambda \sum_{j \in N(i)} \cos(\theta_j - \theta_i)$$

$$r\, e^{i\Psi} = \frac{1}{N} \sum_j e^{i\theta_j}$$




**ER (k=4)**
**>500 samples**

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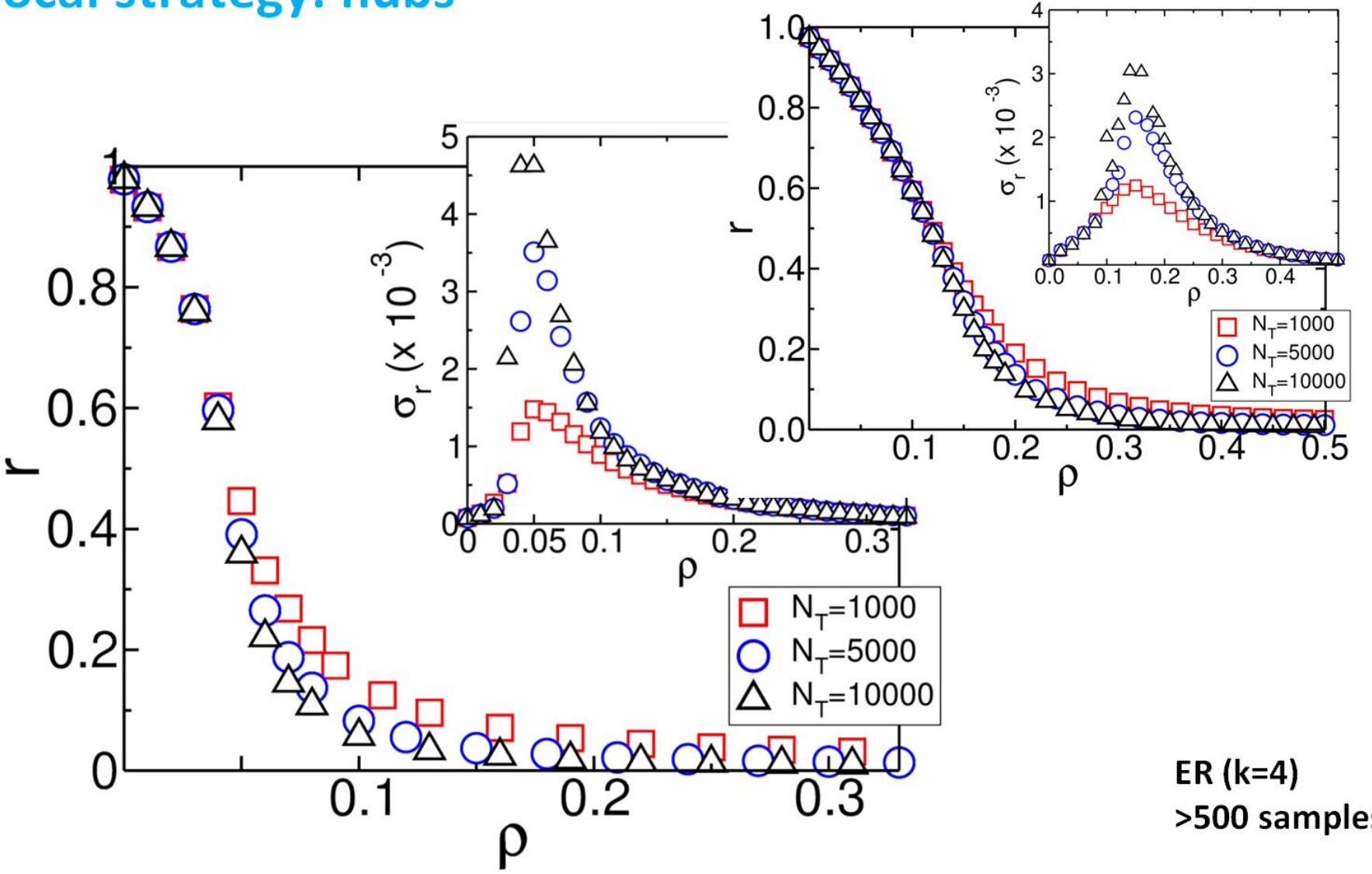
# Contrarians
## local strategy: hubs

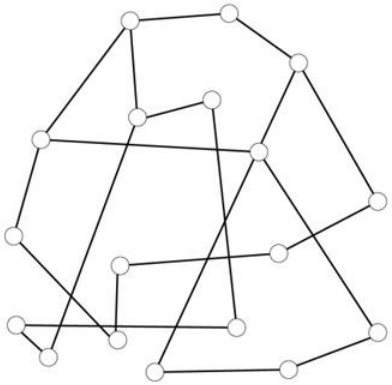



**ER (k=4)**
**>500 samples**

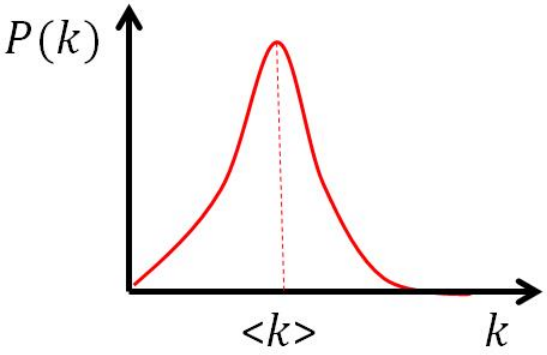
V. H. Louzada, N. A. M. Araújo, J. S. Andrade Jr., H. J. Herrmann. *Scientific Reports* **2**, 658 (2012).

# Contrarians networks

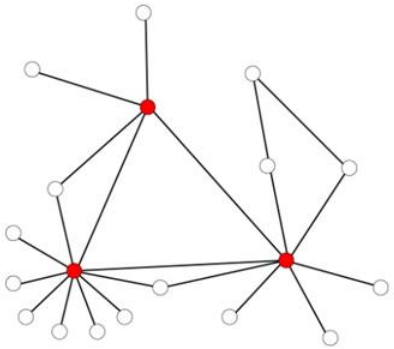
***ER networks***



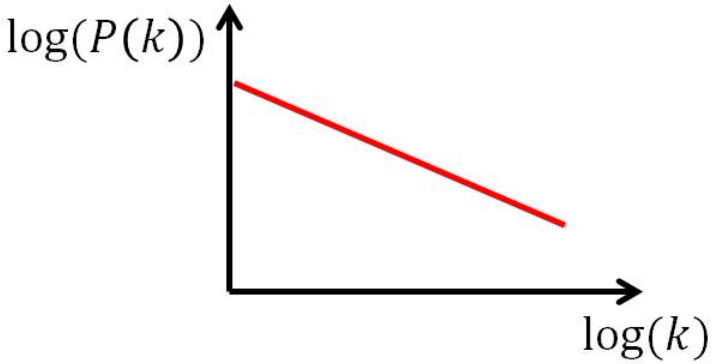
$$P(k) = \exp(-\lambda)\frac{\lambda^k}{k!}$$



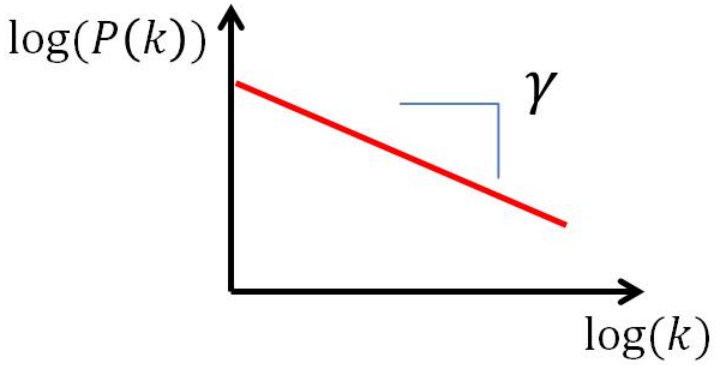

***BA network***

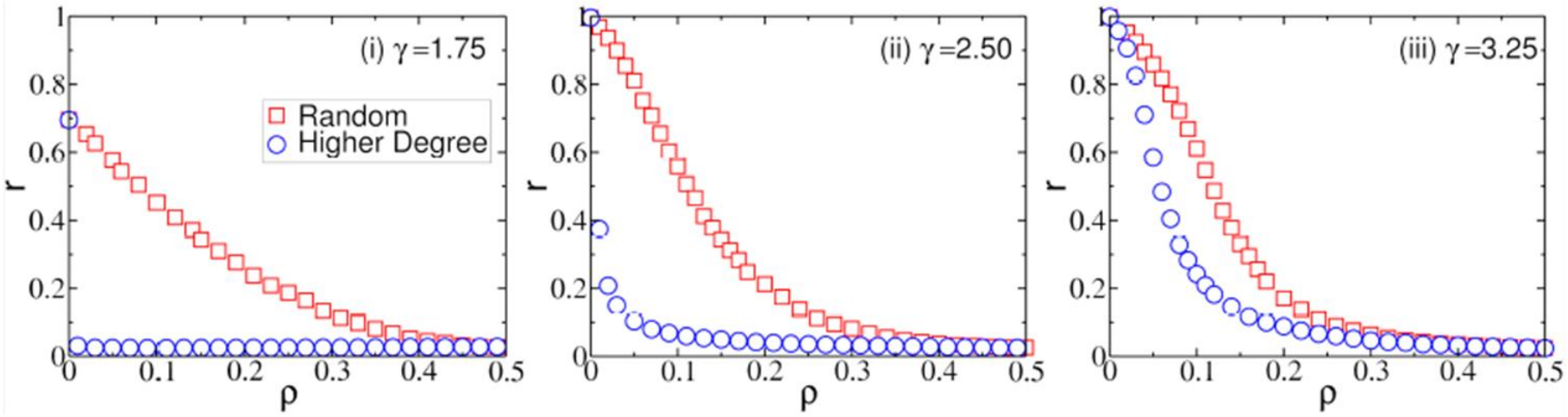


$$P(k) \sim k^{-\gamma}$$



# Contrarians
## scale-free networks

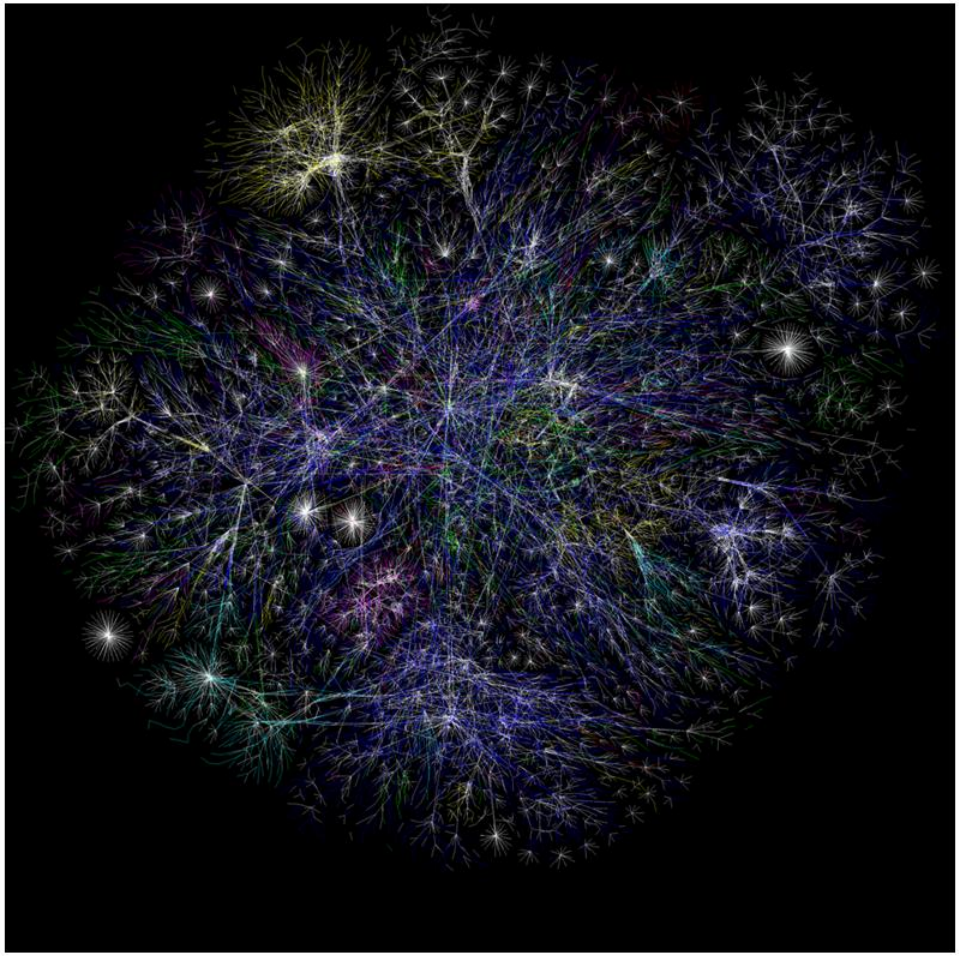



V. H. Louzada, N. A. M. Araújo, J. S. Andrade Jr., H. J. Herrmann. *Scientific Reports* **2**, 658 (2012).

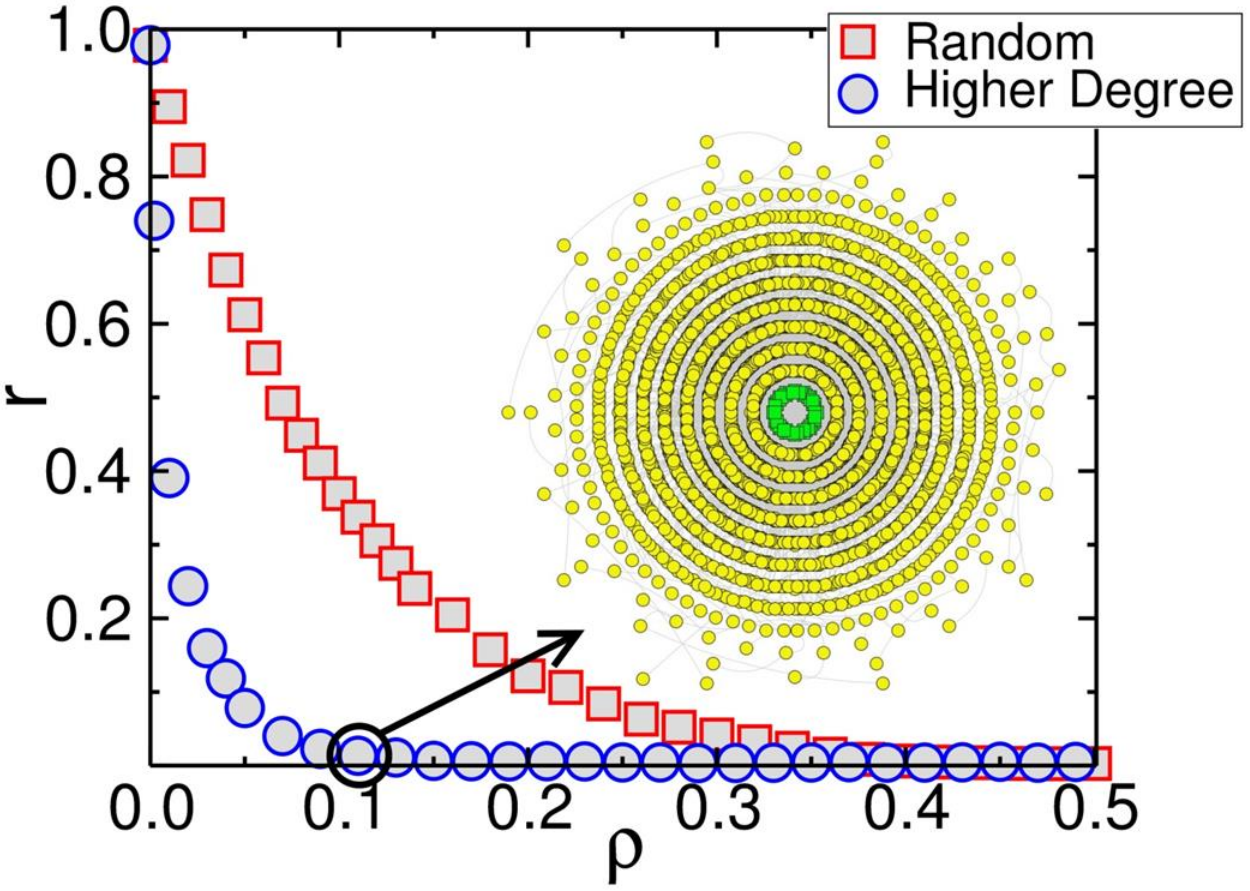
# Contrarians
## local strategy: router

*Internet: Autonomous Systems*
40028 nodes



The Opte project, *http://opte.prg*

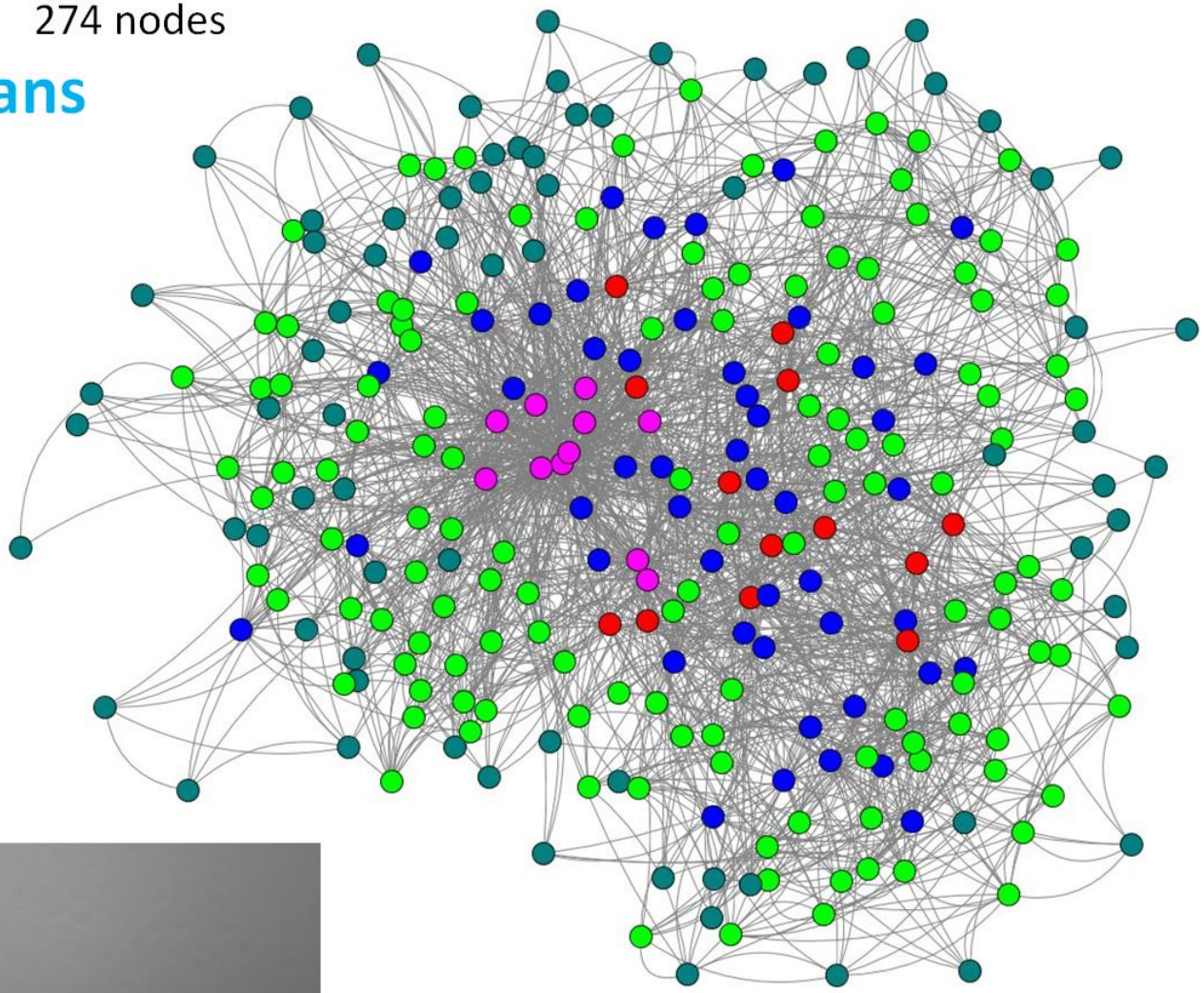
# Contrarians
## local strategy: router




V. H. Louzada, N. A. M. Araújo, J. S. Andrade Jr., H. J. Herrmann. *Scientific Reports* **2**, 658 (2012).

# Contrarians
# local strategy:
# neurons of C. Elegans

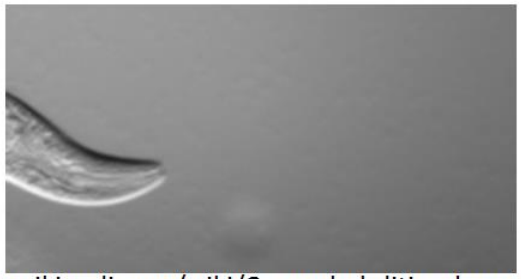
*C. Elegans*
274 nodes



**Degree**

- < 10
- 10 - 20
- 20 - 30
- 30 - 40
- > 40

1 mm

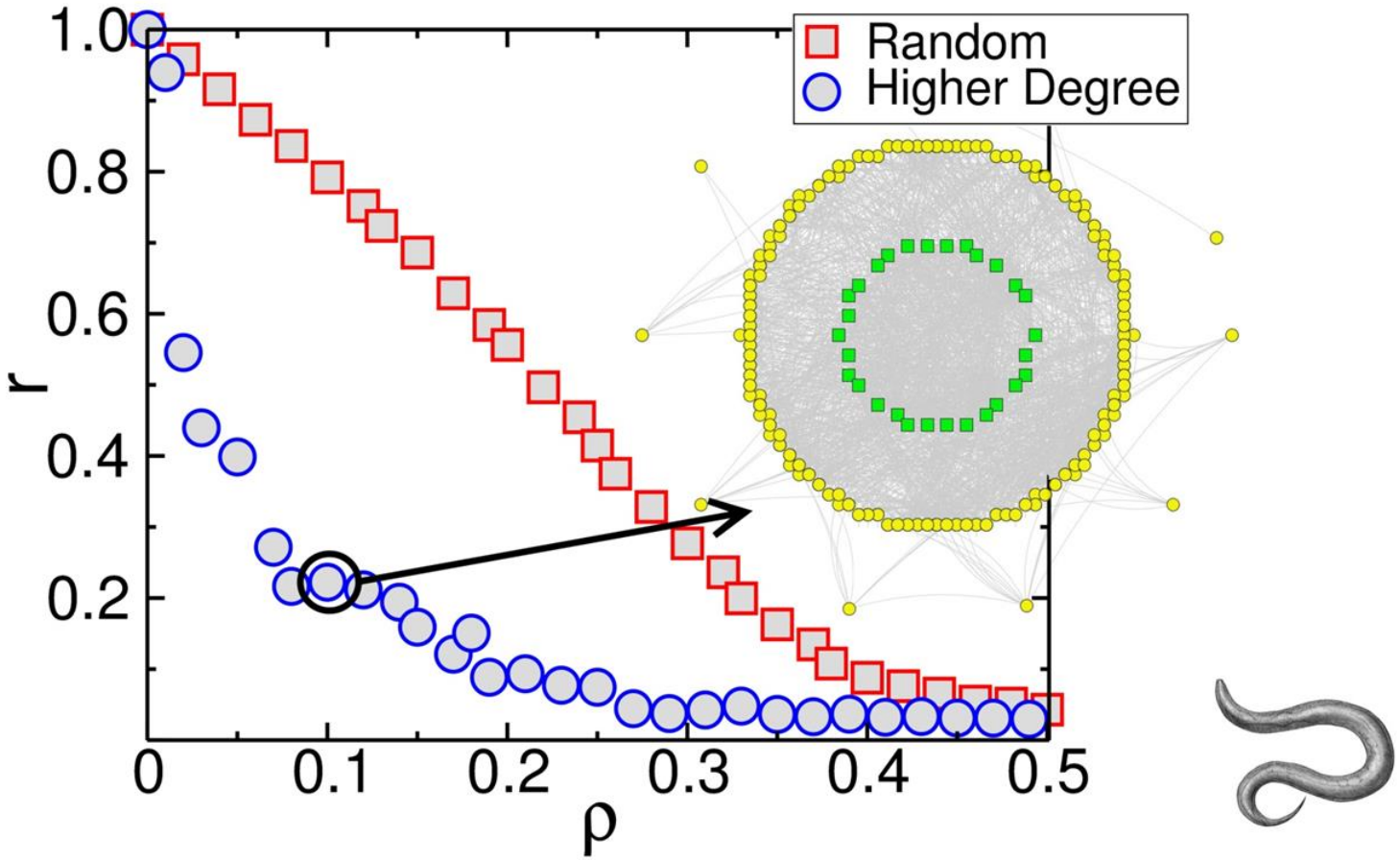


http://en.wikipedia.org/wiki/Caenorhabditis_elegans

*PNAS* **103**, 4723 (2006) *and* http://wormweb.org

# Contrarians

## local strategy: neurons of C. Elegans




V. H. Louzada, N. A. M. Araújo, J. S. Andrade Jr., H. J. Herrmann. *Scientific Reports* **2**, 658 (2012).

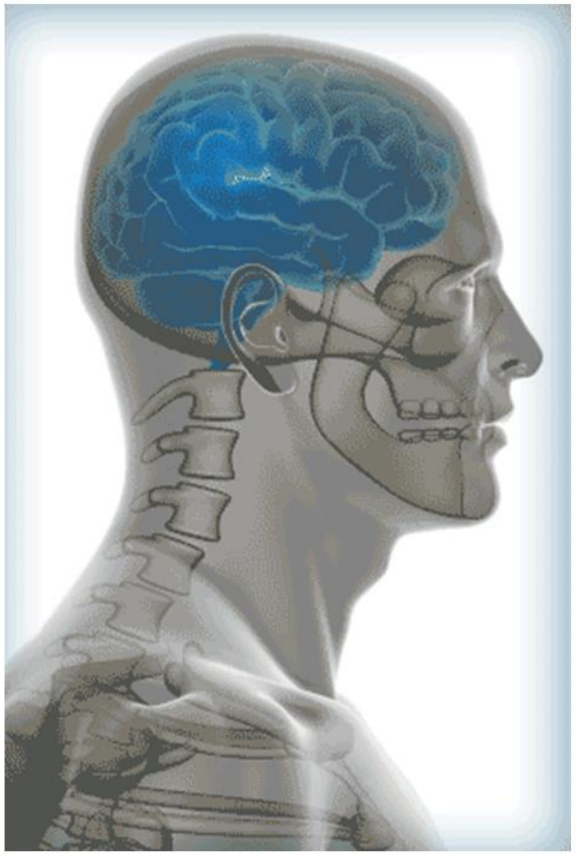
# The «ugly»
# Virtual choir

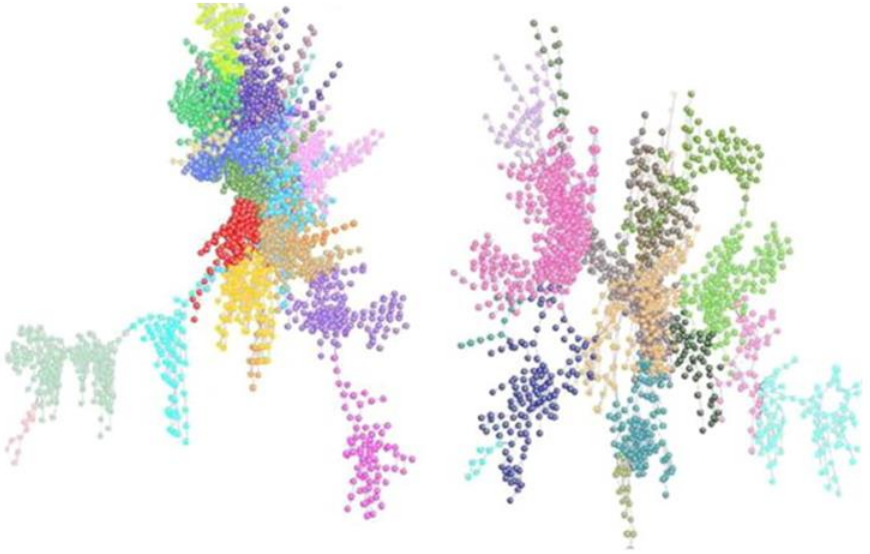
*Eric Whitacre*

# The «ugly»

## Chris Hadfield (International Station) and Barenaked Ladies

# The «ugly» brain





**Brain Modules Identified**

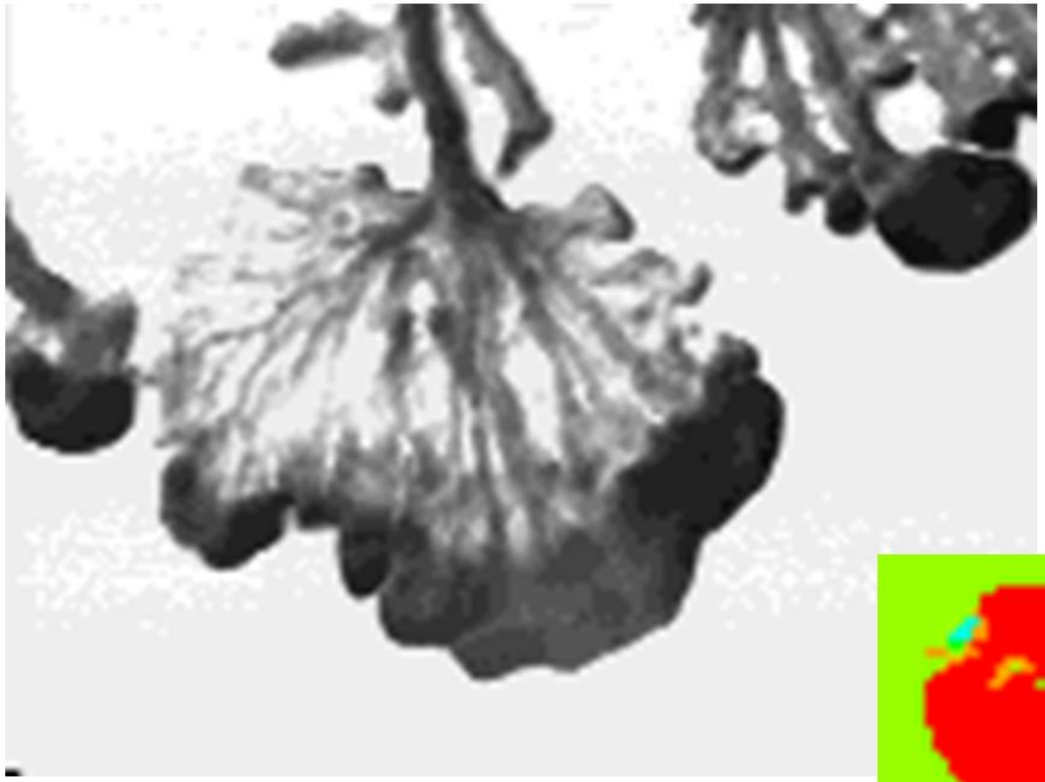
L. K. Gallos, H. A. Makse, and M. Sigman, *Proc. Nat. Acad. Sci. USA* **109**, 2825 (2012).

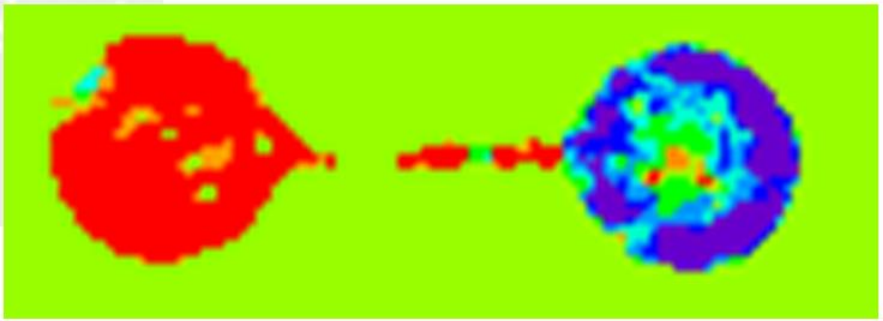
# The «ugly»
# plasmodial slime mol

http://faculty.clintoncc.suny.edu/faculty/michael.gregory



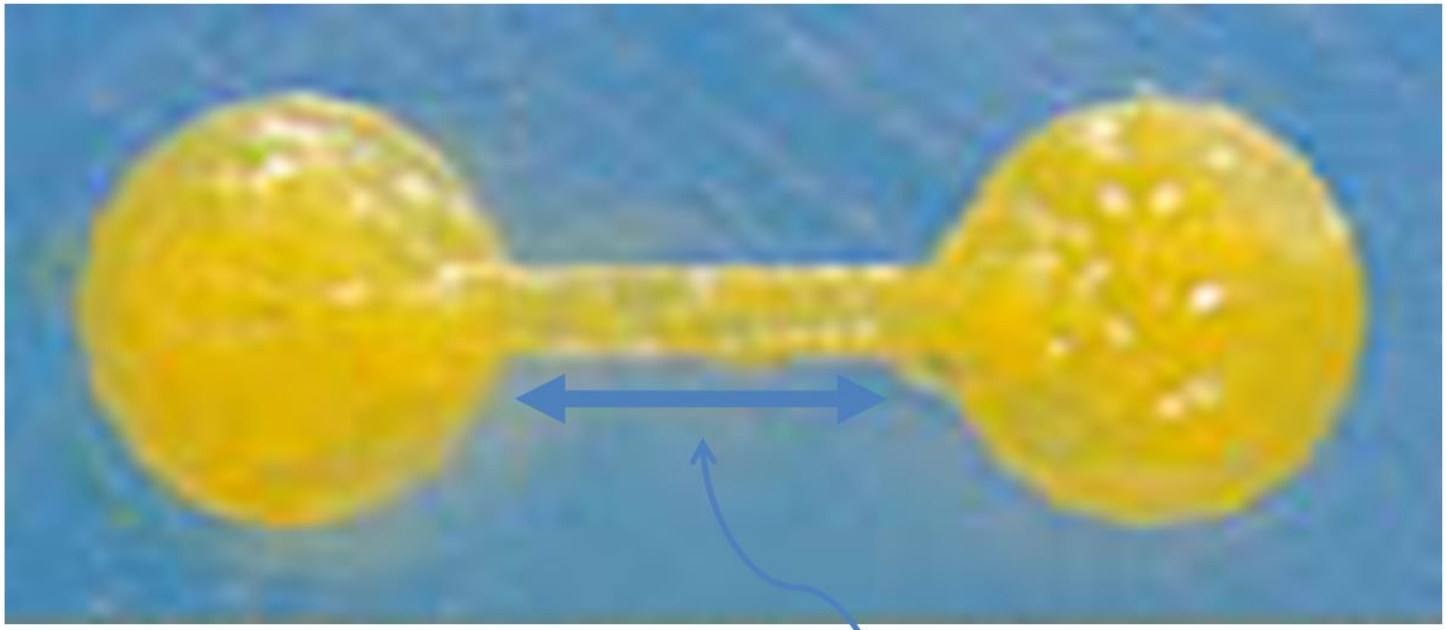
# The «ugly»
# plasmodial slime mol





A. Takamatsu, T. Fujii, and I. Endo, *Phys. Rev. Lett.* **85**, 2026 (2000).

# The «ugly»
# plasmodial slime mol

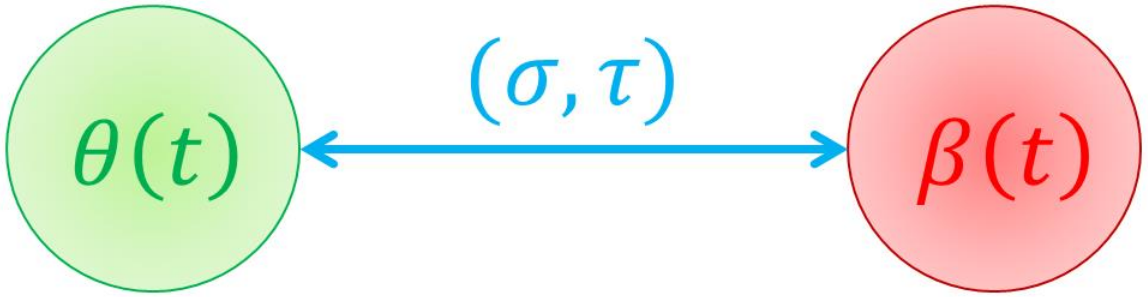


**Coupling strength and delay between sections**

A. Takamatsu, T. Fujii, and I. Endo, *Phys. Rev. Lett.* **85**, 2026 (2000).

# Time delay

## two coupled oscillators: Schuster and Wanger



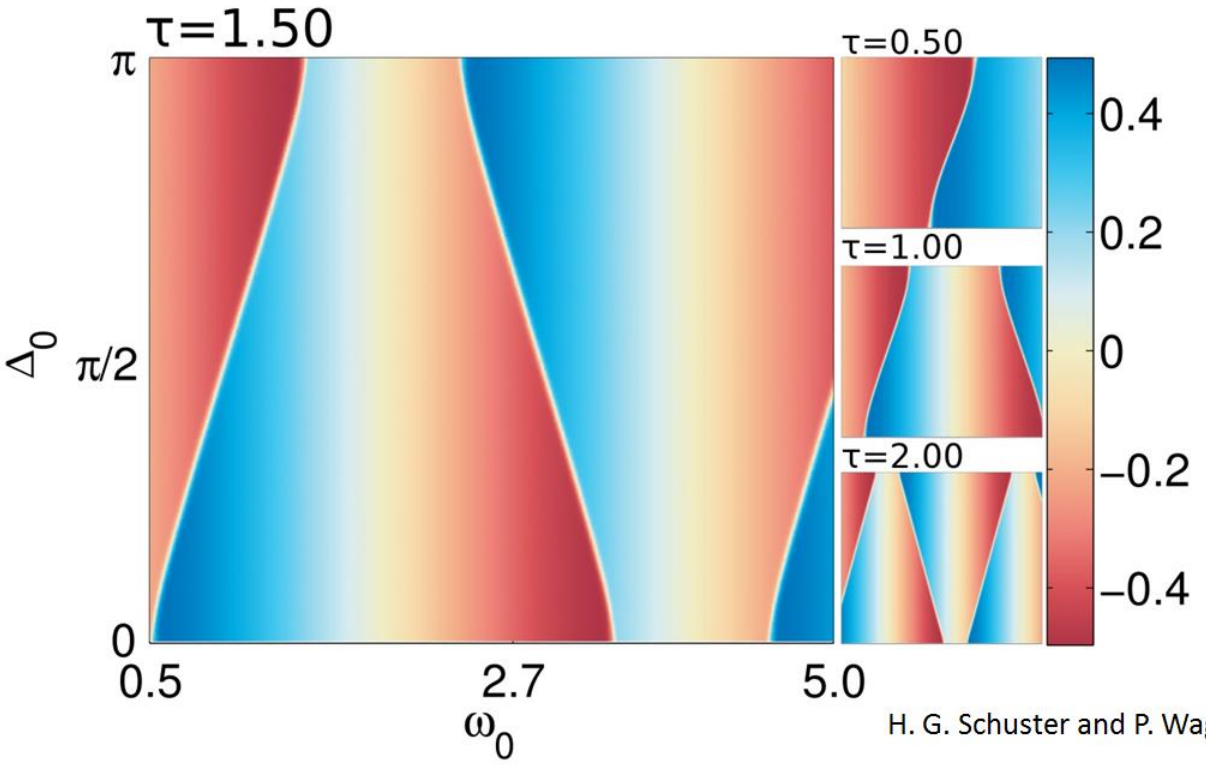

$$\dot{\theta}(t) = \omega_\theta + \sigma \sin[\beta(t-\tau) - \theta(t)]$$

$$\dot{\beta}(t) = \omega_\beta + \sigma \sin[\theta(t-\tau) - \beta(t)]$$

H. G. Schuster and P. Wagner, *Prog. Theor. Phys.* **81**, 939 (1989).

# Time delay
## two coupled oscillators: Schuster and Wanger

$$\omega = \omega_0 - \sigma \sin(\omega\tau)$$



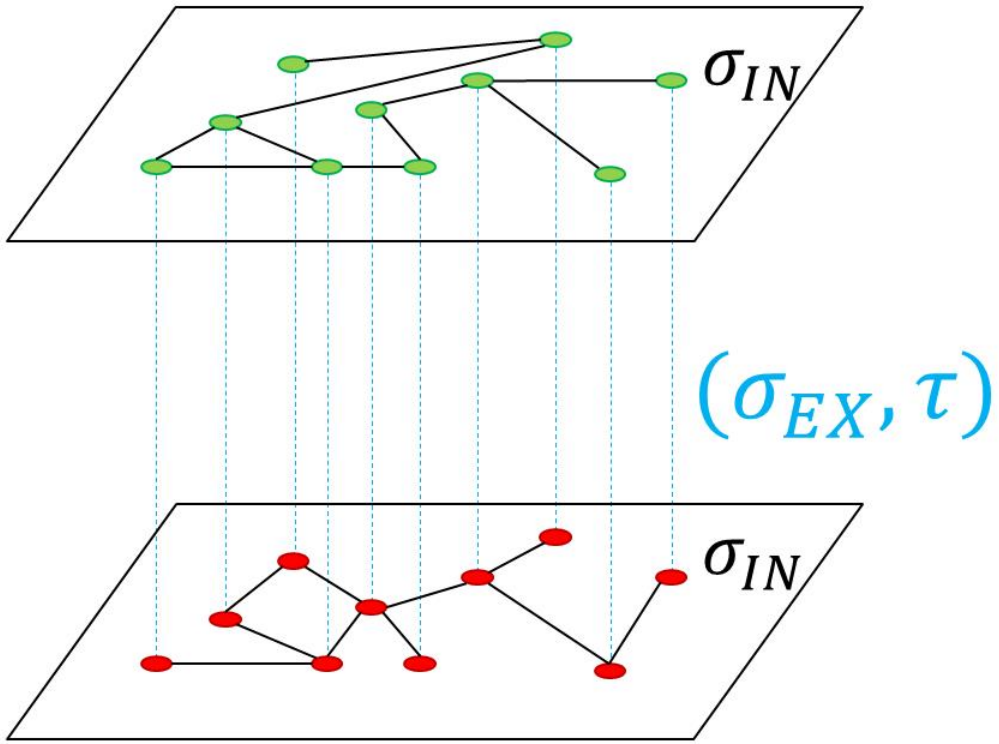

In phase when:

$$\cos(\omega\tau) > 0$$

H. G. Schuster and P. Wagner, *Prog. Theor. Phys.* **81**, 939 (1989).

# Time delay
## multiplex network with time delay

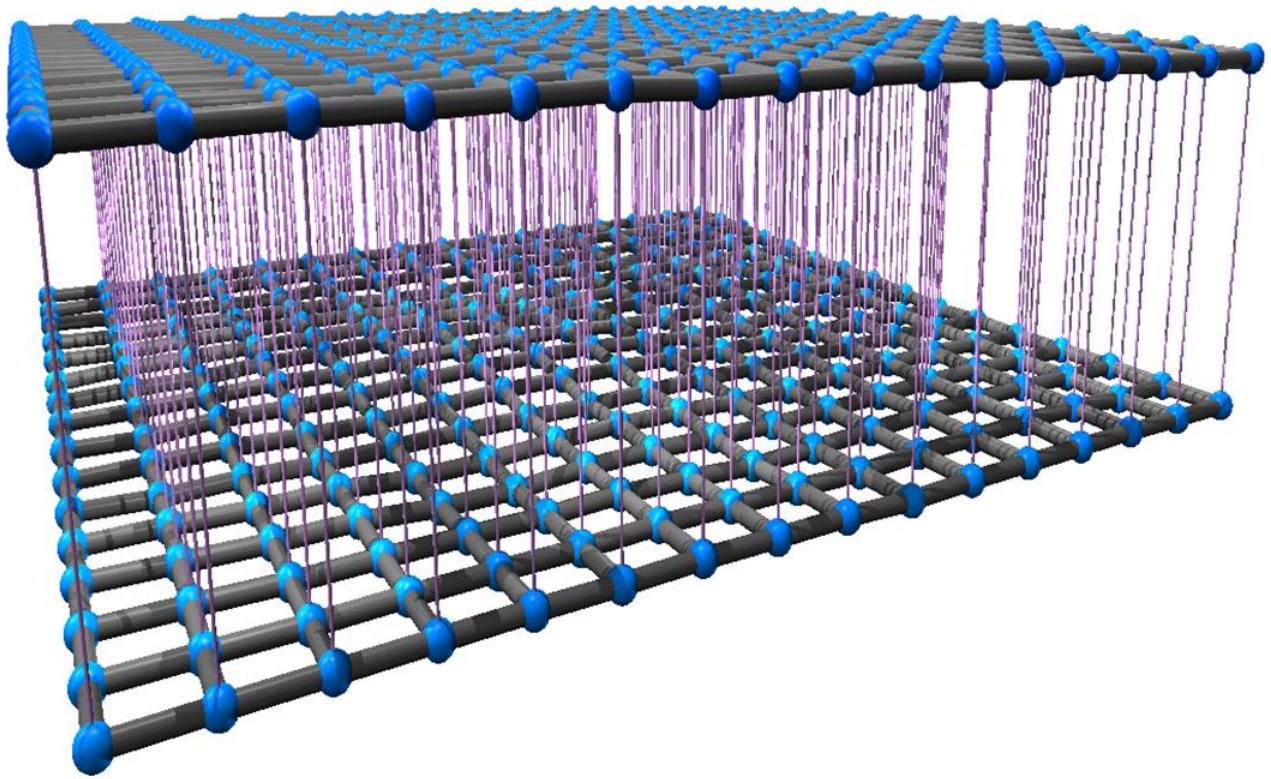
$$\dot{\theta}_i = \omega_i + \sigma_{EX} \sin\left(\beta_j(t-\tau) - \theta_i\right) + \sigma_{IN} \sum_{l \epsilon N(i)} \sin(\theta_l - \theta_i)$$




$$\dot{\beta}_j = \omega_j + \sigma_{EX} \sin\left(\theta_i(t-\tau) - \beta_j\right) + \sigma_{IN} \sum_{l \epsilon N(j)} \sin\left(\beta_l - \beta_j\right)$$

V. H. Louzada, N. A. M. Araújo, J. S. Andrade Jr., H. J. Herrmann. *Scientific Reports* **3**, 3289 (2013).

# Time delay
# two coupled square lattices



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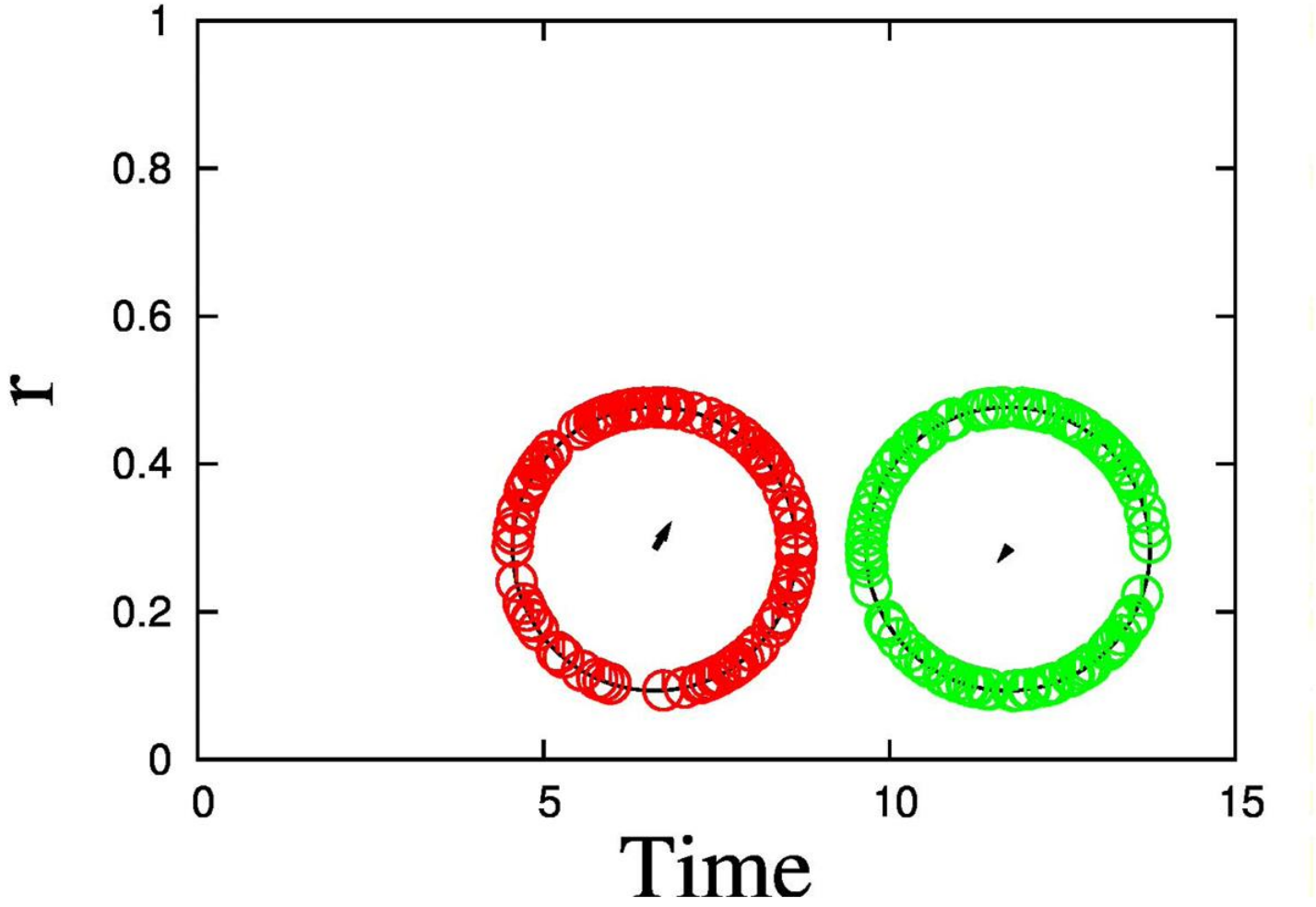
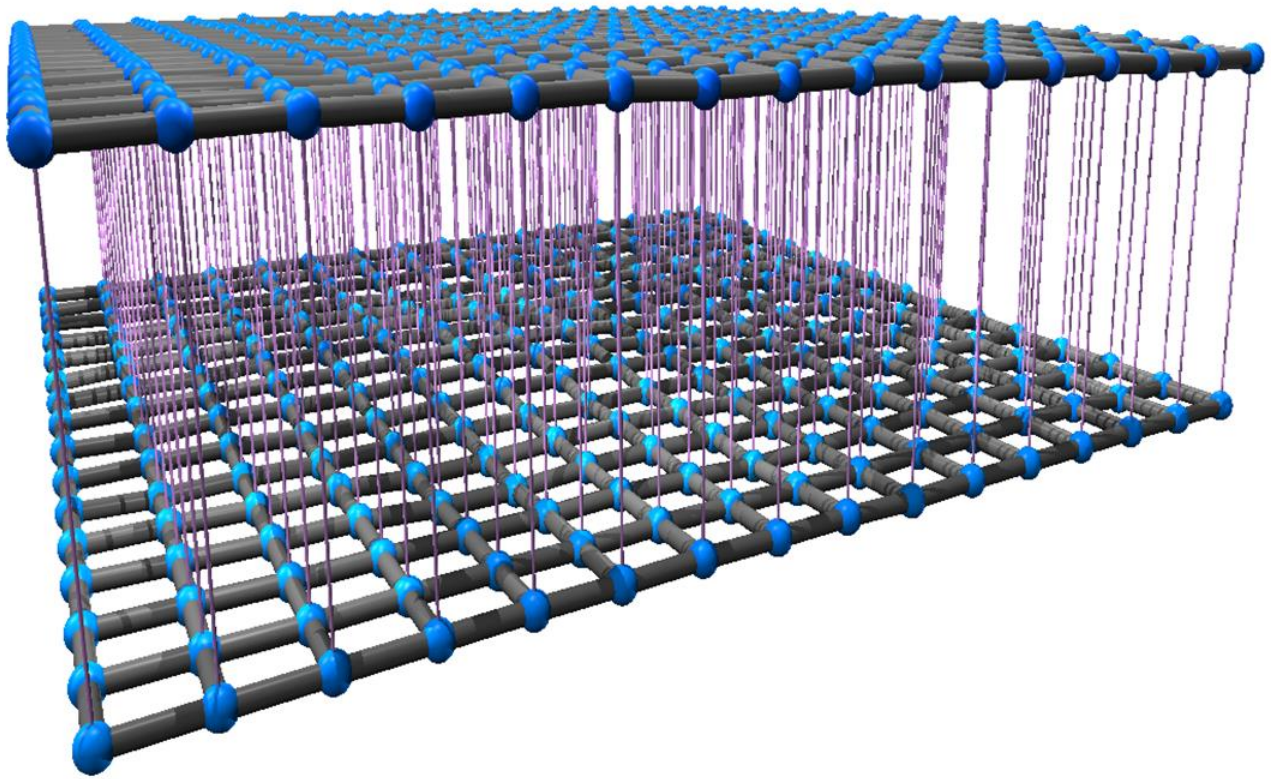
# Time delay
## no external coupling



# Time delay
# two coupled square lattices



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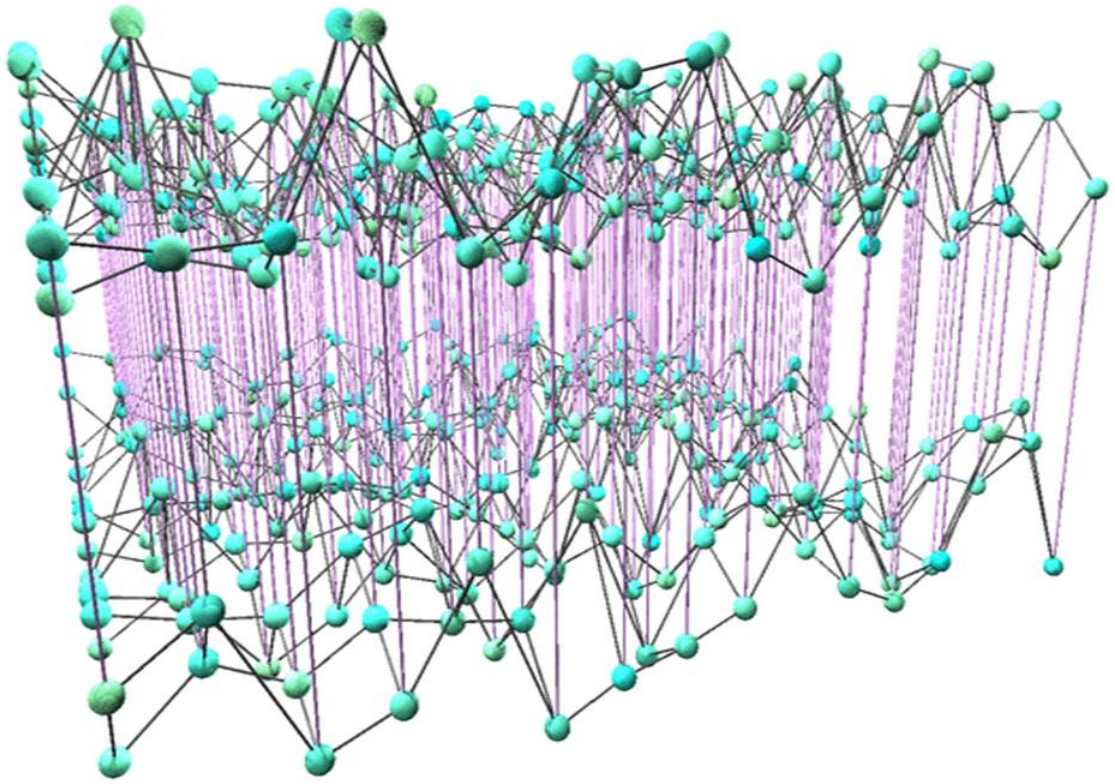
# Time delay
## weak internal coupling



t=0.23

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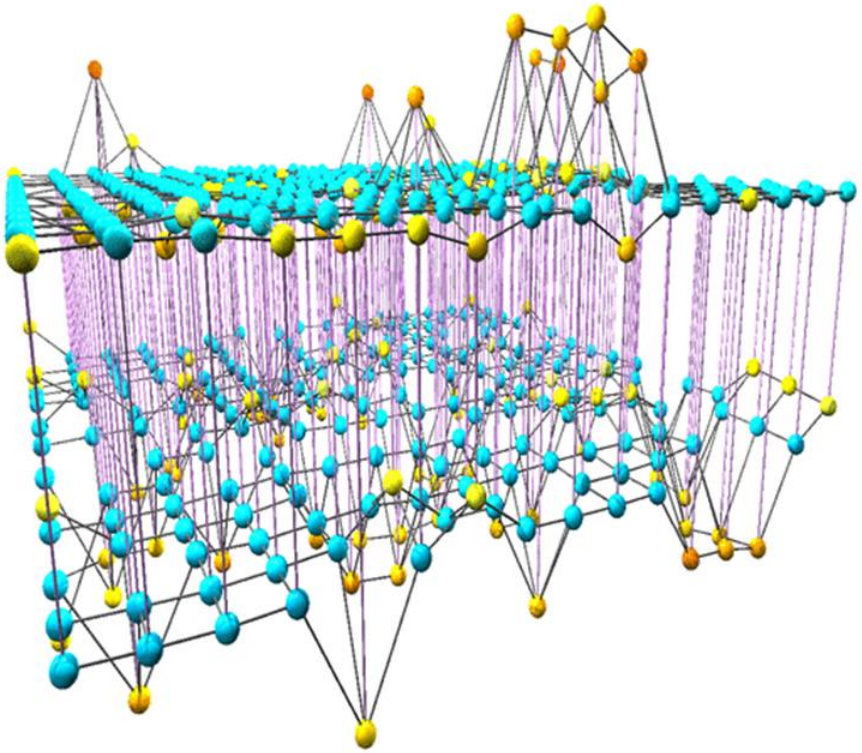
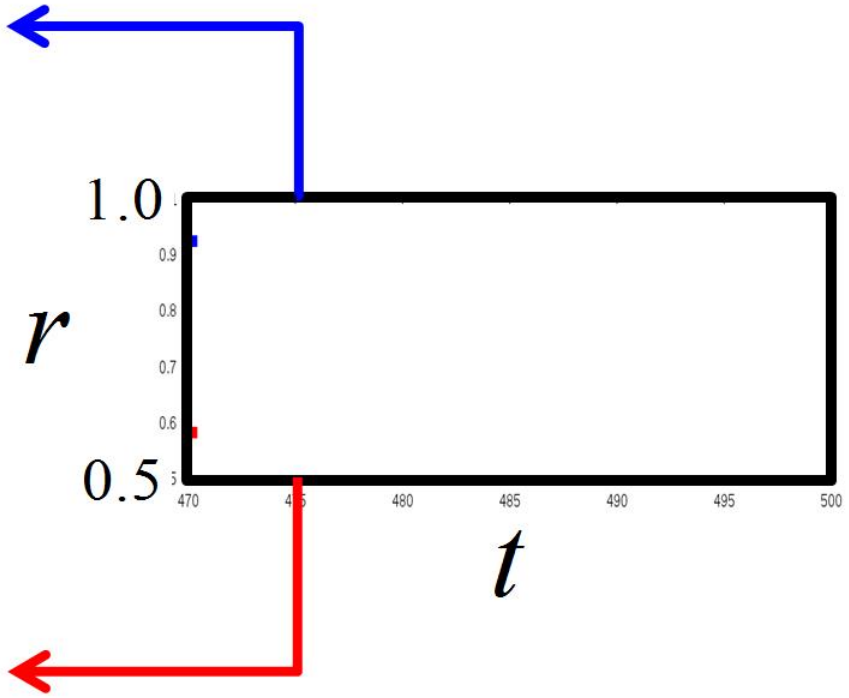
# Time delay
## weak internal coupling: breathing synchronization



t=470.03

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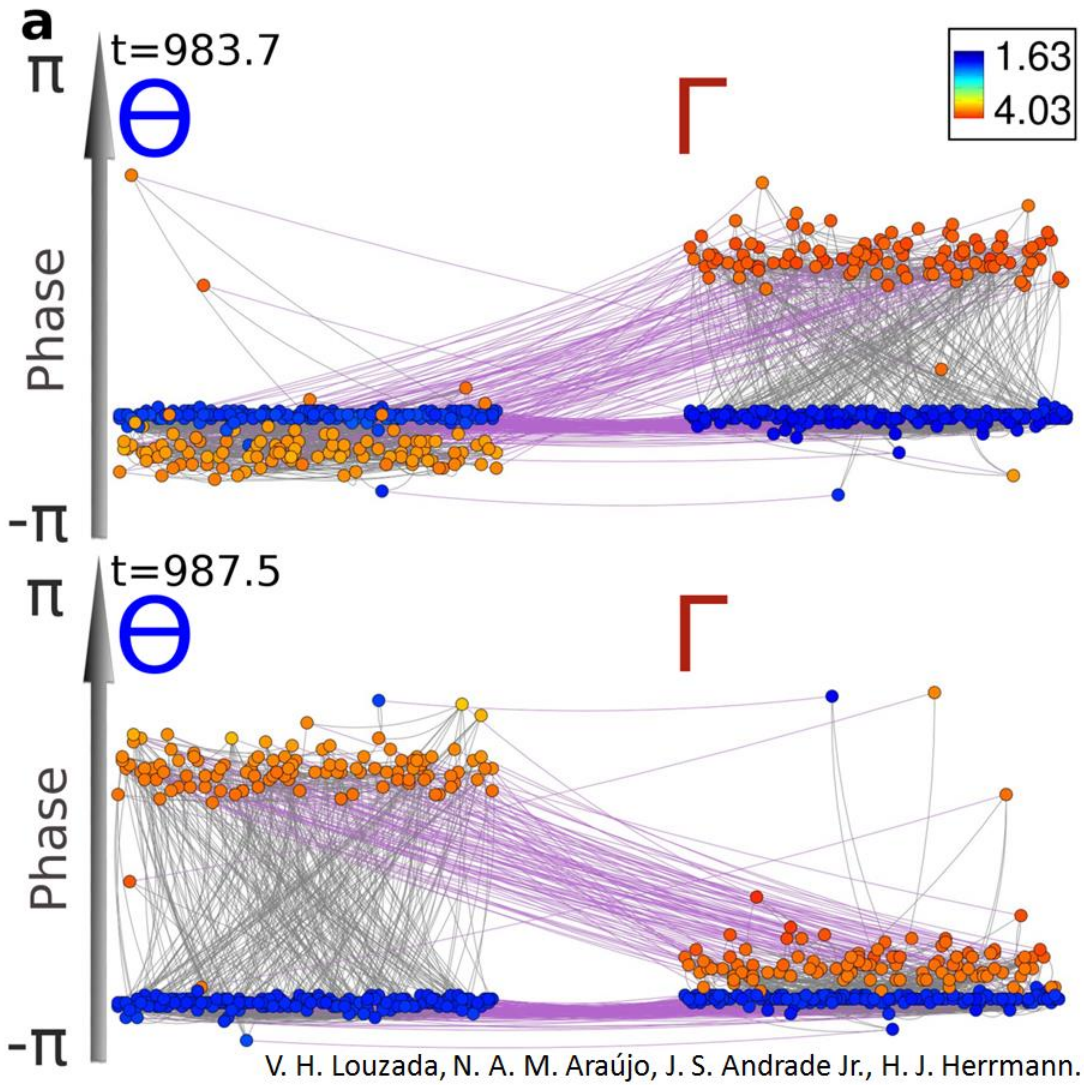
# Time delay

## weak internal coupling: breathing synchronization

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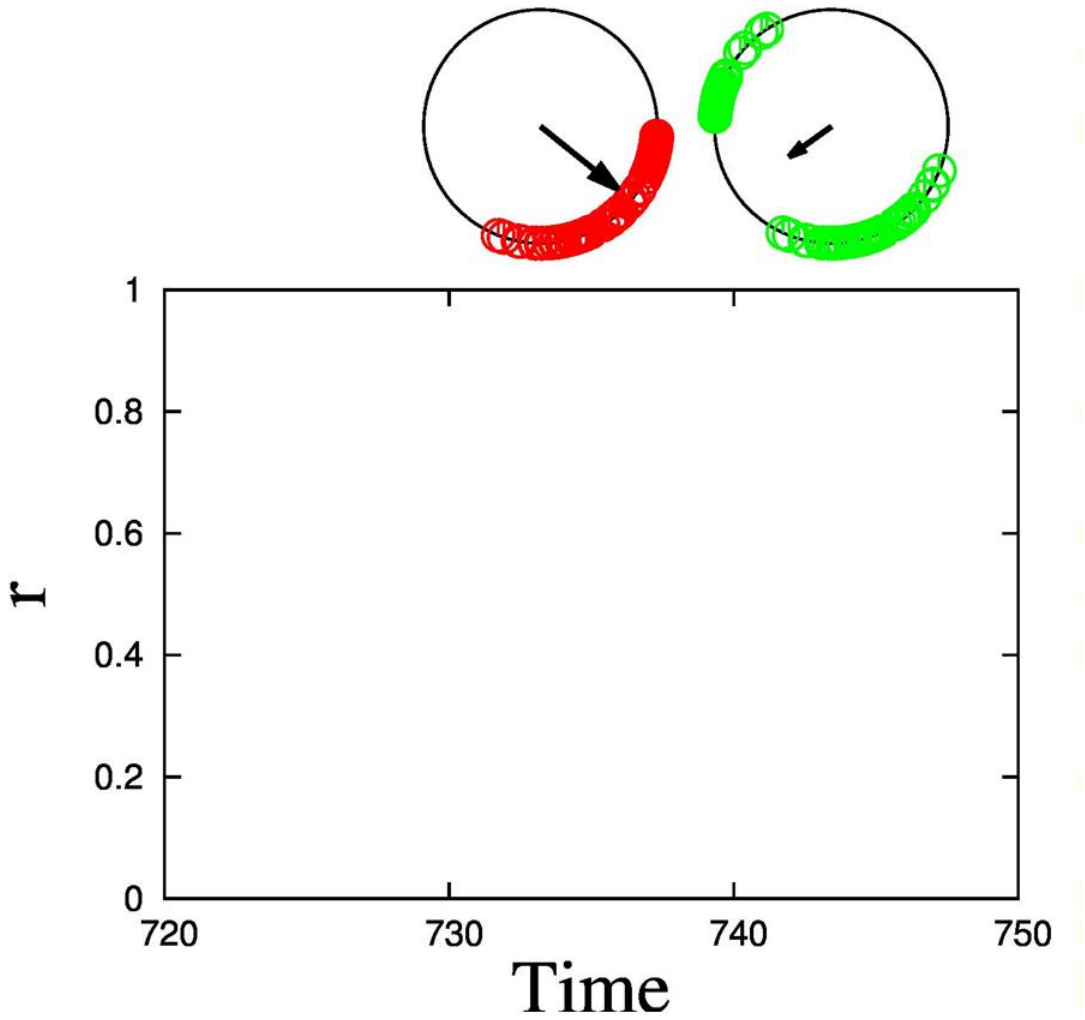
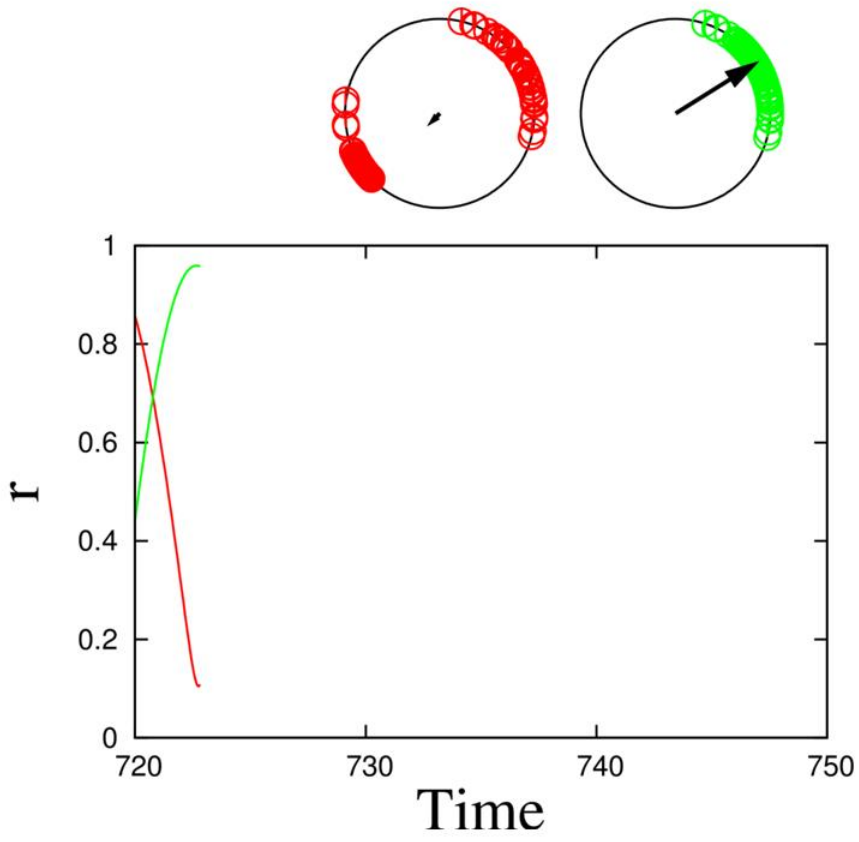
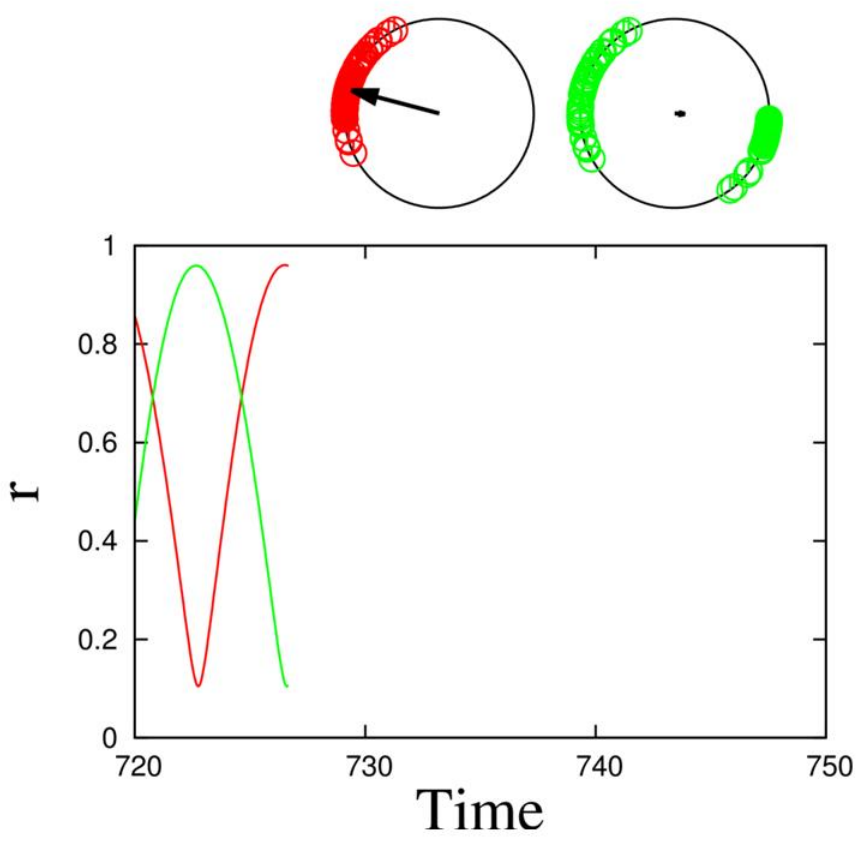
# Time delay

## weak internal coupling: breathing synchronization



# Time delay
## weak internal coupling: breathing synchronization

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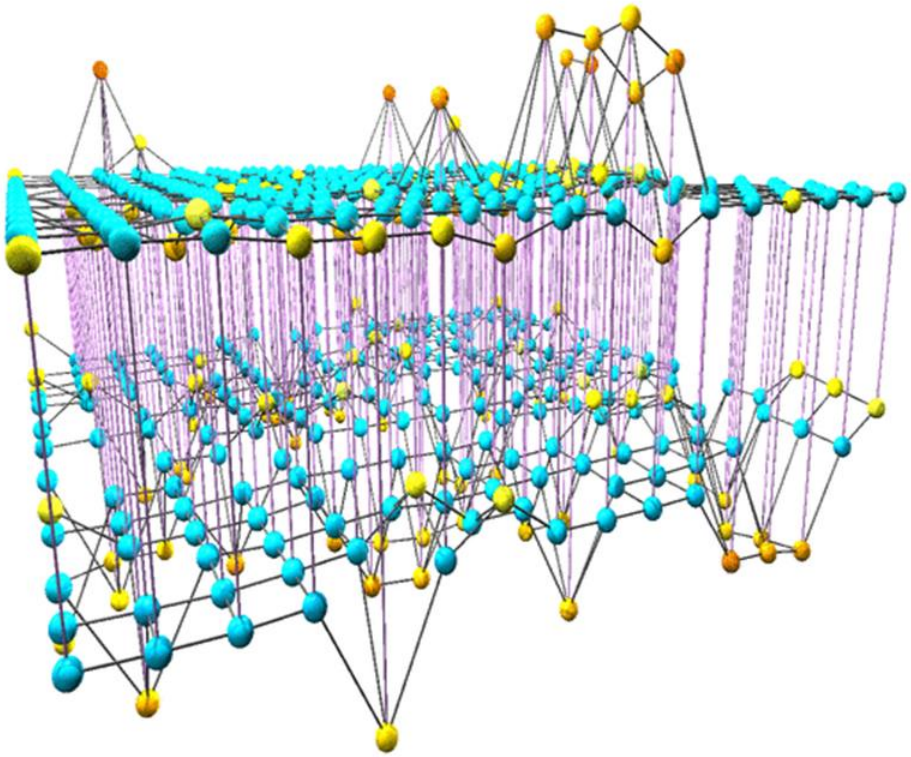
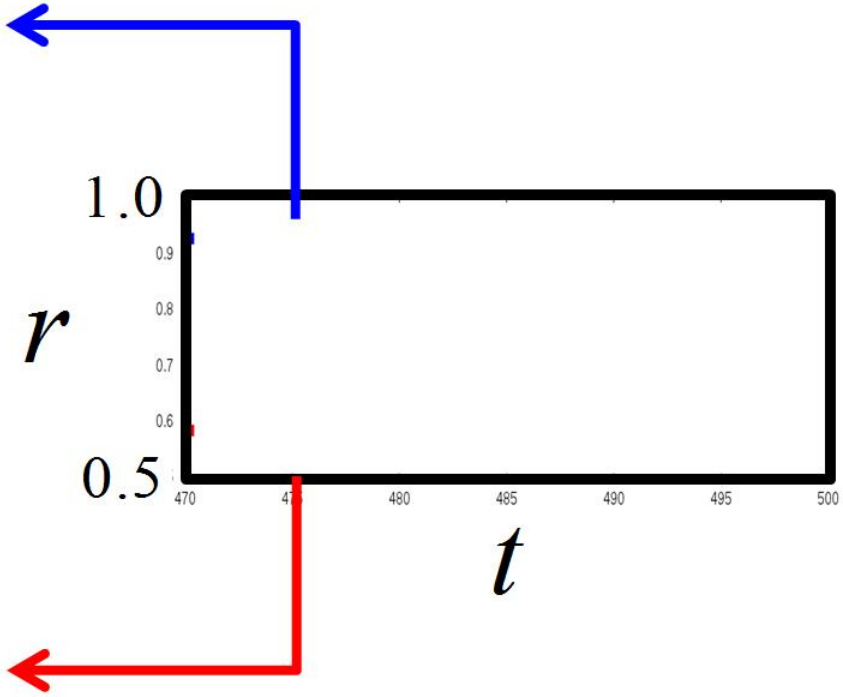
# Time delay
## increasing internal coupling






t=470.03 $\sigma_{IN}$=0.07

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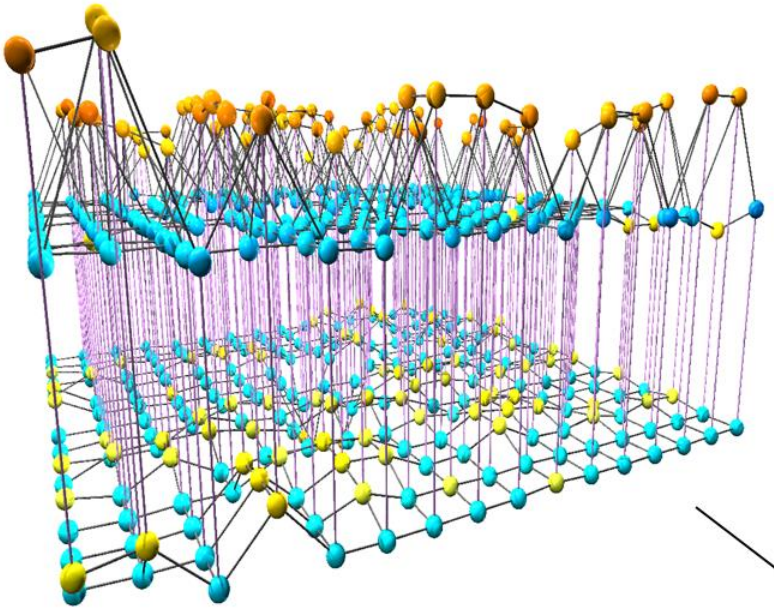
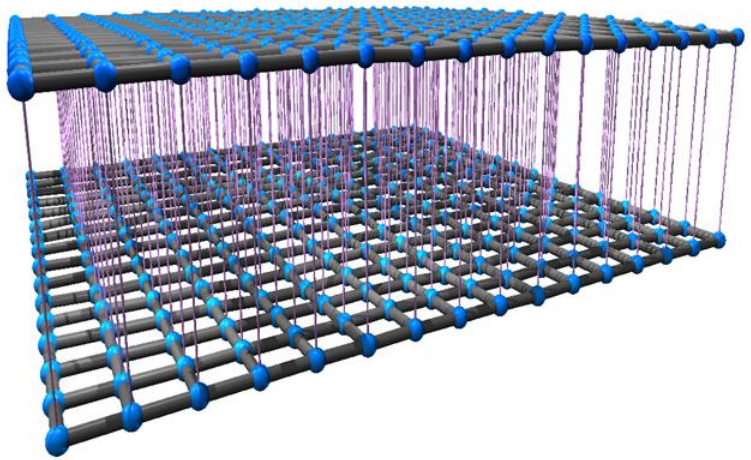
# Time delay
# increasing internal coupling



*Breathing*

*Supernode*



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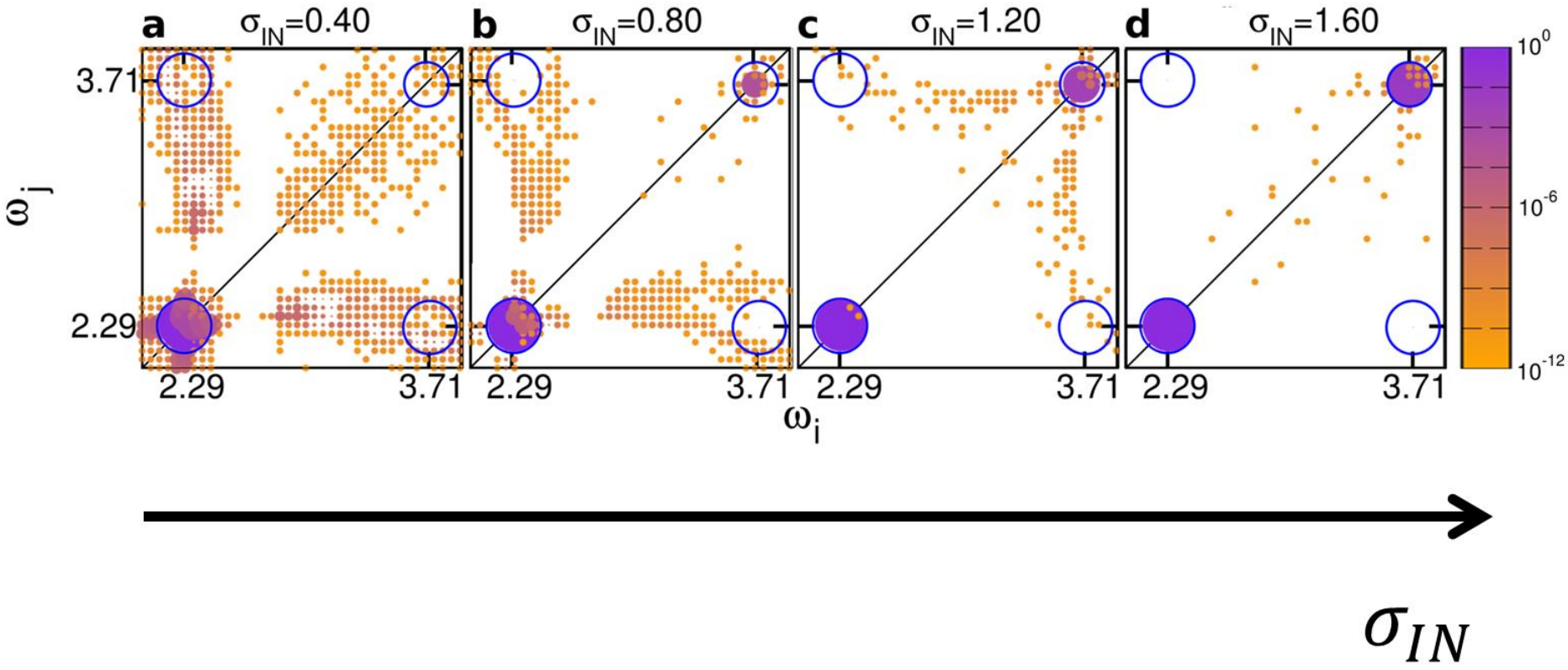
# Time delay
## increasing internal coupling

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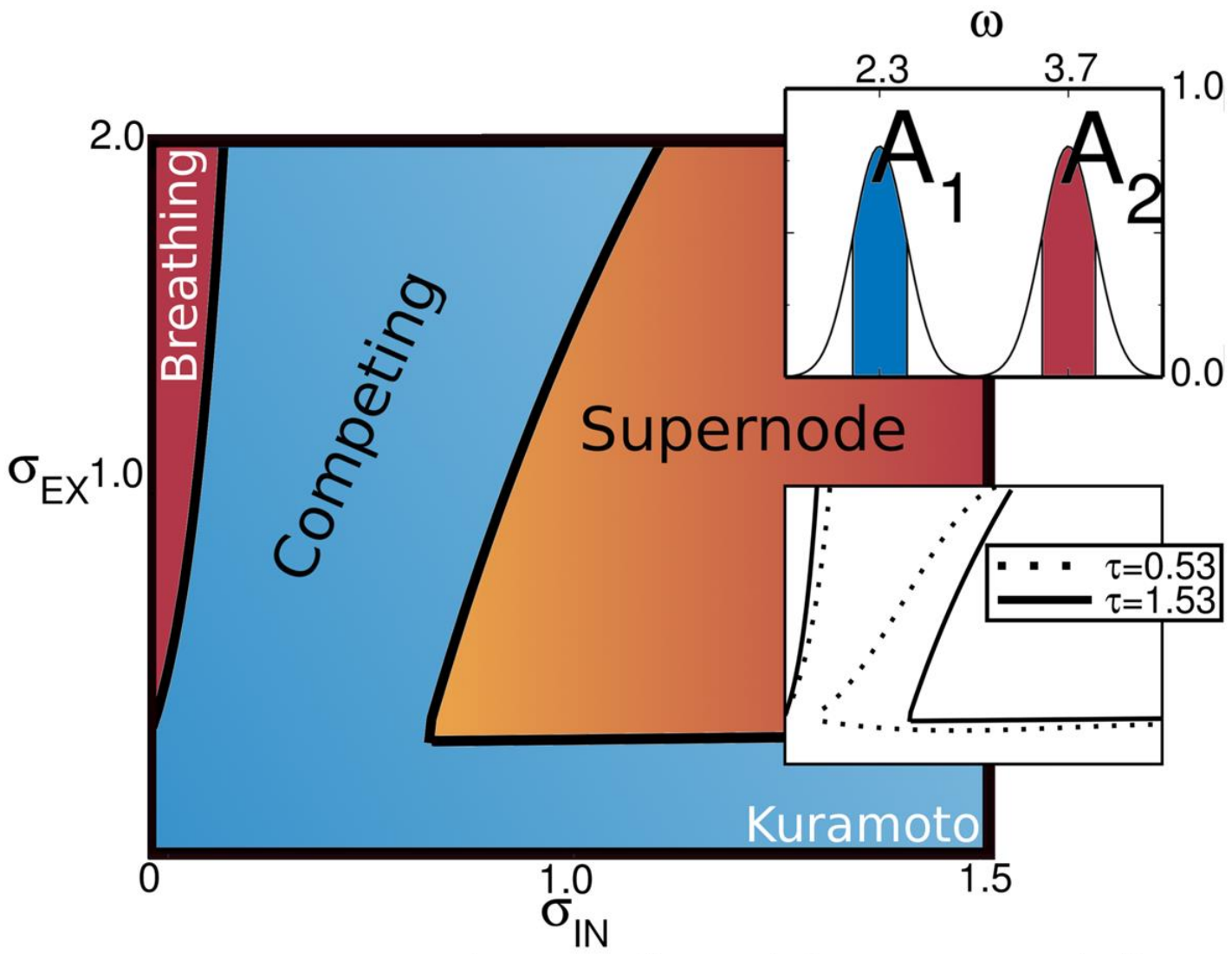
# Time delay diagram

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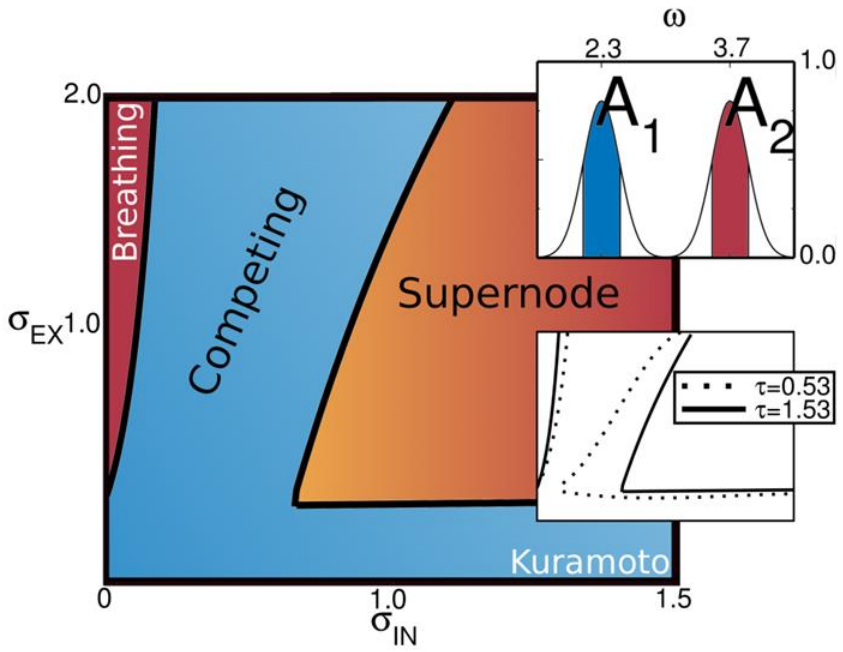
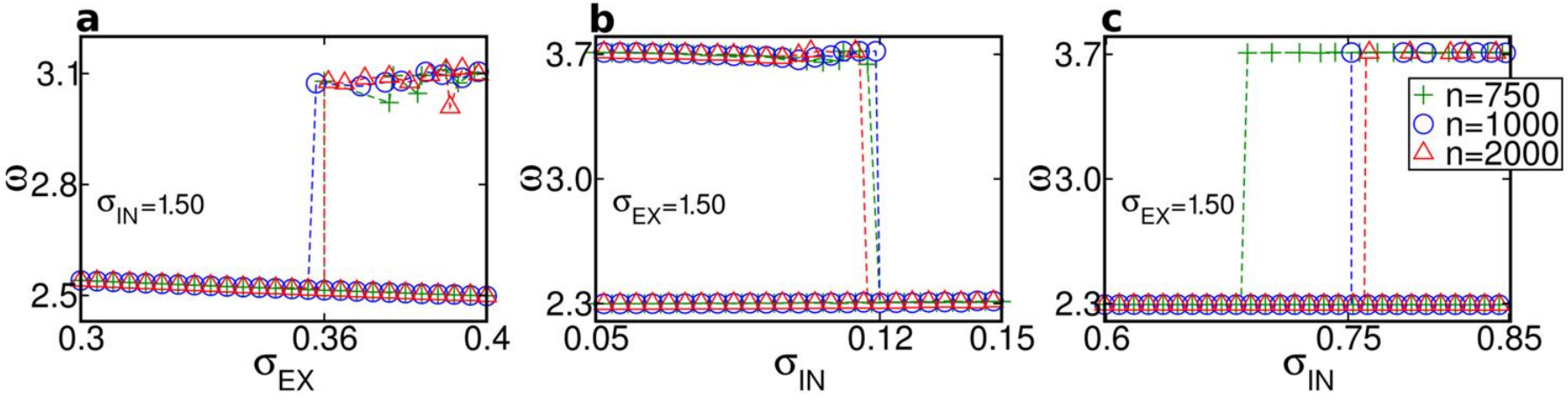
# Time delay
## bifurcation in frequency

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# Conclusions

- **Synchronization** can also lead to **unpleasant** situations
- **Local contrarians** can be used to **suppress synchronization**: no necessity for global knowledge
- A **rich diagram** emerges in the presence of **time delay**.
- For **weak intra-network** coupling **two groups** emerge.
- A **phase-shift** and **two frequencies** leads to **breathing** synchronization.