

# *Structure and robustness of network infrastructures*

Nuno Araújo

Centro de Física Teórica e Computacional, Universidade de Lisboa, Portugal

*School on Complex Networks and  
Applications to Neuroscience*

01/October/2015

<http://www.namaraujo.net>



**INVESTIGADOR FCT**



**Hans Herrmann**  
*ETH Zürich*



**Trivik Verma**  
*ETH Zürich*



**Vitor Louzada**  
*ETH Zürich*

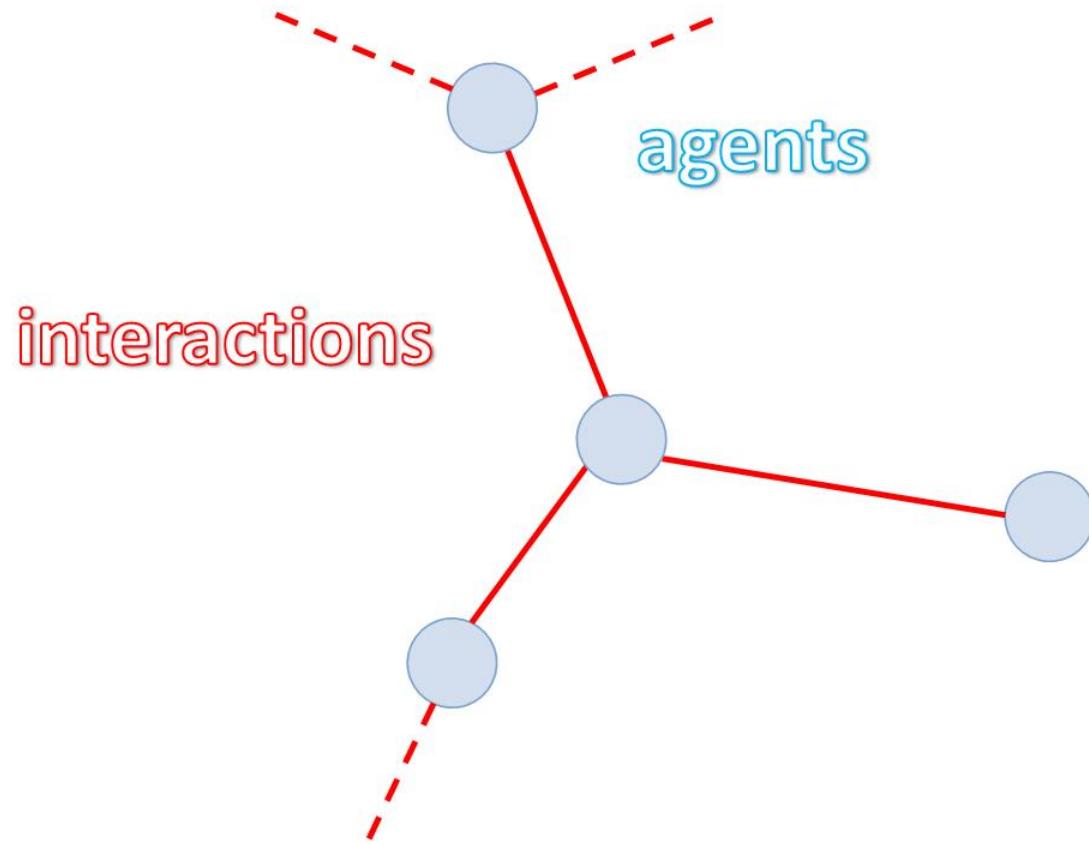


**George Mamede**  
*UNILAB*

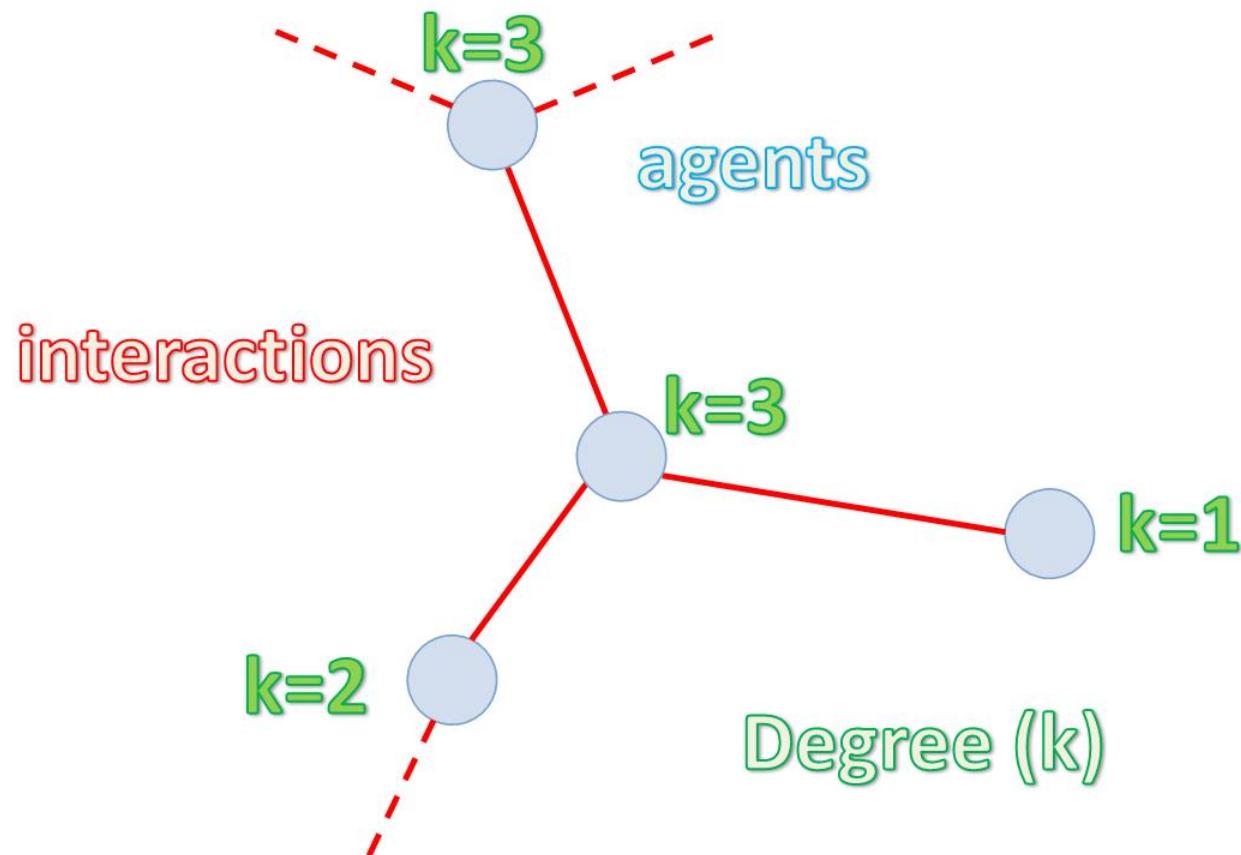


**José Carlos Araújo**  
*Universidade Ceará*

# *Ingredients: **nodes** and **links***

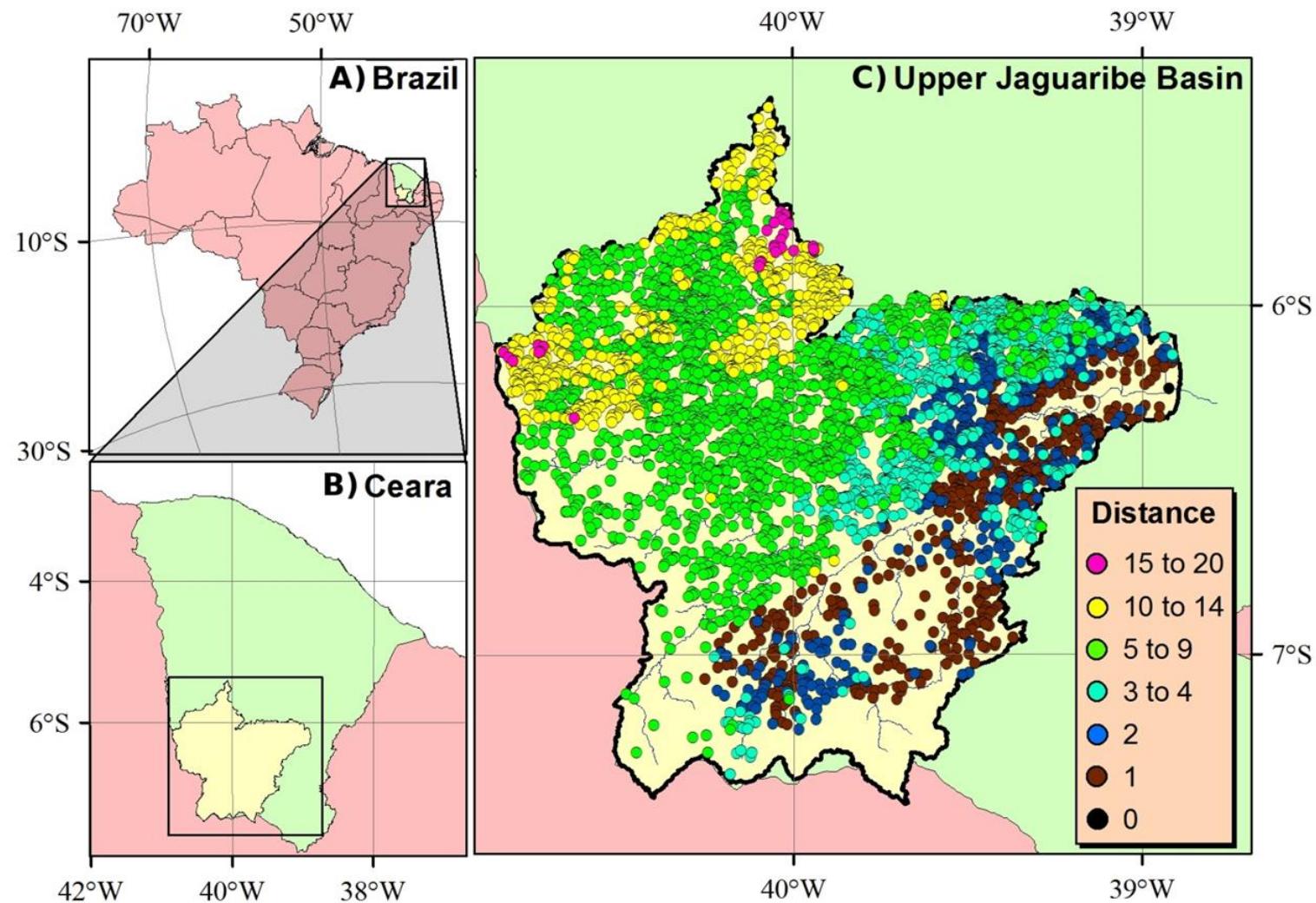


# *Ingredients: nodes and links*

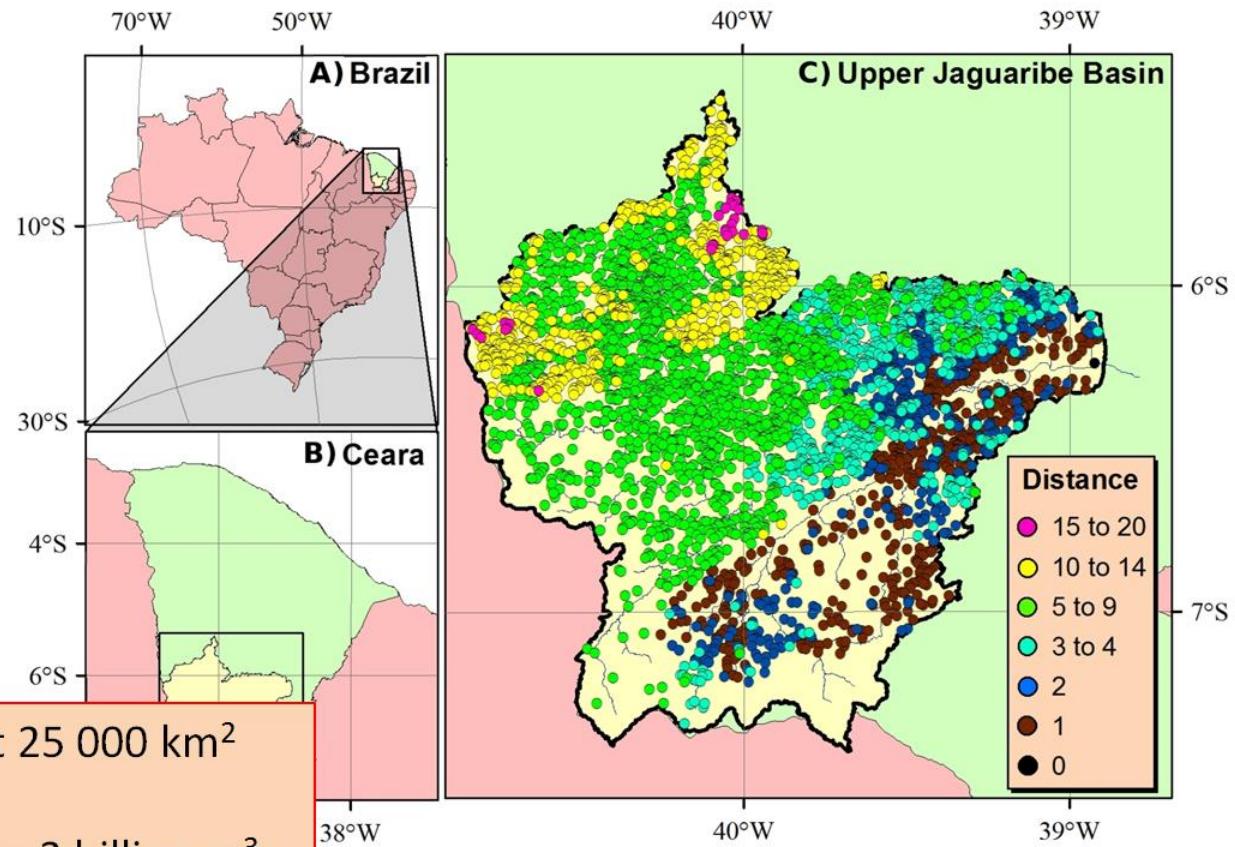


# *Water Reservoirs*

# *The Upper Jaguaribe Basin*

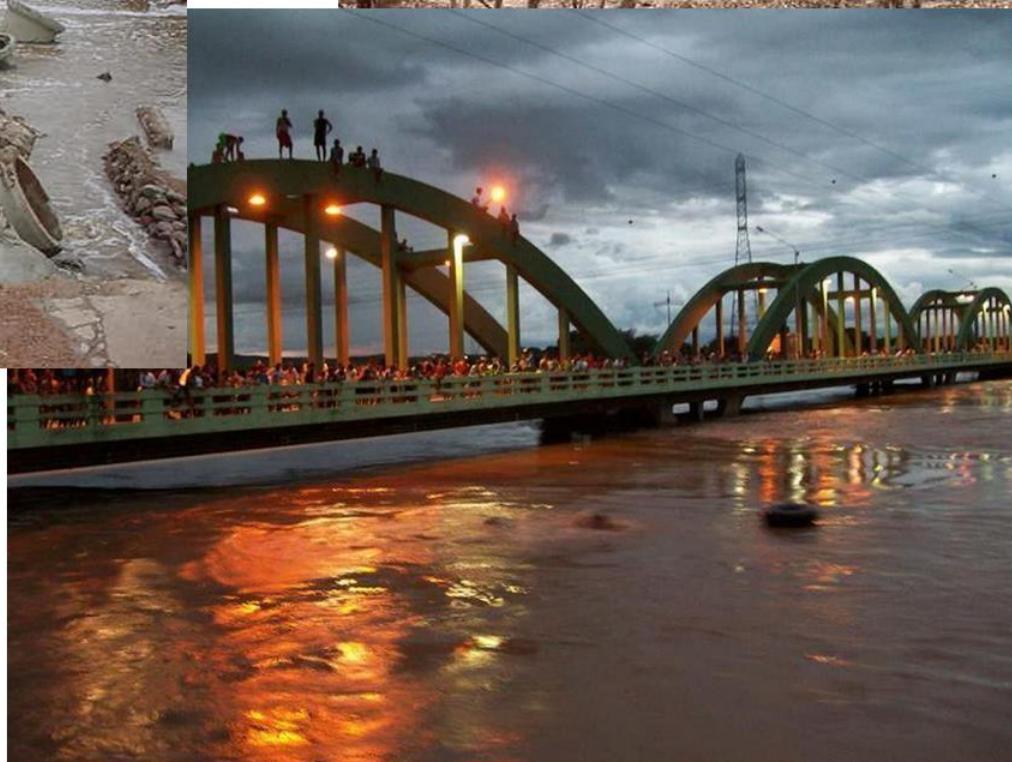


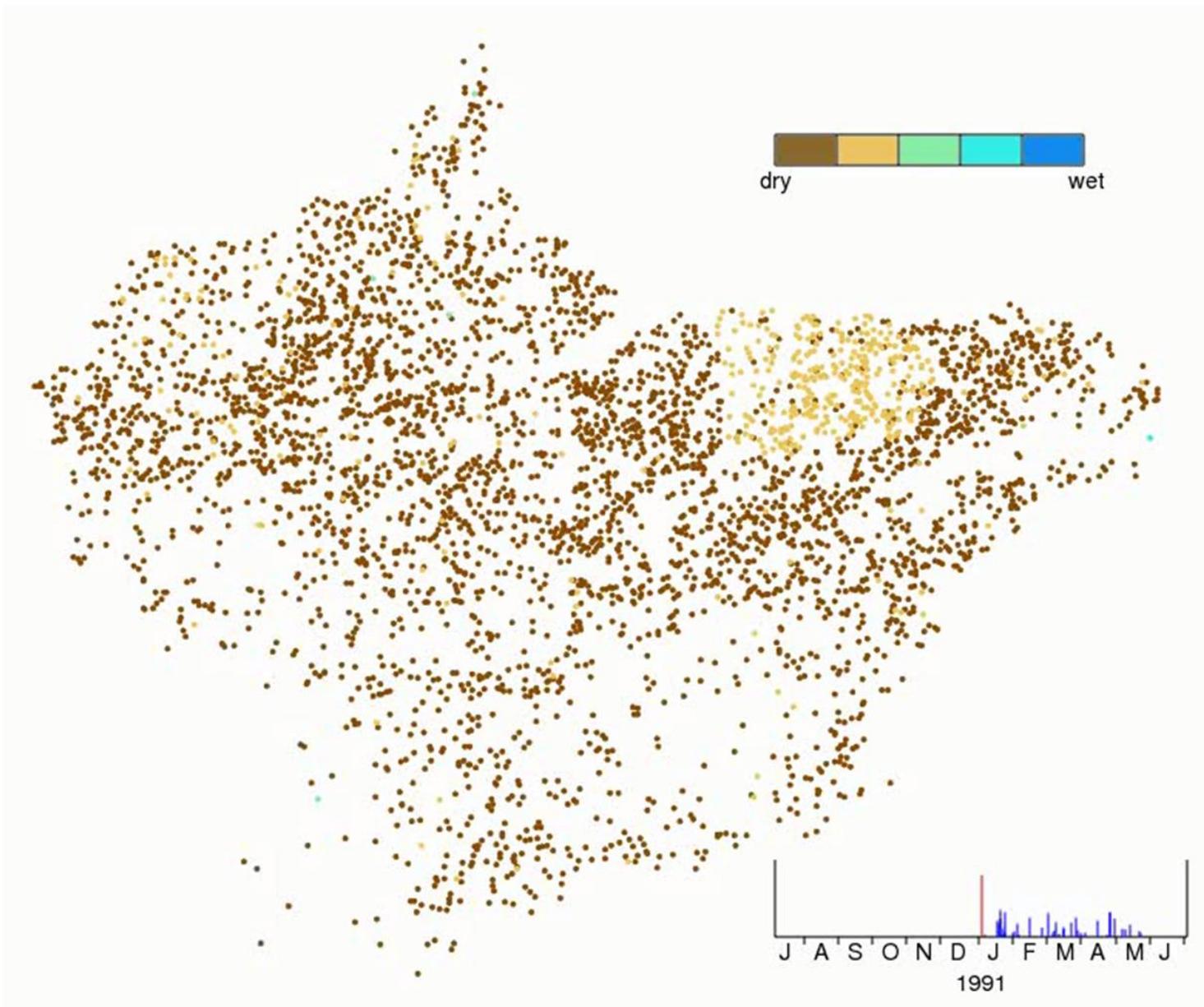
# The Upper Jaguaribe Basin



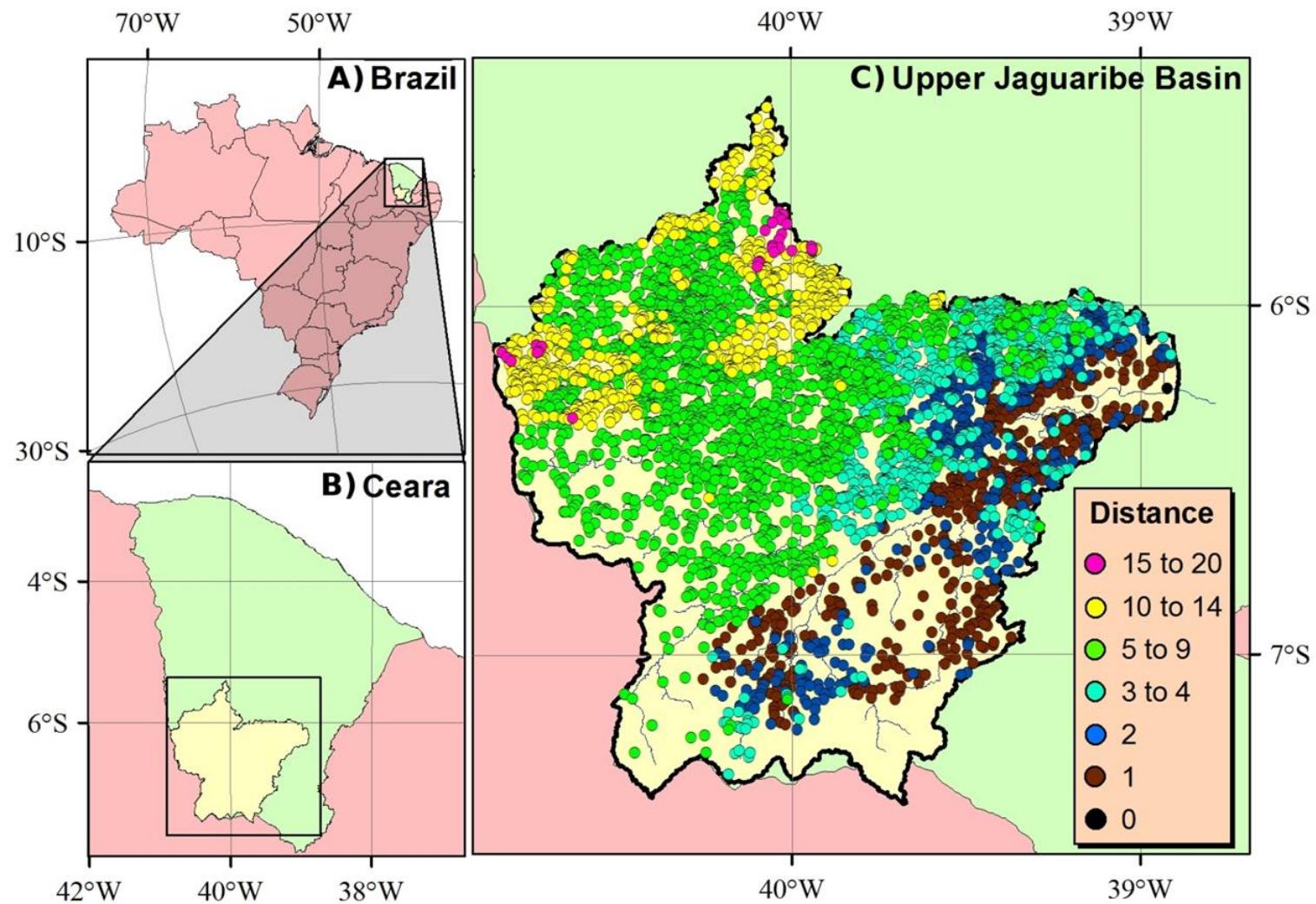
- Catchment area of about 25 000 km<sup>2</sup>
- Almost 4 000 reservoirs
- Capacity from 2 500 m<sup>3</sup> to 2 billion m<sup>3</sup>
- Affects half a million people
- Average annual rainfall: 860 mm
- Evaporation: 2 000 mm

# *The Upper Jaguaribe Basin*

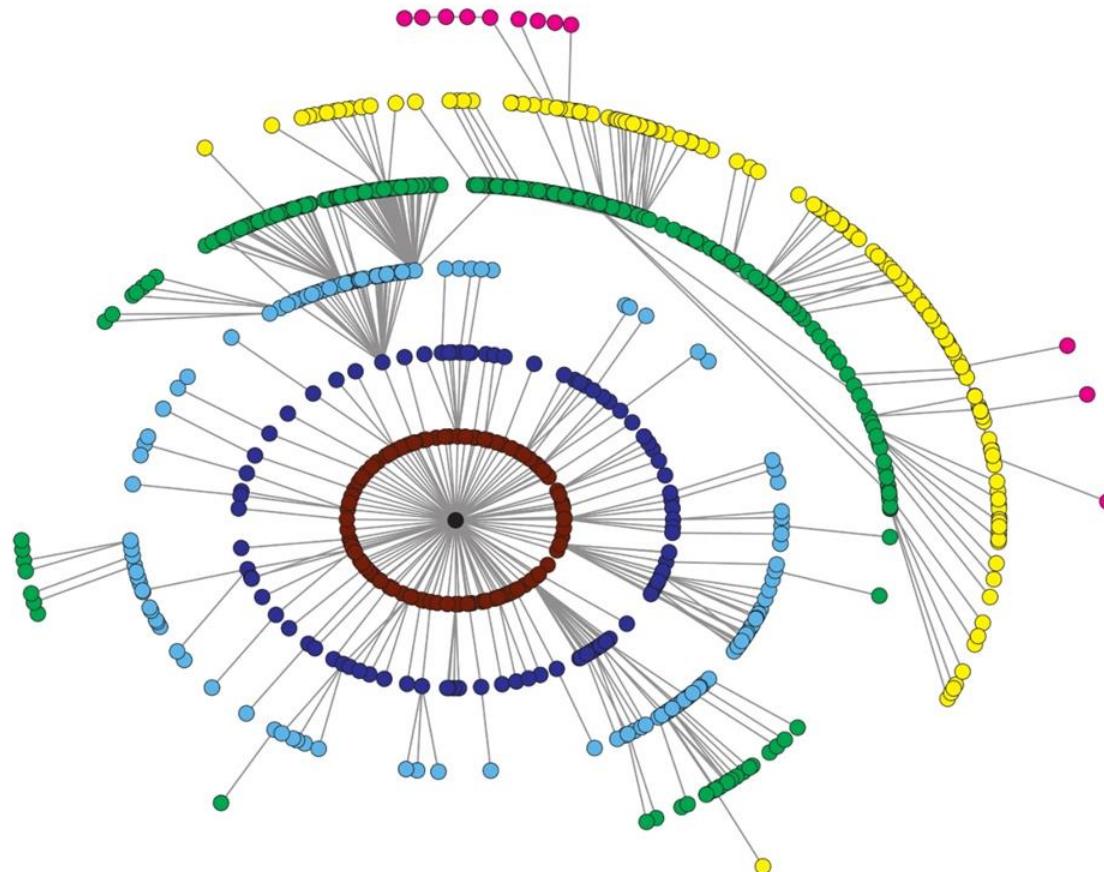




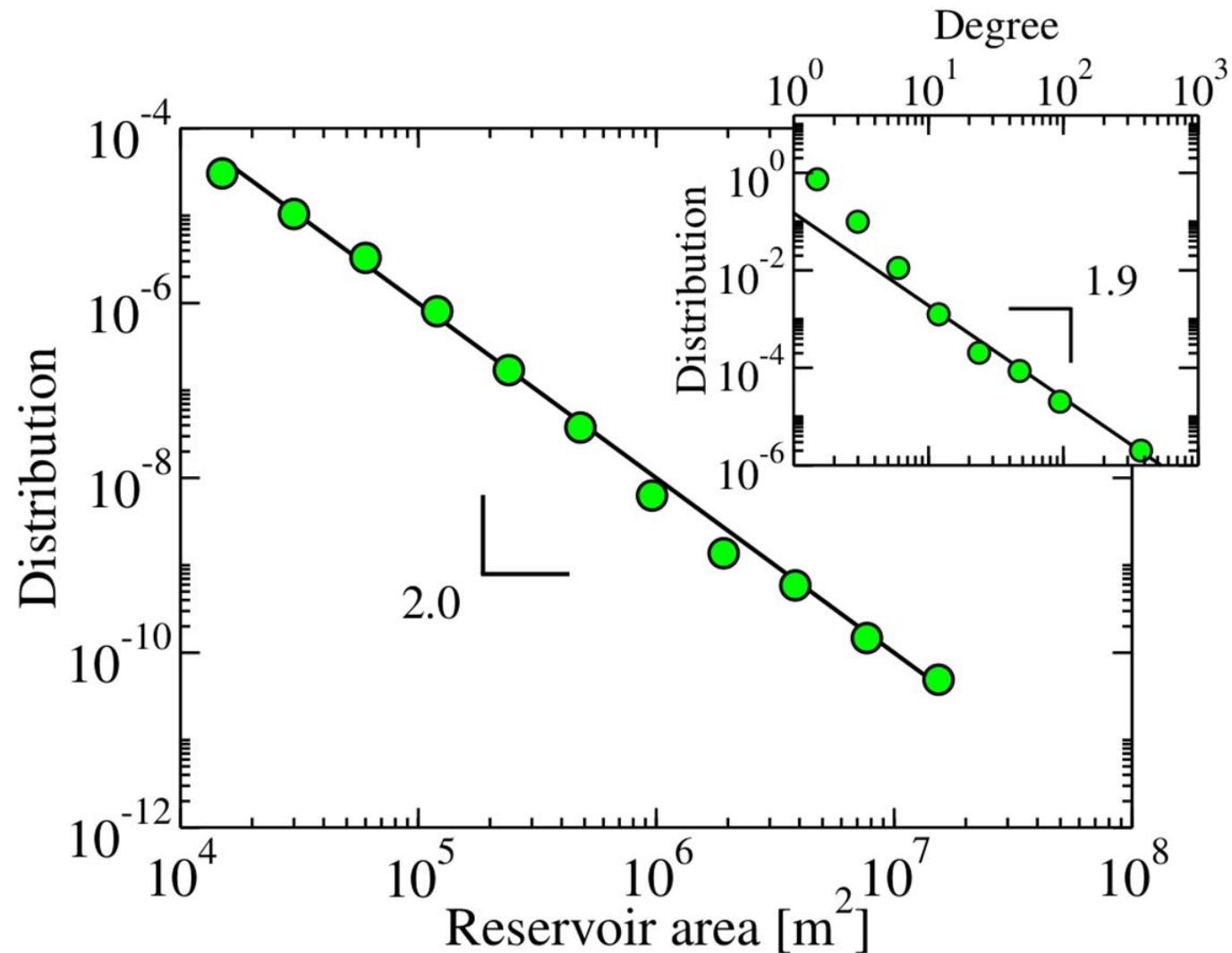
# *The Upper Jaguaribe Basin*



# *The topology of the network*



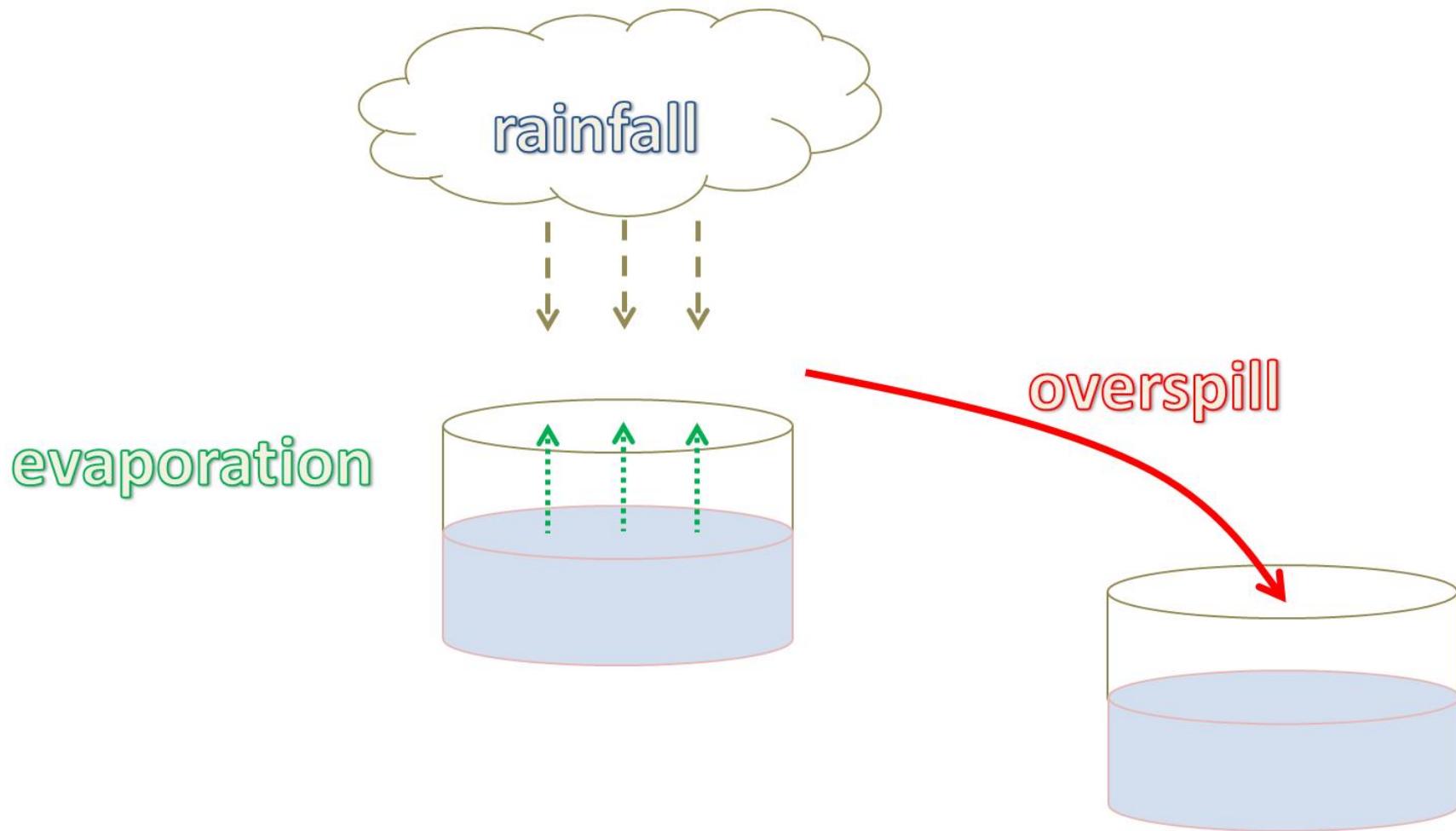
# *Size and degree distributions*



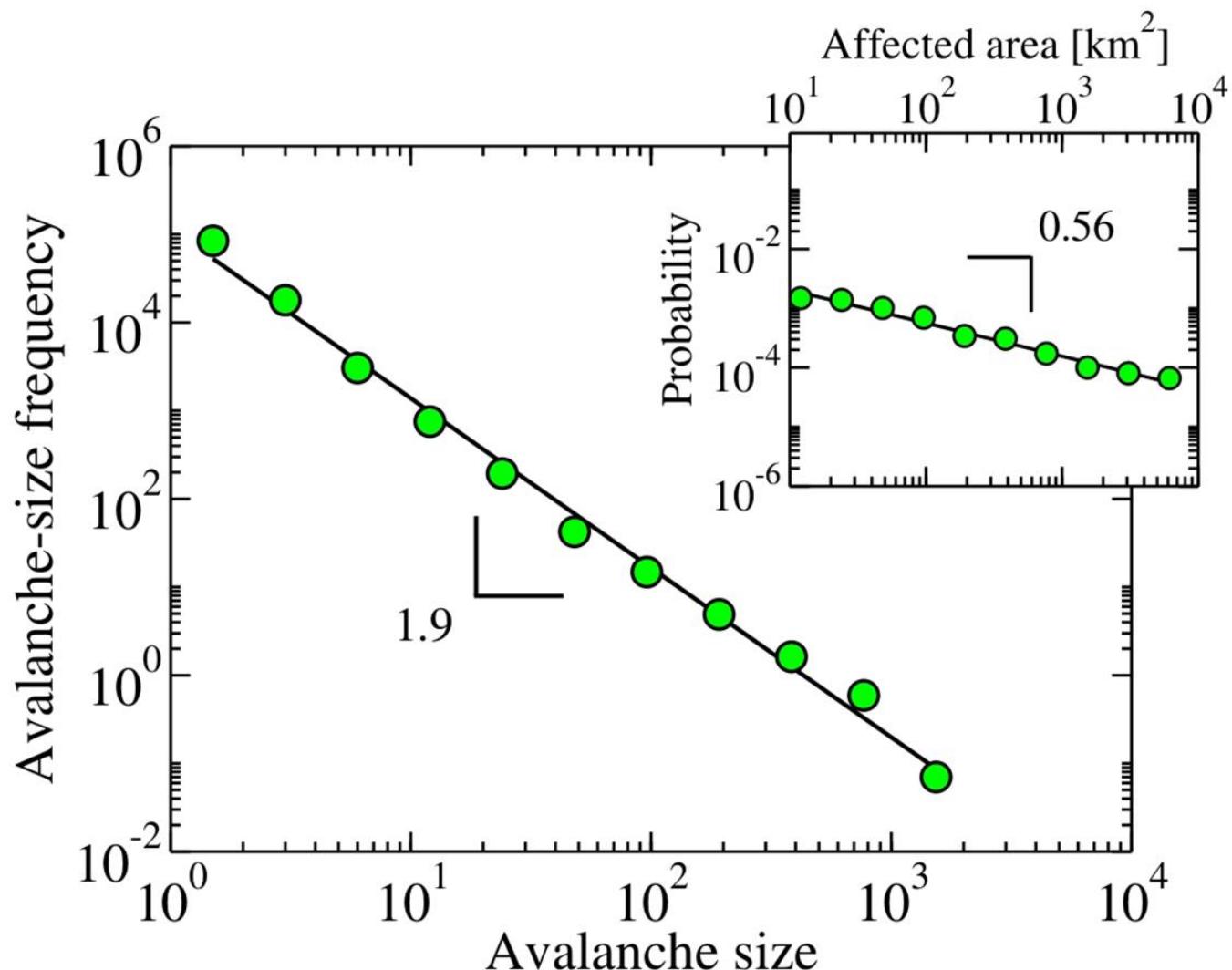
# *Climatic data*



# *The hydrological model*



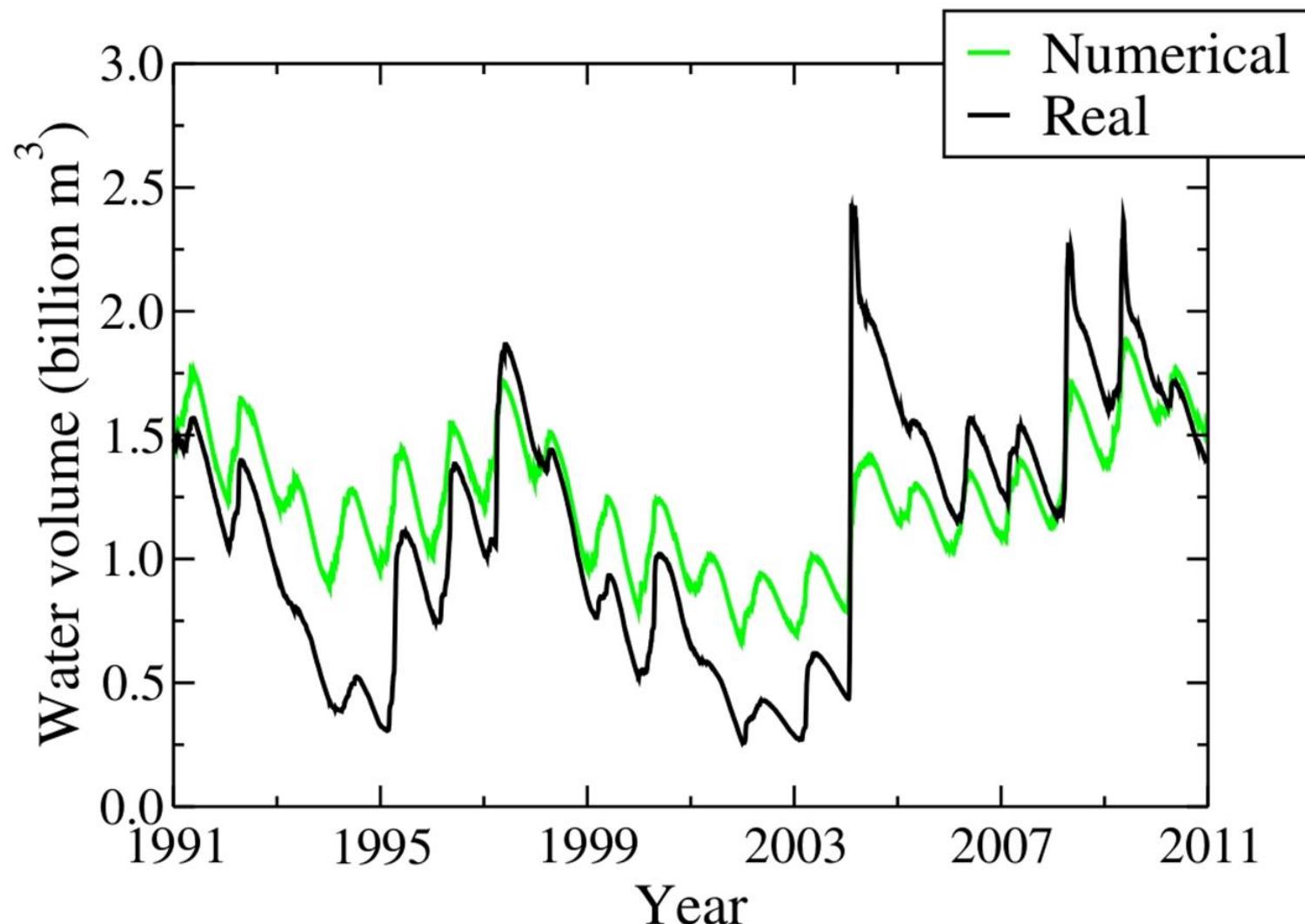
# Avalanche-size distribution



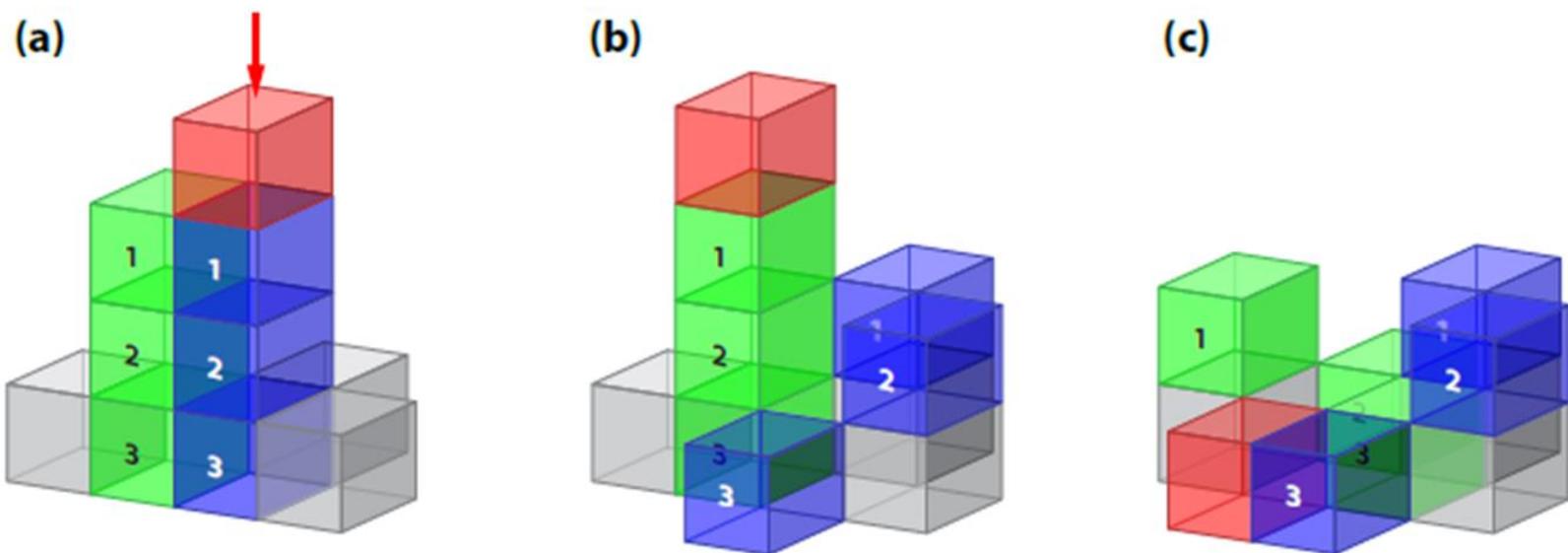
# *Reservoir water level*



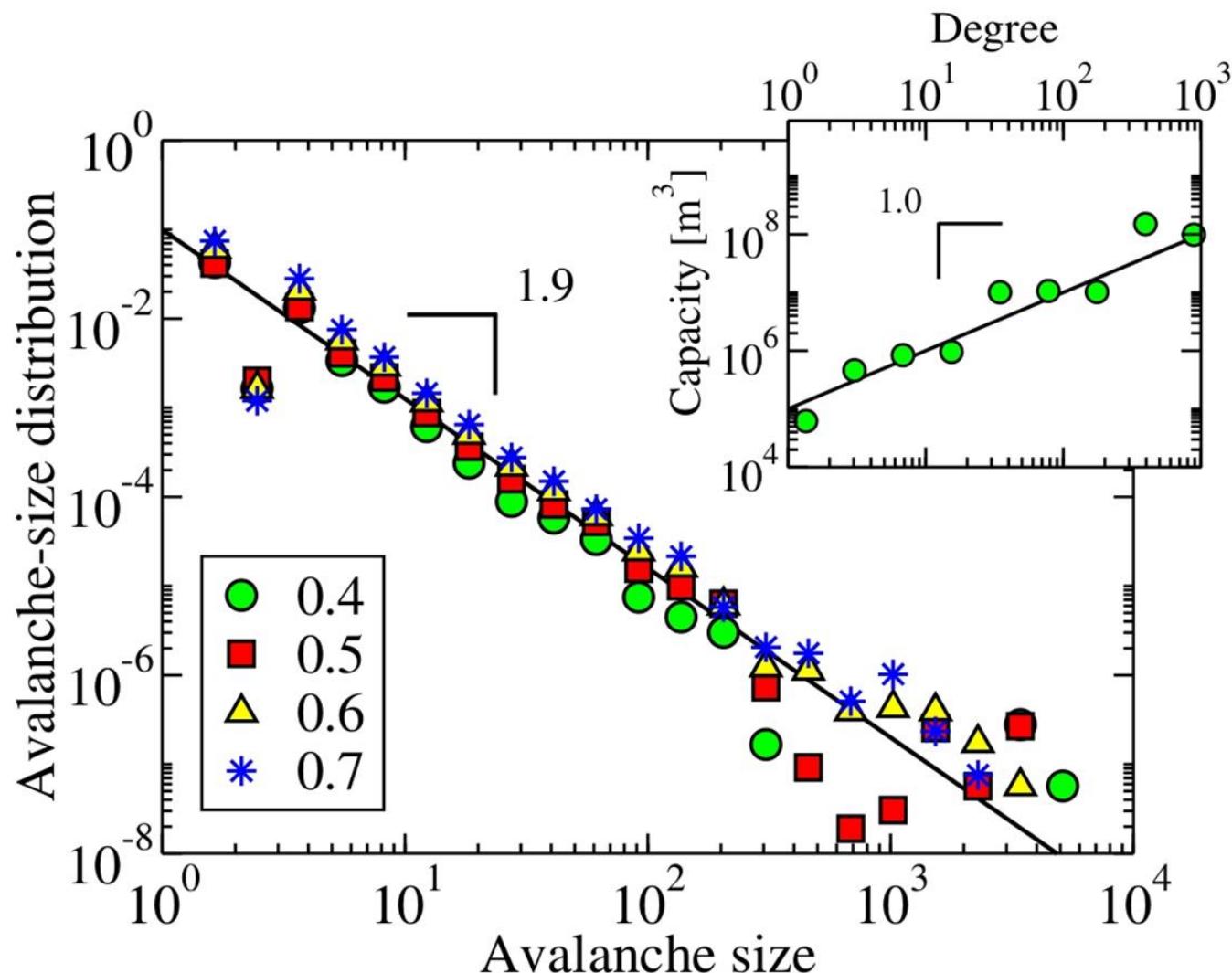
# *Reservoir water level*



# Stochastic model



# Stochastic model



# *Summary*

- We found unexpected **scale-free behavior** in the **dynamics** of water reservoirs;
- The **network** of reservoirs has a **tree-like structure** with **power-law distribution of capacities**;
- The **hydrological model** reveals an excellent **qualitative** and, in most cases, **quantitative agreement**;
- The **stochastic model** **grasps** the **main features**.

*World Airline Network*

# *Networks can fail to work*

## Eyjafjallajökull



107,000 flights cancelled during an 8 day period, accounting for 48% of total air traffic and roughly 10 million passengers

ItoWorld, <http://vimeo.com/11739091>

What is the **structure** of the **WAN** and how **resilient** is it?

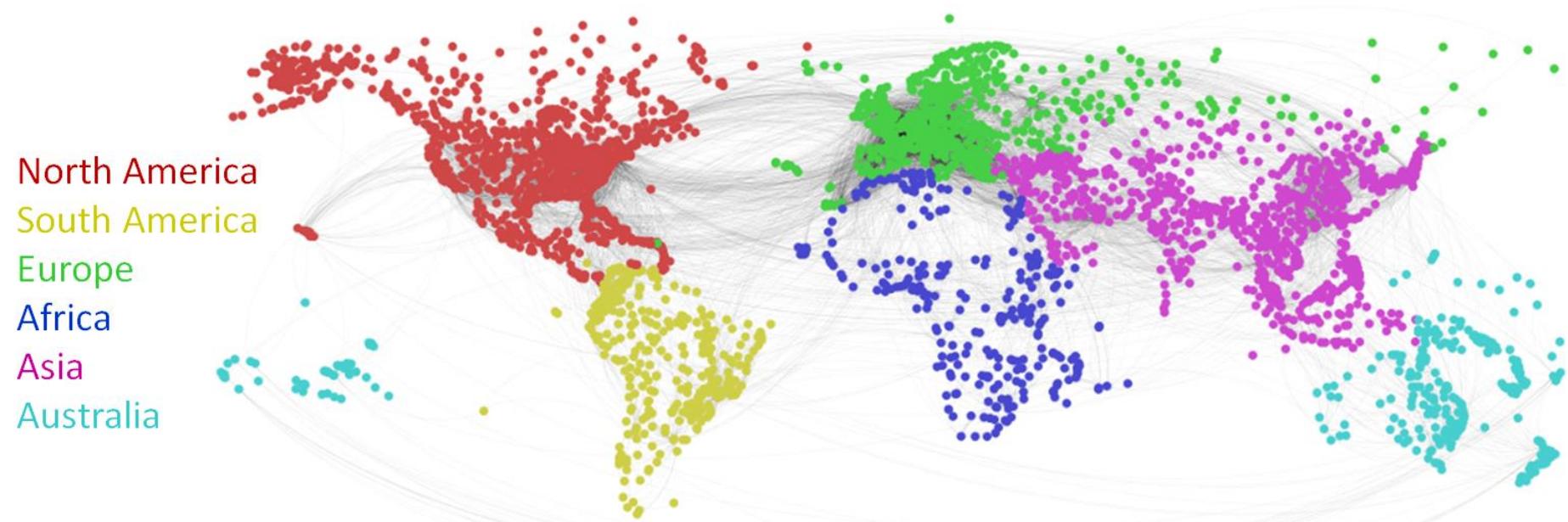
# *World Airline Network (WAN)*

Airports	3237
Connections	18125
Flights	62202

$$\langle k^w \rangle = 19.21$$

$$\langle l \rangle = 4.05$$

$$l_{\max} = 12$$

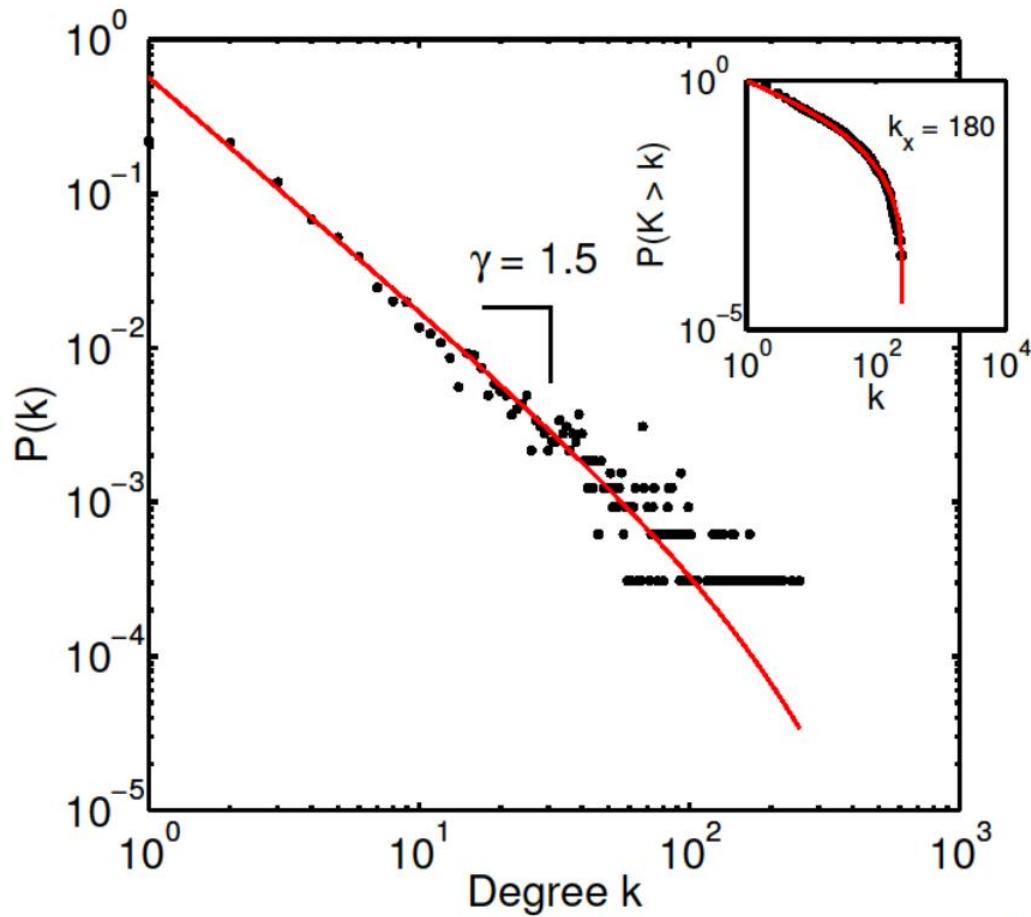


$$\gamma = 1.5 \pm 0.1$$

$$k_x = 180 \pm 5$$

# Degree Distribution

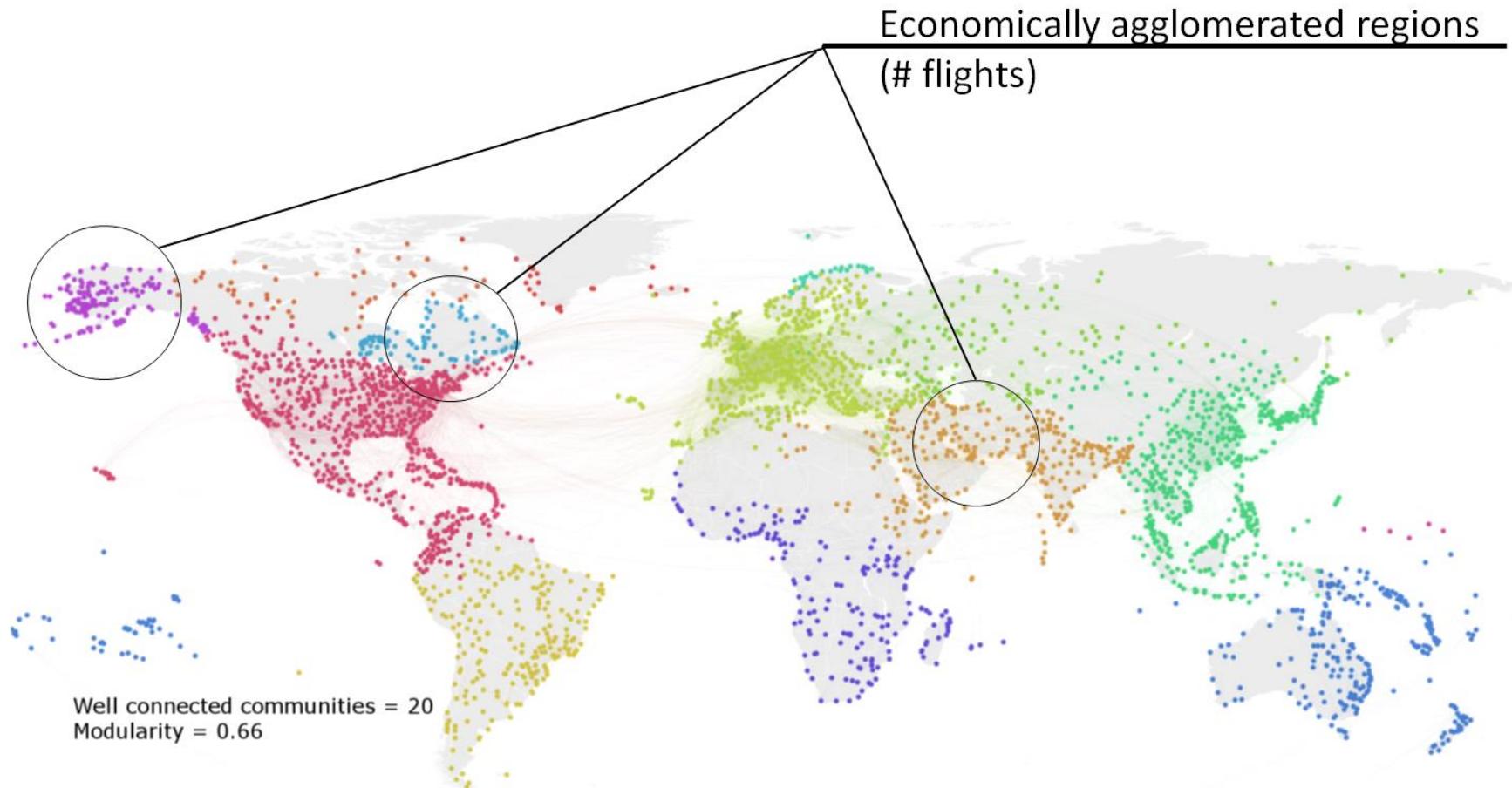
$$P(k) \sim k^{-\gamma} \exp(-k/k_x)$$



$$P(K > k) = 1 - \int_0^k P(k') dk'$$

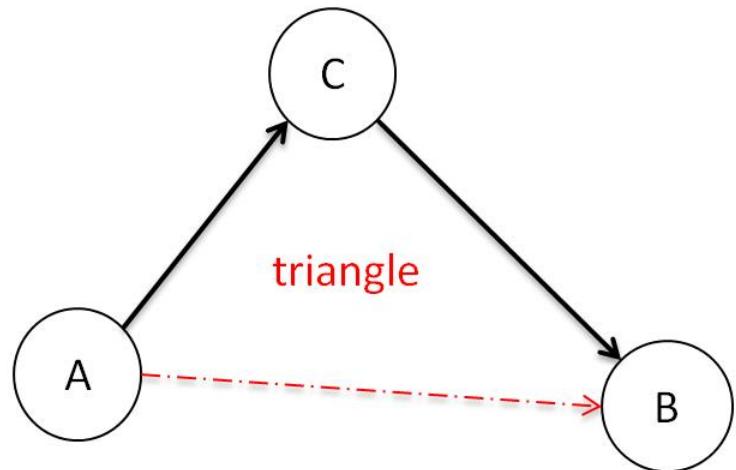
# Communities

*High modularity indicates presence of well formed communities*



- t = 0
- t = 1
- + - ?

# *t*-Core decomposition



# *t*-Core decomposition

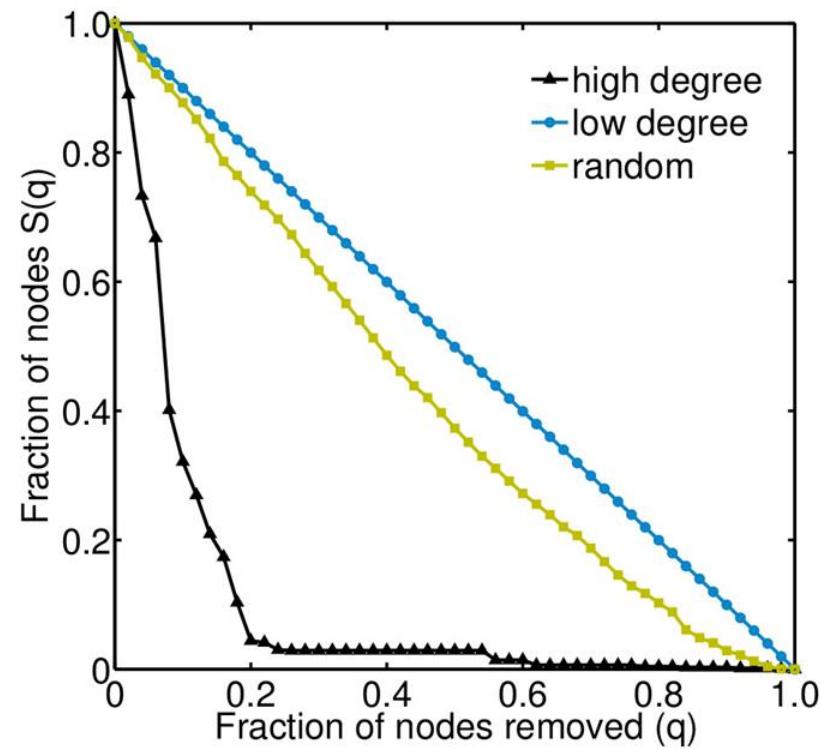
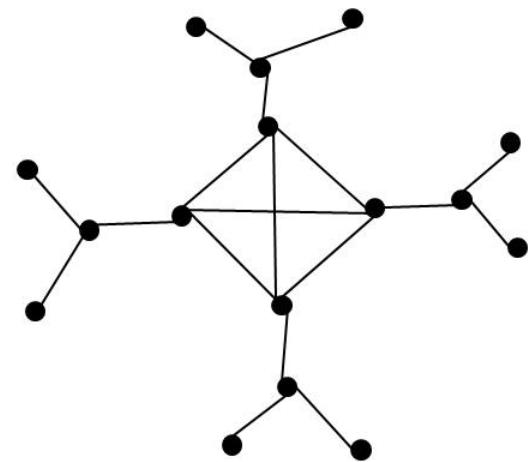


- 8.5% fall out of the cluster  
*Highly redundant connections*

# Disruptions

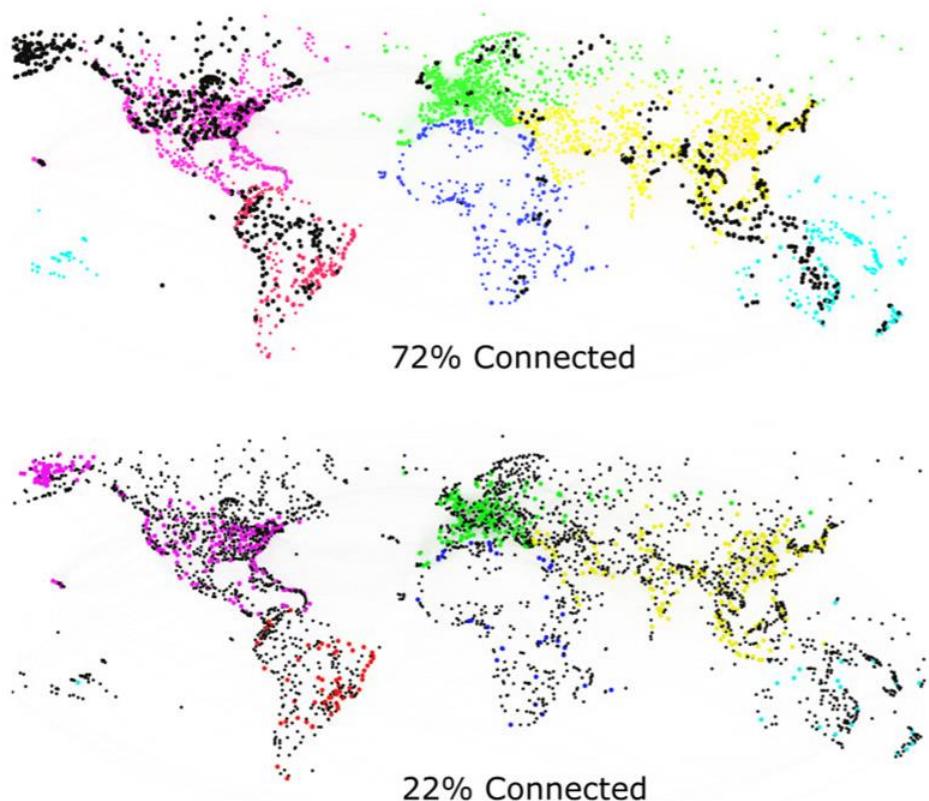
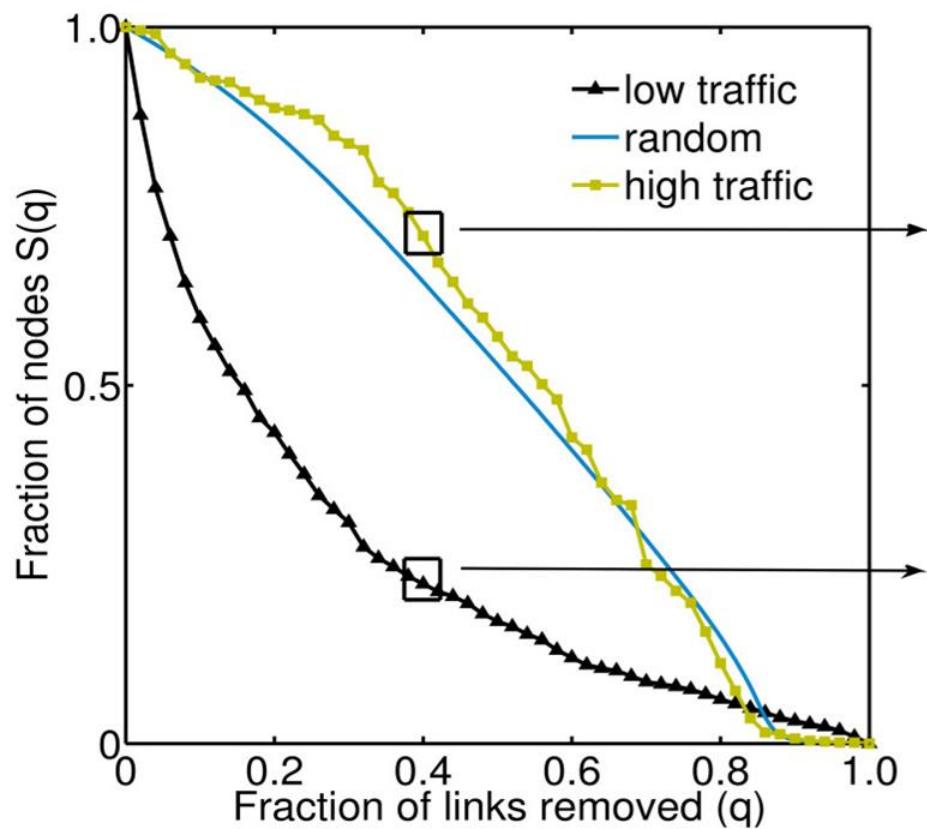
## Global Connectivity – $S(q)$

- Fraction of airports in the largest connected cluster
- $S(0) = 1$ , connected WAN
- $q$  : removal of elements
  - Airports ([usual behavior](#))
  - Connections

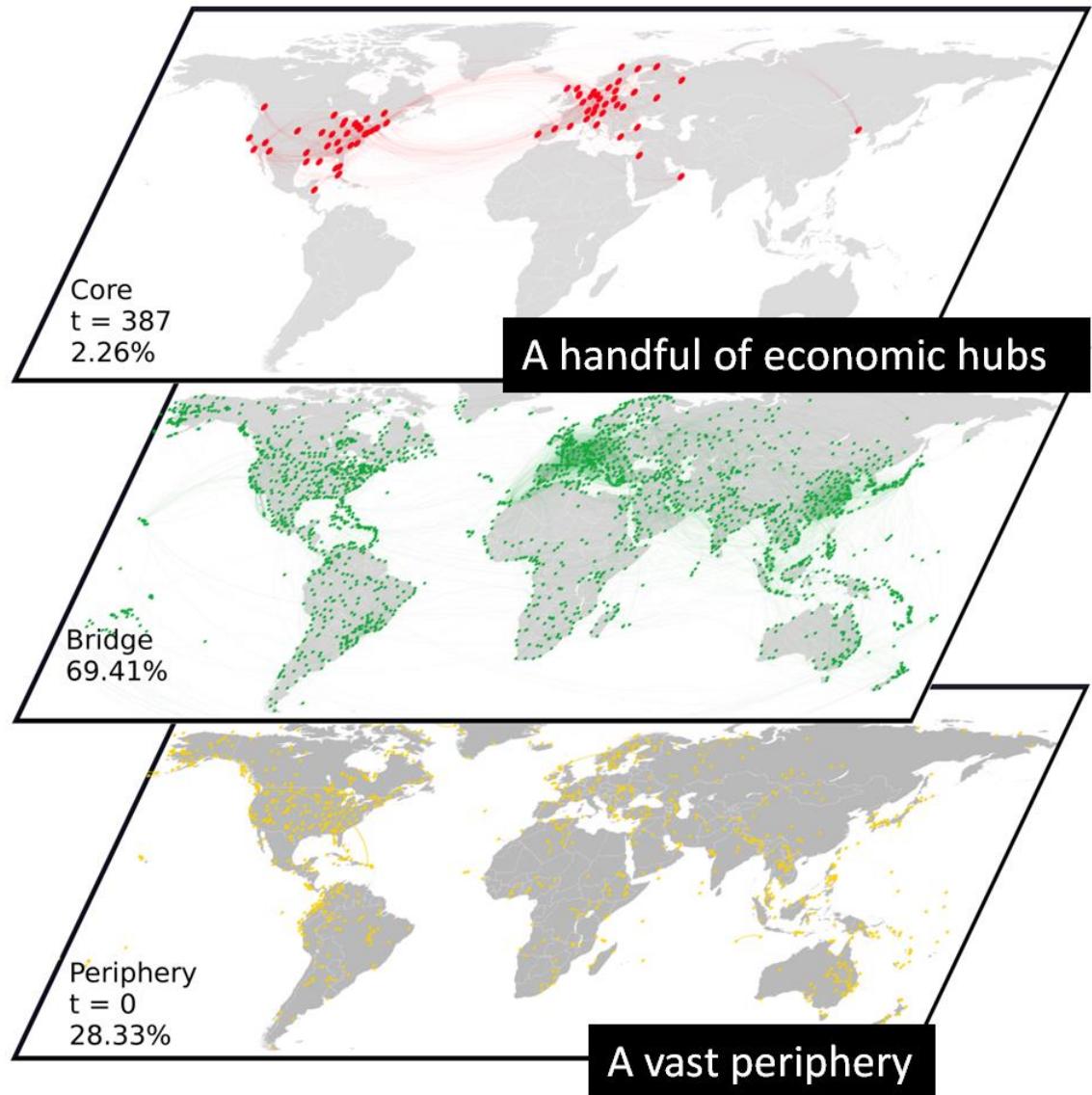


# Results

*Low Traffic* removal causes more damage to the network

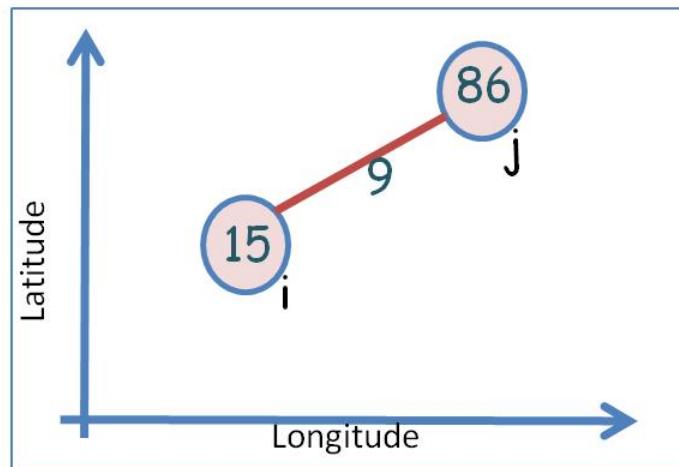


# *The structure of the World Airline Network*

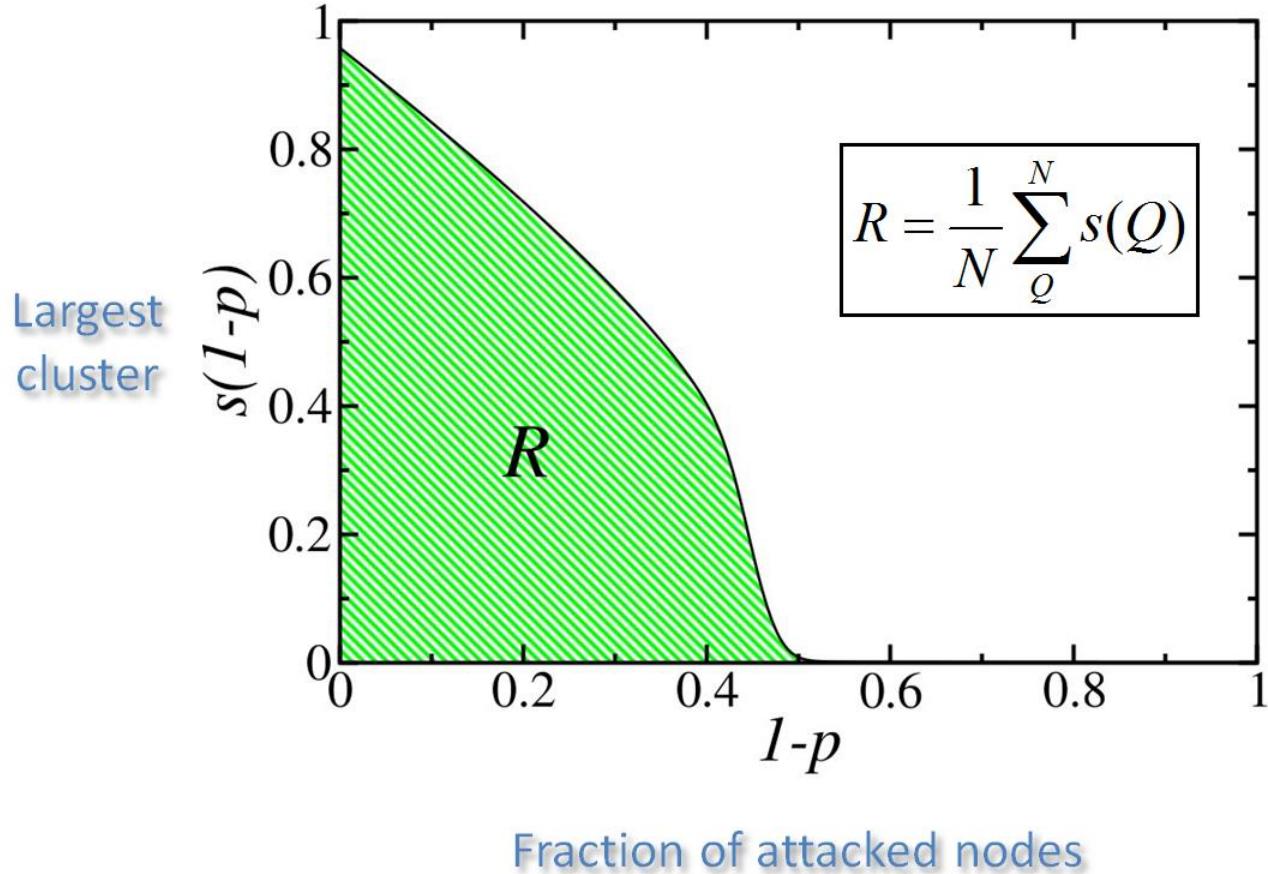


# *Improve the resilience*

- Nodes are airports;
- Node weight is the number of passengers;
- Edge weight is the number of flights;
- We have geographical coordinates.



# Network *Robustness*



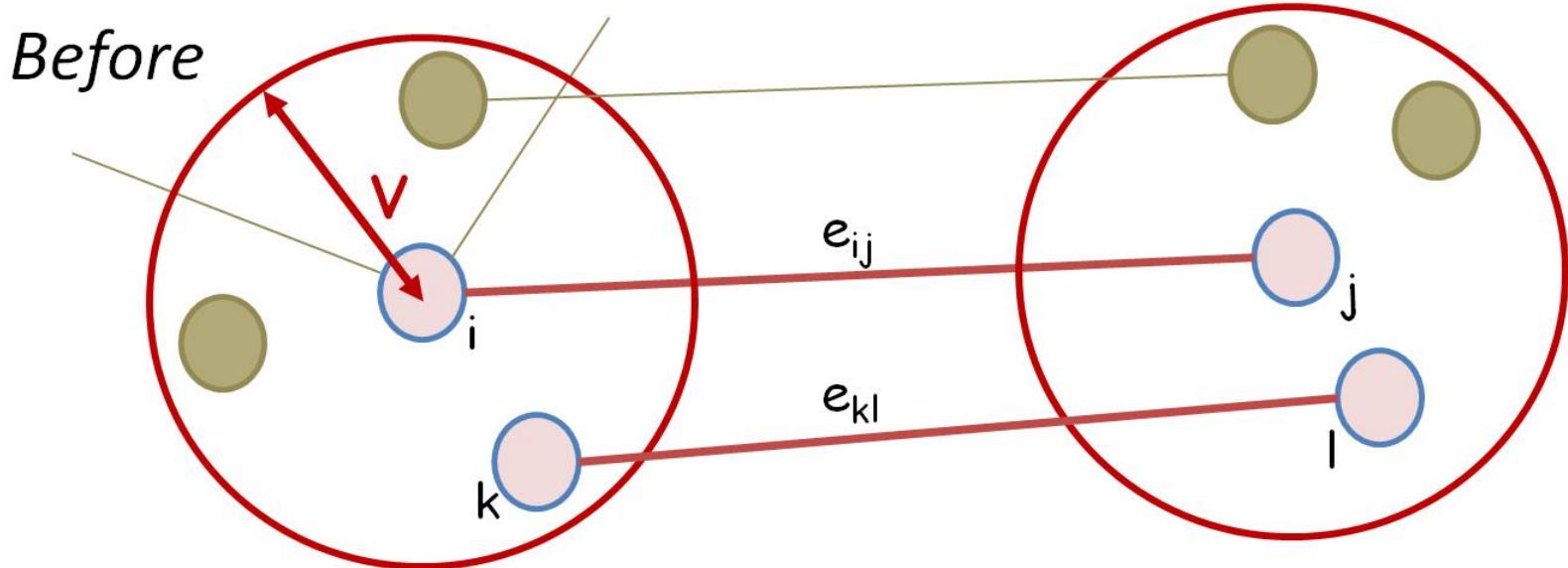
*Robust networks attend to many passengers even in face of a threat*

People in the largest connected component

Deleting the node with highest degree

$$Robustness = \frac{1}{\Pi(0)} \sum_{i=1}^N \Pi(i)$$

Delete the highest connected node (the hub), calculate the number of nodes passengers in the largest connected component, find the new hub, and repeat.

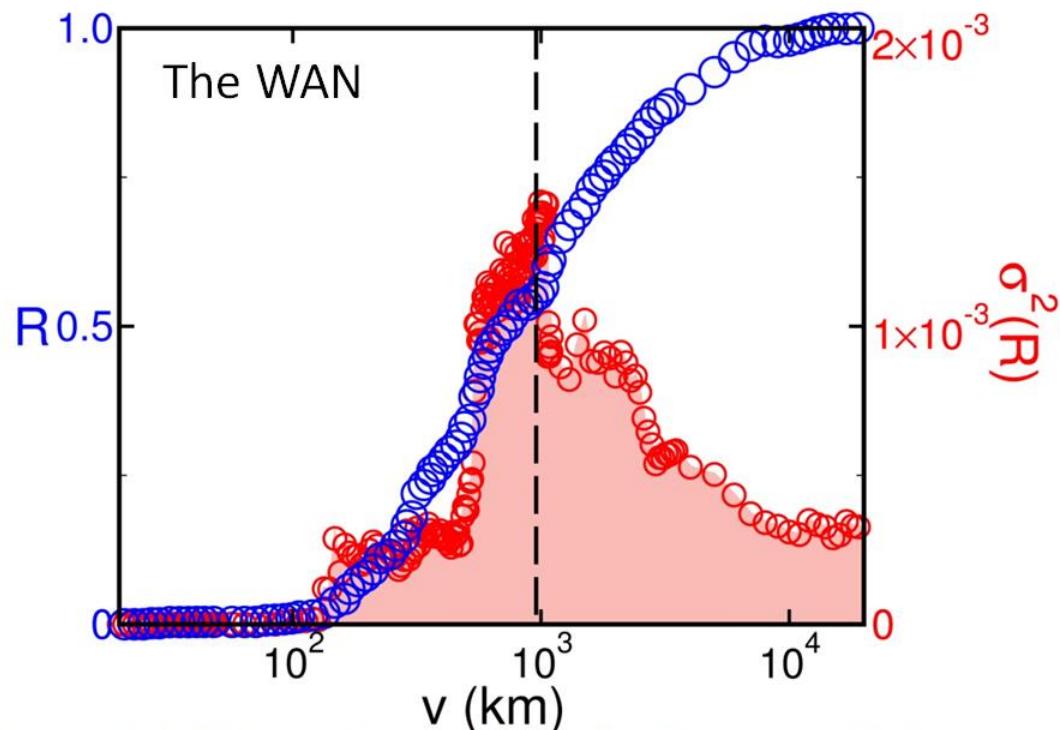
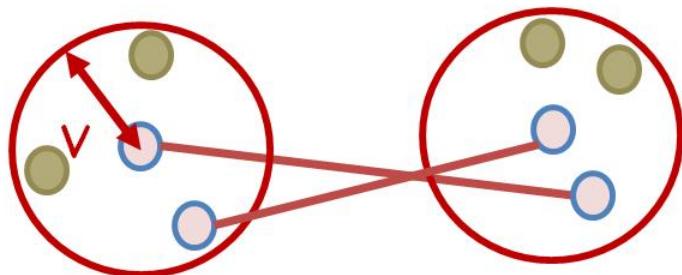


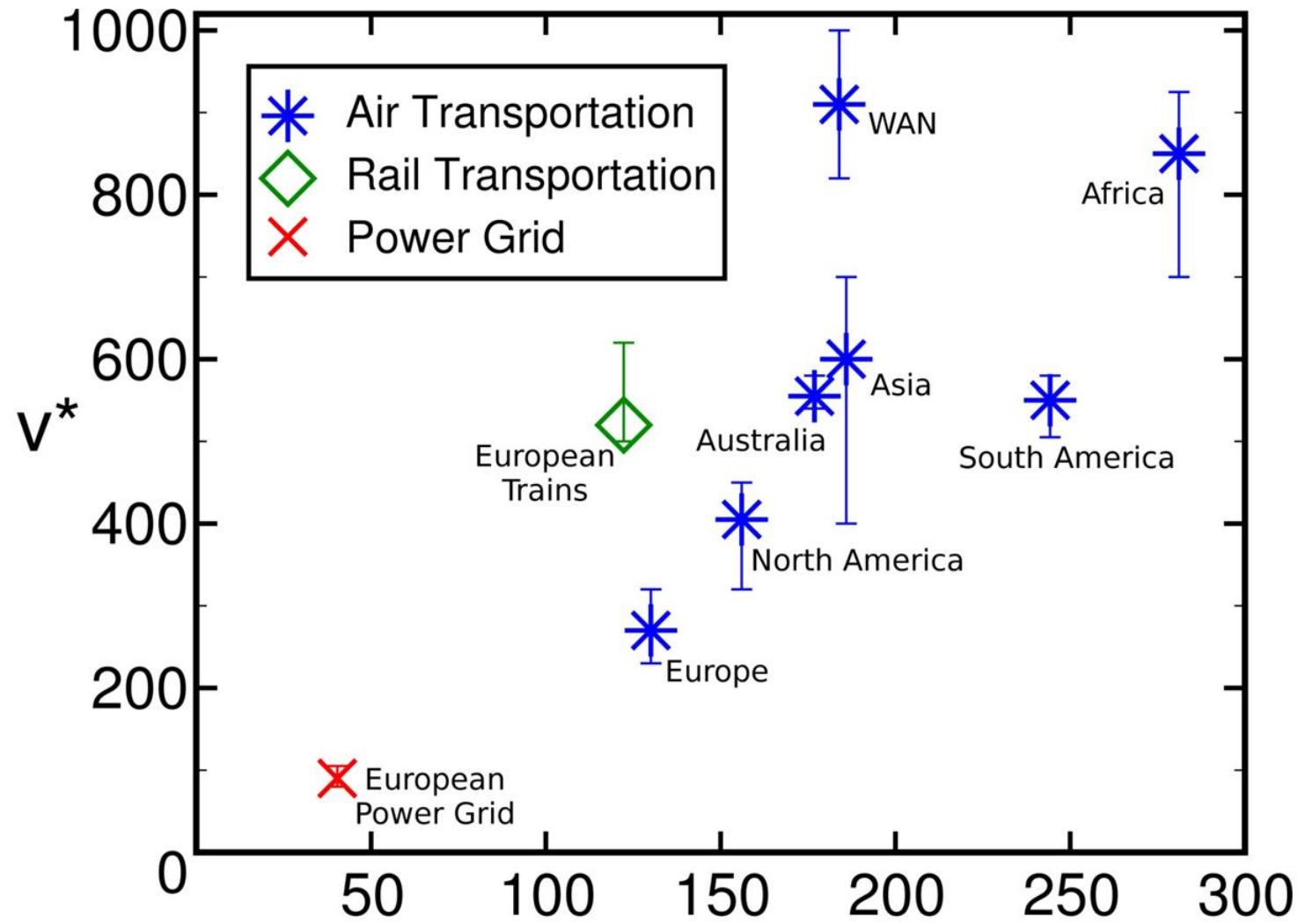
$$P(e_{ij}) \propto \frac{1}{\text{weight}(e_{ij})} \quad P(e_{kl}) \propto \frac{1}{\text{weight}(e_{kl})}$$

*After*

- If  $R_{\text{after}} > R_{\text{before}}$ , keep it. Otherwise, unswap.

- Once you land in a different airport, take a car to the original destination;
- The cooperation range  $v$  should be small in Europe, but large in South America;

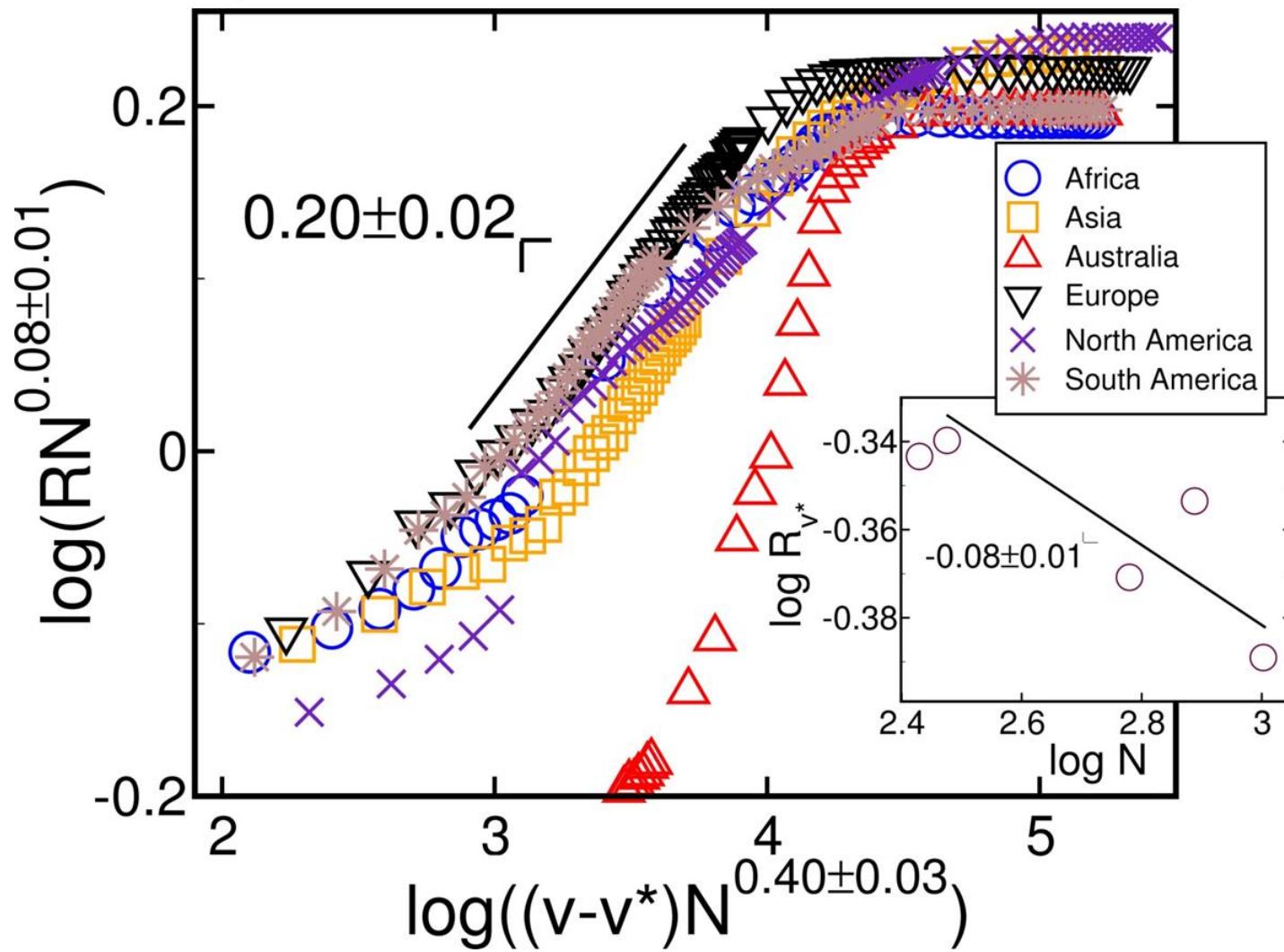




$$\sqrt{\frac{\text{area of the continent}}{\text{number of airports}}}$$

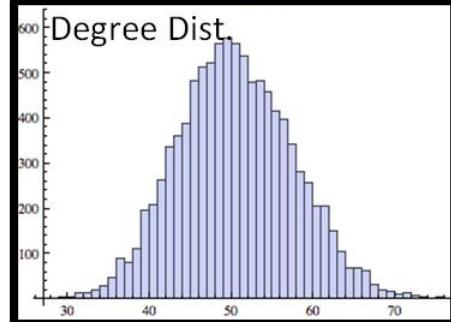
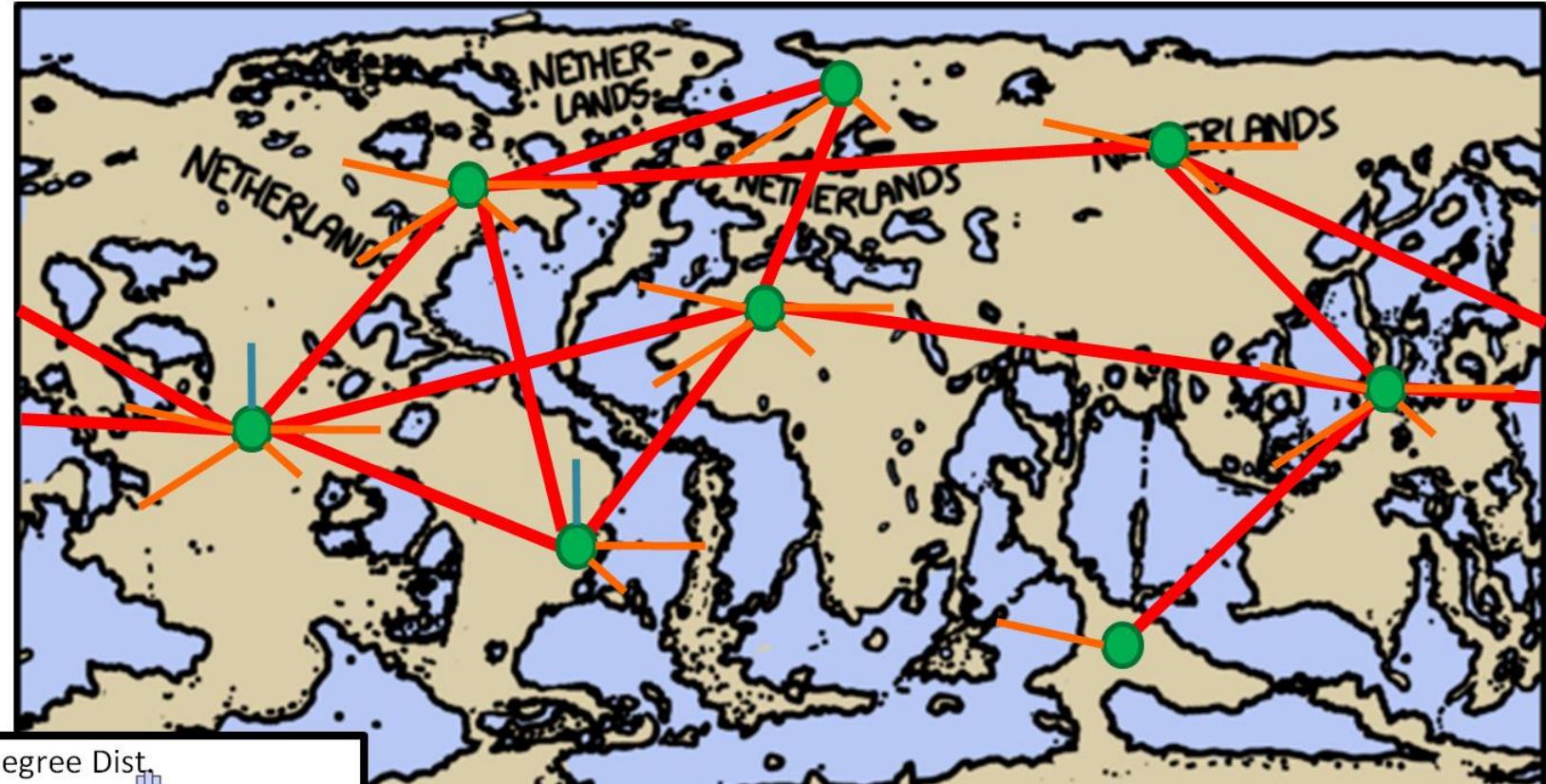
*All continents*

*data collapse «works»*



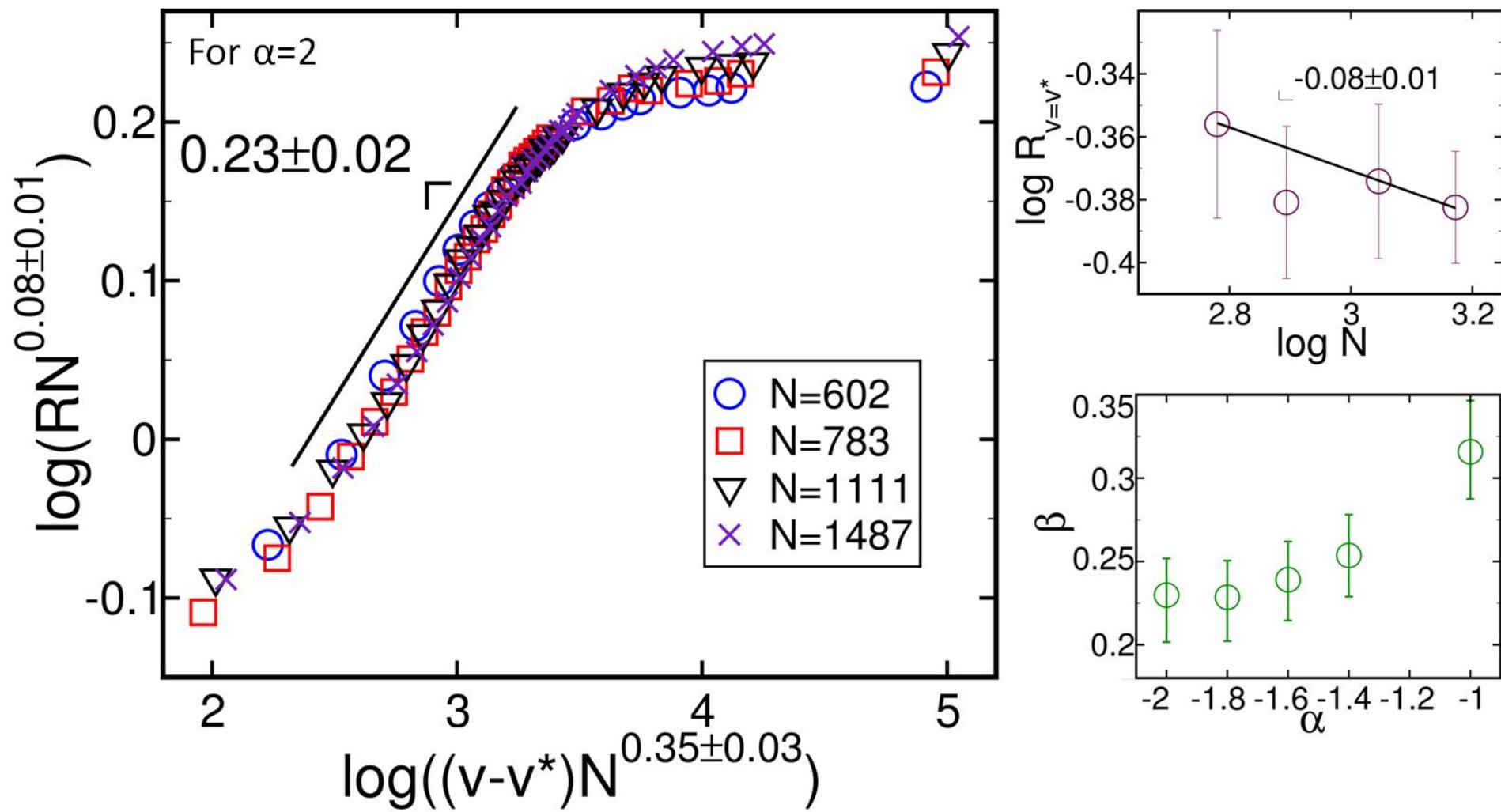
*It turns out that a random model like this:*

A random world created if the oceans were drained 5km

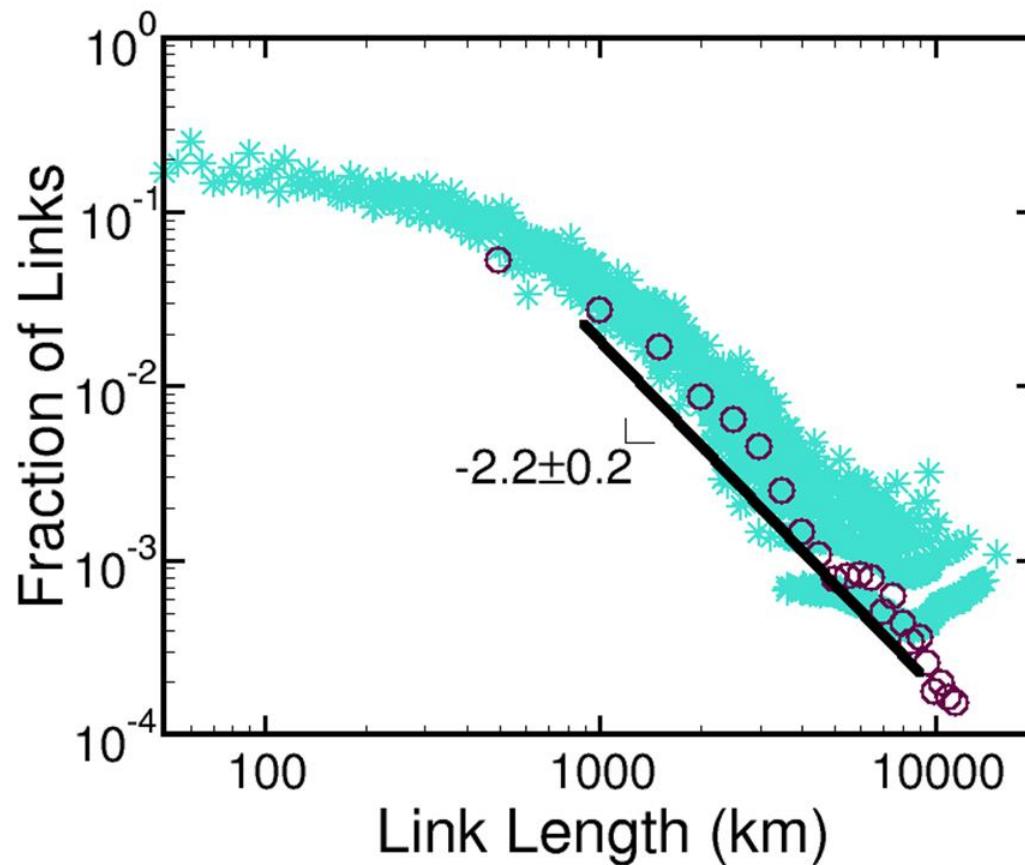


$$P(i, j) \propto \frac{1}{(\text{distance}(i, j))^\alpha}$$

# *Artificial WAN*



$\alpha=2$ : as we also find in the WAN



# Summary

- A small core resilient structure (guarantees intercontinental connections) and many peripheral connections;
- There is a critical cooperation range for effective improvement;
- Swaps are an effective and economically viable way of improving it.