Some multiplex dynamics that I find interesting

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SUPERDIFFUSION
Opinion dynamics in multiplex networks: Reaction-diffusion systems and synchronisation
\[
\frac{dx_i^{[\alpha]}}{dt} = D^{[\alpha]} \sum_j a_{ij}^{[\alpha]} (x_j^{[\alpha]} - x_i^{[\alpha]}) + D_x (x_i^{[\beta]} - x_i^{[\alpha]})
\]
Opinion dynamics

Reaction-diffusion systems

Synchronisation

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Multiplex networks

\[
\frac{dx_i^{[\alpha]}}{dt} = D^{[\alpha]} \sum_j a_{ij}^{[\alpha]} (x_j^{[\alpha]} - x_i^{[\alpha]}) + D_x (x_i^{[\beta]} - x_i^{[\alpha]})
\]
\[ \frac{dx_i^{[\alpha]}}{dt} = D^{[\alpha]} \sum_j a_{ij}^{[\alpha]} (x_j^{[\alpha]} - x_i^{[\alpha]}) + D_x (x_i^{[\beta]} - x_i^{[\alpha]}) \]

\[ \dot{x} = -\mathcal{L}\dot{x} \]

\[ \mathcal{L} = \begin{pmatrix} D^{[\alpha]} L^{[\alpha]} + D_x I & -D_x I \\ -D_x I & D^{[\beta]} L^{[\beta]} + D_x I \end{pmatrix} \]

**Where** \( L^{[\alpha]}, L^{[\beta]} \) **are the Laplacians of the two layers**
\[ \dot{x} = -\mathcal{L} \dot{x} \]

**DIFFUSION TIME-SCALE**

\[ \tau = \frac{1}{\lambda_{\text{min}}} \]

*where* \( \lambda_{\text{min}} \) *is the smallest non-zero eigenvalue of* \( \mathcal{L} \)
Opinion dynamics and reaction-diffusion systems in multiplex networks.
Opinion dynamics and reaction-diffusion systems in Multiplex networks.

\[ D_x \approx 0 \quad \lambda_{\min} = \min(\lambda_2^\alpha, \lambda_2^\beta) \]
Opinion dynamics and reaction-diffusion systems in multiplex networks.

\[ D_x \simeq 0 \quad \lambda_{\text{min}} = \min(\lambda_2^\alpha, \lambda_2^\beta) \]

\[ D_x \to \infty \quad \lambda_{\text{min}} = \frac{\lambda_s}{2} \geq \frac{\lambda_2^\alpha + \lambda_2^\beta}{2} \geq \min(\lambda_2^\alpha, \lambda_2^\beta) \]
ISING MODEL
Opinion dynamics

Reaction-diffusion systems

Synchronisation

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Multiplex networks
SPIN on layer 1

Coherence

SPIN on layer 2

\[ F_i^{[\alpha]} = J_i \sum_{j=1}^{N} a_{ij}^{[\alpha]} s_j^{[\alpha]} + \gamma \frac{\chi_i}{J_i} \sum_{\beta \neq \alpha}^{M} s_i^{[\beta]} + h_i^{[\alpha]} \]

Relative weight of coherence
Opinion dynamics

Reaction–diffusion systems

Synchronisation

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VOTER MODEL
Opinion dynamics Reaction-diffusion systems Synchronisation

Vincenzo Nicosia Multiplex networks
Opinion dynamics, Reaction-diffusion systems, Synchronisation, Multiplex networks.

\[ N(1 - q) \]
Opinion dynamics Reaction-diffusion systems Synchronisation

\[ q \]

\[ \mu^{\text{aggr}}_1 \]

\[ \omega=0.0 \]
\[ \omega=0.3 \]
\[ \omega=0.7 \]
\[ \omega=1.0 \]

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Multiplex networks
Opinion dynamics Reaction-diffusion systems Synchronisation

\[ F(\omega) \]

\[ d=2 \]
\[ d=3 \]
\[ d=4 \]

\[ \omega \]

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AXELROD MODEL
Opinion dynamics Reaction-diffusion systems Synchronisation

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(a) classical social influence model
(b) layered social influence model

active bond frozen bond
Opinion dynamics Reaction-diffusion systems Synchronisation

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size of largest component

log time to steady state

A

B

C

D

0.001 0.01 0.1 1.0

p

0.001 0.01 0.1 1.0

p

0.001 0.01 0.1 1.0

p
TURING PATTERNS
You could have had here a nice slide with hawks and hares, and another slide with some of the beautiful patterns formed by the skin of fish and other animals.

BUT I was actually brutally forced to go out for a dessert yesterday night, and since I don't like desserts, we ended up drinking cerveza nacional and talking of Yom Kippur, knowledge, atheism, Monty Python, and other amenities, while a few of us tried to explain to the waiter that waffles and bananas cannot stay in the same plate......then I fell asleep...
Opinion dynamics

Reaction-diffusion systems

Synchronisation

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Multiplex networks

Activator: $u_i(t)$

Inhibitor: $v_i(t)$
Reaction-diffusion systems

Opinion dynamics

Synchronisation

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\[ \begin{align*}
\frac{du(t)}{dt} &= F(u(t), v(t)) + \sigma[1] L[1] u(t) \\
\frac{dv(t)}{dt} &= G(u(t), v(t)) + \sigma[2] L[2] v(t)
\end{align*} \]
Linear Stability:

\[ J = \begin{pmatrix} L^{[1]} + f_v I & f_v I \\ g_u I & \sigma (L^{[2]} + g_v I) \end{pmatrix} \]
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\[ J = \begin{pmatrix} L^{[1]} + f_u I & f_v I \\ g_u I & \sigma (L^{[2]} + g_v I) \end{pmatrix} \]

\[ \lambda_1 < \lambda_2 < \ldots < \lambda_N = \lambda_M \]
Linear Stability:

\[
J = \begin{pmatrix}
L^{[1]} + f_u I & f_v I \\
g_u I & \sigma (L^{[2]} + g_v I)
\end{pmatrix}
\]

\[\lambda_1 < \lambda_2 < \ldots < \lambda_N = \lambda_M\]

Amplitude:

\[A = \sqrt{\sum_i (u_i - \bar{u})^2 + (v_i - \bar{v})^2}\]
What if we tune inter-layer correlations?

\[ \rho_{\alpha,\beta} = \frac{\sum_i \left(R_i^{[\alpha]} - \overline{R}[\alpha]\right) \left(R_i^{[\beta]} - \overline{R}[\beta]\right)}{\sqrt{\sum_i \left(R_i^{[\alpha]} - \overline{R}[\alpha]\right)^2 \sum_j \left(R_j^{[\beta]} - \overline{R}[\beta]\right)^2}} \]
Opinion dynamics

Reaction-diffusion systems

Synchronisation

\[ |v_i| \]

\( \text{node (i)} \)

\[ 0 200 400 600 800 1000 \]

\[ 0 0.2 0.4 0.6 0.8 \]

\[ |v_i| \]

\( \text{STABLE} \)

\( \text{UNSTABLE} \)

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Multiplex networks
Opinion dynamics

Reaction-diffusion systems

Synchronisation

\[ r_i = \frac{k_i^{[2]}}{k_i^{[1]}} \]

\[ \sim r^{-2.2} \]
SYNCHRONISATION
The "multiplex" brain
The "multiplex" brain

Neurons
(Activity)
The "multiplex" brain

Neurons (Activity) + Blood vessels (Energy)
Opinion dynamics, Reaction-diffusion systems, Multiplex networks.

Activity

Energy transport

Multiplex Prism
Opinion dynamics and reaction–diffusion systems

V. Nicosia, P. S. Skardal, V. Latora, A. Arenas. under review
Opinion dynamics on reaction-diffusion systems

V. Nicosia, P. S. Skardal, V. Latora, A. Arenas, under review

\[ \dot{\varphi}_i(t) = \omega_i + \lambda \sum_j a_{ij} \sin(\varphi_j - \varphi_i) \]
Opinion dynamics in reaction-diffusion systems

Activity

Kuramoto Dynamics

\[ \dot{\phi}_i(t) = \omega_i + \lambda \sum_j a_{ij} \sin(\varphi_j - \varphi_i) \]

Energy transport

Biased Random Walk

\[ p_{i \rightarrow j} \propto e_{ij} f_j \]
Opinion dynamics in reaction-diffusion systems

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Kuramoto Dynamics

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Opinion dynamics in reaction-diffusion systems

\[ \dot{\varphi}_i(t) = \omega_i + \lambda \sum_j a_{ij} \sin(\varphi_j - \varphi_i) \]

(more energy \(\rightarrow\) higher frequency)

\[ \omega_i \propto p_i \]
\[ \dot{\varphi}_i(t) = \omega_i + \lambda \sum_j a_{ij} \sin(\varphi_j - \varphi_i) \]

(more energy -> higher frequency)

\[ \omega_i \propto p_i \]

(More synapses -> more blood)

\[ f_j = k_j^\alpha \]
Opinion dynamics and reaction-diffusion systems

Synchronization Layer

Synchronized cluster

Energy Transport Layer

Energy

Transport

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Kuramoto Dynamics

\[ \dot{\varphi}_i(t) = \omega_i + \lambda \sum_j a_{ij} \sin(\varphi_j - \varphi_i) \]

Biased Random Walk

\[ p_{i \rightarrow j} \propto e_{ij} f_j \]

Node state: \((\varphi_i, p_i)\)

\( \varphi_i \) phase (activity)

\( p_i \) fraction of walkers (energy)
Opinion dynamics in reaction-diffusion systems

\[ r \approx 0 \quad \text{Incoherent} \]

\[ r \approx 1 \quad \text{Synchronized} \]
Opinion dynamics and synchronization of Reaction-diffusion systems

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Opinion dynamics in Reaction-diffusion systems.

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CONCLUSIONS
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- **SOME** dynamical processes are BETTER UNDERSTOOD and studied as multiplex ones
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- **SOMETIMES** multiplex dynamics behave **IN A DIFFERENT WAY** w.r.t. their "monoplex" counterparts

- **SOME** dynamical processes are **BETTER UNDERSTOOD** and studied as multiplex ones

- **IN SOME CASES** multiplex dynamics exhibit original new physics, which is GENUINELY (due to the) **MULTIPLEX**


