Opinion dynamics on networks

Vincenzo (Enzo) Nicosia[1]

[1]School of Mathematical Sciences, Queen Mary University of London, UK

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Outline

1. Introduction
2. Simple models (social imitation)
3. A less simple model (bounded confidence)

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1. Introduction
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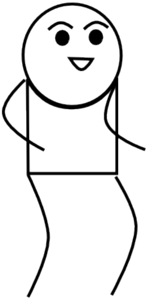

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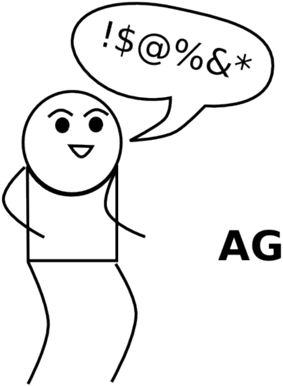

AGENT

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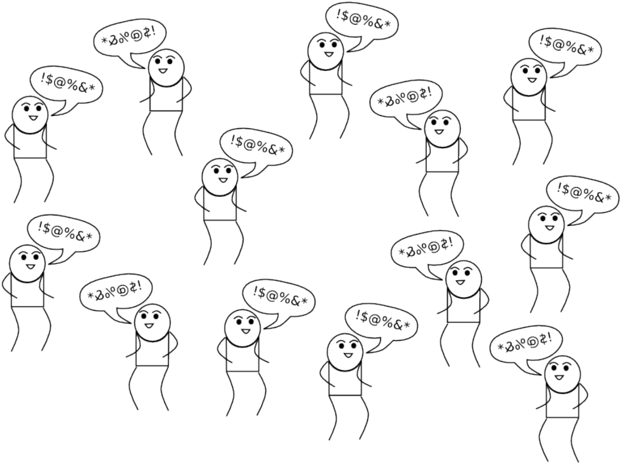

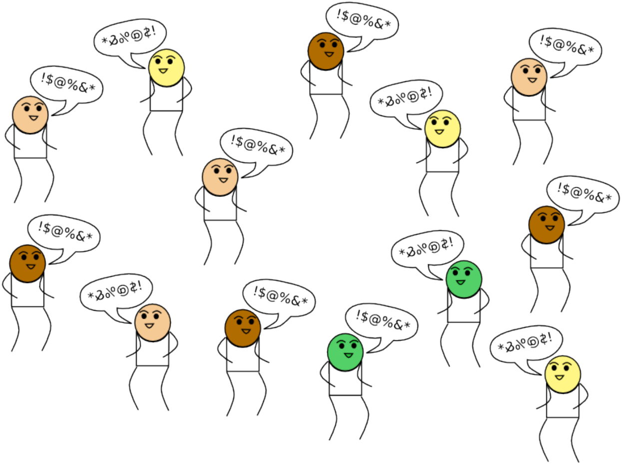

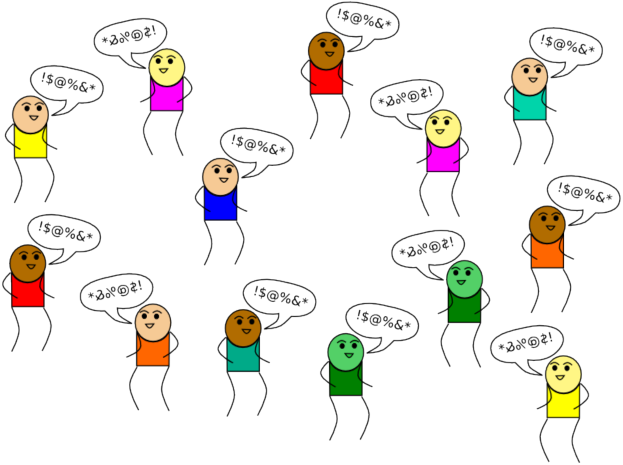

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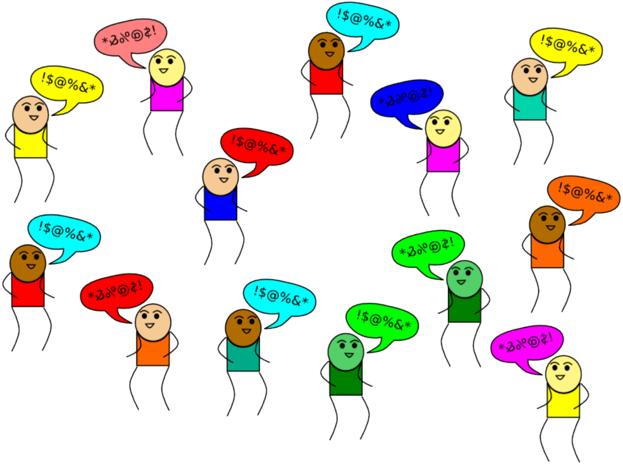
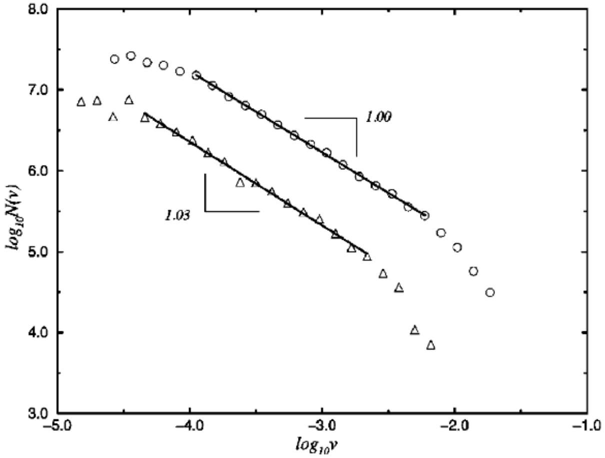
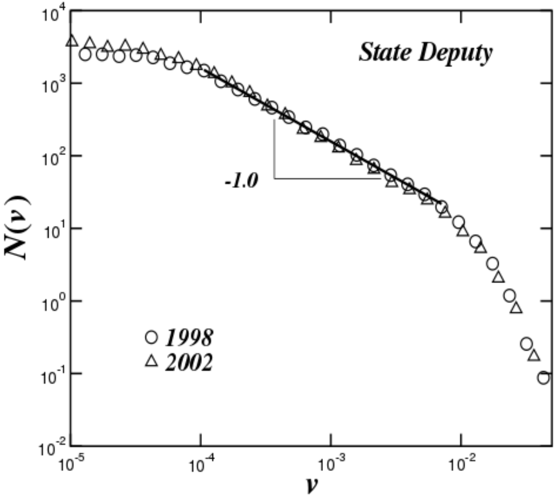
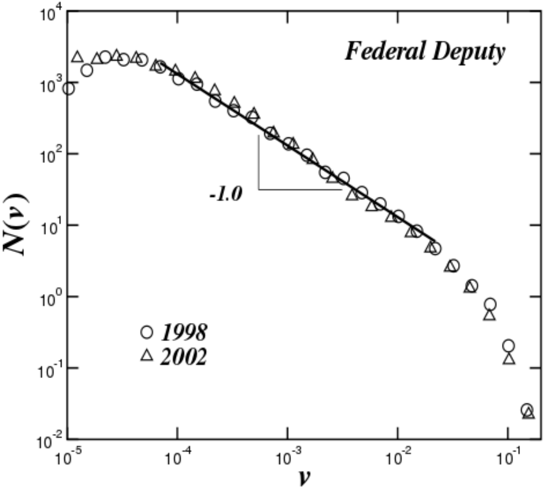
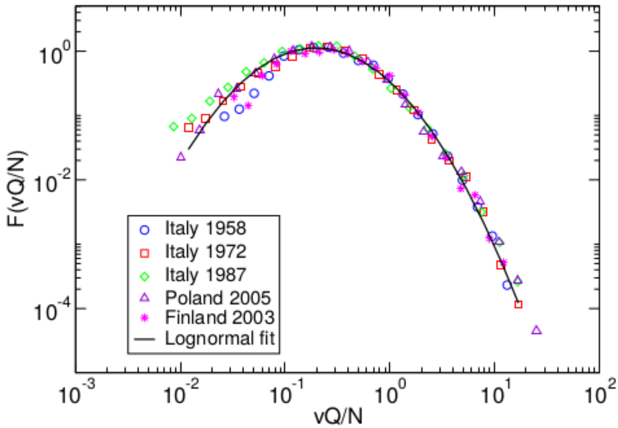




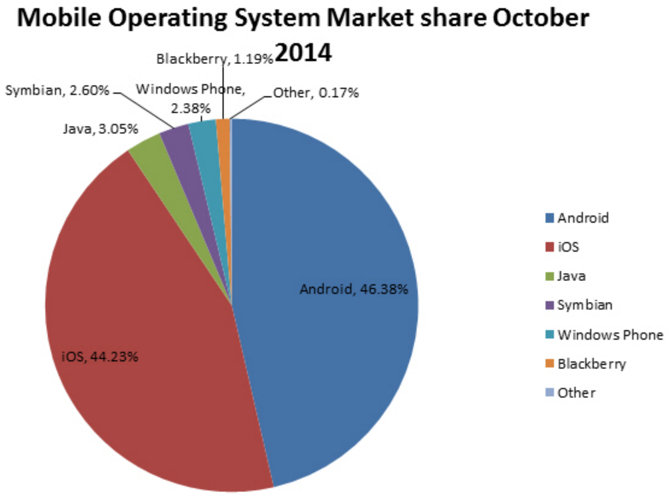
8.0
7.0
6.0
5.0
4.0
3.0
−5.0
−4.0
−3.0
−2.0
−1.0
$log_{10}N(\nu)$
$log_{10}\nu$
1.00
1.03

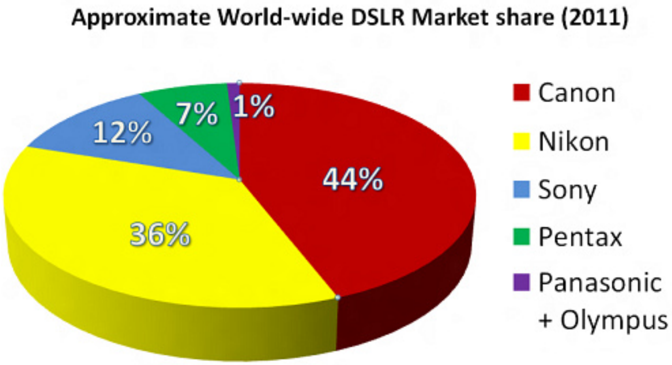

State Deputy
-1.0
1998
2002
N(v)
v
10^4
10^3
10^2
10^1
10^0
10^-1
10^-2
10^-5
10^-4
10^-3
10^-2

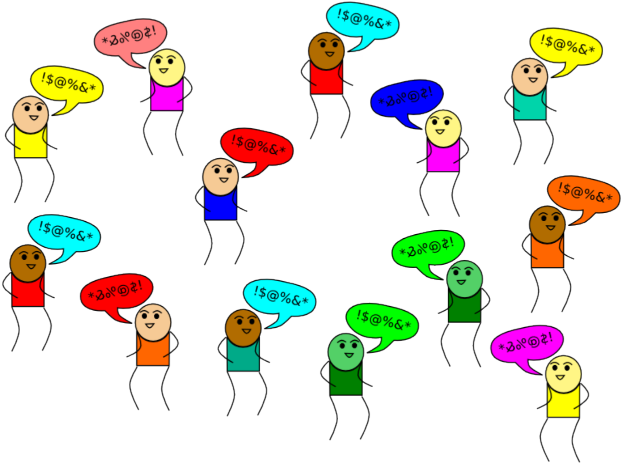

Federal Deputy
-1.0
1998
2002
N(v)
v

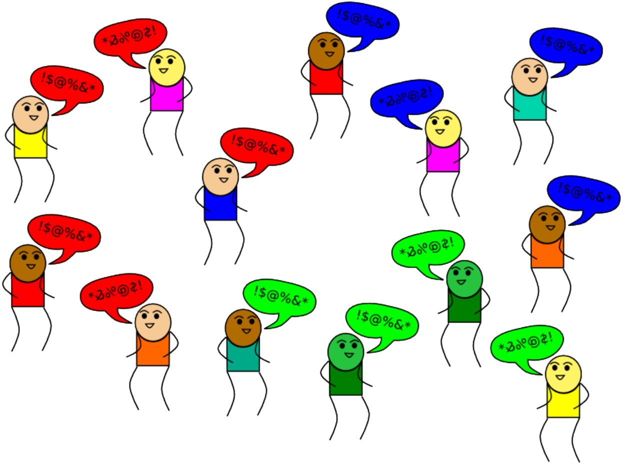


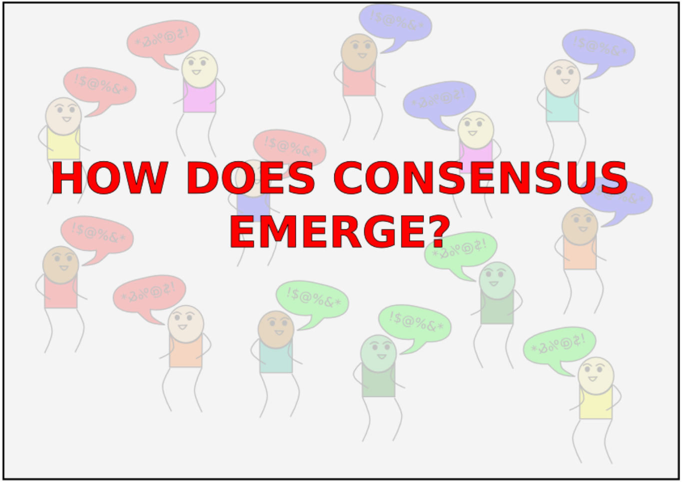
F(vQ/N)
10^0
10^{-2}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^0
10^1
10^2
vQ/N
Italy 1958
Italy 1972
Italy 1987
Poland 2005
Finland 2003
Lognormal fit

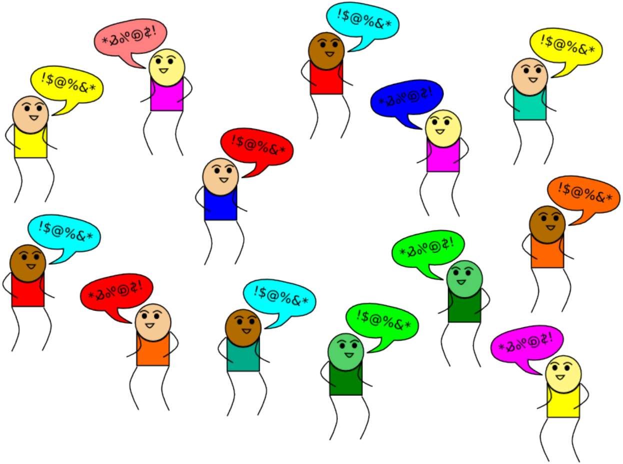

Mobile Operating System Market share October 2014
Blackberry, 1.19%
Symbian, 2.60%
Windows Phone, 2.38%
Other, 0.17%
Java, 3.05%
Android, 46.38%
iOS, 44.23%
Android
iOS
Java
Symbian
Windows Phone
Blackberry
Other


Approximate World-wide DSLR Market share (2011)
44%
36%
12%
7%
1%
Canon
Nikon
Sony
Pentax
Panasonic
+ Olympus






HOW DOES CONSENSUS EMERGE?



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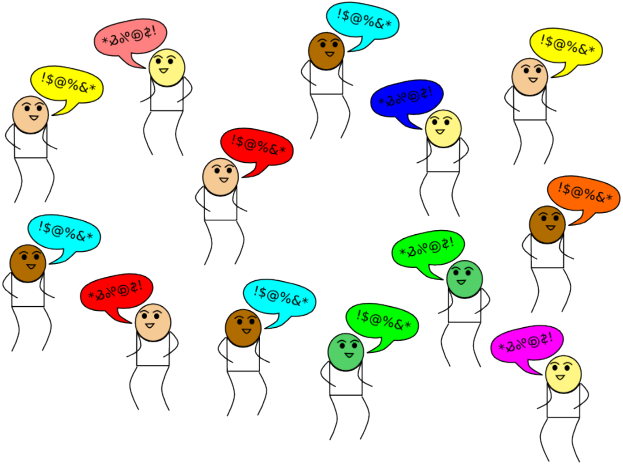

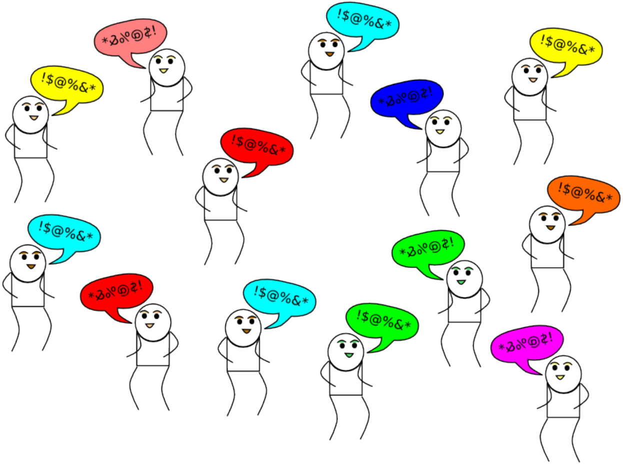

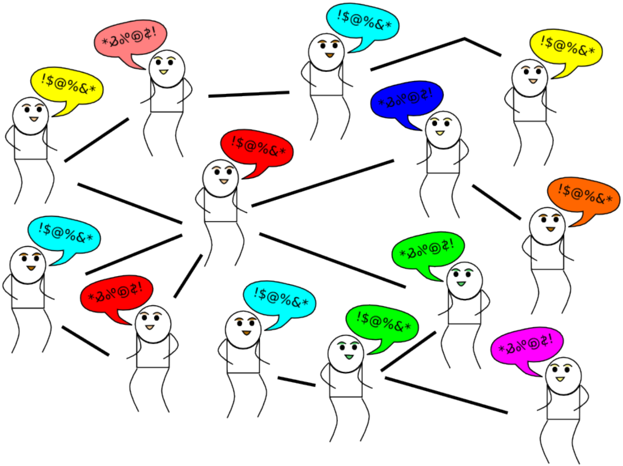

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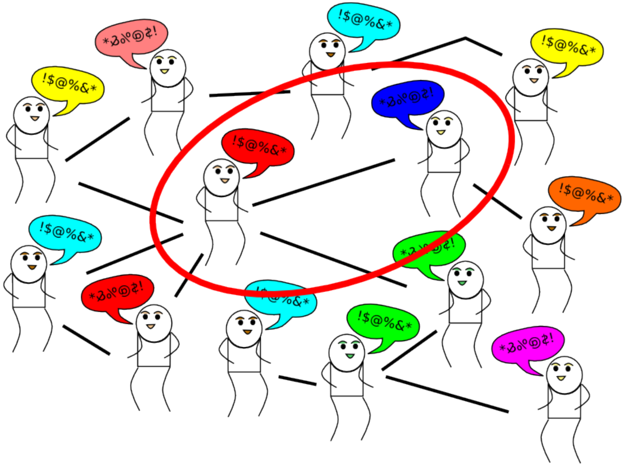
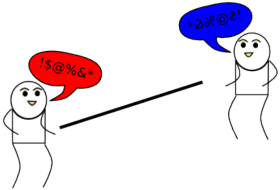
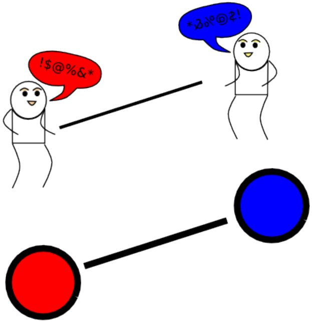
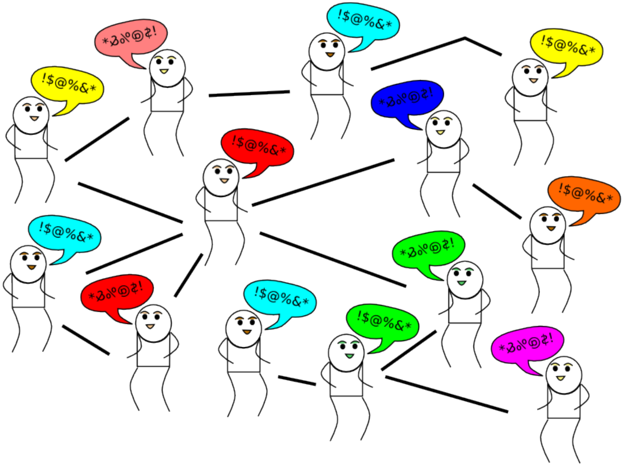
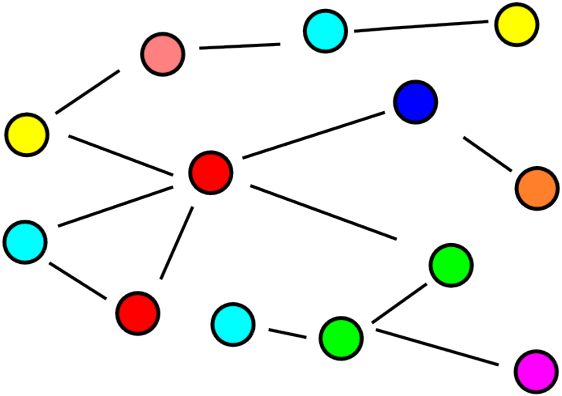
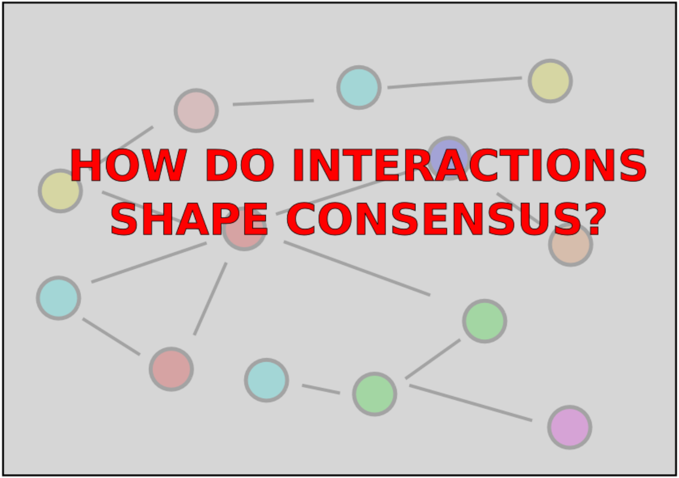











HOW DO INTERACTIONS
SHAPE CONSENSUS?

Outline

1. Introduction
2. Simple models (social imitation)
3. A less simple model (bounded confidence)

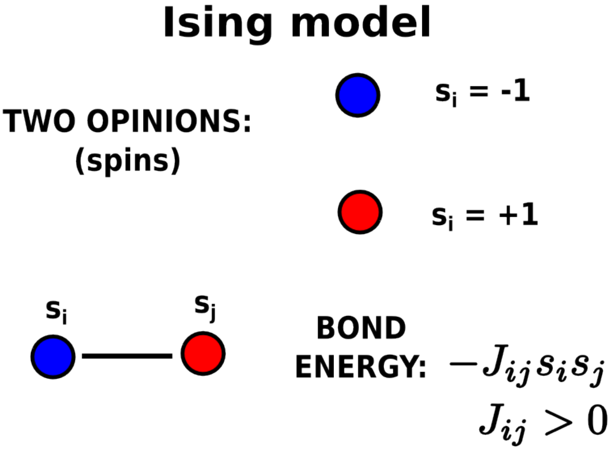
Ising model

Ising model

TWO OPINIONS:
(spins)

$s_i = -1$

$s_i = +1$


Ising model
TWO OPINIONS:
(spins)
s_i = -1
s_i = +1
s_i
s_j
BOND
ENERGY: $-J_{ij} s_i s_j$
$J_{ij} > 0$

Ising model

$\mathbf{S} = \{s_i\}$

Ising model

S = $\{s_i\}$

ENERGY OF STATE S:

$$H(S) = -\sum_{i,j} J_{ij} s_i s_j$$

(Hamiltonian function)

Ising model

$$H(S) = -\sum_{i,j} J_{ij} s_i s_j$$

Ising model

$$H(S) = -\sum_{i,j} J_{ij} s_i s_j$$

PROBABILITY OF STATE S:

$$P(S) = \frac{e^{-\beta H(S)}}{Z_\beta} \qquad Z_\beta = \sum_S e^{-\beta H(S)}$$

$$\beta = (k_B T)^{-1}$$

Ising model

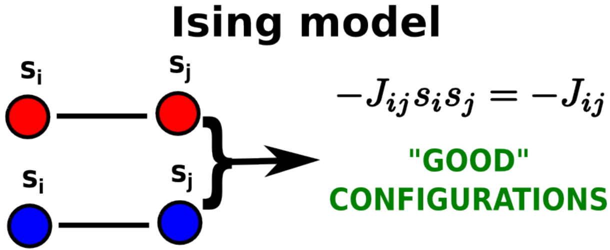
$$H(S) = -\sum_{i,j} J_{ij} s_i s_j$$

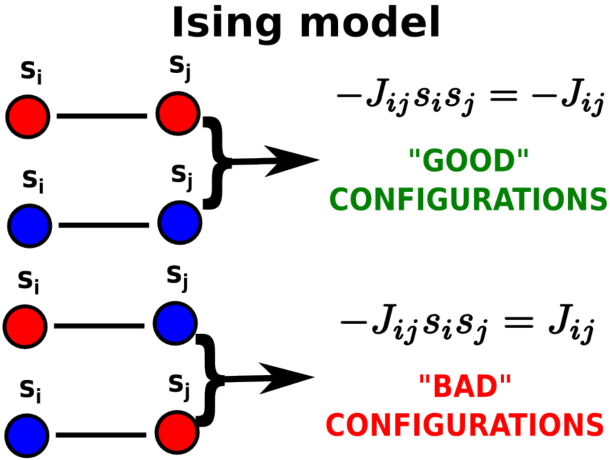
PROBABILITY OF STATE S:

$$P(S) = \frac{e^{-\beta H(S)}}{Z_\beta} \qquad Z_\beta = \sum_S e^{-\beta H(S)}$$

$$\beta = (k_B T)^{-1}$$

HIGH-ENERGY STATES ARE LESS PROBABLE


Ising model
s_i
s_j
s_i
s_j
$-J_{ij}s_is_j = -J_{ij}$
"GOOD"
CONFIGURATIONS


Ising model
s_i
s_j
$-J_{ij}s_is_j = -J_{ij}$
"GOOD"
CONFIGURATIONS
$-J_{ij}s_is_j = J_{ij}$
"BAD"
CONFIGURATIONS

Ising model

ORDER PARAMETER:

$$M = \frac{1}{N} \sum_i s_i$$

Ising model

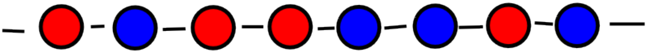
ORDER PARAMETER:

$$M = \frac{1}{N} \sum_i s_i$$

$M \simeq 0$ **Disordered Phase**

$|M| \geq 0$ **CONSENSUS**

Ising Model - 1D lattices



NO ORDERED PHASE

$$M \simeq 0 \quad \forall \beta$$

(NO CONSENSUS)

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Ising Model - lattices with $D \geq 2$ (and networks)

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Ising Model - lattices with $D \geq 2$ (and networks)

Small β (Large T) → $M \simeq 0$

(NO CONSENSUS)

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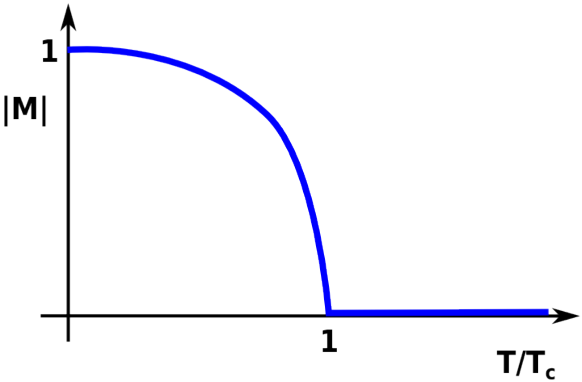
Ising Model - lattices with $D \geq 2$ (and networks)

Small β (Large T) → $M \simeq 0$ **(NO CONSENSUS)**

Large β (Small T) → $M > 0$ **(CONSENSUS)**


1
|M|
1
T/T_c

Social interpretation of "Temperature"

Metropolis algorithm:

1) $H(S) = -\sum_{i,j} J_{ij} s_i s_j$

Social interpretation of "Temperature"

Metropolis algorithm:

1) $H(S) = -\sum_{i,j} J_{ij} s_i s_j$

2) Spin flip: 

Social interpretation of "Temperature"

Metropolis algorithm:

1) $H(S) = -\sum_{i,j} J_{ij} s_i s_j$

2) Spin flip:

3) $H(S') = -\sum_{i,j} J_{ij} s_i s_j$

Social interpretation of "Temperature"

The flip:

is accepted with probability:

$$p = \begin{cases} 1 & \text{if } H(S') < H(S) \\ e^{-\frac{[H(S') - H(S)]}{T}} & \text{if } H(S') \geq H(S) \end{cases}$$

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Social interpretation of "Temperature"

$$e^{-\frac{[H(S')-H(S)]}{T}}$$

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Social interpretation of "Temperature"

$$e^{-\frac{[H(S')-H(S)]}{T}}$$

If $T \to 0$ **Unfavorable states are never visited**

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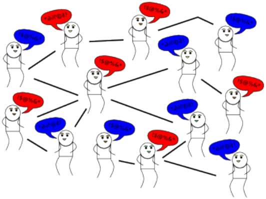
Social interpretation of "Temperature"

$$e^{-\frac{[H(S')-H(S)]}{T}}$$

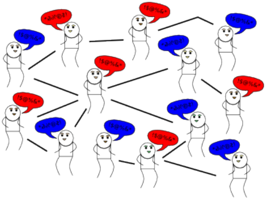
If $T \to 0$ **Unfavorable states are never visited**

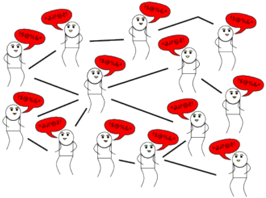
Temperature -> information inaccuracy (noise)

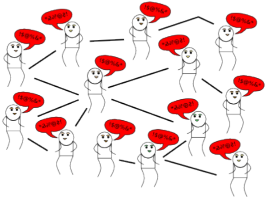


$$T < T_c$$

(accurate information)

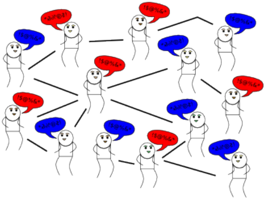


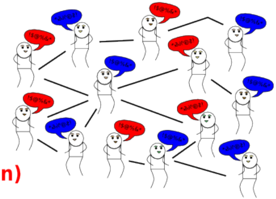




$$T < T_c$$

(accurate information)





$$T > T_c$$

(inaccurate information)

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"Ising says":

CONSENSUS can emerge IF

- agents copy their neighbours' opinions (imitation)

- the information about the state of a neighbour is accurate enough (low noise)

Voter Model

TWO OPINIONS:
(spins)

$s_i = -1$

$s_i = +1$

Voter Model

At each step:

Voter Model

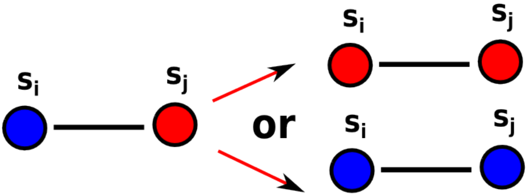
At each step:

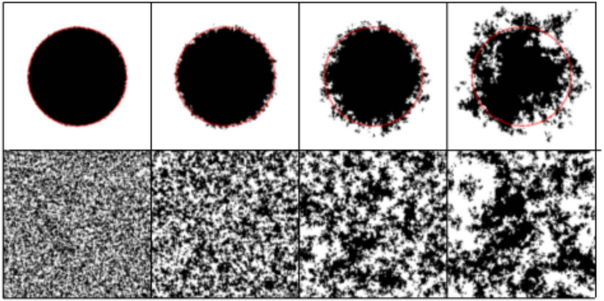
1) **Select an edge at random**
2) **Equate the states of its endpoints**

Voter Model

At each step:

1) **Select an edge at random**
2) **Equate the states of its endpoints**



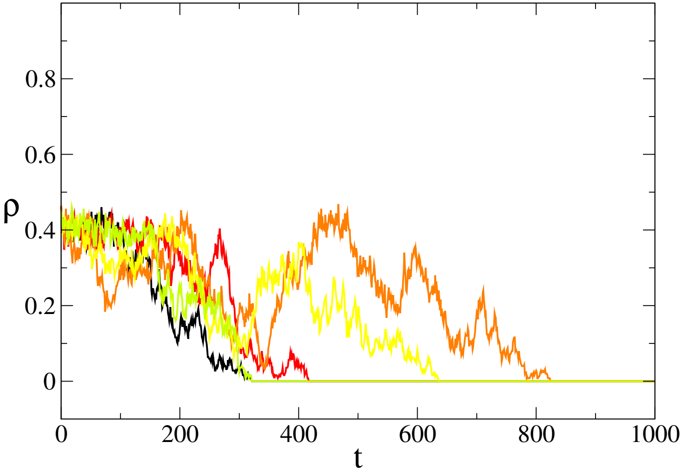


Interface Density

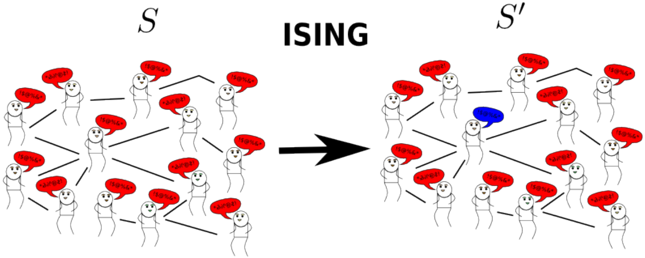
Fraction of edges whose endpoints have different opinions:

$$\rho = \frac{1}{4K} \sum_{i,j} a_{ij} |s_i - s_j|$$

CONSENSUS WHEN $\rho \to 0$

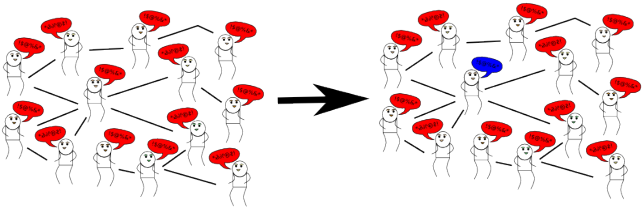

ρ
0.8
0.6
0.4
0.2
0
0
200
400
600
800
1000
t

Ising Model vs Voter



Ising Model vs Voter

S **ISING** S'



POSSIBLE, with probability

$$e^{-\frac{[H(S')-H(S)]}{T}}$$

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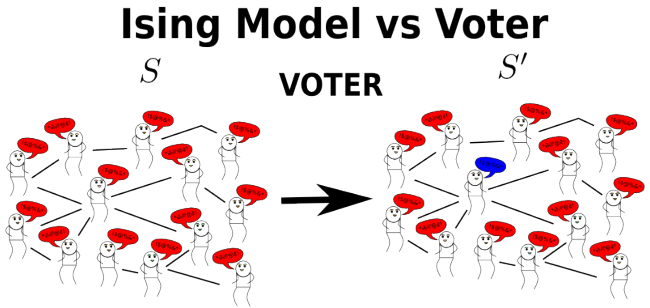

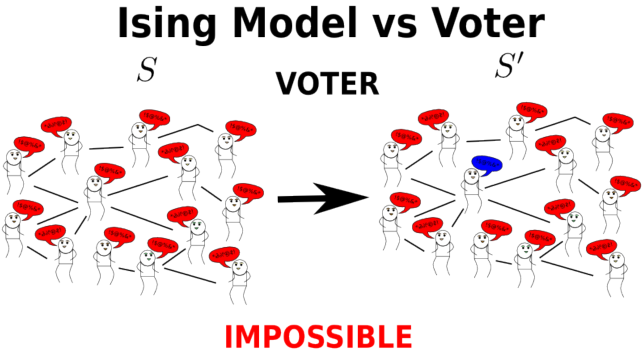
Ising Model vs Voter
S
VOTER
S'
IMPOSSIBLE

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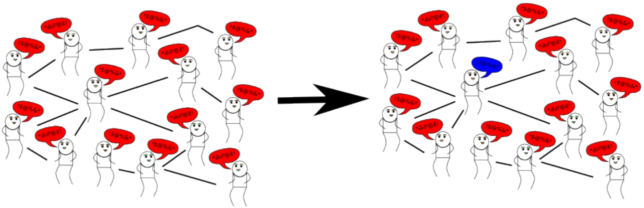
Ising Model vs Voter

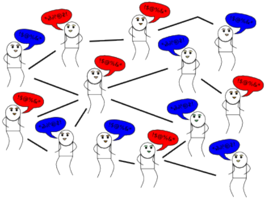
S **VOTER** S'



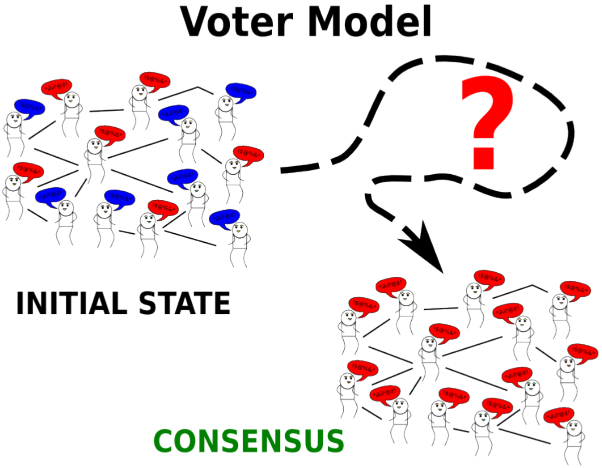
IMPOSSIBLE

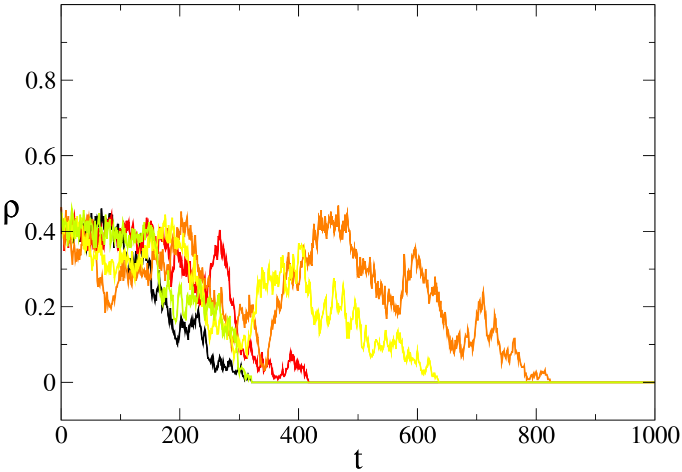
→ **CONSENSUS is an ABSORBING STATE**

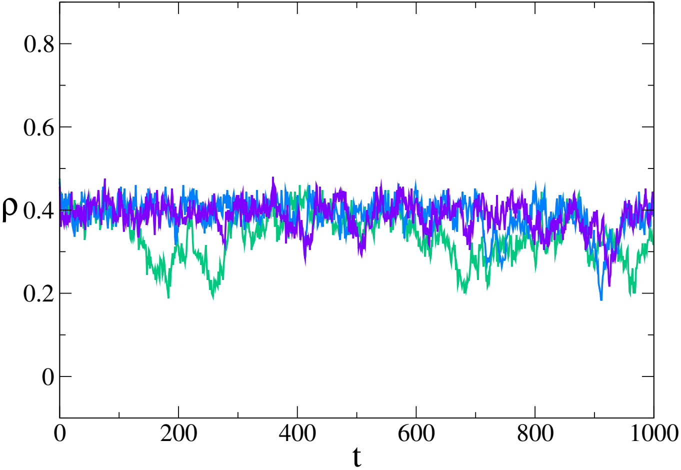
Voter Model



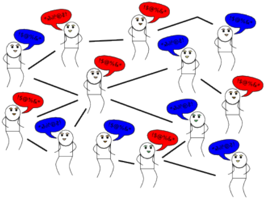
INITIAL STATE


Voter Model
?
INITIAL STATE
CONSENSUS


ρ
0.8
0.6
0.4
0.2
0
0
200
400
600
800
1000
t


ρ
0.8
0.6
0.4
0.2
0
0
200
400
600
800
1000
t

Ordering Time

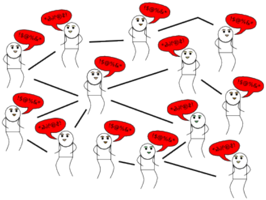


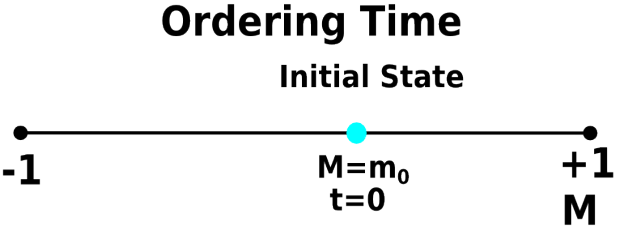
INITIAL STATE

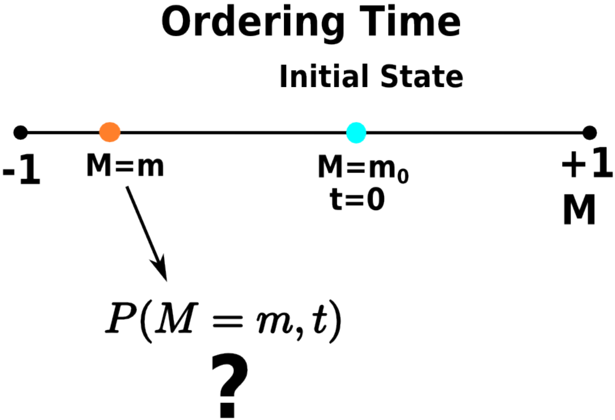
$$M = m$$

CONSENSUS

$$|M| = 1$$




Ordering Time
Initial State
-1
M=m0
t=0
+1
M


Ordering Time
Initial State
-1
M=m
M=m0
t=0
+1
M
P(M = m, t)
?

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Ordering Time

$$\frac{\partial P(m,t')}{\partial t'} = \frac{\partial^2}{\partial m^2}\left[(1-m^2)P(m,t')\right]$$

(Fokker-Planck equation for P(m,t'))

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Ordering Time

$$\frac{\partial P(m,t')}{\partial t'} = \frac{\partial^2}{\partial m^2}\left[(1-m^2)P(m,t')\right]$$

(Fokker-Planck equation for P(m,t'))

$$t' = \frac{t}{\tau} \qquad \tau = \frac{(\mu-1)\mu^2 N}{(\mu-2)\mu_2}$$

$$\mu = \sum_k kp(k) \qquad \mu_2 = \sum_k k^2 p(k)$$

Ordering Time

$$\frac{\partial P(m,t')}{\partial t'} = \frac{\partial^2}{\partial m^2}\left[(1-m^2)P(m,t')\right]$$

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$T(m_0)$ **Time to reach consensus when starting from** $M = m_0$

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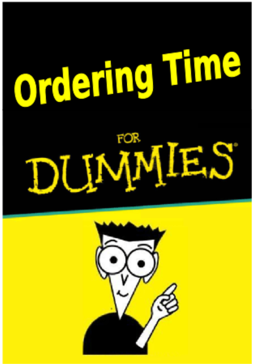
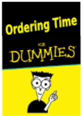
$$T(m_0) \sim \tau = \frac{(\mu-1)\mu^2}{(\mu-2)\mu_2} N$$


Ordering Time
FOR
DUMMIES

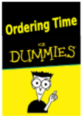
$$T(m_0) \sim \tau = \frac{(\mu-1)\mu^2}{(\mu-2)\mu_2} N$$



If your system gets...

Ordering time gets...

$$T(m_0) \sim \tau = \frac{(\mu-1)\mu^2}{(\mu-2)\mu_2} N$$



If your system gets...

LARGER

Ordering time gets...

LARGER

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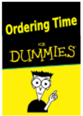
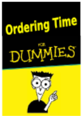
$$T(m_0) \sim \tau = \frac{(\mu-1)\mu^2}{(\mu-2)\mu_2} N$$



If your system gets...	Ordering time gets...
LARGER	**LARGER**
DENSER	**LARGER**

$$T(m_0) \sim \tau = \frac{(\mu-1)\mu^2}{(\mu-2)\mu_2} N$$



If your system gets...	Ordering time gets...
LARGER	**LARGER**
DENSER	**LARGER**
HETEROGENOUS	**SMALLER**

Voter Model: Observations

$$T(m_0) \sim \tau = \frac{(\mu - 1)\mu^2}{(\mu - 2)\mu_2} N$$

Voter Model: Observations

$$T(m_0) \sim \tau = \frac{(\mu-1)\mu^2}{(\mu-2)\mu_2} N$$

Finite systems → **CONSENSUS** **(if you wait enough time)**

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Voter Model: Observations

$$T(m_0) \sim \tau = \frac{(\mu-1)\mu^2}{(\mu-2)\mu_2} N$$

Finite systems → **CONSENSUS** **(if you wait enough time)**

Scale-free graphs

$p(k) \sim k^{-\gamma}, 2 < \gamma < 3$ → **CONSENSUS** **(and we have to wait less time...)**

μ_2 **diverges**

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Voter Model: Observations

$$T(m_0) \sim \tau = \frac{(\mu-1)\mu^2}{(\mu-2)\mu_2} N$$

What if $N \to \infty$ **???**

Is CONSENSUS still possible?

Inifinte Systems

$N \to \infty$ **THE SYSTEM REMAINS ACTIVE**

Inifinte Systems

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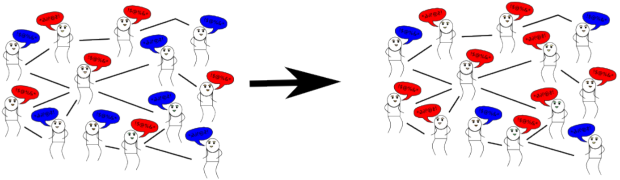
$$\boxed{\rho(t) > 0 \forall t}$$

Inifinte Systems

$N \to \infty$ **THE SYSTEM REMAINS ACTIVE**

$$\rho(t) > 0 \forall t$$



Interface density for $N \to \infty$

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Interface density for $N \to \infty$

$$\lim_{t\to\infty} \rho(t) = \frac{(\mu-2)}{3(\mu-1)}$$

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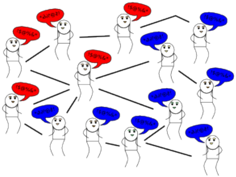

"Voter Model" says:

FINITE SYSTEMS: CONSENSUS

LARGE SYSTEMS: SLOWER

INFINITE SYSTEMS: PARTIAL CONSENSUS

(ACTIVE PHASE)

Outline

1. Introduction
2. Simple models (social imitation)
3. A less simple model (bounded confidence)

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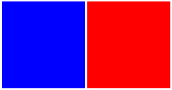
ISING MODEL

VOTER MODEL



REALITY?

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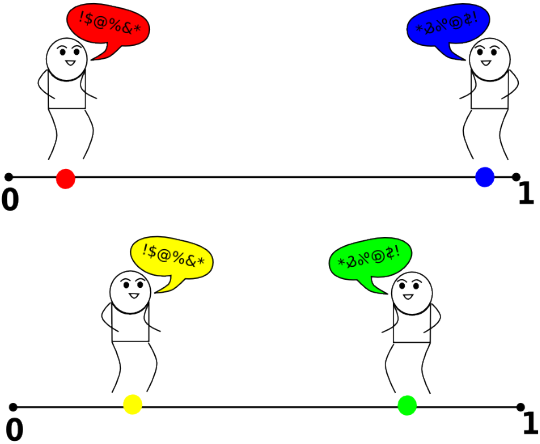
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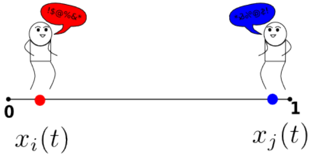
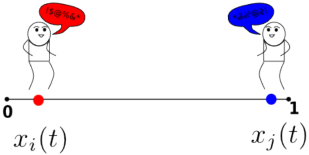
DEFFUANT MODEL



DEFFUANT MODEL



IF $|x_i(t) - x_j(t)| > \epsilon$ **Nothing happens**

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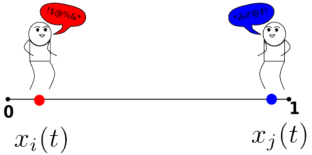
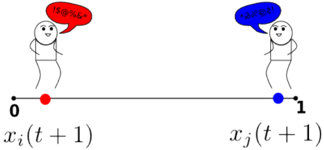
DEFFUANT MODEL



IF $|x_i(t) - x_j(t)| > \epsilon$ **Nothing happens**



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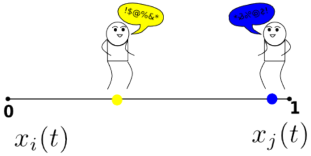
DEFFUANT MODEL



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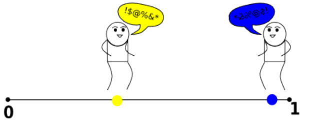
DEFFUANT MODEL



$x_i(t)$ $x_j(t)$

IF $|x_i(t) - x_j(t)| < \epsilon$ **Agents get "closer"**

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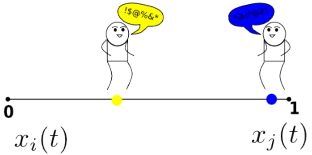
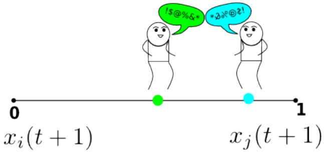
DEFFUANT MODEL



IF $|x_i(t) - x_j(t)| < \epsilon$ **Agents get "closer"**



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DEFFUANT MODEL

$$|x_i(t) - x_j(t)| > \epsilon$$

$$x_i(t+1) = x_i(t)$$
$$x_j(t+1) = x_j(t)$$

DEFFUANT MODEL

$$|x_i(t) - x_j(t)| > \epsilon$$

$$x_i(t+1) = x_i(t)$$

$$x_j(t+1) = x_j(t)$$

$$|x_i(t) - x_j(t)| < \epsilon$$

$$x_i(t+1) = x_i(t) + \mu[x_j(t) - x_i(t)]$$

$$x_j(t+1) = x_j(t) + \mu[x_i(t) - x_j(t)]$$

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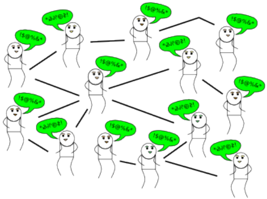
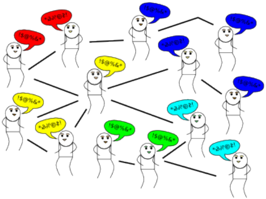
REACHING "CONSENSUS"

$\epsilon > \frac{1}{2}$ **CONSENSUS at** $\overline{x} = \frac{1}{2}$



REACHING "CONSENSUS"

$\epsilon < \frac{1}{2}$ **MANY OPINIONS SURVIVE**



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