Phase transitions in network growth

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Sept. 29th 2015
Outline

1. Phase transitions in street patterns
2. Phase transitions in neural networks
3. Summing up
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(Sub-)Urban street patterns

(Sub-)Urban street patterns


GROANE

125 km²
29 Urban centers
14 Municipalities
Phase transitions in street patterns

(Sub-)Urban street patterns


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(Sub-)Urban street patterns

Network phase transitions

No centralised urban planning

- 29 urban centers within 14 municipalities
Shadows of a lost empire

- Backbone: Roman and Medieval roads.
Shadows of a lost empire

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Shadows of a lost empire

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Shadows of a lost empire

- Backbone: Roman and Medieval roads.
New towns just out of the blue

- New villages and towns appear and grow around crossings of old roads.
New towns just out of the blue

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New towns just out of the blue

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The big squeeze

- Small villages become larger and are merged together into a compact urban area
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1914
The big squeeze

- Small villages become larger and are merged together into a compact urban area
Small villages become larger and are merged together into a compact urban area.
Small villages become larger and are merged together into a compact urban area.

1980
Small villages become larger and are merged together into a compact urban area.
The big squeeze

- Small villages become larger and are merged together into a compact urban area.
Phase transitions in street patterns

Denser \(\rightarrow\) More fragmented

- The land is subsequently fragmented into smaller cells
- The shape and size of some cells remain constant over time
Denser $\Longrightarrow$ More fragmented

- The land is subsequently fragmented into smaller cells
- The shape and size of some cells remain constant over time

1833
Denser $\rightarrow$ More fragmented

- The land is subsequently fragmented into smaller cells
- The shape and size of some cells remain constant over time
Phase transitions in street patterns

Denser \implies More fragmented

The land is subsequently fragmented into smaller cells.

The shape and size of some cells remain constant over time.
Denser $\Rightarrow$ More fragmented

- The land is subsequently fragmented into smaller cells
- The shape and size of some cells remain constant over time

1933
Denser $\implies$ More fragmented

- The land is subsequently fragmented into smaller cells
- The shape and size of some cells remain constant over time

1955
Denser $\implies$ More fragmented

- The land is subsequently fragmented into smaller cells.
- The shape and size of some cells remain constant over time.

1980
Denser $\Longrightarrow$ More fragmented

- The land is subsequently fragmented into smaller cells
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Denser $\rightarrow$ More fragmented

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2007
A quantitative approach (1)

The diagram shows the growth in population (Pop) and the number of networks (N) over the years 1850 to 2000. The population grows significantly from 1850 to 2000, while the number of networks shows a more linear increase.
**A quantitative approach (1)**

- \( N \) grows linearly with the number of inhabitants
A quantitative approach (1)

- $N$ grows linearly with the number of inhabitants
- There are on average 51 inhabitants per node (!)
A quantitative approach (2)

**Edge Betweenness**

$$b(e) = \frac{\text{# of shortest paths through } e}{\text{total # of shortest paths}}$$
A quantitative approach (2)

**Edge Betweenness**

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b(e) = \frac{\text{# of shortest paths through } e}{\text{total # of shortest paths}}
\]

A quantitative approach (2)

**Edge Betweenness**

\[ b(e) = \frac{\text{# of shortest paths through } e}{\text{total # of shortest paths}} \]

A quantitative approach (3)

\[ \delta_b(e) = \frac{\overline{b}(G_t) - \overline{b}(G_t \setminus \{e\})}{\overline{b}(G_t)} \]

**Impact on Betweenness**

**A quantitative approach (3)**

**IMPACT ON BETWEENNESS**

\[
\delta_b(e) = \frac{\bar{b}(G_t) - \bar{b}(G_t \setminus \{e\})}{\bar{b}(G_t)}
\]

A quantitative approach (3)

**Densification**

\[ \delta_b(e) = \frac{\bar{b}(G_t) - \bar{b}(G_t \setminus \{e\})}{\bar{b}(G_t)} \]

**Impact on Betweenness**

A quantitative approach (3)

\[
\delta_b(e) = \frac{\bar{b}(G_t) - \bar{b}(G_t \setminus \{e\})}{\bar{b}(G_t)}
\]

Impact on Betweenness

Densification

Exploration

Phase transitions in street patterns

A quantitative approach (4)

A quantitative approach (4)

\[
\phi = \frac{A}{A'}
\]

A quantitative approach (4)

**Shape Factor**

\[ \phi = \frac{A}{A'} \]

A quantitative approach (4)

\[ \phi = \frac{A}{A'} \]

**SHAPE FACTOR**

![Map of a street pattern before WWII](image)

Phase transitions in street patterns

A quantitative approach (4)


**Shape Factor**

\[ \phi = \frac{A}{A'} \]

**Before WWII**

**After WWII**

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Phase transitions in street patterns

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Animal neural networks

C. ELEGANS

Animal neural networks

C. ELEGANS

NEURAL NETWORK

~300 NEURONS (NODES)
~2400 SYNAPSES (EDGES)

Animal neural networks

C. Elegans

Neural Network

~300 Neurons (nodes)
~2400 Synapses (edges)

Animal neural networks

C. Elegans

Neural Network

~300 Neurons
(nodes)

~2400 Synapses
(edges)

Name: Caenorhabditis
Surname: Elegans
Length: 1.1 mm
Occupation: lab worm
Particular features: transparent, blind
Abilities: Moving, eating, having sex with just about 300 neurons
Featured on: Nature, Science, PNAS, PRL
Phase transitions in neural networks

**Small-World**

- **Regular**: High L, High C
- **Small World**: Low L, High C
- **Random**: Low L, Low C

Increasing random connectivity
Phase transitions in neural networks

Phase transitions in neural networks

Phase transitions in neural networks

Nodes (N)

Edges (K)

\[ K \sim N^2 \]

Quadratic Increase

\[ K \sim N \]

Linear increase

Phase transitions in neural networks

Nodes (N) Edges (K)

Quadratic Increase

Linear increase

K \sim N^2

K \sim N

Phase transitions in neural networks

$$p_{i \rightarrow j} = \frac{1}{N(t)}$$

Accelerated growth

Quadratic increase

Phase transitions in neural networks

\[ p_{i \to j} \propto k_j(t) \]

Linear increase

Phase transitions in neural networks


*Network phase transitions*

*Sept. 29th 2015*
The brain is spread throughout the body!

Short-range connections = small wiring cost

Long-range connections (to hubs) = high efficiency

Phase transitions in neural networks

Network efficiency

Wiring cost

+HUBS

+SHORT LINKS

Phase transitions in neural networks

\[ p_{i \rightarrow j} \propto h_j e^{-d_{ij}(t)/\delta} \]

degree in the adult worm

Phase transitions in neural networks

\[ p_{i \rightarrow j} \propto h_j e^{-d_{ij}(t)/\delta} \] 

space + time

Phase transitions in neural networks

\[ p_{i \rightarrow j} \propto h_j e^{-d_{ij}(t) / \delta} \]

Efficiency  Wiring cost

Economical Spatio-temporal growth

- Quadratic growth
- Linear growth
- Phase transition

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