Phase transitions in network growth

V. Nicosia

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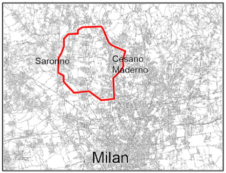
Outline

1. Phase transitions in street patterns
2. Phase transitions in neural networks
3. Summing up

Outline

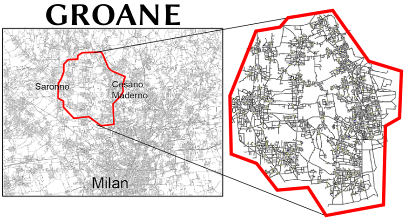
(Sub-)Urban street patterns

GROANE



E. Strano, V. Nicosia, V. Latora, S. Porta, M. Barthélemy, *Nature Sci. Rep.* 2, 296 (2012)

(Sub-)Urban street patterns



125 km^2

29 Urban centers

14 Municipalities

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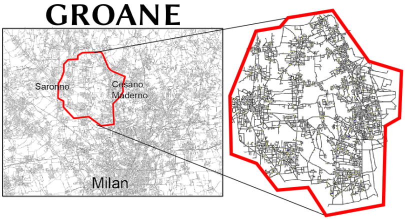

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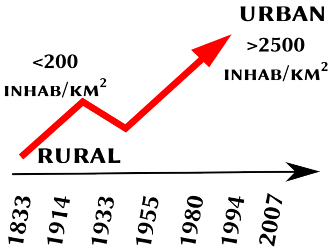

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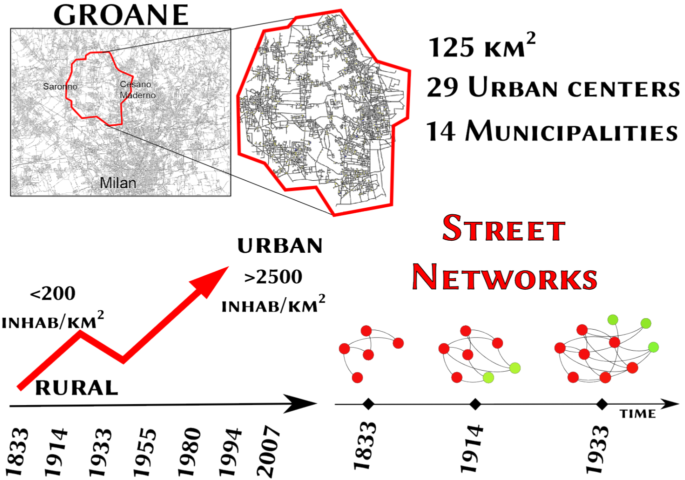

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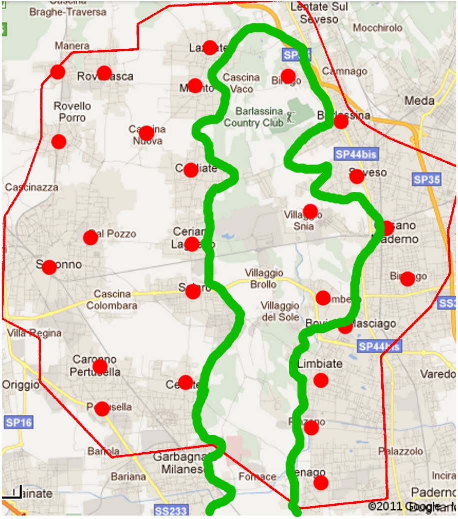
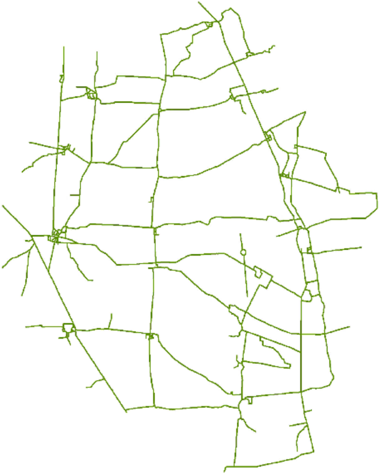
No centralised urban planning

- 29 urban centers within 14 municipalities



Shadows of a lost empire

- Backbone: Roman and Medieval roads.



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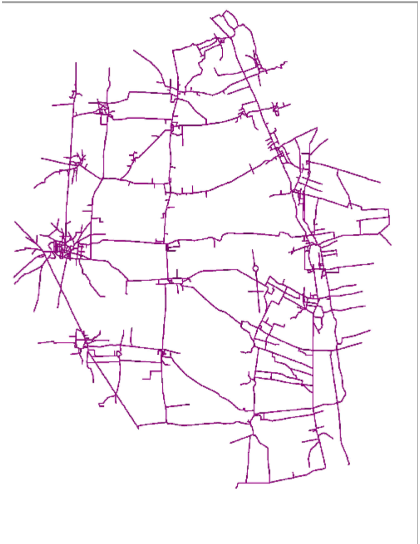

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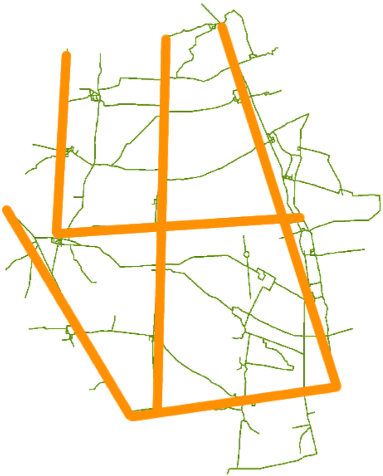

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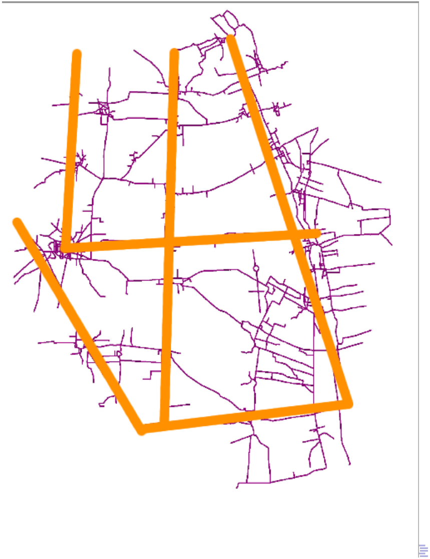
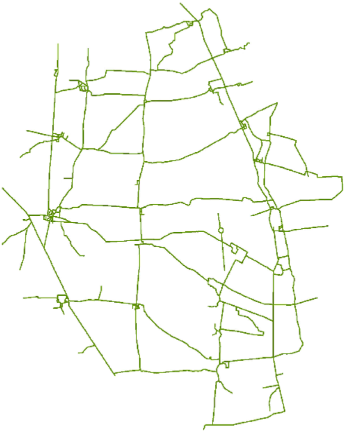


New towns just out of the blue

- New villages and towns appear and grow around crossings of old roads.



1833

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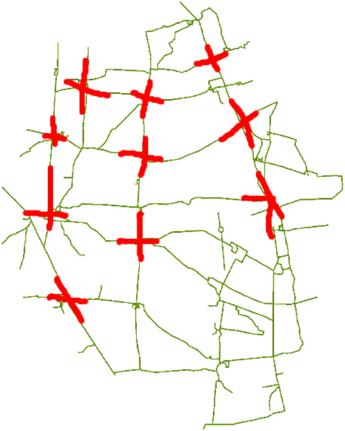

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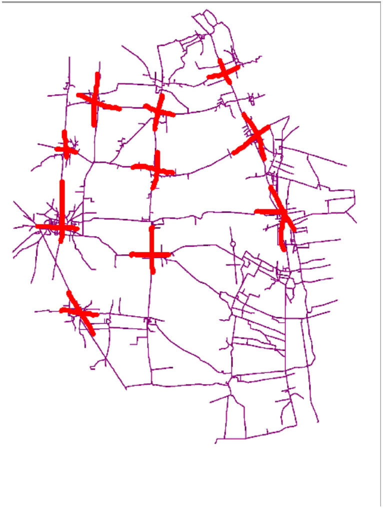

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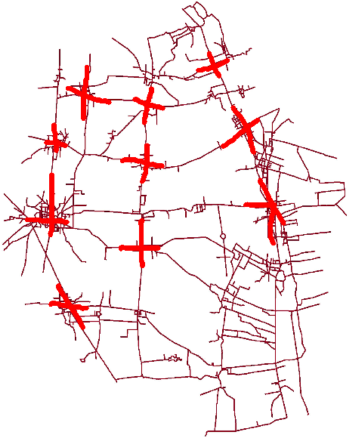

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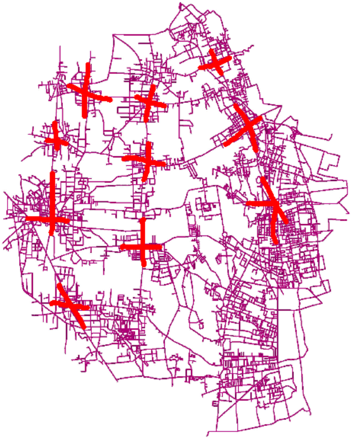

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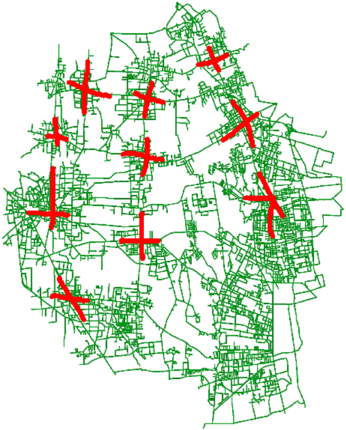
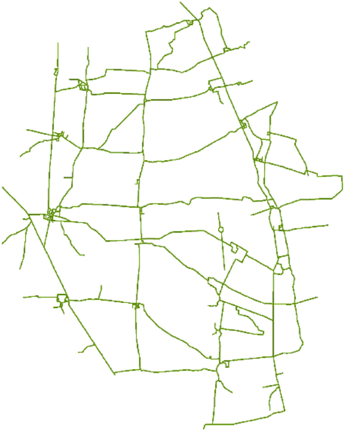


2007

The big squeeze

- Small villages become larger and are merged together into a compact urban area



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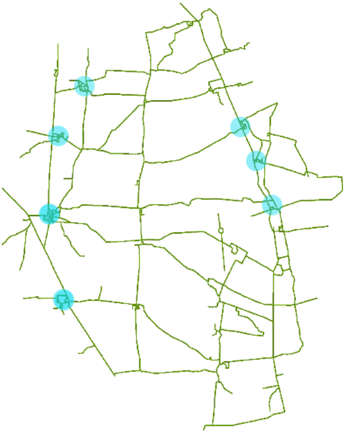

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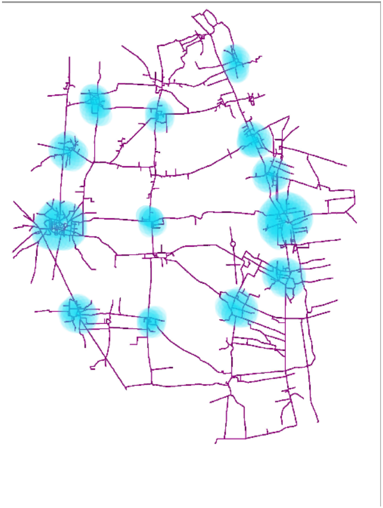

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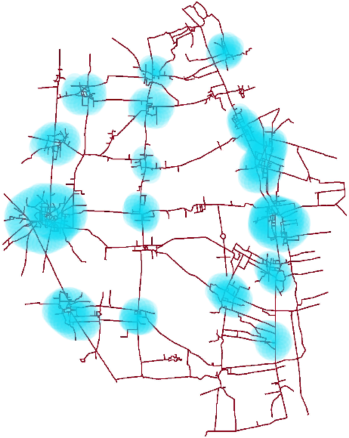

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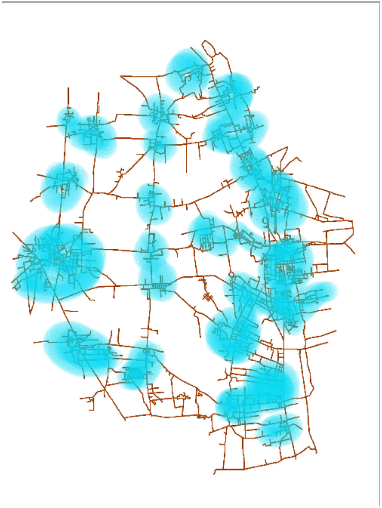

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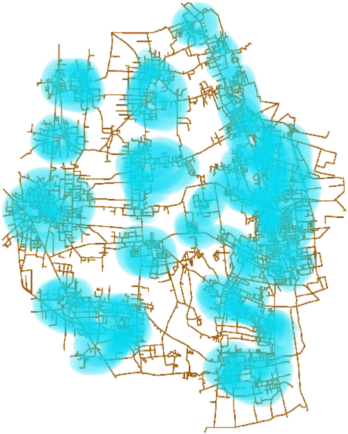

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Denser $\Longrightarrow$ More fragmented

- The land is subsequently fragmented into smaller cells
- The shape and size of some cells remain constant over time

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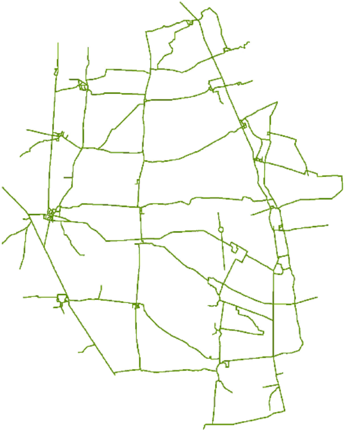

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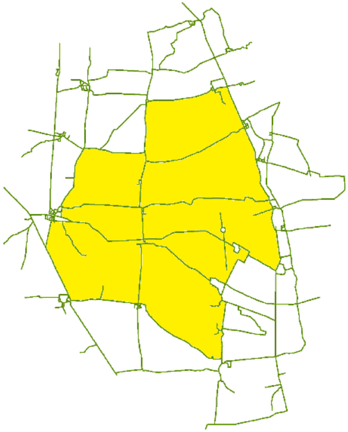

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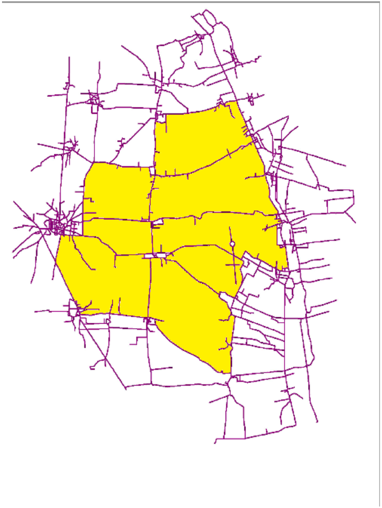

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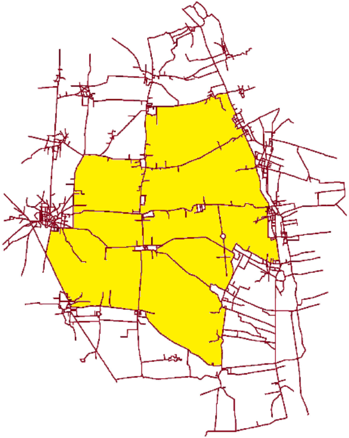

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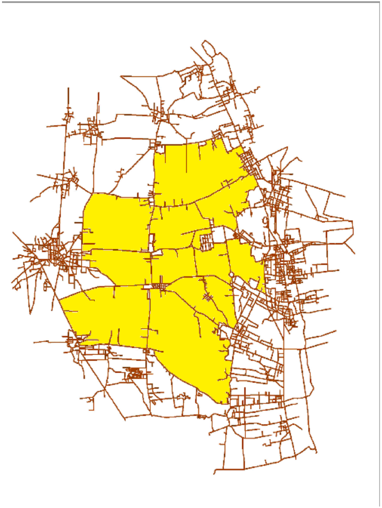

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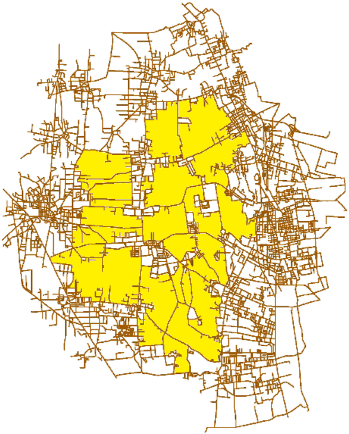

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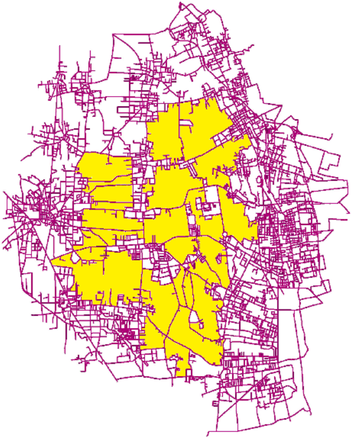

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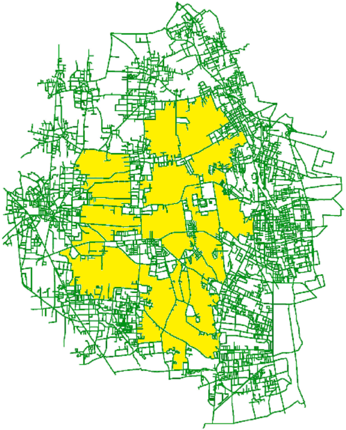

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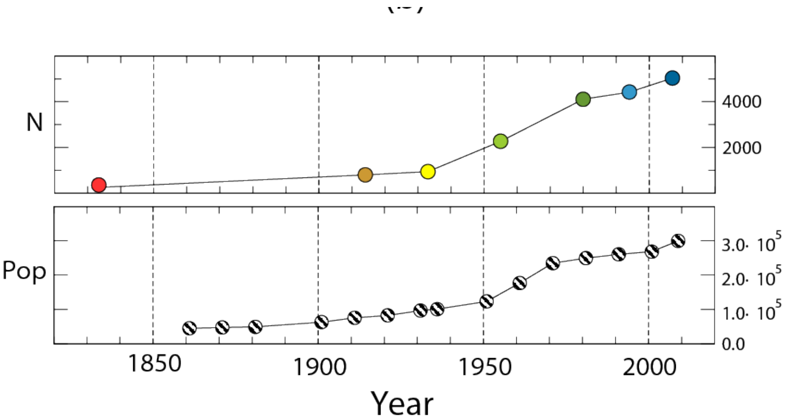
A quantitative approach (1)



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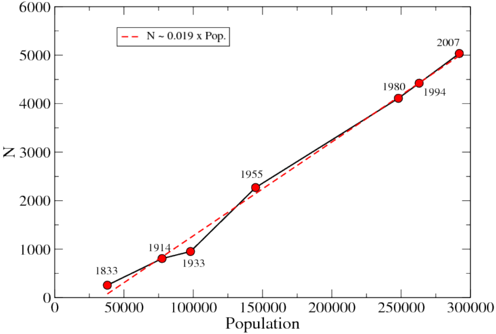

- N grows linearly with the number of inhabitants

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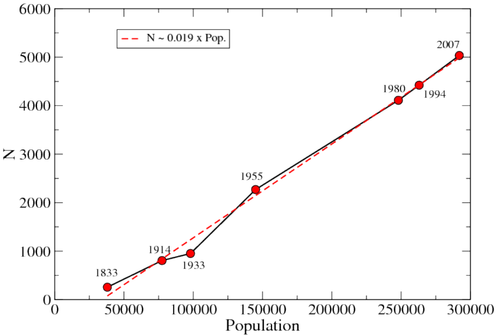
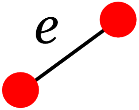


- N grows linearly with the number of inhabitants
- There are on average 51 inhabitants per node (!)

A quantitative approach (2)



Edge Betweenness

$$b(e) = \frac{\text{\# of shortest paths through e}}{\text{total \# of shortest paths}}$$

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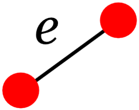

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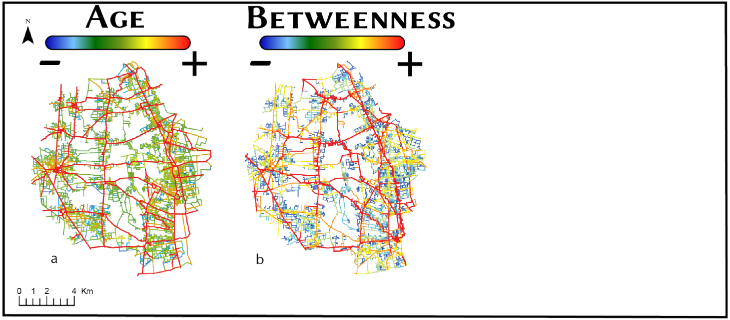

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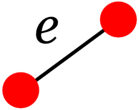

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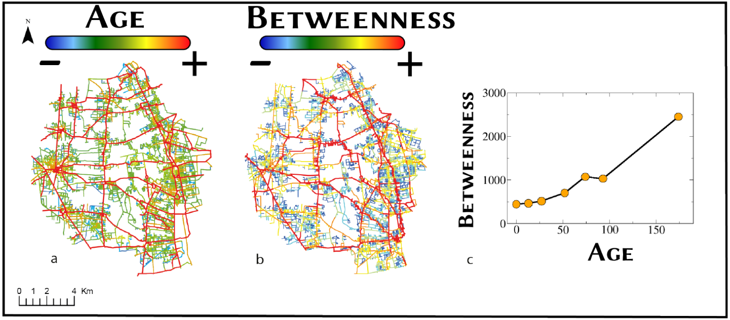

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A quantitative approach (3)

Impact on Betweenness

$$\delta_b(e) = \frac{\left[\bar{b}(G_t) - \bar{b}(G_t \setminus \{e\})\right]}{\bar{b}(G_t)}$$

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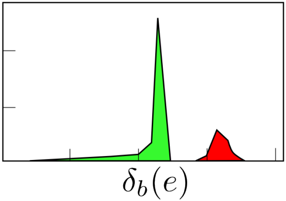

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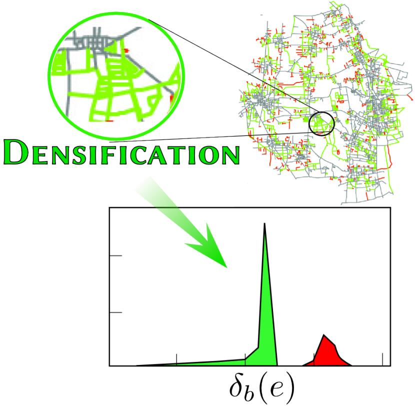

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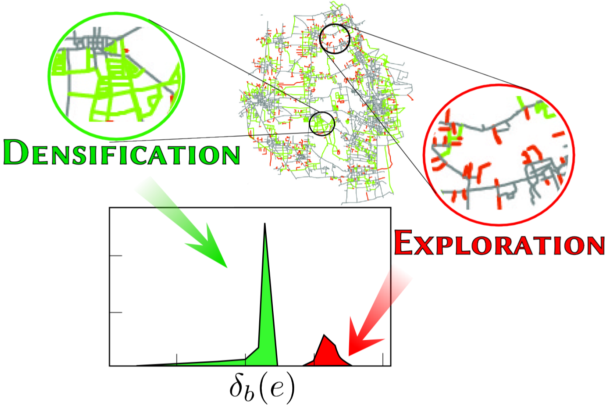

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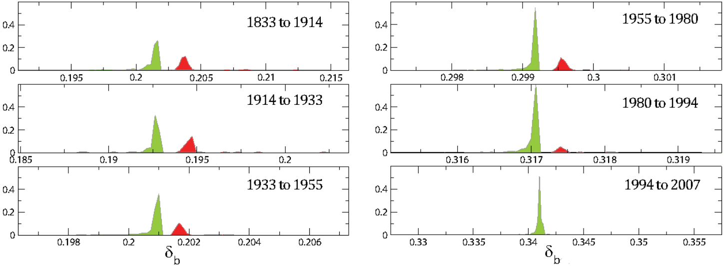

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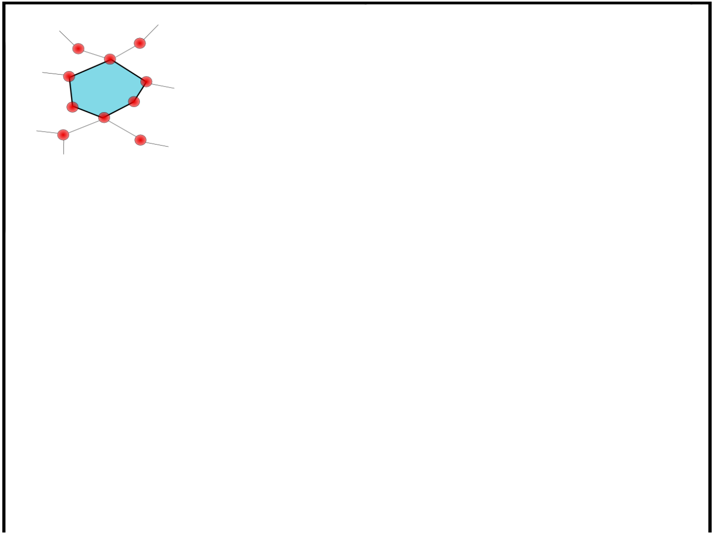
A quantitative approach (4)



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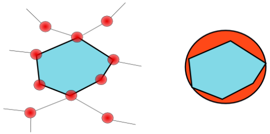

Shape Factor $\phi = \frac{A}{A'}$

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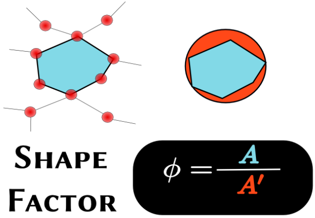

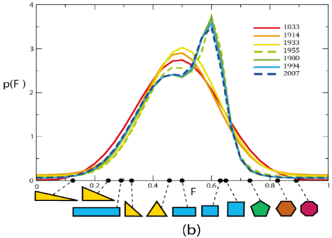

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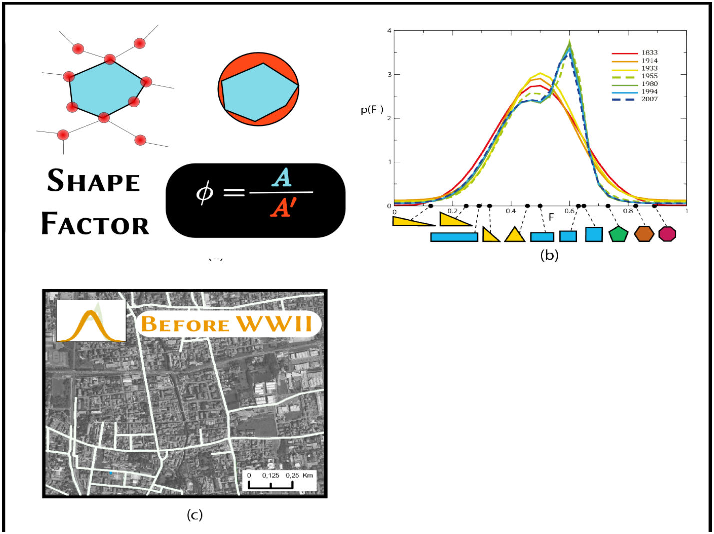

(b)

(c)

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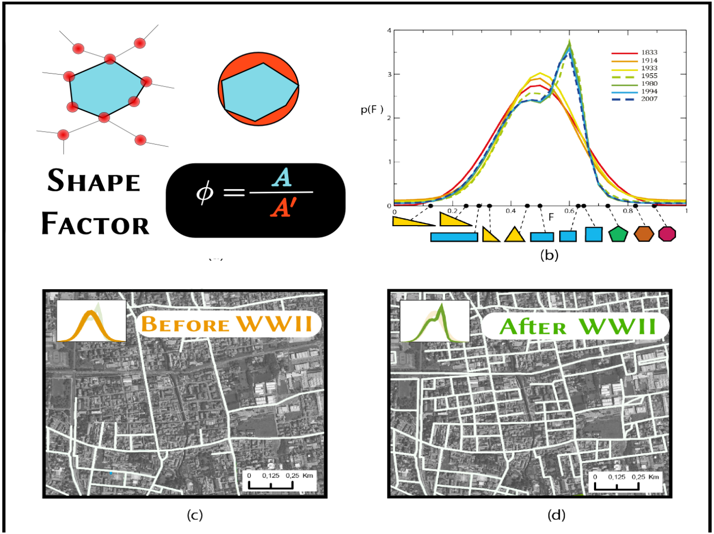

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(d)

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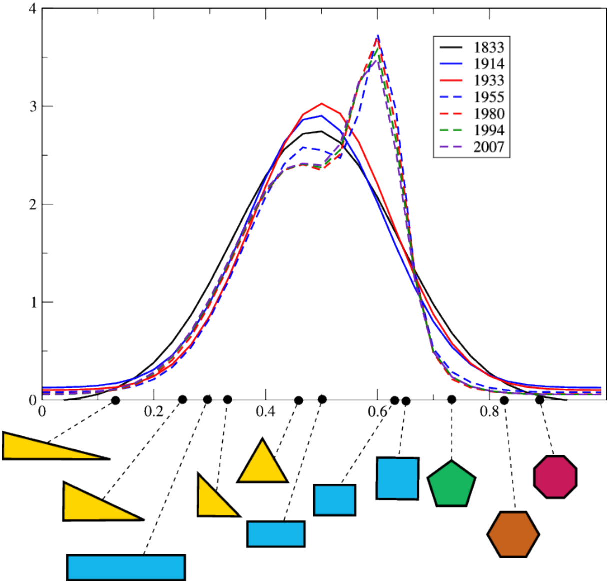

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Outline

Animal neural networks

C. Elegans

V. Nicosia, P. Vertes, W. Schafer, V. Latora, E. Bullmore, *Proc. Natl. Acad. Sci. USA* 110 (19), 7880-7885 (2013)

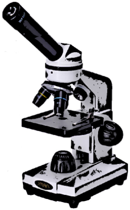
Animal neural networks

C. Elegans

Neural Network

~300 Neurons (nodes)

~2400 Synapses (edges)



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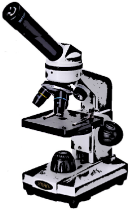

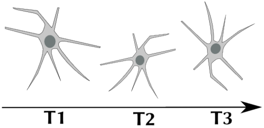

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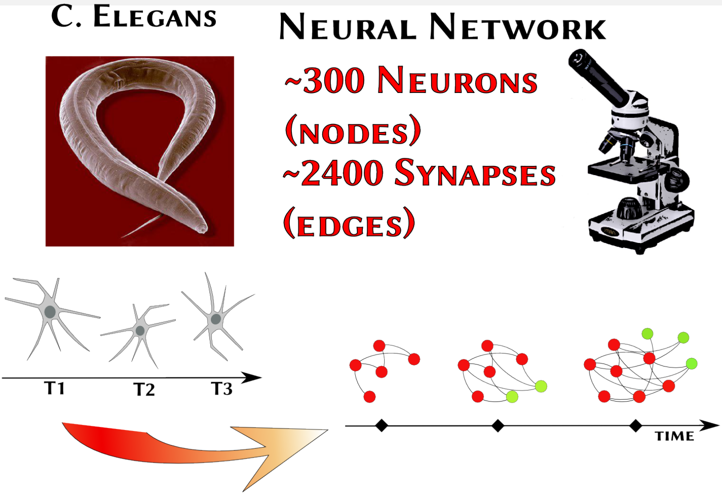

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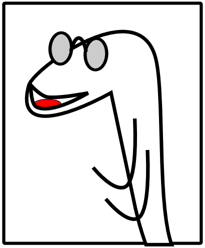


Name: Caenorhabditis

Surname: Elegans

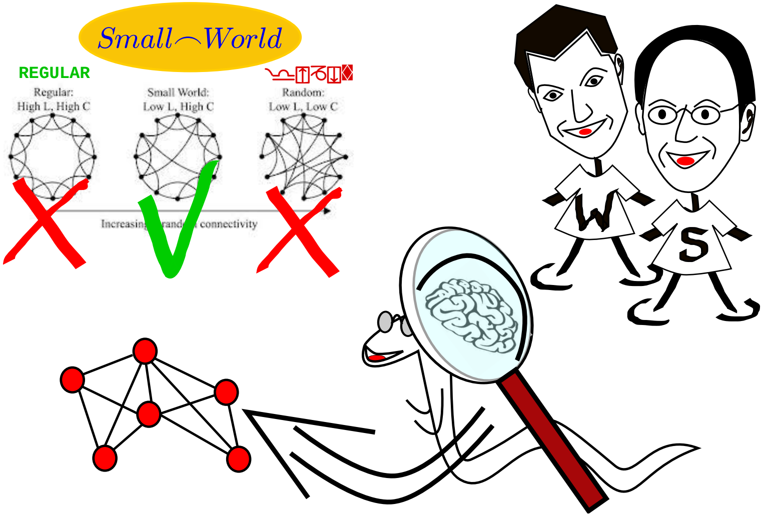
Length: 1.1 mm

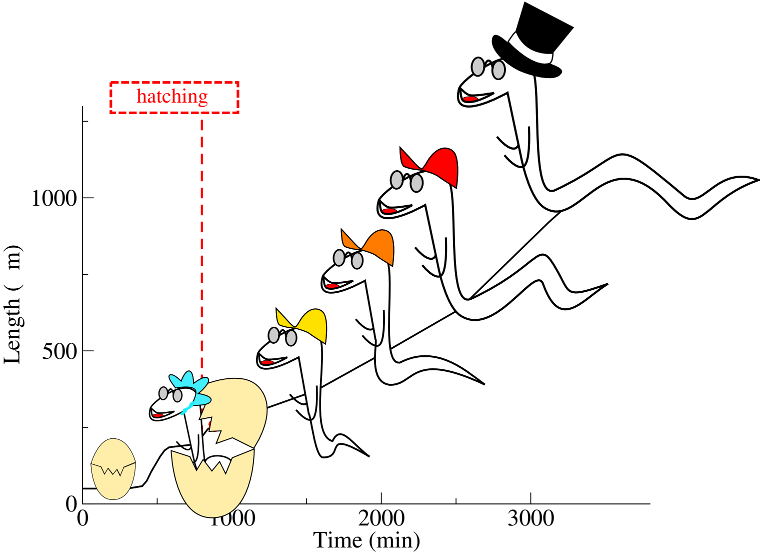
Occupation: lab worm

Particular features: transparent, blind

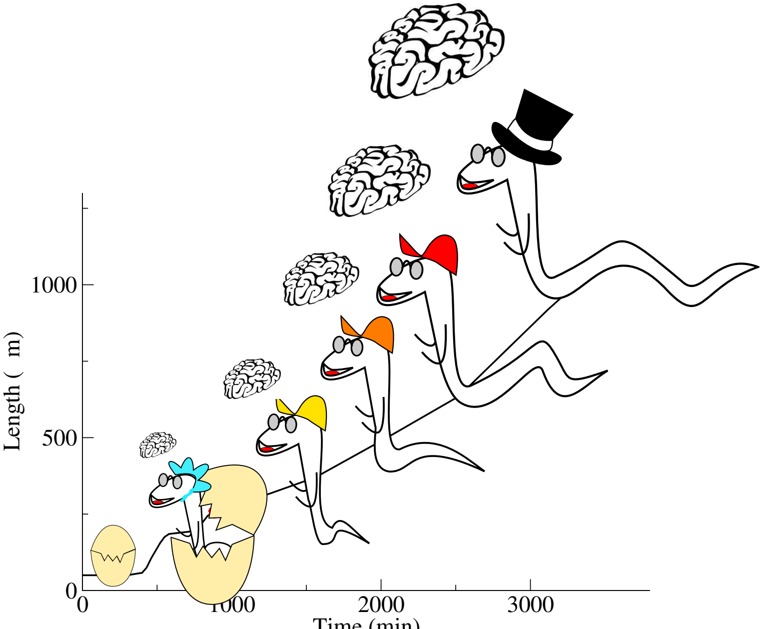
Abilities: Moving, eating, having sex with just about 300 neurons

Featured on: Nature, Science, PNAS, PRL


Small⌢World
REGULAR
Regular:
High L, High C
Small World:
Low L, High C
Random:
Low L, Low C
connectivity
W
S

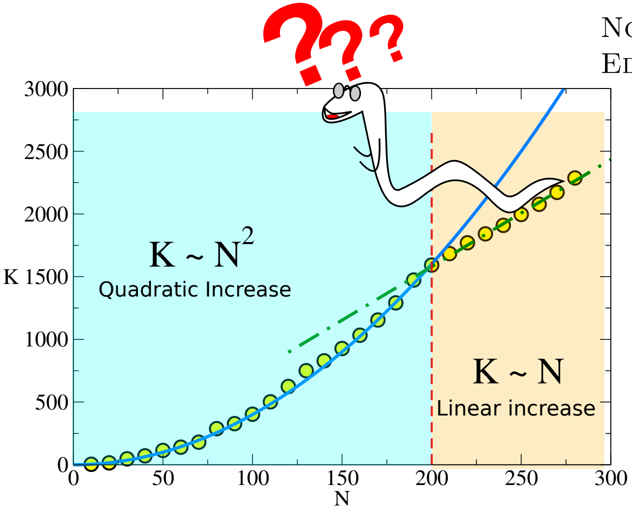


V. Nicosia, P. Vertes, W. Schafer, V. Latora, E. Bullmore, *Proc. Natl. Acad. Sci. USA* 110 (19), 7880-7885 (2013)



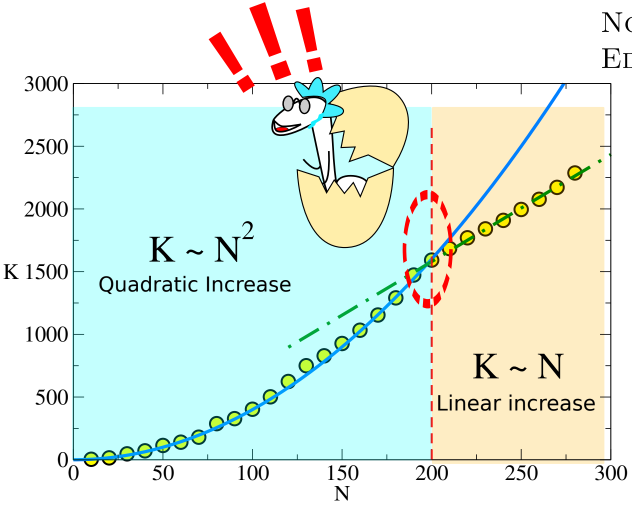
V. Nicosia, P. Vertes, W. Schafer, V. Latora, E. Bullmore, *Proc. Natl. Acad. Sci. USA* 110 (19), 7880-7885 (2013)

NODES (N)
EDGES (K)

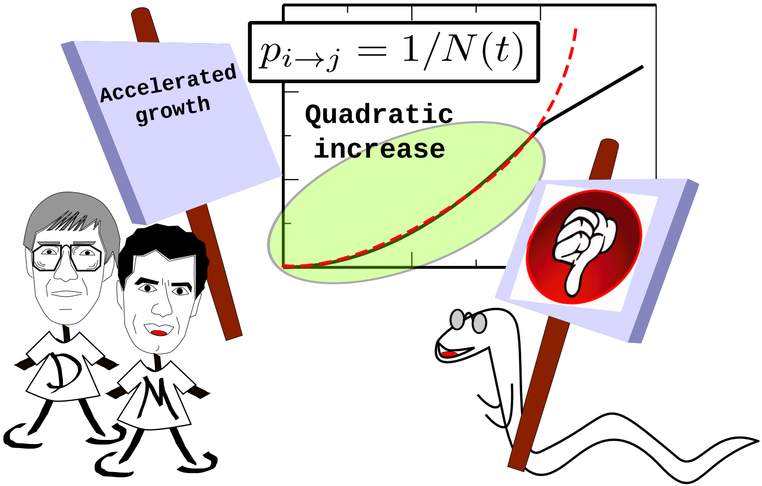


V. Nicosia, P. Vertes, W. Schafer, V. Latora, E. Bullmore, *Proc. Natl. Acad. Sci. USA* 110 (19), 7880-7885 (2013)

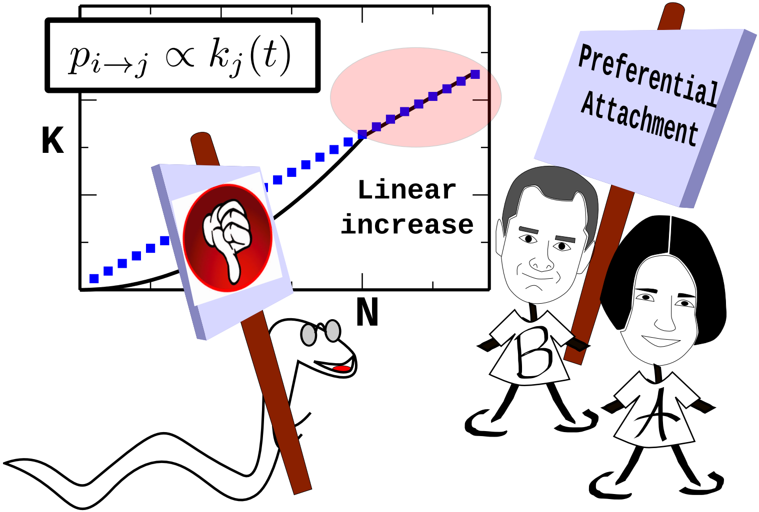
NODES (N)
EDGES (K)



V. Nicosia, P. Vertes, W. Schafer, V. Latora, E. Bullmore, *Proc. Natl. Acad. Sci. USA* 110 (19), 7880-7885 (2013)

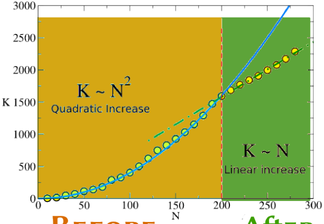


V. Nicosia, P. Vertes, W. Schafer, V. Latora, E. Bullmore, *Proc. Natl. Acad. Sci. USA* 110 (19), 7880-7885 (2013)

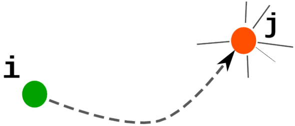


V. Nicosia, P. Vertes, W. Schafer, V. Latora, E. Bullmore, *Proc. Natl. Acad. Sci. USA* 110 (19), 7880-7885 (2013)

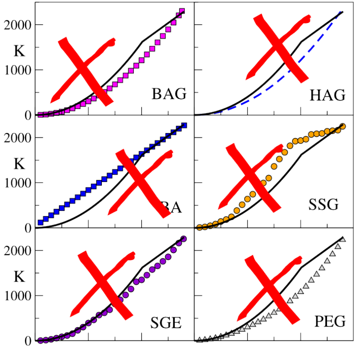
NODES (N)
EDGES (K)



BEFORE HATCHING **AFTER HATCHING**

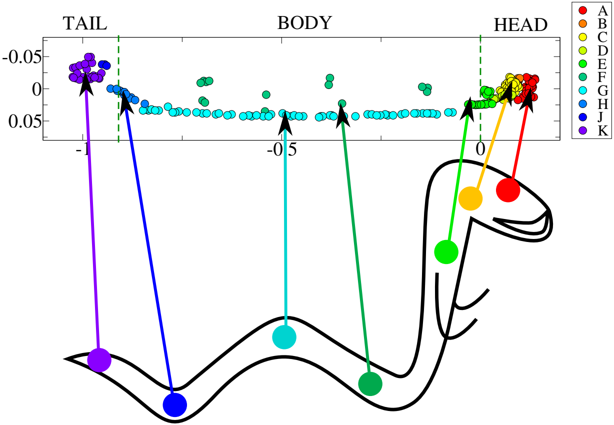


$$P_{i \to j}$$



V. Nicosia, P. Vertes, W. Schafer, V. Latora, E. Bullmore, *Proc. Natl. Acad. Sci. USA* 110 (19), 7880-7885 (2013)

The brain is spread throughout the body!



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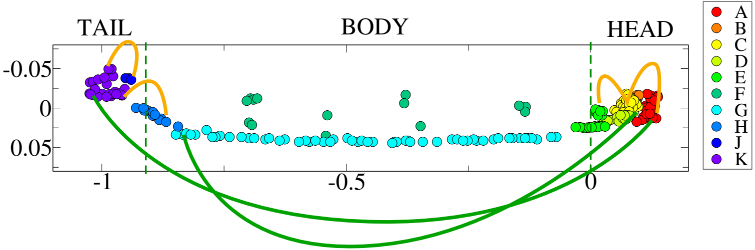
Short-range connections = small wiring cost



Long-range connections (to hubs) = high efficiency

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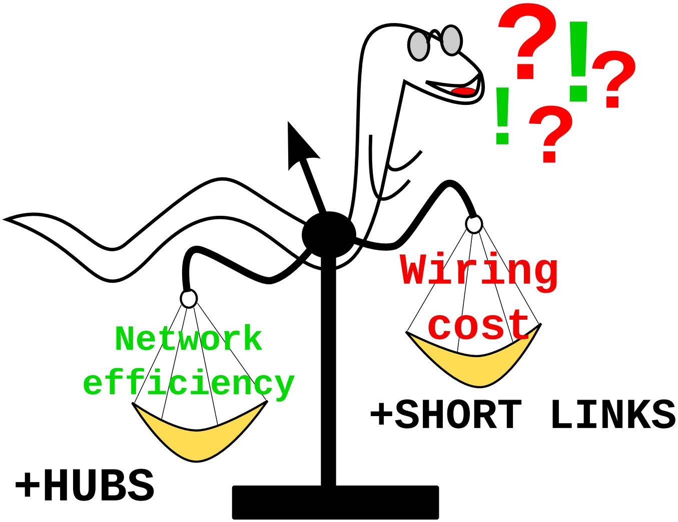
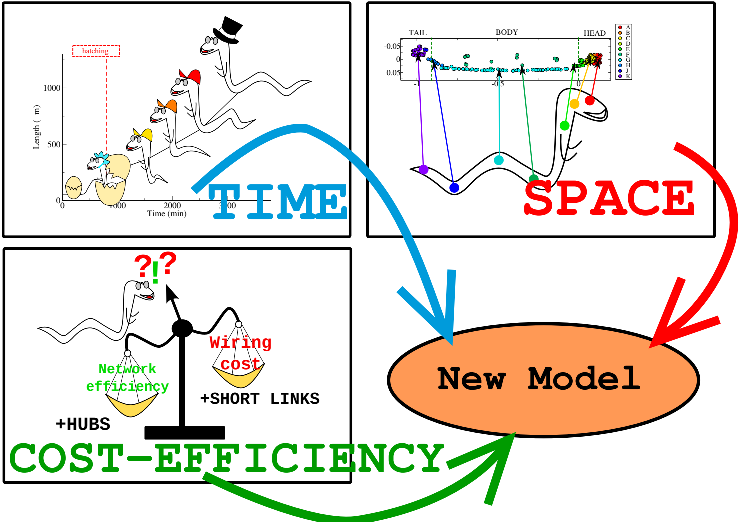
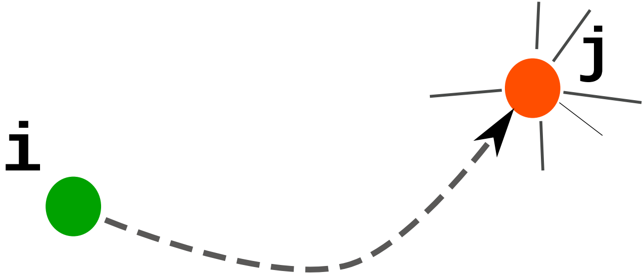


V. Nicosia, P. Vertes, W. Schafer, V. Latora, E. Bullmore, *Proc. Natl. Acad. Sci. USA* 110 (19), 7880-7885 (2013)



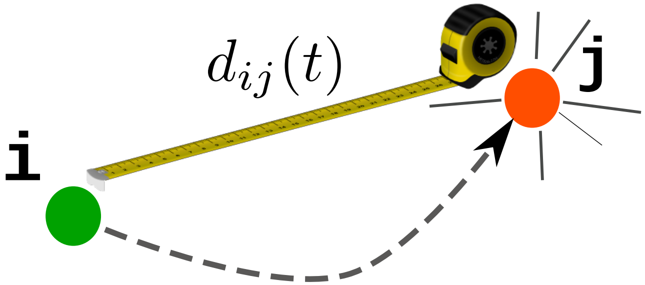
V. Nicosia, P. Vertes, W. Schafer, V. Latora, E. Bullmore, *Proc. Natl. Acad. Sci. USA* 110 (19), 7880-7885 (2013)



$$p_{i\to j} \propto h_j e^{-d_{ij}(t)/\delta}$$

degree in the adult worm

V. Nicosia, P. Vertes, W. Schafer, V. Latora, E. Bullmore, *Proc. Natl. Acad. Sci. USA* 110 (19), 7880-7885 (2013)



$$p_{i\to j} \propto h_j e^{-d_{ij}(t)/\delta}$$

space + time

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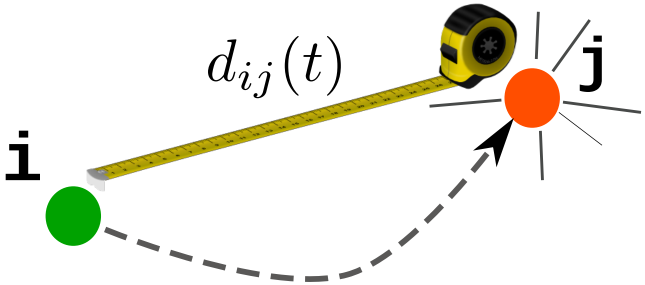
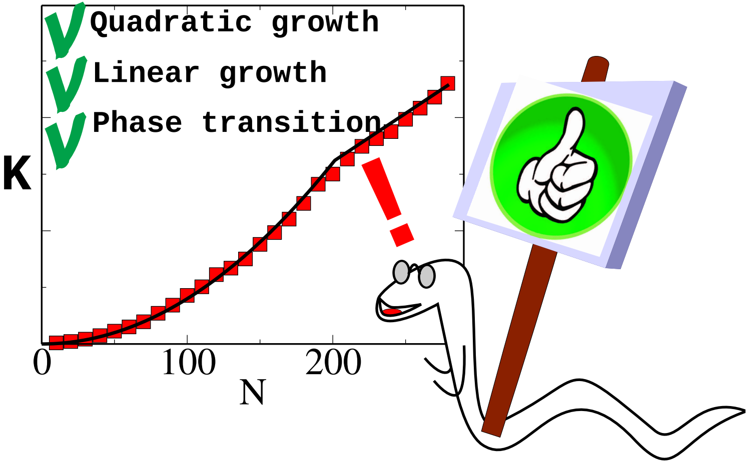


$$p_{i\to j} \propto h_j e^{-d_{ij}(t)/\delta}$$

Efficiency **Wiring cost**

V. Nicosia, P. Vertes, W. Schafer, V. Latora, E. Bullmore, *Proc. Natl. Acad. Sci. USA* 110 (19), 7880-7885 (2013)

Economical Spatio-temporal growth



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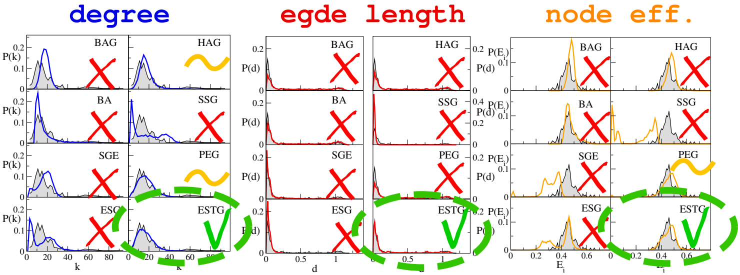

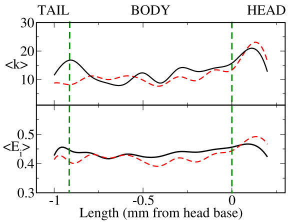

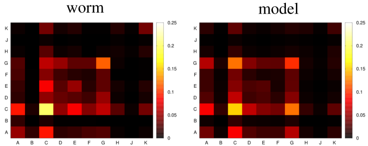

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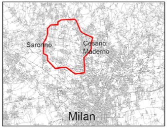
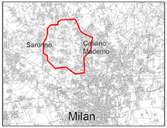
Outline



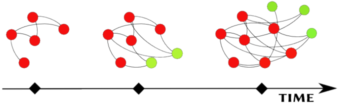
GROANE

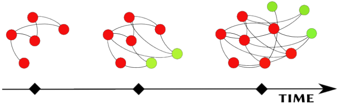
C.ELEGANS

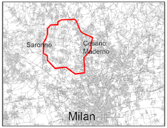


GROANE

C.ELEGANS







GROANE

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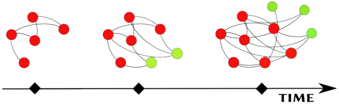
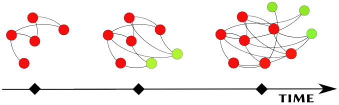
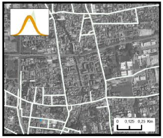
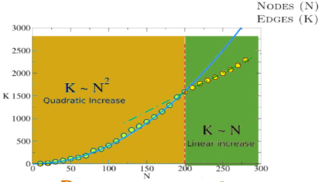






Before WWII **After WWII**



Before Hatching **After Hatching**

References

- E. Strano, V. Nicosia, V. Latora, S. Porta, M. Barthelemy, "Elementary processes governing the evolution of road networks", Nature Sci. Rep. 2, 296 (2012).
- V. Nicosia, P. E. Vertes, W. Schafer, V. Latora, E. Bullmore, "Phase transition in the economically modeled growth of a cellular nervous system", Proc. Natl. Acad. Sci. USA, 110 (19) 7880-7885 (2013)