Haldane phase and magnetic end-states in 1D topological Kondo insulators

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Workshop on Next Generation Quantum Materials

ICTP-SAIFR, Sao Paulo, April 2016

- *Lobos et al, Phys. Rev. X* ***5****, 021017 (2015)*
- *Mezio et al, Phys. Rev. B* ***92****, 205128 (2015)*

Collaborators

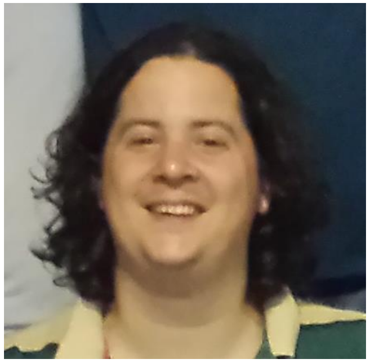
- *Analytical part (Bosonization)*

Ariel Dobry (IFIR-Rosario, Argentina)

Victor Galitski (UMD-CMTC, USA)

- *Numerics (Density Matrix Renormalization Group)*

Claudio Gazza (IFIR-Rosario, Argentina)



Alejandro Mezio (IFIR-Rosario, Argentina)
→Moving to University of Queensland, Australia

Where is Rosario?

Outline

- Introduction
  - Topological insulators
  - Topological Kondo insulators
  - The Haldane chain
- Main theoretical results using bosonization
- Numerical (DMRG) results
- Summary and perspectives

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- Introduction
    - **Topological insulators**
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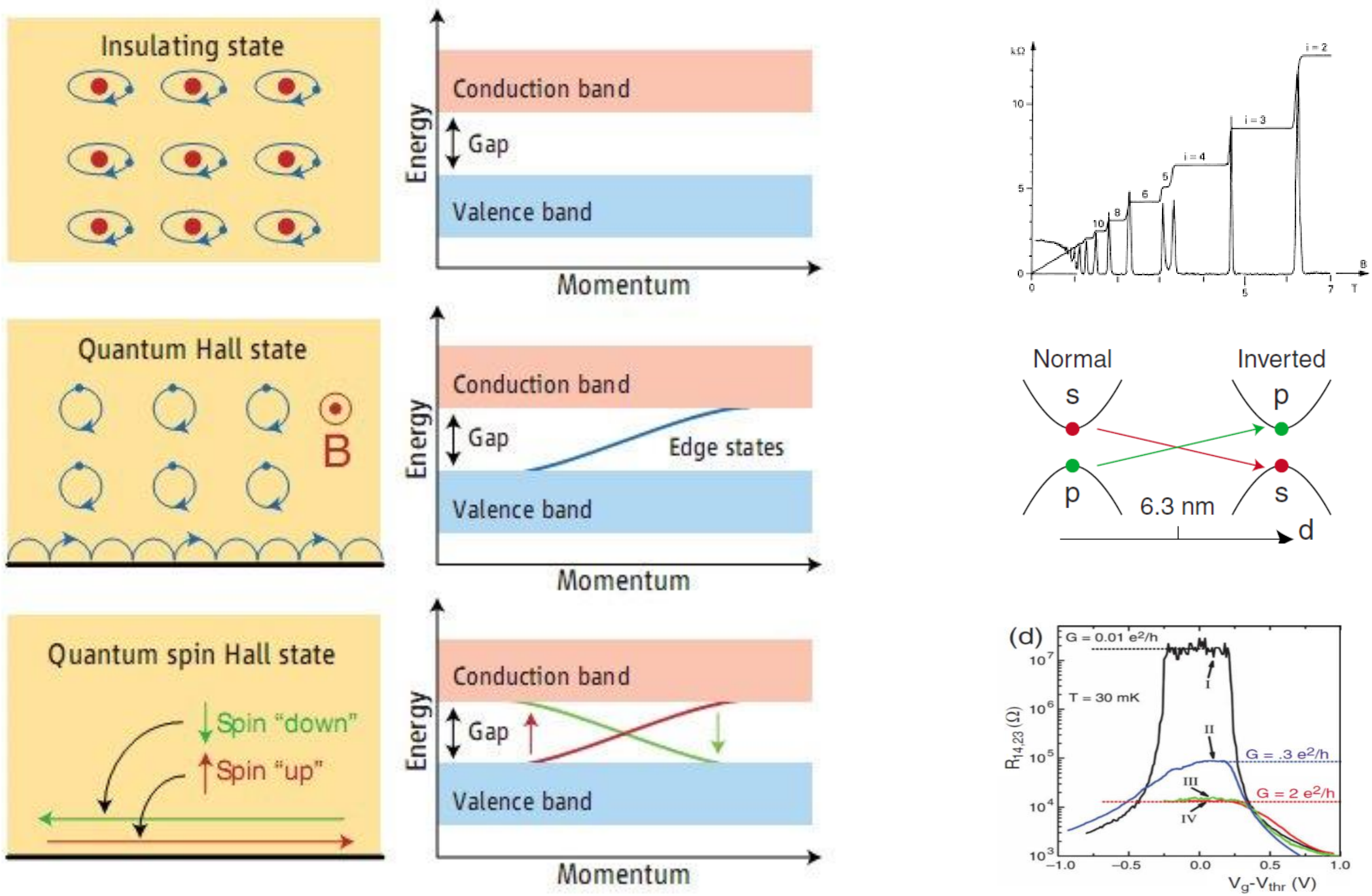
Topological insulators

- A new “zoo” of quantum phases of insulating materials classified by the topology of the band structure



- What distinguishes these insultators is the topology (topological invariant)
- **What is the effect of strong interactions?**

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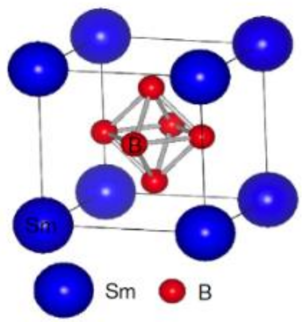
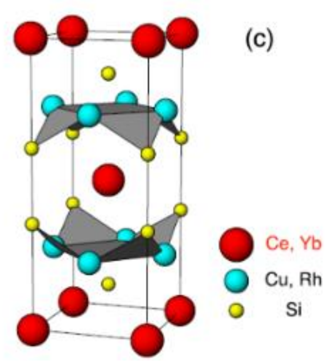
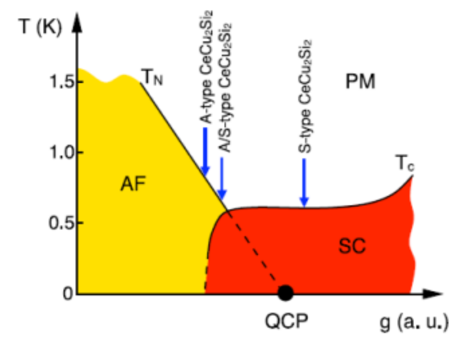
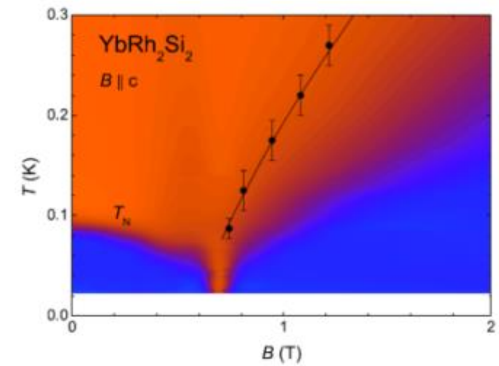
Heavy-fermions and Kondo insulators

- Heavy-fermions (SmB_6, YbB_{12}, $Ce_3Bi_4Pt_3$, $Ce_3Pt_3Sb_3$, CeNiSn, CeRhSb, etc.)

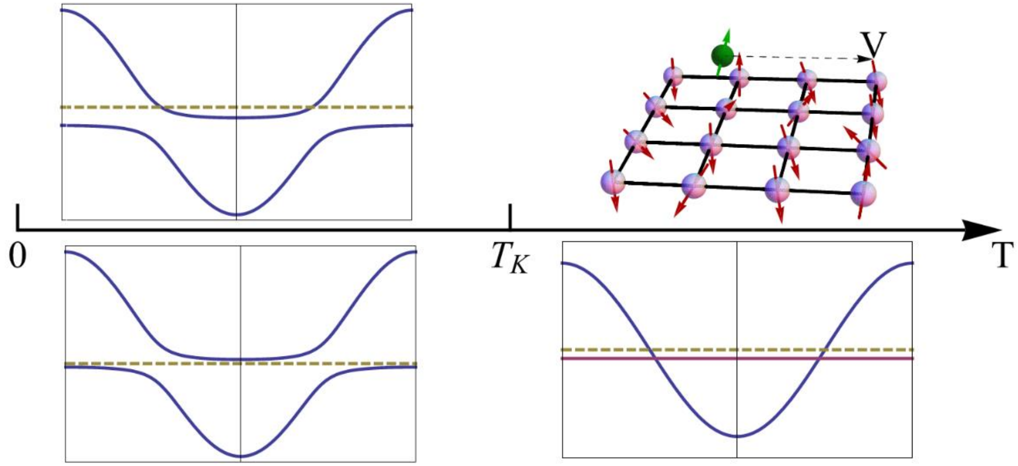






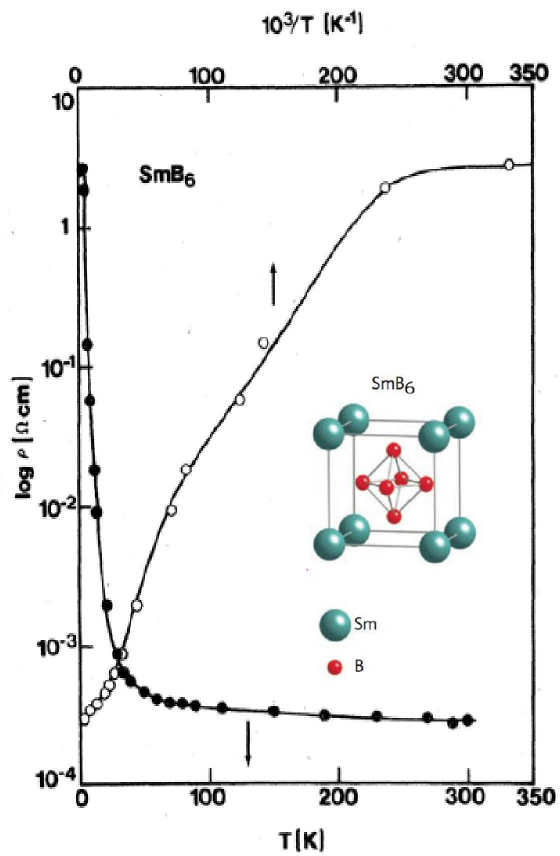


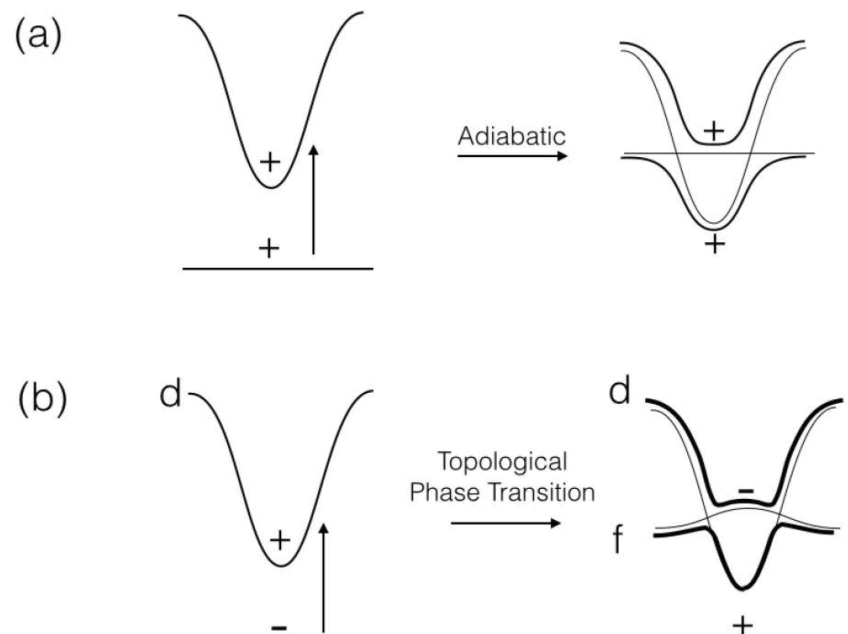
- Opening of a gap due to **strong correlations**



Topological Kondo insulators

- SmB_6: a Kondo insulator
- Strange feature: saturation of the resistivity at low T





- J. M. Allen, B. Batlogg and P. Wachter, PRB **20**, 4807 (1979)
- M. Dzero, K. Sun, V. Galitski and P. Coleman, PRL **104**, 106408 (2010)

Topological Kondo insulators in 1D

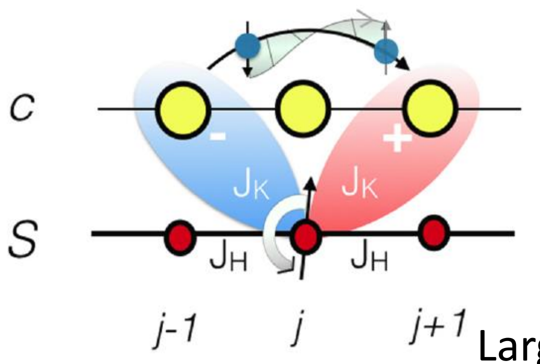
- Interplay of topology and strong correlation

PHYSICAL REVIEW B **90**, 115147 (2014)

End states in a one-dimensional topological Kondo insulator in large-*N* limit

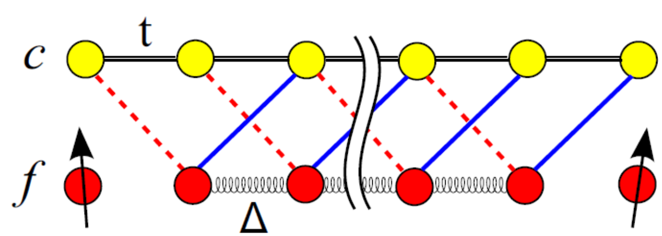
Victor Alexandrov[1] and Piers Coleman[1,2]
[1]*Center for Materials Theory, Department of Physics and Astronomy, Rutgers University, Piscataway, New Jersey 08854-8019, USA*
[2]*Department of Physics, Royal Holloway, University of London, Egham, Surrey TW20 0EX, UK*
(Received 27 March 2014; revised manuscript received 20 August 2014; published 26 September 2014)

- A toy model for a 1D TKI: The ***"p*-wave" Kondo-Heisenberg model** in 1D



Large-N mean-field approximation

Topological insulator
(Topological band theory)



Topologically protected end states

Topological *sp* chains

Contents lists available at ScienceDirect

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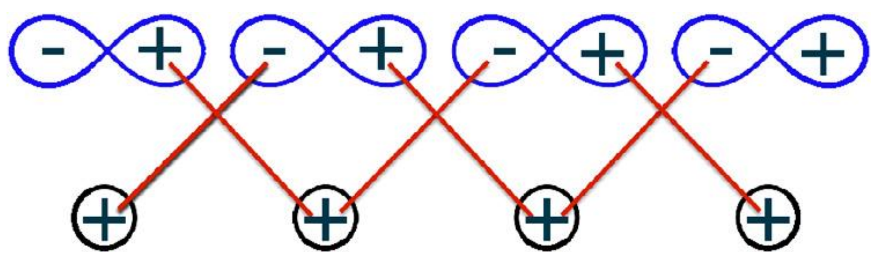
Topological states in normal and superconducting *p*-wave chains

Mucio A. Continentino [a,*], Heron Caldas [b], David Nozadze [c], Nandini Trivedi [c]

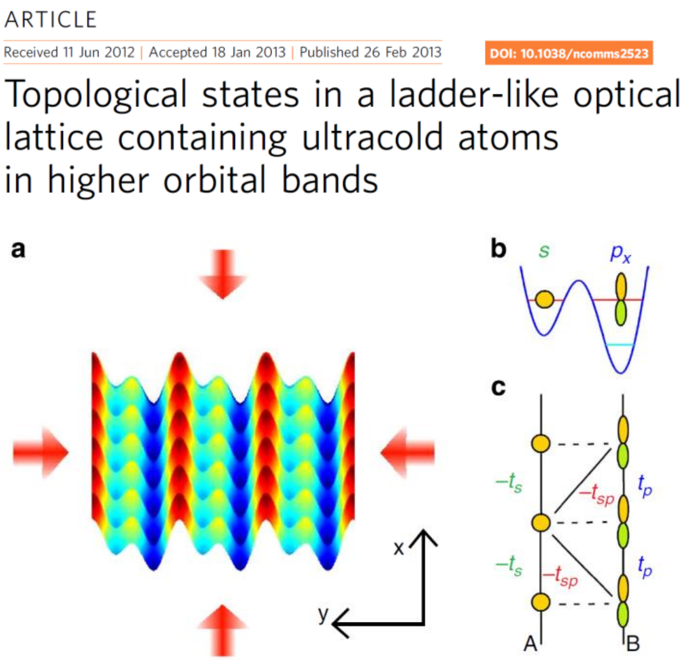
[a] Centro Brasileiro de Pesquisas Físicas, Rua Dr. Xavier Sigaud, 150, Urca 22290-180, Rio de Janeiro, RJ, Brazil
[b] Departamento de Ciências Naturais, Universidade Federal de São João Del Rei, 36301-000, São João Del Rei, MG, Brazil
[c] Department of Physics, The Ohio State University, Columbus, OH 43210, USA



- Non-interacting topological insulator in 1D sp chains
- Schottky-like Surface states

Experimental motivation: ultra-cold gases with higher p-bands

- **X. Li et at, Nature Comm. (2012)**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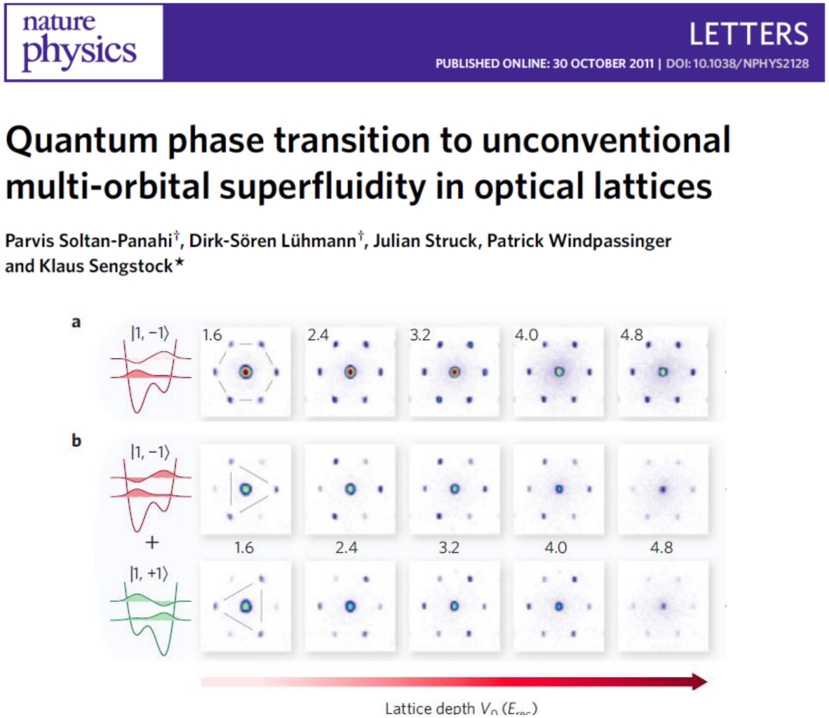
Received 11 Jun 2012 | Accepted 18 Jan 2013 | Published 26 Feb 2013 DOI: 10.1038/ncomms2523

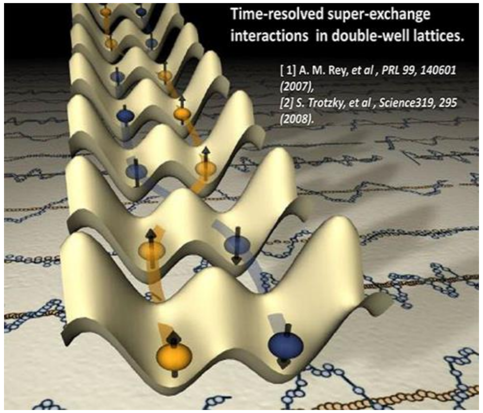
Topological states in a ladder-like optical lattice containing ultracold atoms in higher orbital bands

- **Soltan-Panahi et al, Nat. Phys. (2011)**


nature physics LETTERS
PUBLISHED ONLINE: 30 OCTOBER 2011 | DOI: 10.1038/NPHYS2128

Quantum phase transition to unconventional multi-orbital superfluidity in optical lattices

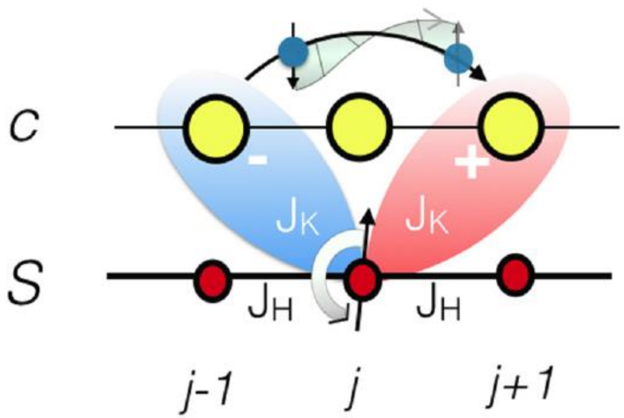
Parvis Soltan-Panahi†, Dirk-Sören Lühmann†, Julian Struck, Patrick Windpassinger and Klaus Sengstock*



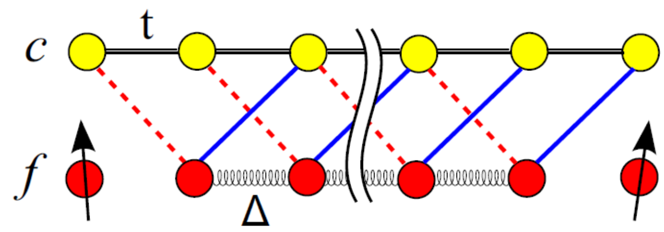
- **A. M. Rey et al, PRL 99, 146001 (2007)**

Main results of this work

- Mean-field is not correct in 1D



~~Large-N mean-field approximation~~



~~Topological band theory~~

- Main result (using Abelian bosonization and DMRG)

1D Topological Kondo insulator ⟷ **Haldane insulator**

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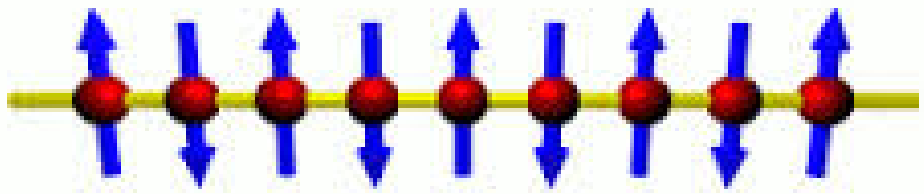
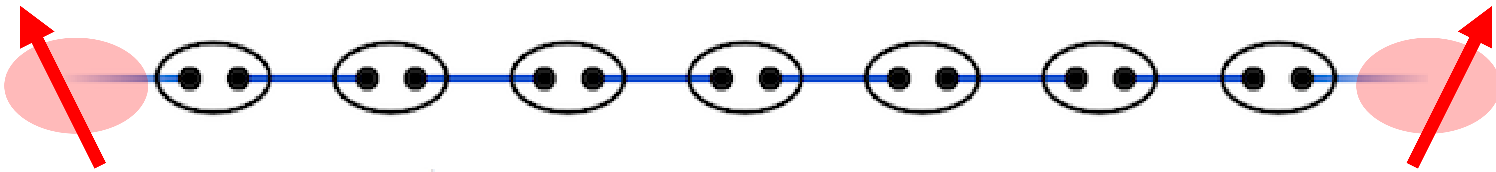
The Haldane chain



$$H = J \sum_j \mathbf{S}_j . \mathbf{S}_{j+1}$$

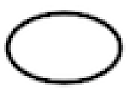
- Haldane conjecture for Heisenberg model: Half-integer spin chains are gapless
  Integer spin chains are gapped

Affleck-Kennedy-Tasaki-Lieb (AKTL) construction



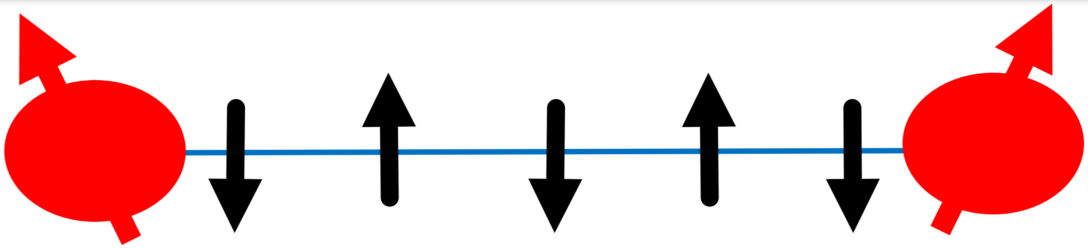
Spin-1/2 end-states

 $= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$

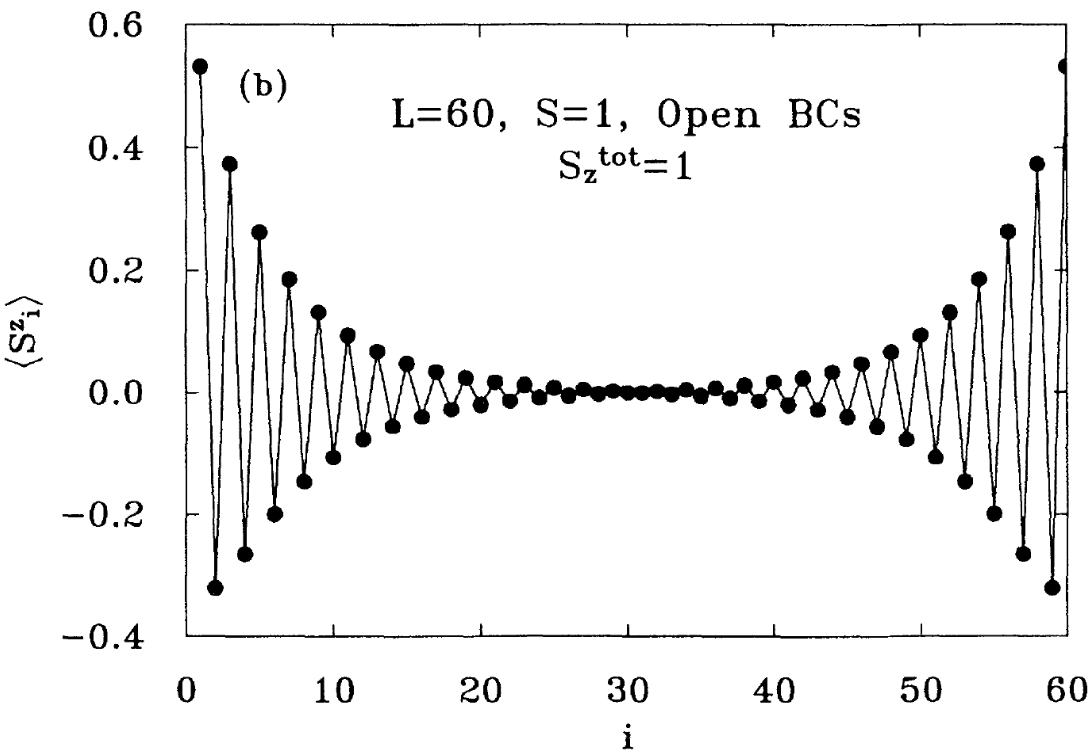
 $= |+\rangle\langle\uparrow\uparrow| + |0\rangle\frac{\langle\uparrow\downarrow| + \langle\downarrow\uparrow|}{\sqrt{2}} + |-\rangle\langle\downarrow\downarrow|$

- Strongly-correlated topological phase in 1D
- "String" order parameter $\mathcal{O}^{\alpha}_{\mathrm{string}}(l-m) \equiv -\left\langle T^{\alpha}_{l} e^{i\pi\sum_{l<j<m} T^{\alpha}_{j}} T^{\alpha}_{m}\right\rangle$
- Degeneracy in the entanglement-entropy spectrum (Pollman *et al*, PRB **81**, 064439, 2010)

Open-end Haldane S=1 chain. DMRG calculations



Magnetic S=1/2 end-states



Steven White, Phys. Rev B **48**, 10345 (1993)

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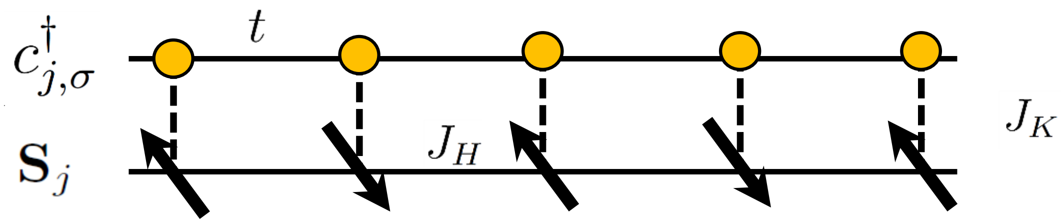
The “standard” 1D Kondo-Heisenberg model

PHYSICAL REVIEW B, VOLUME 63, 205104

Staggered liquid phases of the one-dimensional Kondo-Heisenberg lattice model

Oron Zachar
ICTP, 11 Strada Costiera, Trieste 34100, Italy
(Received 20 November 2000; published 13 April 2001)



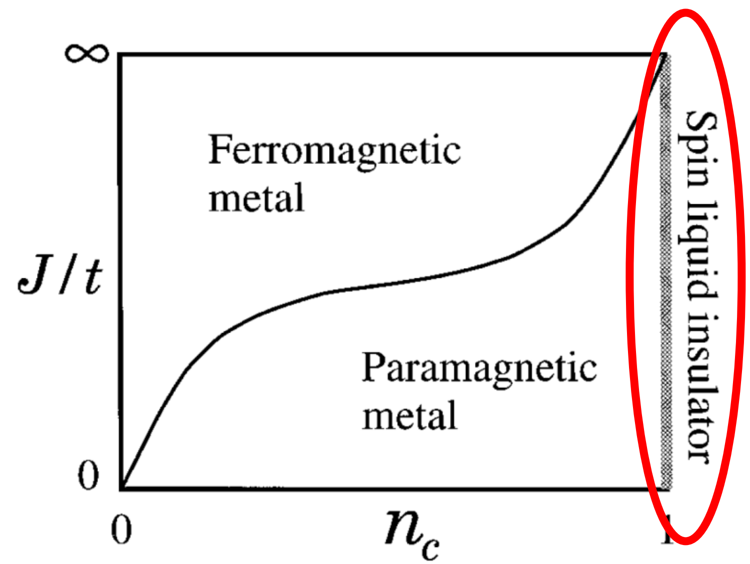
Original motivation:

- Heavy-fermion compounds
- Stripe phase in HTSC

$$H_1 = \sum_{j,\sigma} \left[-t \left(c^\dagger_{j,\sigma} c_{j+1,\sigma} + \mathrm{H.c.} \right) - \mu n_j \right]$$

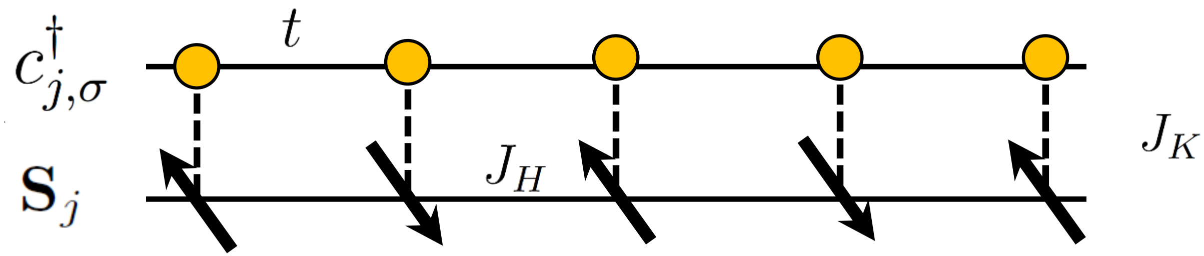
$$H_2 = J_H \sum_j \mathbf{S}_j . \mathbf{S}_{j+1}$$

$$H_K^{(\mathrm{s})} = J_K \sum_j \mathbf{S}_j . c^\dagger_{j,\alpha} \left(\frac{\boldsymbol{\sigma}_{\alpha\beta}}{2} \right) c_{j,\beta}$$



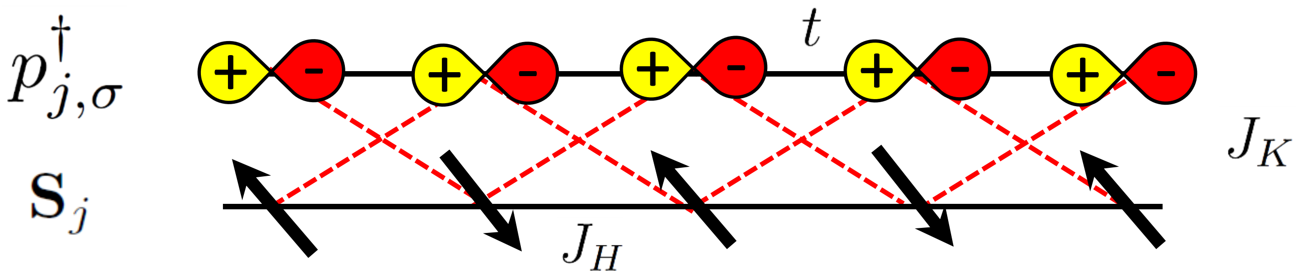
Standard vs p-wave Kondo-Heisenberg model

- **"Standard" Kondo-Heisenberg model**



$$H_K^{(\mathrm{s})} = J_K \sum_j \mathbf{S}_j . c^\dagger_{j,\alpha} \left(\frac{\boldsymbol{\sigma}_{\alpha\beta}}{2} \right) c_{j,\beta}$$

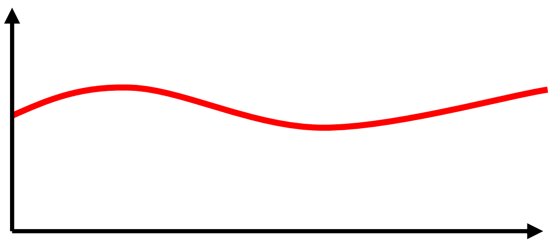
- ***"p-wave"* Kondo-Heisenberg model** (Alexandrov & Coleman, PRB 2014)

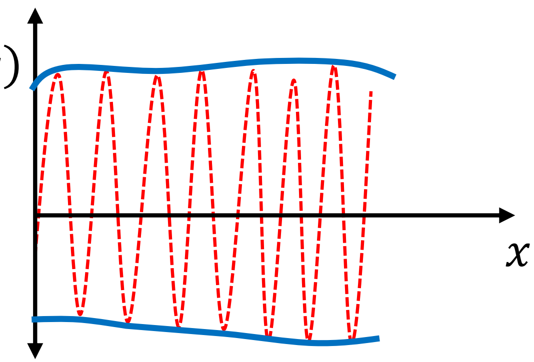


$$H_K = J_K \sum_{j=1}^{L} \mathbf{S}_j \cdot \boldsymbol{\pi}_j \qquad \boldsymbol{\pi}_j \equiv \sum_{\alpha,\beta} \Big(\frac{p^\dagger_{j+1,\alpha} - p^\dagger_{j-1,\alpha}}{\sqrt{2}} \Big) \Big(\frac{\boldsymbol{\sigma}_{\alpha\beta}}{2} \Big) \Big(\frac{p_{j+1,\beta} - p_{j-1,\beta}}{\sqrt{2}} \Big)$$

p-wave combination of conduction-electron orbitals

- Decomposition of the spin densities





$$\frac{\mathbf{S}_j}{a} \sim \overbrace{\mathbf{J}_{2R}(x_j) + \mathbf{J}_{2L}(x_j)}^{\text{Uniform magnetization field}} + \overbrace{(-1)^j\, \mathbf{N}_2(x_j)}^{\text{“Staggered” field}}$$

$$\frac{\boldsymbol{\pi}_j}{a} \sim 2\left[\mathbf{J}_{1R}(x_j) + \mathbf{J}_{1L}(x_j) - (-1)^j\, \mathbf{N}_1(x_j)\right]$$

$\boxed{\text{Sign!}}$

Field-theory description

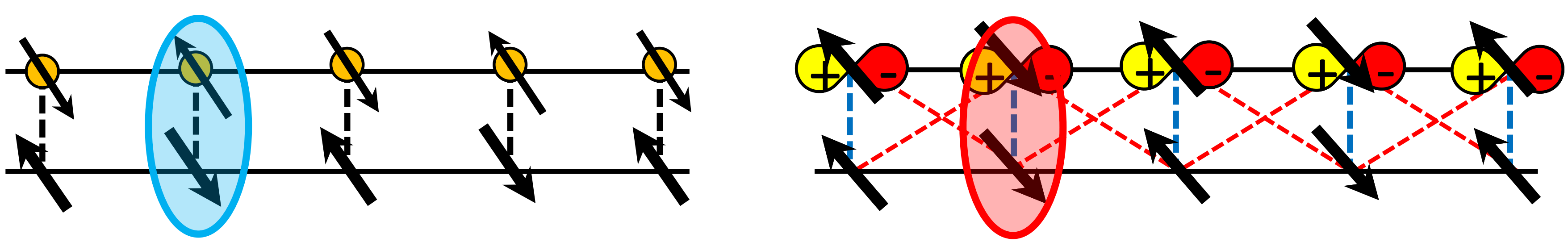
- Antiferromagnetic Kondo interaction $(J_K > 0)$

$$H_K = J_K \sum_j \mathbf{S}_j . \boldsymbol{\pi}_j$$

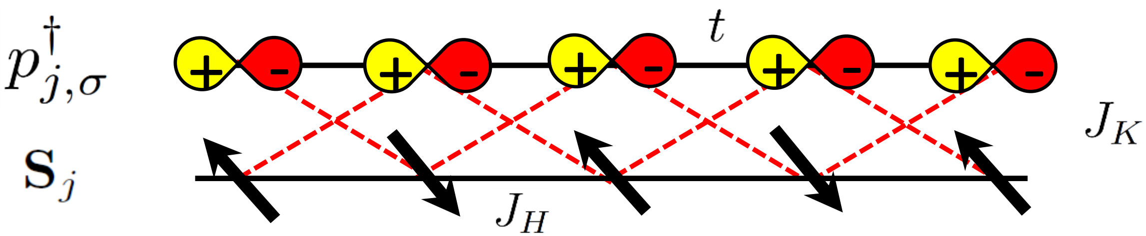
$$\sim 2J_K a \int_0^L dx \; \left[\mathbf{J}_{2R}(x) + \mathbf{J}_{2L}(x) + (-1)^j \mathbf{N}_2(x)\right] . \left[\mathbf{J}_{1R}(x) + \mathbf{J}_{1L}(x) - (-1)^j \mathbf{N}_1(x)\right]$$

$$\sim -2J_K a \int_0^L dx \; \mathbf{N}_1(x) . \mathbf{N}_2(x) + (\text{less relevant terms } \sim \mathbf{J}_1(x) . \mathbf{J}_2(x))$$

- **Effect of topology**: Effective (local) **ferromagnetic** interaction $(-J_K < 0)$



Bosonization



- Bosonized Hamiltonian

Charge sector (conduction band)

Spin sector (both chains)

$$H_K = -\frac{2J_K m_2}{\pi^2 a}\int_0^L dx\, \boxed{\cos\left(\sqrt{2}\phi_{1c}\right)} \times \left[\cos\left(\sqrt{2}\theta_{1s}\right)\cos\left(\sqrt{2}\theta_{2s}\right) + \sin\left(\sqrt{2}\theta_{1s}\right)\sin\left(\sqrt{2}\theta_{2s}\right) + \sin\left(\sqrt{2}\phi_{1s}\right)\sin\left(\sqrt{2}\phi_{2s}\right)\right].$$

- Perturbative RG equations *(Lobos et al, Phys. Rev. X, **5**, 021017 (2015))*

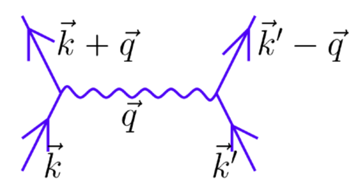
Luttinger parameters K →

Umklapp →

Kondo parameter →

$$\frac{dG_{2c}}{dl} = G_3^2 + \frac{3}{4}G_K^2$$
$$\frac{dG_3}{dl} = G_{2c}G_3 + \frac{3}{4}G_K^2$$
$$\frac{dG_K}{dl} = \frac{G_K}{2} + \frac{3}{4}G_{2c}G_K + \frac{3}{4}G_3G_K$$

Dinamically generated *U>0* in the band



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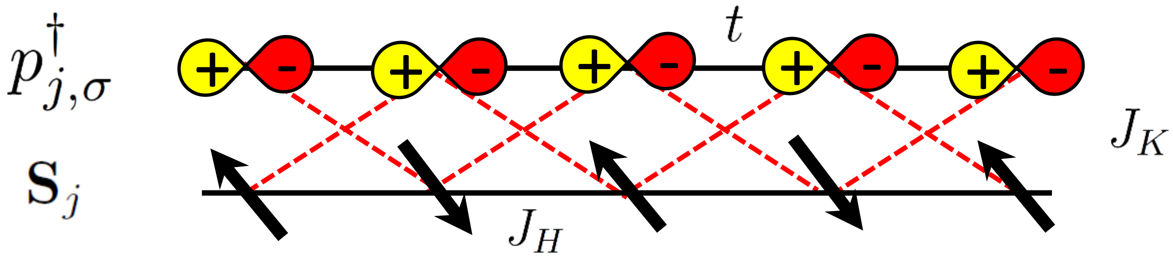
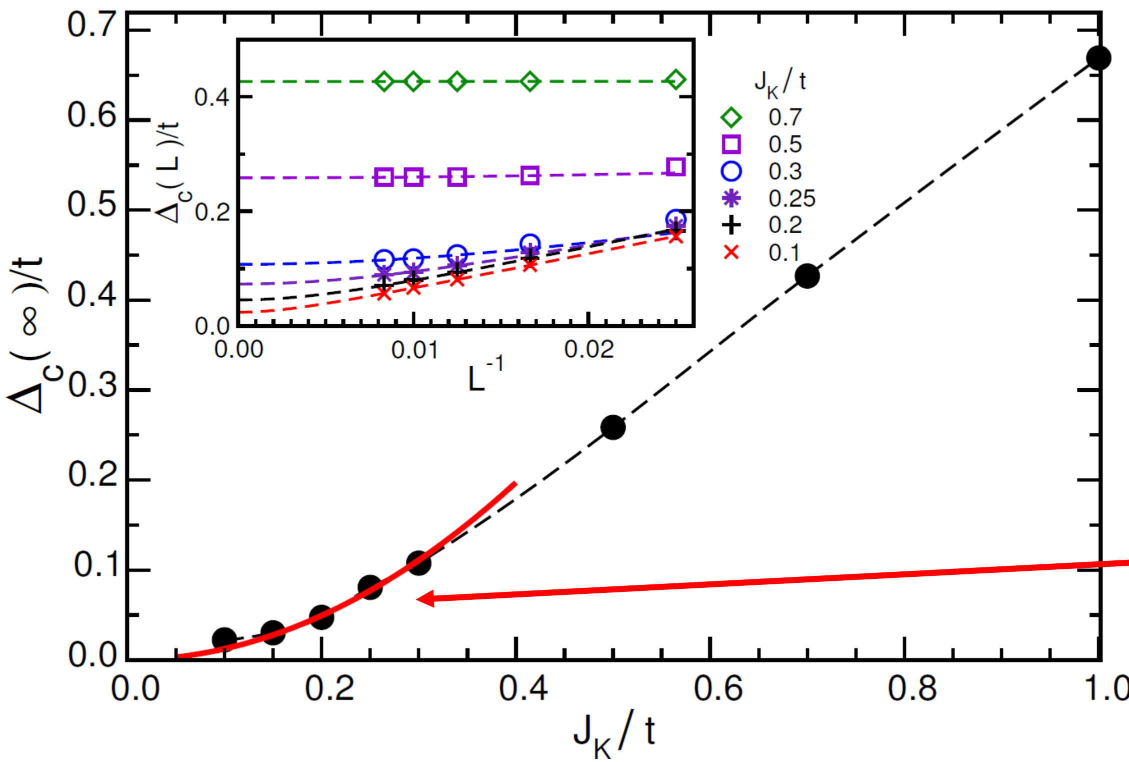
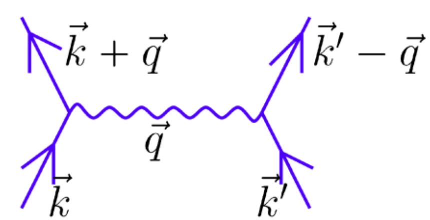
DMRG results: Gap in the charge sector (Mott insulator)





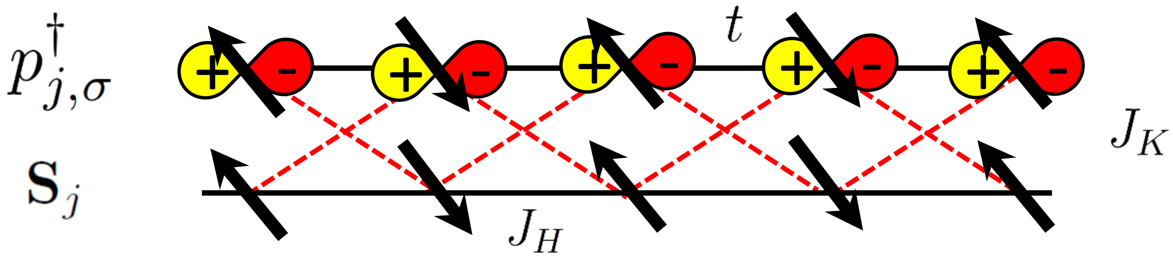
~ J_K^2 behavior (as predicted by bosonization)



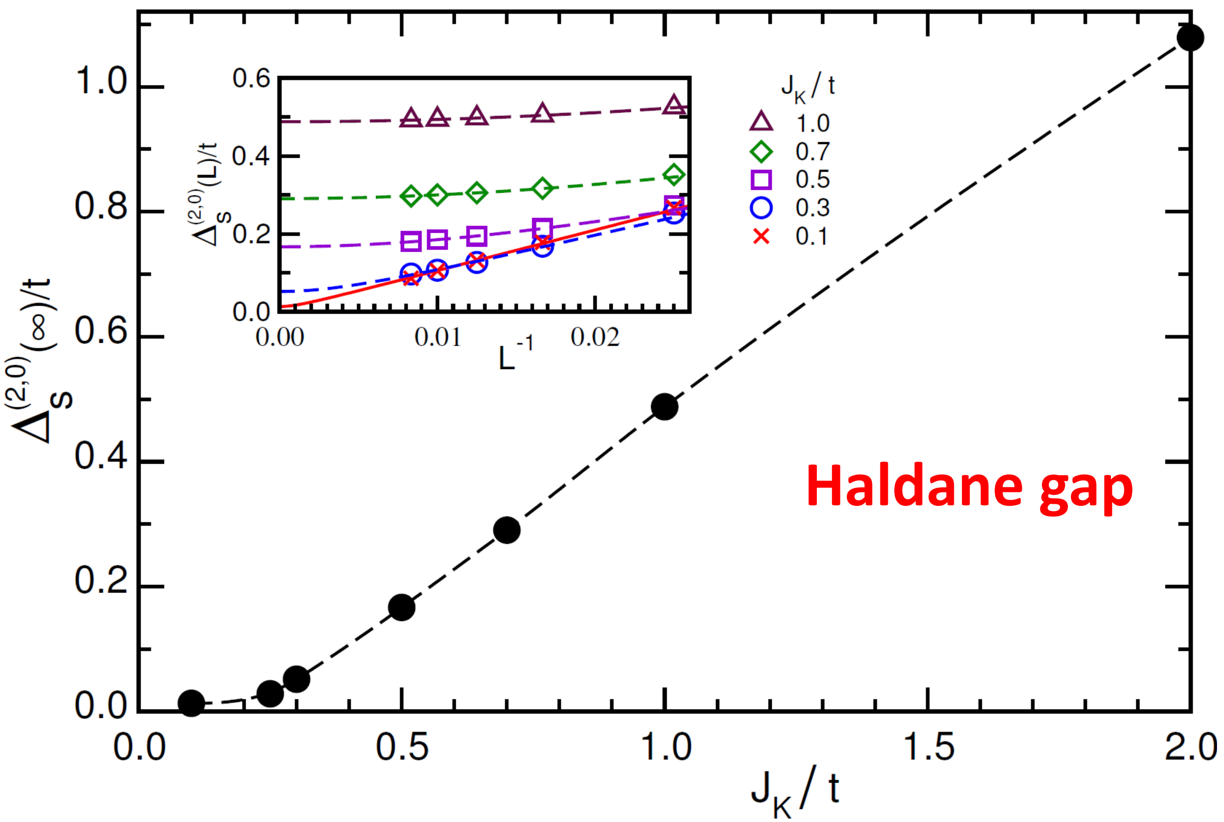
Insulating behavior: Mott gap (even for *U*=0)

*Mezio et al, Phys. Rev. B **92**, 205128 (2015)*

DMRG results. Haldane gap

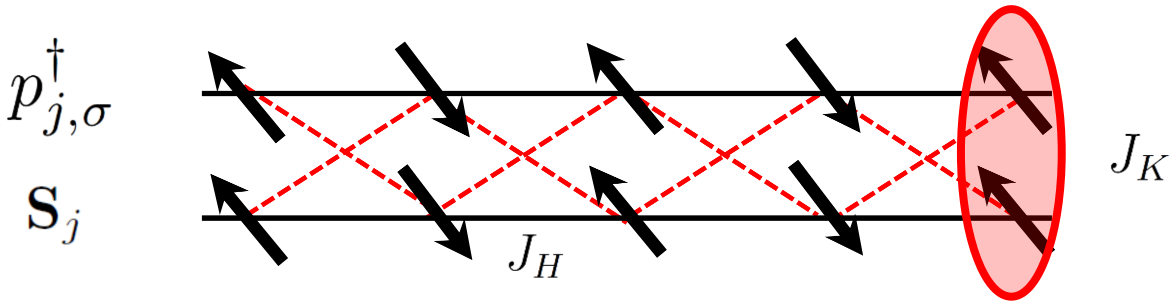


- Effective spin-1/2 FM ladder → Effective spin-1 chain



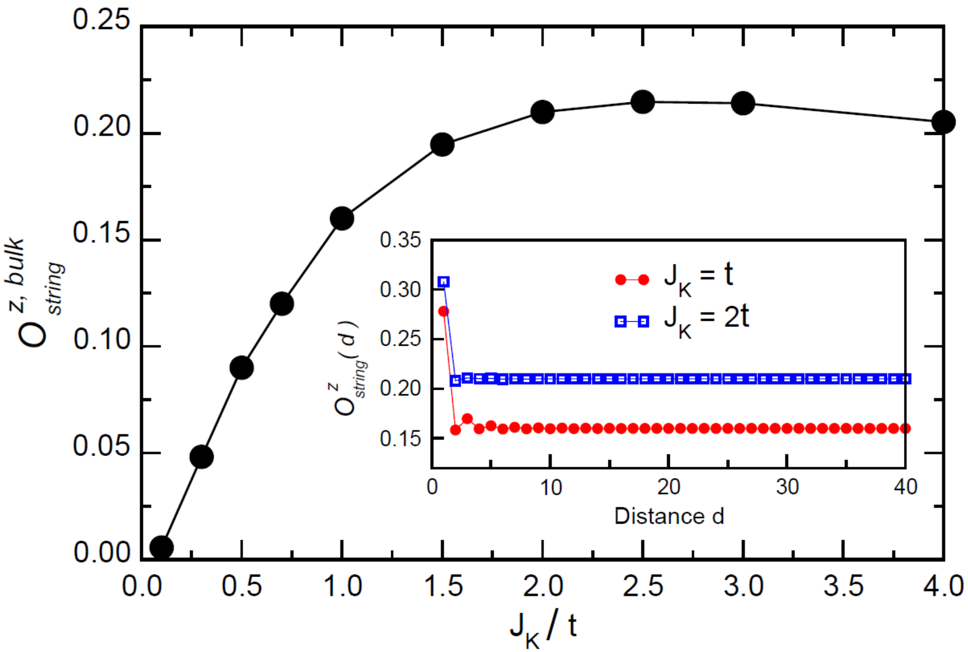
*Mezio et al, Phys. Rev. B **92**, 205128 (2015)*

DMRG results. String order parameter



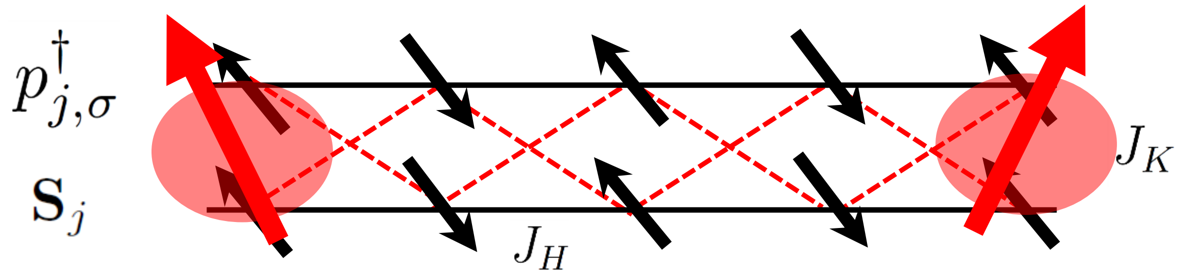
$$\mathbf{T}_j \equiv \mathbf{S}_j + \mathbf{s}_j$$

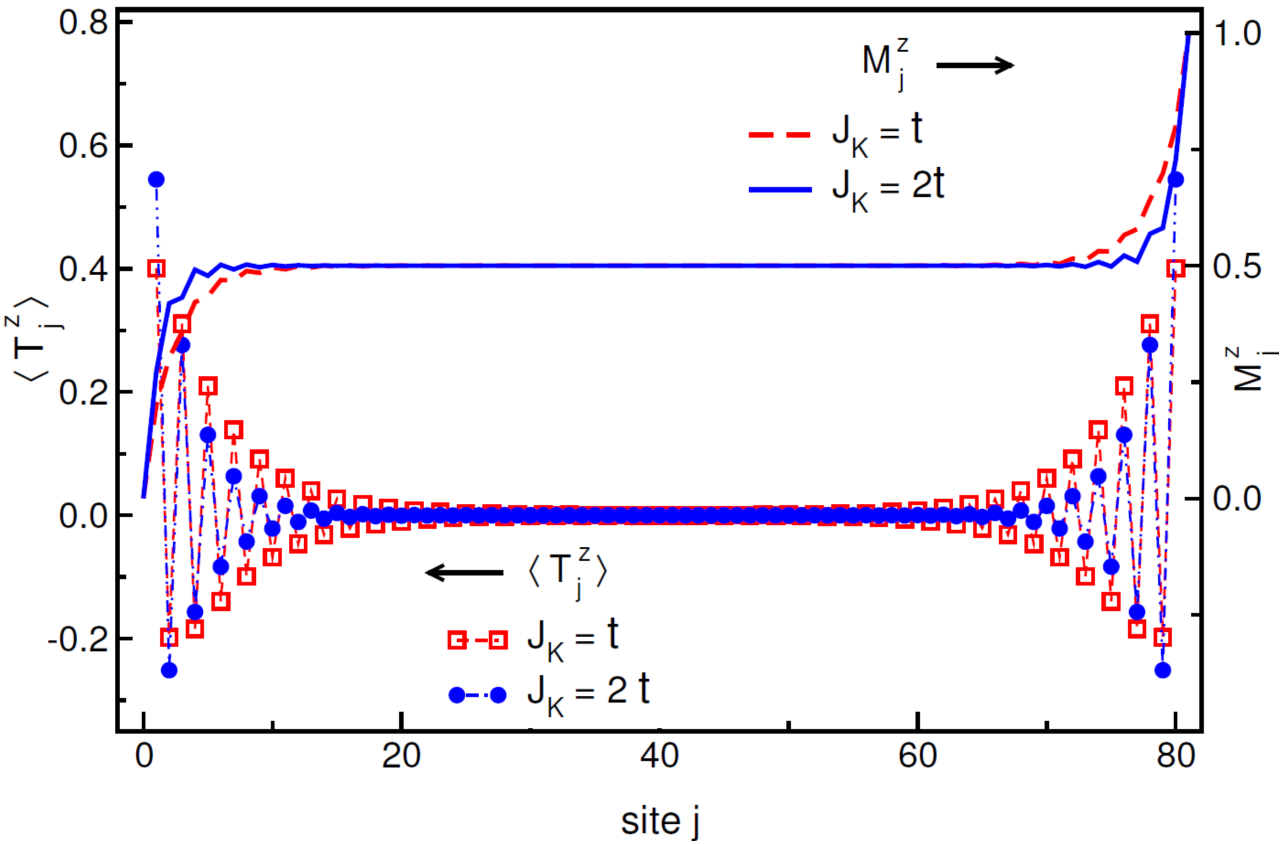
- "String" order parameter $\mathcal{O}^{\alpha}_{\text{string}}(l-m) \equiv -\left\langle T^{\alpha}_l e^{i\pi \sum_{l<j<m} T^{\alpha}_j} T^{\alpha}_m \right\rangle$



*Mezio et al, Phys. Rev. B **92**, 205128 (2015)*

DMRG results. Spin ½ end-states



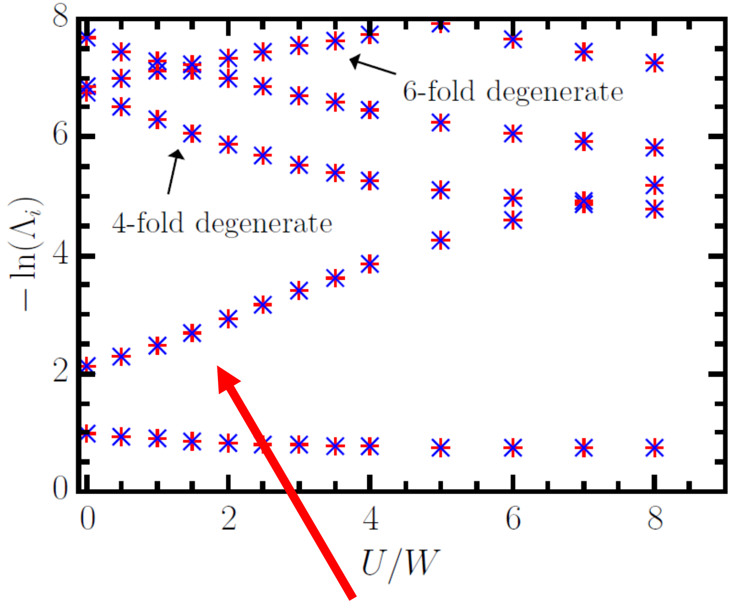


*Mezio et al, Phys. Rev. B **92**, 205128 (2015)*

Independent confirmation of the Haldane phase

Characterization of a correlated topological Kondo insulator in one dimension

I. Hagymási and Ö. Legeza
Strongly Correlated Systems "Lendület" Research Group, Institute for Solid State Physics and Optics, MTA Wigner Research Centre for Physics, Budapest H-1525 P.O. Box 49, Hungary
(Dated: January 19, 2016)



- DMRG results showing the degeneracy in the entanglement-entropy spectrum
- "True" signature of the Haldane phase

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Summary and perspectives

- Summary
  - The groundstate of a 1D topological Kondo insulator is a Haldane phase
  - Robust spin-1/2 magnetic end-states of topological origin
- Perspectives
  - How these results fit in a more general classification scheme (i.e., symmetry-protected topological phases)?
  - Higher dimensions?
  - What is the relation to the non-interacting limit?

Thanks!