Effective Field Theory for Cold Atoms II

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EMMI

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DFG

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Agenda

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1. EFT for Ultracold Atoms I: Effective Field Theories & Universality
2. EFT for Ultracold Atoms II: Cold Atoms & the Unitary Limit
3. EFT for Ultracold Atoms III: Weak Coupling at Finite Density
4. EFT for Ultracold Atoms IV: Few-Body Systems in the Unitary Limit
5. Beyond Ultracold Atoms: Halo Nuclei and Hadronic Molecules

Literature

G.P. Lepage, TASI Lectures 1989, arXiv:hep-ph/0506330

D.B. Kaplan, arXiv:nucl-th/9506035

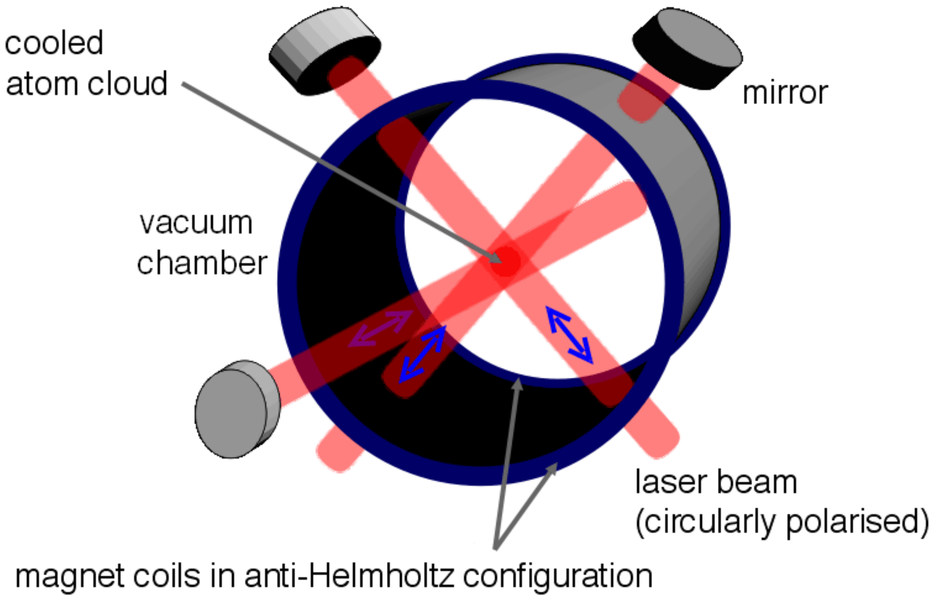
E. Braaten, HWH, Phys. Rep. **428** (2006) 259 [arXiv:cond-mat/0410417]

Cold Atom Physics

- Atoms can be trapped and cooled using EM fields and lasers

- Example:

  Magneto-Optical-Trap (MOT)



- Nobel Prize 1997: Chu, Cohen-Tannoudji, Phillips

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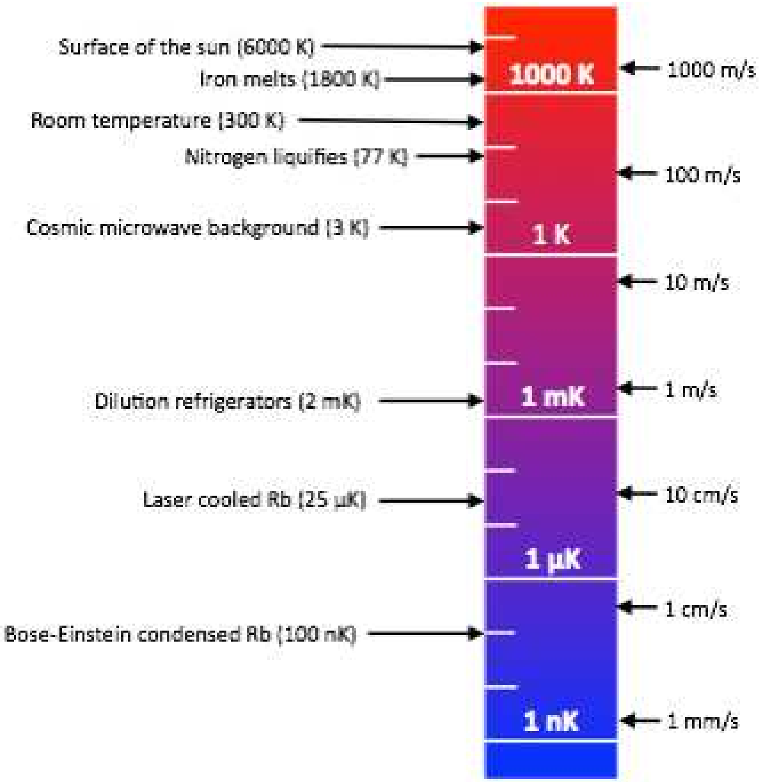
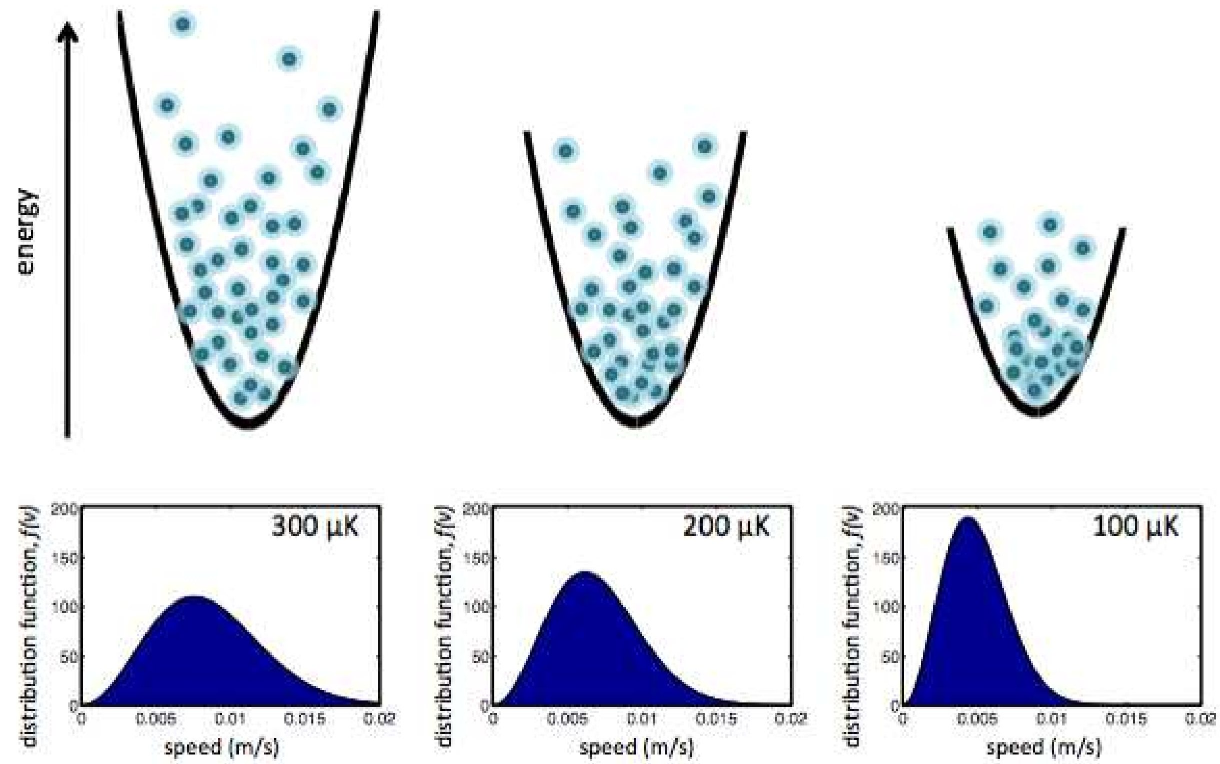
- Temperature further reduced by evaporative cooling

  $\Longrightarrow$ “coldest matter in the universe”





average velocity for Rb atoms

L.J. Leblanc, U. Alberta

Bose-Einstein Condensation

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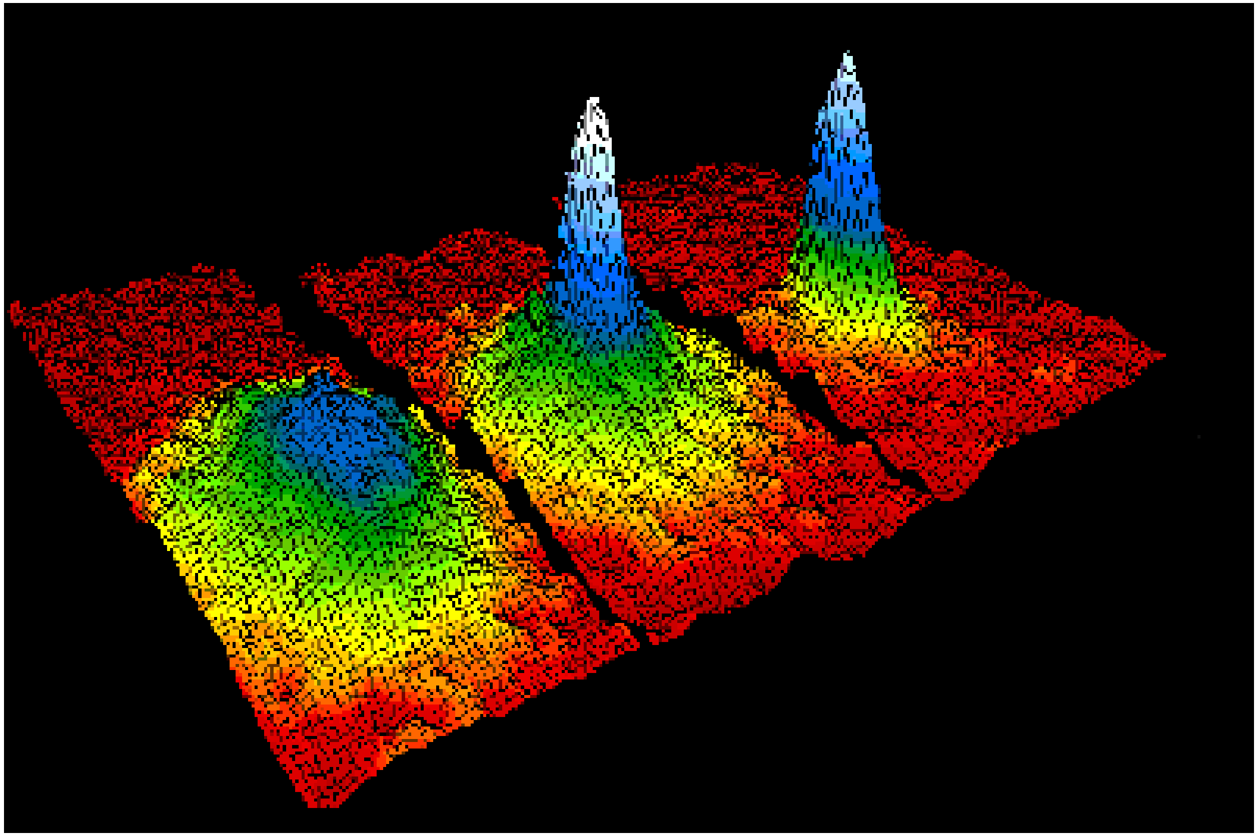
- Velocity distribution of ^{87}Rb atoms ($T =$ 400 nK, 200 nK, 50 nK)

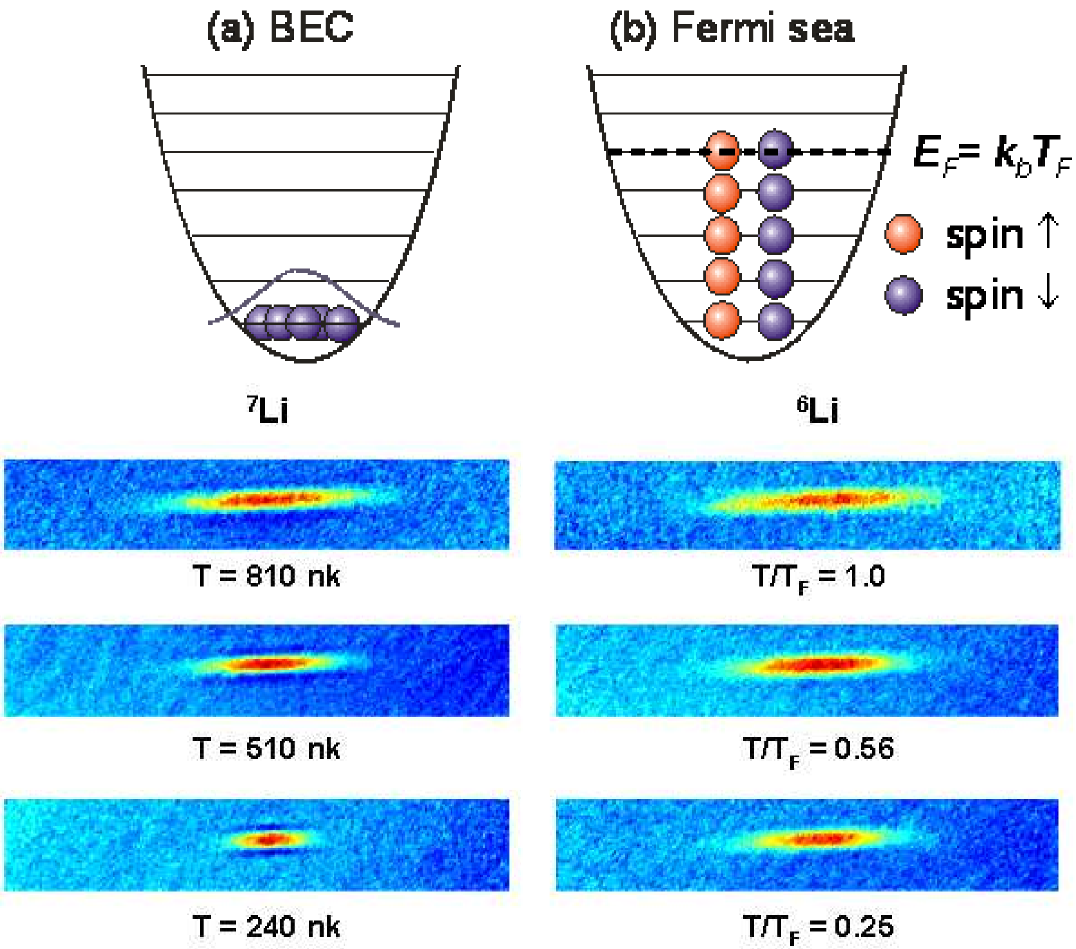


http://jilawww.colorado.edu/bec/

- Nobel Prize 1997: Cornell, Wieman, Ketterle

Many-Body Ground State

- Ground state of a many-body system



Hulet group, Rice University

Atom Interactions

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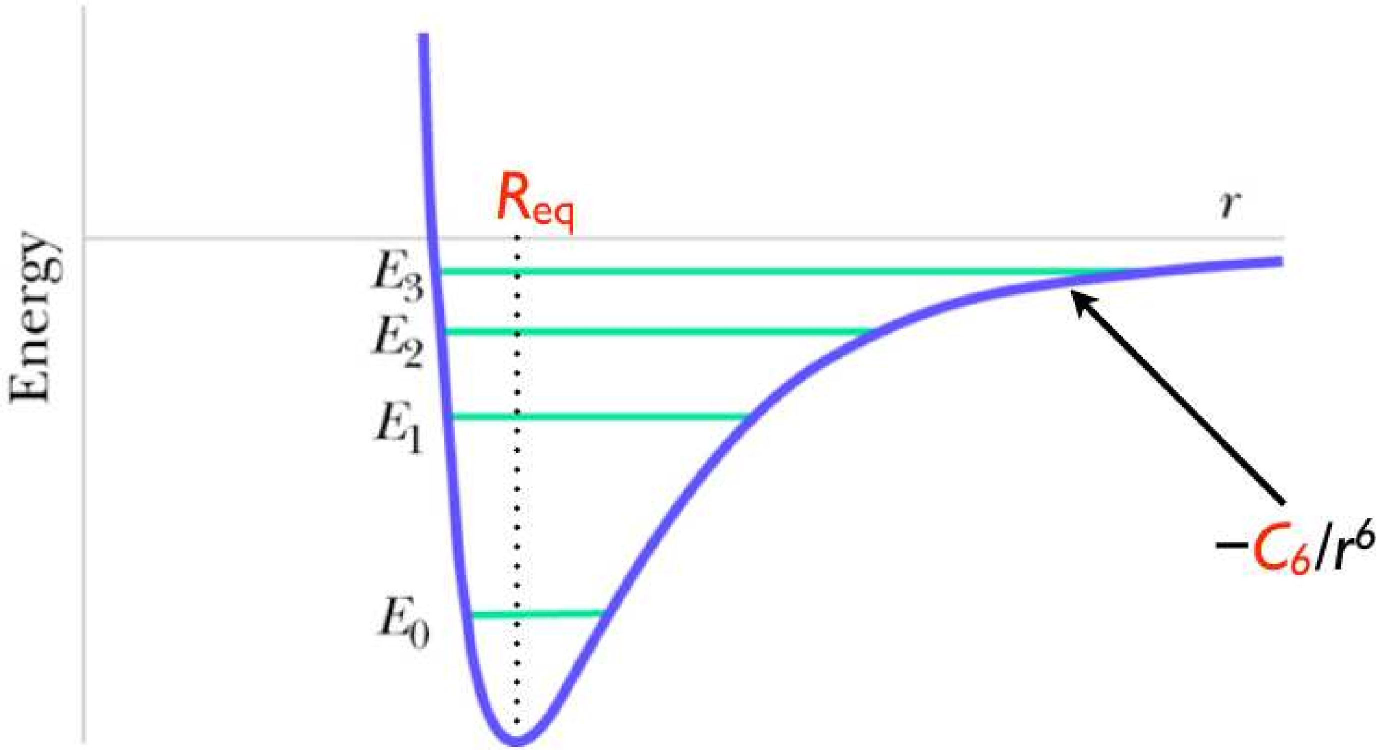
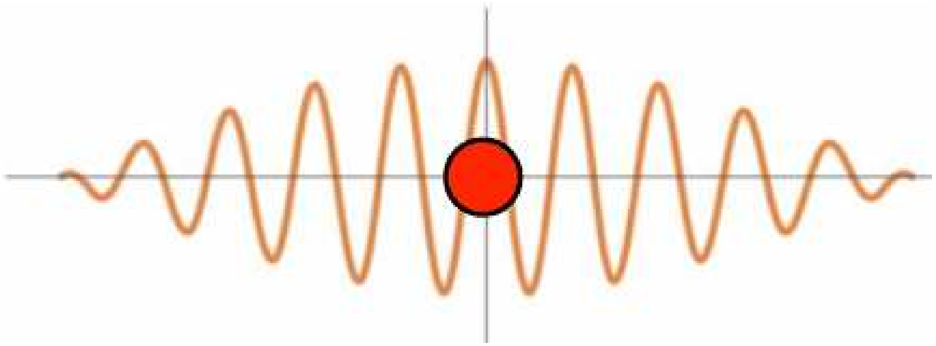


- Size of the atoms: $R_{eq} \sim 0.4$ nm (Rb)
- Range of interaction: $R_{vdW} = (mC_6/\hbar^2)^{1/4} \sim 8$ nm (Rb)

Atom Interactions

- Thermal de Broglie wavelength: $\lambda_{th} = (2\pi\hbar^2/mk_BT)^{1/2}$

T	λ_{th} (Rb)
1 K	0.2 nm
1 mK	6 nm
1 μK	20 nm
1 nK	600 nm



- Size of atoms: $R_{eq} \sim 0.4$ nm (Rb)
- Range of interaction: $R_{vdW} \sim 8$ nm (Rb)
- $T < 1$ K: atoms behave as point particles
- $T < 1$ mK: only S-wave interactions, finite range unresolved

  $\Longrightarrow$ use contact interactions (zero range)

Atom Interactions

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- Effective range expansion

$$T(k) \propto \Big[\underbrace{-1/a + r_e k^2/2 + \ldots}_{k \cot \delta} - ik\Big]^{-1}, \qquad k = \sqrt{mE}$$

- Natural case: $R_{vdW} \sim a,\ r_e,\ ..$

  $\Longrightarrow \quad k \ll 1/R_{vdW}$: expand in ka, kr_e, ..
- Interactions at low energies determined by scattering length a

  $\Rightarrow$ scattering cross section at threshold: $\sigma = 4\pi a^2$
- Ultracold atoms: control variables ρ, T

$$\Longrightarrow \rho^{-1/3}, \lambda_{th} \gg R_{vdW} \sim a,\ r_e,\ ..$$

- Physics captured by perturbative EFT with contact interactions
- Applications to many-body systems of Bosons & Fermions

  (cf. Braaten, Nieto (1997); HWH, Furnstahl (2000), ...)

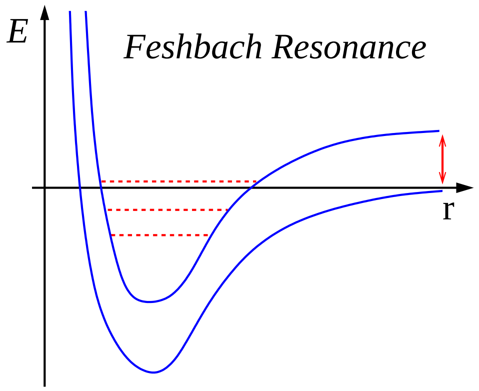
Variable Scattering Length

- Ultracold atoms: control variables ρ, T, a

$$\Longrightarrow \rho^{-1/3}, \lambda_{th}, |a| \gg R_{vdW}$$

- Feshbach(Fano) resonance:

  energy of molecular state in closed channel close to energy of scattering state



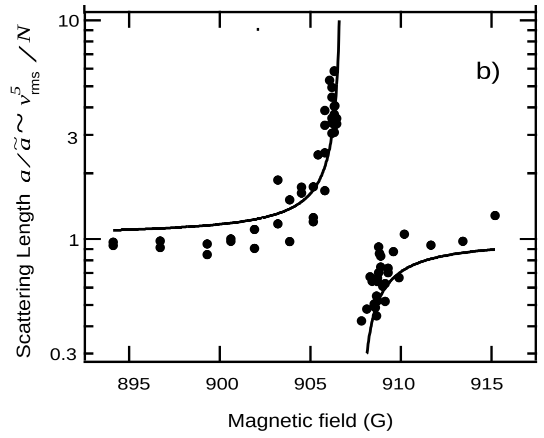
- Tune scattering length via external magnetic field

  (Tiesinga, Verhaar, Stoof, 1993)

- Observation in a Na BEC

  (Inouye et al. (MIT), 1998)

$$\frac{a(B)}{a_0} = 1 + \frac{\Delta}{B_0 - B}$$



Physics Near the Unitary Limit

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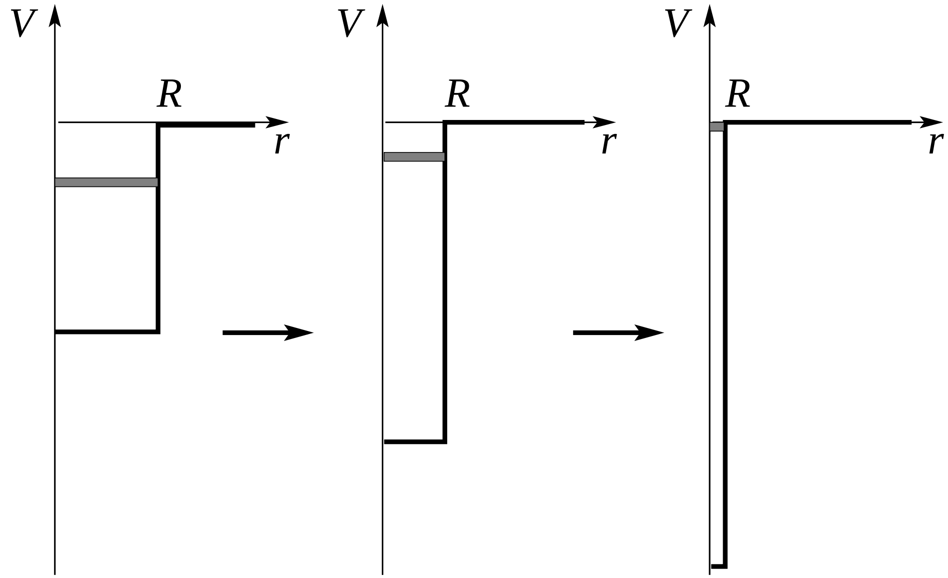
- Consider system with short-ranged, resonant interactions
- Unitary limit: $a \to \infty$, $R \sim r_e \to 0$ (cf. Bertsch problem, 2000)



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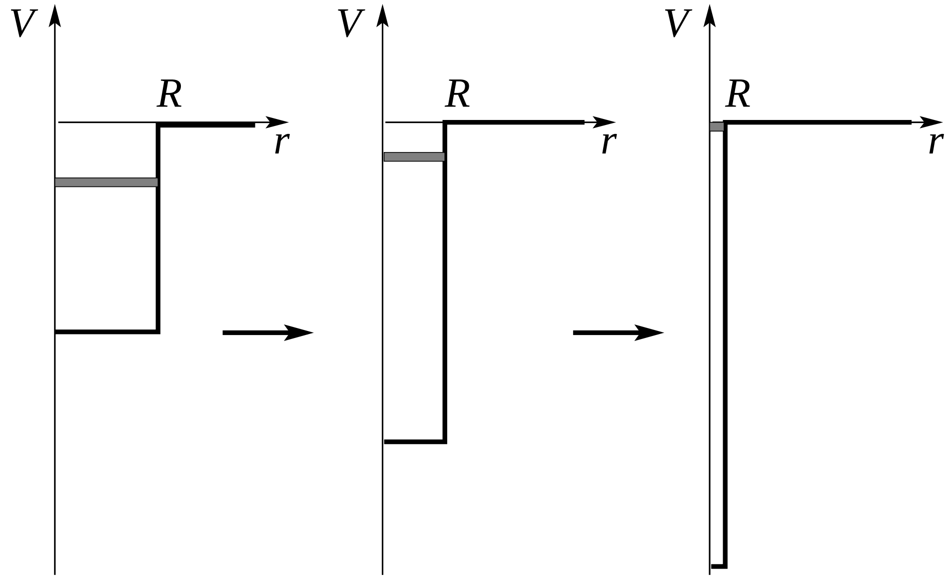

$$\mathcal{T}_2(k,k) \propto \Big[\underbrace{k \cot \delta}_{-1/a + r_e k^2/2 + \ldots} - ik \Big]^{-1} \Longrightarrow i/k$$

- Scattering amplitude scale invariant, saturates unitarity bound

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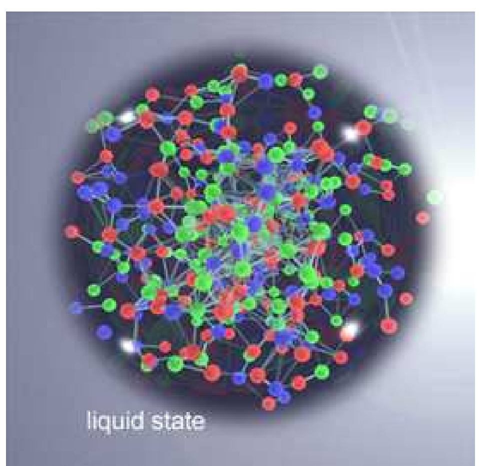
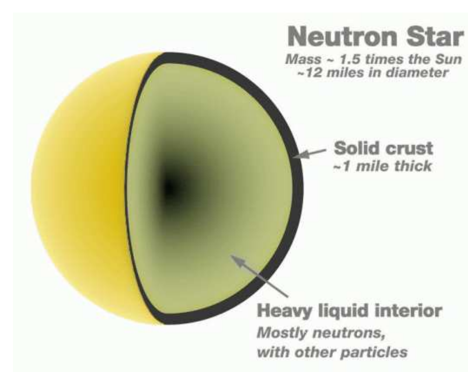
  - can be realized with cold atoms close to Feshbach resonance
- Interesting many-body physics: BEC/BCS crossover, universal viscosity bound $\Rightarrow$ perfect liquid, ...





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  - can be realized with cold atoms close to Feshbach resonance
- Interesting many-body physics: BEC/BCS crossover, universal viscosity bound ⇒ perfect liquid, ...
- Use as starting point for description of few-body properties
  - Large scattering length: $|a| \gg R \sim r_e, R_{vdW}, \ldots$
  - Natural expansion parameter: $R/|a|$, kR,...
  - Universal dimer with energy $E_d = -1/(ma^2)$ $(a > 0)$

    size $\langle r^2 \rangle^{1/2} \sim a$ $\Rightarrow$ halo state

Short-Range Interactions

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- Two-Body System (S-Waves)
- Large scattering length: $a \gg R \sim r_e, .. \Longrightarrow$ universal properties

$$T(k) \propto \frac{-a}{1+ika}\left[1+\frac{r_e a k^2/2}{1+ika}+\ldots\right], \qquad 1/a \sim k \ll 1/r_e$$

- Shallow bound/virtual states: $B_d = \frac{\hbar^2}{ma^2}\left(1 + r_e/a + \ldots\right)$
- Nonperturbative resummation in EFT (van Kolck, Kaplan, Savage, Wise)

  $\Longrightarrow$ resum leading-order contact interaction to generate shallow bound/virtual state
- Nontrivial physics in few- and many-body systems

Physical Systems with Large a

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- Natural expansion parameter: $R/|a|$, kR,... $(R \sim r_e, R_{vdW}, ...)$
- Nuclear Physics: S-wave NN scattering, halo nuclei,...
  - $^1S_0$, $^3S_1$: $|a| \gg r_e \sim 1/m_\pi \longrightarrow B_d \approx 2.2$ MeV
  - neutron matter
  - halo nuclei
- Atomic Physics:
  - ^{4}He atoms: $a \approx 104$ Å $\gg r_e \approx 7$ Å $\sim l_{vdW}$, $\longrightarrow B_d \approx 100$ neV
  - ultracold atoms near a Feshbach resonance
- Particle Physics
  - Is the $X(3872)$ a $|D^0\bar{D}^{0*} + \bar{D}^0 D^{0*}\rangle$ molecule? $(J^{PC} = 1^{++})$

$$E_X = m_D + m_{D^*} - m_X = (0.1 \pm 0.2) \text{ MeV}$$

Summary

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- Cold atoms provide controlled environment for experiments in few- and many-body systems (T, ρ)
- "Simple", tunable interactions at low energies

  $\Rightarrow$ weak and strong interactions can be realized via Feshbach resonances
- Amenable to Effective (Field) Theory description
- Test of theoretical models $\Longrightarrow$ Quantum Simulations
- Example: ground state energy of the unitary Fermi gas

$$E/N = \xi \, E_{free} = \xi \, \frac{3}{5} \, \frac{k_F^2}{2m} \,, \qquad \xi \approx 0.37$$

Additional Transparencies