Beyond Ultracold Atoms: Halo Nuclei and Hadronic Molecules

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School on Effective Field Theory across Length Scales, ICTP-SAIFR, Sao Paulo, Brazil, 2016
Agenda

1. EFT for Ultracold Atoms I: Effective Field Theories & Universality
2. EFT for Ultracold Atoms II: Cold Atoms & the Unitary Limit
3. EFT for Ultracold Atoms III: Weak Coupling at Finite Density
4. EFT for Ultracold Atoms IV: Few-Body Systems in the Unitary Limit
5. Beyond Ultracold Atoms: Halo Nuclei and Hadronic Molecules

Literature

Effective Theory

- Separation of scales:
  \[ \frac{1}{k} = \lambda \gg R \]

- Limited resolution at low energy:
  \[ \rightarrow \text{expand in powers of } kR \]
Effective Theory

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  \[ \longrightarrow \text{capture in low-energy constants using renormalization} \]
  \[ \longrightarrow \text{include long-range physics explicitly} \]

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Effective Theory

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- Systematic, model independent
- Very low energies: only short range interactions
- Exploit cluster substructures \[ \implies \text{Halo EFT} \]
- Universal properties
Halo Nuclei

- Low separation energy of valence nucleons: $B_{\text{valence}} \ll B_{\text{core}}, E_{\text{ex}}$

  $\rightarrow$ close to “nucleon drip line” $\rightarrow$ scale separation $\rightarrow$ EFT

EFT for halo nuclei

(Bertulani, HWH, van Kolck, 2002; Bedaque, HWH, van Kolck, 2003; ...)

C.-B. Moon, Wikimedia Commons
Scales and Antisymmetrization

- **Scales:** \( R_{\text{halo}} \gg R_{\text{core}} \sim \ell \)

- **Antisymmetrization** with respect to neutrons in core?

- **Core neutrons not active dof in halo EFT**

- **Physics:** exchange of core nucleon and halo nucleon only contributes to observables if there is spatial overlap between wave functions of core and halo nucleon

  \[ \Rightarrow \quad \text{small for } R_{\text{core}} \ll R_{\text{halo}} \]

- **Effects subsumed in low-energy constants, included perturbatively in expansion in** \( R_{\text{core}} / R_{\text{halo}} \)
Effective Lagrangian (schematically)

\[ \mathcal{L}_d = \psi^\dagger \left( i \partial_t + \frac{\vec{\nabla}^2}{2m} \right) \psi + \frac{g_2}{4} \bar{d}d - \frac{g_2}{4} (\bar{d} \psi^2 + (\psi^\dagger)^2 d) - \frac{g_3}{36} \bar{d} \psi^\dagger \psi + .. \]

2- and 3-body interaction at leading order: \( g_2, g_3 \) enhanced!

2-body amplitude:

\[ \begin{array}{c}
\text{Structure of 2-neutron halo nuclei} \rightarrow \text{energies, matter form factors, radii}
\end{array} \]
Renormalization

- Observables are independent of regulator/cutoff $\Lambda$

$\Rightarrow$ Running coupling $H(\Lambda) \propto \Lambda^2 g_3(\Lambda)$

- $H(\Lambda)$ periodic: limit cycle

  $\Lambda \rightarrow \Lambda e^{n\pi/s_0} \approx \Lambda(22.7)^n$

- Discrete scale invariance

- Efimov effect (Efimov, 1970)

- Observed in ultracold atoms
  (Krämer et al., 2006; ...)

- Relevant in halo nuclei?
Efimov Physics in Halo Nuclei

- **Efimov effect in halo nuclei?** (Fedorov, Jensen, Riisager, 1994)
  
  $\implies$ excited states obeying scaling relations

- **Correlation plot:** $E_{nn} \leftrightarrow E_{nc}$ (Amorin, Frederico, Tomio, 1997)

![Correlation plot diagram](image)

Efimov Physics in $^{22}\text{C}$

- **Matter radius from $^{22}\text{C} + p$ & Glauber:** $\langle r_0^2 \rangle^{1/2} = 5.4(9) \text{ fm}$

- **Halo EFT analysis of impact on other observables in $^{22}\text{C}$**

![Plots for $\langle r_0^2 \rangle^{1/2} = 4.5, 5.4, 6.3 \text{ fm}$](image)

- **Excited Efimov states in $^{22}\text{C}$ appear to be ruled out**
Efimov Physics in $^{62}$Ca


- The Many and the Few: emergence of effective halo degrees of freedom
- Coupled cluster calculations of $^{60}$Ca and $^{61}$Ca using chiral N2LO two-body force and schematic three-body force:
  - $^{61}$Ca is a weakly bound S-wave state (or virtual state)
- Quantitative estimate: $S_n = B_{nc} = 5\ldots8$ keV
- Scattering Parameters:
  - $a_{cn} = 54(1)$ fm, $r_{cn} = 9.0(2)$ fm $\Rightarrow r_{cn}/a_{cn} \approx 1/6$
- Investigate consequences for $^{62}$Ca using halo EFT
- Prospects for excited Efimov states in $^{62}$Ca:
  - $S_{\text{deep}} = 1/(\mu_{cn}r_{cn}^2) \approx 500$ keV, scaling factor $\lambda_0 \approx 16$
  - $\Rightarrow$ possible if $S_{2n} \gtrsim 230$ keV
Efimov Physics in $^{62}\text{Ca}$

- Universal correlations between $S_{2n}$, $^{61}\text{Ca}-n$ scattering length, $^{62}\text{Ca}$ matter, and charge radii

![Graph showing the relationship between $S_{2n}$ and scattering length and matter radius for $^{62}\text{Ca}$](image)

$S_{2n}^{(62}\text{Ca)} [\text{keV}]$

- Excited Efimov state appears around $S_{2n} \approx 230 \text{ keV}$
- Matter radii of order tens of Fermi possible

How to study excited Efimov states experimentally?  
(A. Macchiavelli, Few-Body Syst. 56, 773 (2015)

Consider transfer reactions for candidate nucleus $^{A}Z_{N}$

(a) One-neutron transfer: $(^{A-1}Z_{(N-1)})(d, p)^{A}Z_{N}$
(b) Two-neutron transfer: $(^{A-2}Z_{(N-2)})(t, p)^{A}Z_{N}$

Back-of-the-envelope estimate $\Rightarrow$ (a) is most promising

Reaction calculation in Halo EFT would be useful
Hadronic Molecules

- New $c\bar{c}$ states at B factories: $X$, $Y$, $Z$
- Example: $X(3872)$ (Belle, CDF, BaBar, D0)
- No ordinary $c\bar{c}$-state
  - Decays violate isospin
  - Measured mass depends on decay channel

$$m_X = (3871.69 \pm 0.17) \text{ MeV} \quad \Gamma < 1.2 \text{ MeV} \quad J^{PC} = 1^{++}$$

- Nature of $X(3872)$?
  - $D^0 D^{0*}$-molecule, tetraquark, charmonium hybrid, ...
- Molecular nature $\Rightarrow$ interaction of $X(3872)$ with $D^0$, $\bar{D}^0$, $D^{0*}$, $\bar{D}^{0*}$ determined by large scattering length
Nature of $X(3872)$

- Nature of $X(3872)$ not finally resolved
- Assumption: $X(3872)$ is weakly-bound $D^0-\bar{D}^0*$-molecule

\[ |X\rangle = (|D^0\bar{D}^0*\rangle + |\bar{D}^0D^0*\rangle)/\sqrt{2}, \quad B_X = (0.11 \pm 0.21) \text{ MeV} \]

\[ \implies \text{universal properties} \quad \text{(cf. Braaten et al., 2003-2008, ...)} \]

- Explains isospin violation in decays of $X(3872)$ \Rightarrow superposition of $I = 1$ and $I = 0$
- Different masses due to different line shapes in decay channels

- Large scattering length to LO determines interaction of $X(3872)$ with $D^0$ and $D^{0*}$
- Higher orders: EFT with perturbative pions $\Rightarrow$ XEFT

(Fleming, Kusunoki, Mehen, van Kolck, 2007; Fleming, Mehen, 2008)

(Braaten, HWH, Mehen, 2010; ...)

EFT for Cold Atoms – p. 16
EFT for $X(3872)$

- **Effective Lagrangian**

\[
\mathcal{L} = \sum_{j=D^0, D^{*0}, \bar{D}^0, \bar{D}^{*0}} \psi_j^\dagger \left( i\partial_t + \frac{\nabla^2}{2m_j} \right) \psi_j + \Delta X^\dagger X
\]

\[-\frac{g}{\sqrt{2}} \left( X^\dagger \left( \psi_{D^0} \psi_{\bar{D}^{*0}} + \psi_{D^{*0}} \psi_{\bar{D}^0} \right) + \text{H.c.} \right) + \ldots,
\]

- **Propagator of the $X(3872)$**

\[
\quad = \ldots + \quad + \quad + \ldots,
\]

- **Three-body integral equation**

\[
\quad = \quad + \quad
\]
Predictions for scattering amplitude/cross section


Three-body scattering lengths:

\[ a_{D^0X} = -9.7a \approx -85 \text{ fm} \]
\[ a_{D^*0X} = -16.6a \approx -146 \text{ fm} \]
Experimental Observation?

- Behavior of $X(3872)$ produced in isolation should be distinguishable from its behavior when in the presence of $D^0, D^{*0}, \bar{D}^0, \bar{D}^{*0}$

- Final state interaction of $D, D^*$ mesons in $B_c$-decays

- Example: quark-level $B_c$ decay yielding three charmed/anticharmed quarks in final state

\[
\begin{align*}
\bar{c} & \quad \rightarrow \quad \bar{c} \\
b & \quad \rightarrow \quad c \\
W & \quad \rightarrow \quad \bar{c} \\
& \quad \rightarrow \quad q
\end{align*}
\]

- Process in principle accessible at the LHC
Summary

- Cluster EFT for halo nuclei
  - Large scattering length/shallow states
  - Controlled, systematic approach → error estimates
  - Straightforward inclusion of external currents
- Universal theory has applications in atomic, nuclear, and particle physics
- Universality predicts correlations between observables
  - Input from theory or experiment
- Excited Efimov state possible in $^{62}$Ca
- Calculations of EM structure and reactions