Emergent symmetries in disordered quantum spin chains

Eduardo Miranda
State University of Campinas, Brazil

Victor Quito
Pedro L. S. Lopes
José Abel Hoyos
Univ. São Paulo

Workshop on Next Generation Quantum Materials
São Paulo, April 5, 2016
Symmetry in nature

Usual scenario: physical systems are less symmetric at low temperatures/energies (via phase transitions):

- crystals vs gas/liquid
- (anti-)ferromagnets vs paramagnets
- liquid crystals vs normal liquids
- superconductors vs normal metals
- superfluids vs normal fluids
- $\text{SU}(2) \times \text{U}(1)$ vs $\text{U}_{\text{em}}(1)$….

1. Condensed matter physicists spend money to cool things down and find broken symmetries.
2. High energy physicists spend (a lot more) money to “heat things up” and access more symmetric states.
Less common: emergent symmetries

Emergent symmetries: the low-energy sector is more symmetric than the high-energy one (via crossovers)

- Quantum spin-2 chains: SU(3) (P. Chen et al., PRL ‘15)
- Certain quantum critical points: gauge symmetry (Senthil, Vishwanath, Balents, Sachdev, Fisher, Science 303, 1490 (2004))....
- Symmetry-protected topological states: fermions and gauge fields (Xiao-Gang Wen)........
Less common: emergent symmetries

1. Known examples are few and far between.
2. Generic mechanism not known. General ideas:
   1. Stable low-T fixed point with group $G$
   2. Irrelevant operators break $G$ into $g$ ($g \subset G$)
3. No recipe for its construction: case by case…

(C. Itoi, S. Qin, and I. Affleck, PRB 61, 6747 (2000))
Disordered Heisenberg chain

\[ H = \sum_{i} J_i S_i \cdot S_{i+1} \]

\( J_i > 0 \): distributed according to \( P(J;\Omega) \)
\( \Omega \) is the high energy cutoff
Strong disorder RG method

\[ H = \sum_i J_i S_i \cdot S_{i+1} \quad J_i > 0 \text{ -- distributed according to } P_0(J;\Omega); \]
\[ \Omega \text{ is the high energy cutoff} \]

Decimation procedure:

1. Find the strongest coupling \( \Omega = \text{max} \{ J_i \} \).

\[
\begin{array}{c}
\text{Diagonalize} \\
\Omega \quad \text{Diagonalize} \\
S_2 \quad S_3
\end{array}
\]

\[
\begin{array}{c}
S_2 \\
S_3
\end{array}
\]

\[
\begin{array}{c}
\text{Net result: } S_2 \text{ and } S_3 \text{ disappear and new coupling between } S_1 \text{ and } S_4 \text{ appears} \\
\text{Topology of the (infinite) chain is preserved.}
\end{array}
\]

2. Treat \( H' = J_1 S_1 \cdot S_2 + J_3 S_3 \cdot S_4 \) in 2nd order perturbation theory

\[
\tilde{J} = \frac{J_1 J_3}{2\Omega}
\]

Note \( \tilde{J} < J_1, J_2 \)

**Universality**

All initial distributions have the same fate

The effective disorder increases without limit!
The method is asymptotically exact: the wider the distribution, the more accurate the decimations.

\[ P^*(J, \Omega) = \frac{\alpha}{\Omega} \left( \frac{\Omega}{J} \right)^{1-\alpha} \]

\[ \alpha = \frac{1}{\ln(\Omega_0/\Omega)} \rightarrow 0 \]

\[ \frac{\Delta J}{\langle J \rangle} \sim \frac{1}{\sqrt{\alpha}} \rightarrow \infty \]
Spatial distribution of strong bonds

Decimation procedure:
Spatial distribution of strong bonds

Decimation procedure:

Find the strongest coupling
Spatial distribution of strong bonds

Decimation procedure:
Spatial distribution of strong bonds

Decimation procedure:

Renormalize
Spatial distribution of strong bonds

Decimation procedure:
Spatial distribution of strong bonds

Decimation procedure:

\[ \Omega \]
Spatial distribution of strong bonds

Decimation procedure:
Spatial distribution of strong bonds

Decimation procedure:
Spatial distribution of strong bonds

Decimation procedure:
Ground state

Random Singlet phase

Well-separated, strongly bound spin pairs
Excitations are localized: breakup of long bonds. Energy of an excitation of length $L$:

$$\Omega \sim e^{-L^\psi}, \quad \psi = \frac{1}{2}$$

‘Activated dynamical scaling’

With this scaling, we can get exact results for low-energy properties (susceptibility, specific heat), which I will not discuss.

The correlation function at $T=0$

$$\langle S_i \cdot S_{i+r} \rangle \approx 0$$

Typical pairs are weakly correlated

$$\langle S_i \cdot S_{i+r} \rangle_{typ} \sim \exp(-r^\psi)$$

$$\psi = \frac{1}{2}$$

The correlation function at $T=0$

$$\langle S_i \cdot S_{i+r} \rangle \approx O(1)$$

But average value is dominated by rare singlets

$$\langle S_i \cdot S_{i+r} \rangle_{av} \sim \frac{(-1)^r}{r^\phi}$$

$\phi = 2$

Disordered spin-1 chains

The most general disordered spin-1 chain with global SU(2) invariance.

\[ H_{JD} = \sum_i \left[ J_i S_i \cdot S_{i+1} + D_i (S_i \cdot S_{i+1})^2 \right] \]

Note: the two terms are linearly dependent for spin-1/2, but not for spin-1.

\[ H_{JD} = \sum_i E_i \left[ \cos \theta_i S_i \cdot S_{i+1} + \sin \theta_i (S_i \cdot S_{i+1})^2 \right] \]

\[ E_i = \sqrt{J_i^2 + D_i^2}; \quad \tan \theta_i = \frac{D_i}{J_i} \]

May be experimentally realized in optical lattices loaded with cold $^{23}$Na


What is the behavior at strong disorder?
RG step for generic spin-1 chains

\[ H_0 = J_2 \mathbf{S}_2 \cdot \mathbf{S}_3 + D_2 (\mathbf{S}_2 \cdot \mathbf{S}_3)^2 \]

Singlet (S=0) ground state
Triplet (S=1) ground state
Quintuplet (S=2) ground state

Boechat, Saguia & Continentino
Solid State Commun. '96
Yang & Bhatt, PRL '98
We first consider the case of random $E_i$ but fixed $\theta$.

$$H_{JD} = \sum_i E_i \left[ \cos \theta S_i \cdot S_{i+1} + \sin \theta (S_i \cdot S_{i+1})^2 \right]$$
Random singlet pairs

Singlet (S=0) ground state

arctan(1/3)

\(\frac{3\pi}{4}\)

\(\frac{\pi}{4}\)

1

2

FM

\(\frac{\pi}{2}\)

\(-\frac{3\pi}{4}\)
In phase 2, there is a conventional random singlet phase: asymptotically, only singlet-forming decimations occur.

\[
\Omega \sim e^{-L \psi}
\]

\[
\langle S_i \cdot S_j \rangle_{av} \sim \frac{e^{iq|i-j|}}{|i-j|^{\phi}}
\]

\[
\psi_M = \frac{1}{2}, \quad \phi_M = 2
\]
Random singlet **trios**

In phase 1, the ground state is made of random spin trios (and less frequent sextets, etc.). At each step both singlets and spins-1 are formed.
Random singlet **trios**

In phase 1, the ground state is made of random spin trios (and less frequent sextets, etc.). At each step both singlets and spins-1 are formed.
In phase 1, the ground state is made of random spin trios (and less frequent sextets, etc.). At each step both singlets and spins-1 are formed.

\[ \Omega \sim e^{-L\psi} \]

\[ \langle S_i \cdot S_j \rangle_{av} \sim \frac{e^{iq(i-j)}}{|i-j|^\phi} \]

\[ \psi_B = \frac{1}{3} \quad \phi_B = \frac{4}{3} \]
Emergent SU(3) symmetry
Special SU(3) points

\[ H_{JD} = \sum_i E_i \left[ \cos \theta S_i \cdot S_{i+1} + \sin \theta (S_i \cdot S_{i+1})^2 \right] \]
Where did SU(3) come from?

SU(N): group of N x N unitary matrices with determinant equal to 1

\[ U = e^{i \mathbf{H}} \]  if H is Hermitian and traceless

For N=2, the Pauli matrices are a complete basis for traceless Hermitian matrices:

\[ U = e^{i \theta \cdot \mathbf{\sigma}} \]

For N=3, the following 8 spin-1 operators form an analogous complete set:

\[ \Lambda_4 = S_x S_y + S_y S_x, \]  
\[ \Lambda_5 = S_x S_z + S_z S_x, \]  
\[ \Lambda_6 = S_y S_z + S_z S_y, \]  
\[ \Lambda_7 = S_x^2 - S_y^2, \]  
\[ \Lambda_8 = \frac{1}{\sqrt{3}} \left( 2 S_z^2 - S_x^2 - S_y^2 \right). \]

These 8 operators are the generators of the fundamental (‘quark’) representation of SU(3).
**Where did SU(3) come from?**

SU(N): group of N x N unitary matrices with determinant equal to 1

\[ U = e^{iH} \quad \text{if H is Hermitian and traceless} \]

For N=2, the Pauli matrices are a complete basis for traceless Hermitian matrices:

\[ U = e^{i\theta \cdot \sigma} \]

For N=3, the following 8 spin-1 operators form an analogous complete set:

\[
\begin{align*}
\Lambda_1 &= S_x, & \Lambda_5 &= -S_x S_z + S_z S_x, \\
\Lambda_2 &= S_y, & \Lambda_6 &= -S_y S_z + S_z S_y, \\
\Lambda_3 &= S_z, & \Lambda_7 &= -S^2_x - S^2_y, \\
\Lambda_8 &= -\frac{1}{\sqrt{3}} \left(2S_z^2 - S_x^2 - S_y^2\right).
\end{align*}
\]

If we change the sign of \( \Lambda_a \) (\( a=4,5,6,7,8 \)) we have the ‘antiquark’ one.
**Special SU(3) points**

\[ H \left( \theta = \frac{\pi}{4} \right) = \sum_{i, a} E_i \Lambda_{i,a} \Lambda_{(i+1),a} = \sum_i E_i \Lambda_i \cdot \Lambda_{i+1} \]

SU(3) - (quark-quark)

Quarks disguised as spins!
Special SU(3) points

\[ H \left( \theta = -\frac{\pi}{2} \right) = \sum_{i,a} E_i \Lambda_i,a \Lambda_{(i+1),a} = \sum_i E_i \Lambda_i \cdot \Lambda_{i+1} \]

Quarks/antiquarks disguised as spins!

SU(3) - (quark-antiquark)
Special SU(3) points

SU(3) - (quark-quark)

\[ \psi_B = \frac{1}{3}, \quad \phi_B = \frac{4}{3} \]

J. A. Hoyos and E. M., PRB 70, 180401(R) (2004)
Special SU(3) points

J. A. Hoyos and E. M., PRB 70, 180401(R) (2004)

SU(3) - (quark-antiquark)

$$\frac{\pi}{4}$$

$$\frac{-3\pi}{4}$$

$$\frac{-\pi}{2}$$

$$\psi_M = \frac{1}{2} \quad \phi_M = 2$$
Emergent SU(3) symmetry

At long length scales, the state is the same throughout region 2, including $-\pi/2$: emergent SU(3)

The pair singlet is angle-independent: it is both an SU(2) and an SU(3) singlet.
Emergent SU(3) symmetry

At long length scales, the state is the same throughout region 1, including $\pi/4$: emergent SU(3)

The trio singlet is also angle-independent: it is both an SU(2) and an SU(3) singlet.
Baryonic random singlet phase

\[ H = \sum_{i} E_i \left[ \cos \theta \mathbf{S}_i \cdot \mathbf{S}_{i+1} + \sin \theta (\mathbf{S}_i \cdot \mathbf{S}_{i+1})^2 \right] \]
Mesonic random singlet phase

\[ H = \sum_i E_i \left[ \cos \theta \mathbf{S}_i \cdot \mathbf{S}_{i+1} + \sin \theta \left( \mathbf{S}_i \cdot \mathbf{S}_{i+1} \right)^2 \right] \]

Random quark-antiquark SU(3) singlets (mesons):

mesonic random singlet phase

Again, the response of the state is SU(3) symmetric
Emergent SU(3) symmetry

\[ \langle S_i \cdot S_j \rangle_{av} \sim \frac{e^{iq(i-j)}}{|i-j|^{\phi}} \Rightarrow \langle \Lambda_{ai} \Lambda_{aj} \rangle_{av} \sim \frac{e^{iq(i-j)}}{|i-j|^{\phi}} \quad (a = 1, \ldots, 8) \]

\[
\begin{align*}
\Lambda_1 &= S_x, \\
\Lambda_2 &= S_y, \\
\Lambda_3 &= S_z, \\
\Lambda_4 &= S_x S_y + S_y S_x, \\
\Lambda_5 &= S_x S_z + S_z S_x, \\
\Lambda_6 &= S_y S_z + S_z S_y, \\
\Lambda_7 &= S_x^2 - S_y^2, \\
\Lambda_8 &= \frac{1}{\sqrt{3}} \left(2S_z^2 - S_x^2 - S_y^2\right).
\end{align*}
\]

But note:
- The exponents are all the same.
- The numerical pre-factors are the same only at the SU(3) points or at very strong initial disorder. The emergent SU(3) only appears asymptotically, the procedure is inaccurate at the beginning of the flow.
Allowing for spatial fluctuations of \( \theta \) in the initial distribution
Can this be a more general phenomenon?


$$H = \sum_i \alpha_i^1 (S_i \cdot S_{i+1}) + \alpha_i^2 (S_i \cdot S_{i+1})^2 + \ldots + \alpha_i^{2S} (S_i \cdot S_{i+1})^{2S}$$

- Conventional random singlet phases with SU($2S+1$) symmetry: only spin pairs (mesons), $\psi = \frac{1}{2}$,…
- Phases with $\psi = 1/3$, but no emergent higher symmetry.

Can we find baryonic phases (more than two spins per singlet) with emergent symmetries?
Can this be a more general phenomenon?

But $S = 1$ is also the fundamental representation of SO(3).
• So instead of

  \[ \text{SU(2) with } S=1 \rightarrow \text{SU(2) with } S > 1 \]

• we tried

  \[ \text{SO(3)} \rightarrow \text{SO(N) (with the fundamental repres.)} \]

(V. L. Quito, P. L. S. Lopes, J. A. Hoyos, E.M., in progress)
Can this be a more general phenomenon?

\[ H_{SO(N)} = \sum_i \left[ J_i \sum_{a < b} L_{ab}^i L_{ab}^{i+1} + D_i \left( \sum_{a < b} L_{ab}^i L_{ab}^{i+1} \right)^2 \right] \]

- Most general \( SO(N) \) invariant Hamiltonian.
- \( L^{ab} \) generates rotations is the \( ab \) plane

\[ L^{ab} |b\rangle = i |a\rangle \]
\[ L^{ab} |a\rangle = -i |b\rangle \]
\[ L^{ab} |c\rangle = 0 \quad c \neq a, b \]

Note that \( SO(5) \) symmetry can be realized in \( S=3/2 \) fermionic cold atoms (C. Wu PRL 95, 155115 (2005))
**Emergent SU(2N+1) in SO(2N+1) chains**

Baryonic phase with $\psi = 1/(2N+1)$ and emergent SU(2N+1) symmetry

**SO(5) case**

The case of SO(2N) is more involved and is still in progress…
Conclusions

• Infinite effective disorder in spin-1/2 and spin-1 chains.
• Emergence of $SU(3)$ symmetry in an $SU(2)$-invariant system: Hadrons in condensed matter physics.
• Emergence of $SU(2N+1)$ symmetry in $SO(2N+1)$ chains: Composite singlets with $2N+1$ constituents.