COSMOLOGICAL HIGGS-AXION INTERPLAY FOR A NATURALLY SMALL EW SCALE

ICTP-SAIFR
São Paulo, Brazil
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THE RELAXiON

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OUTLINE

- The relaxion solution to the hierarchy problem
  - Graham-Kaplan-Rajendran model(s)
  - General issues
- Our improved version
  - Some implications
    - Phenomenology
    - Cosmology
- Conclusions and outlook

REFERENCES

04/15 Cosmological Relaxation of the EW Scale
P.W. Graham, D.E. Kaplan, S. Rajendran (GKR)

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04/15 Cosmological Relaxation of the EW Scale
P.W. Graham, D.E. Kaplan, S. Rajendran (GKR)

Previous work with similar ideas

'84 A Mechanism for Reducing the Cosmological Constant
L.F. Abbott

'03 Cosmic Attractors and Gauge Hierarchy
G. Dvali, A. Vilenkin

'04 Large Hierarchies from Attractor Vacua
G. Dvali

REFERENCES

04/15 Cosmological Relaxation of the EW Scale
P.W. Graham, D.E. Kaplan, S. Rajendran (GKR)

Follow-up papers :

06/15 Cosmological Higgs-Axion Interplay for a Naturally Small EW Scale
J.R.E, C. Grojean, G. Panico, A. Pomarol, O. Pujolàs, G. Servant

07/15 EW Relaxation from Finite Temperature
E. Hardy

07/15 Relaxing the EW scale: the Role of Broken dS Symmetry
S.P. Patil, P. Schwaller

09/15 Is the Relaxion an Axion?
R.S. Gupta, Z. Komargodski, G. Pérez, L. Ubaldi

REFERENCES

09/15 Natural Heavy Supersymmetry
B. Batell, G.F. Giudice, M. McCullough

09/15 Mirror Cosmological Relaxation of the EW Scale
O. Matsedonskyi

11/15 Realizing the Relaxion from Multiple Axions and its UV Completion with High Scale Supersymmetry
K. Choi, S.H. Im

11/15 A Clockwork Axion
D.E. Kaplan, R. Rattazzi

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04/15 Cosmological Relaxation of the EW Scale
P.W. Graham, D.E. Kaplan, S. Rajendran (GKR)

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NATURAL SOLUTIONS TO HIERARCHY PROBLEM

Supersymmetry
Composite Higgs $\Big\}$ $\Delta m_H^2 \sim \mathcal{O} \cdot \Lambda^2$

⇨ Predict BSM at ~ TeV
not to spoil naturalness

⇨ Main argument for new physics at LHC

Common lore:

Stabilizing the EW scale requires
new particles near the TeV

THE RELAXION IDEA

New solution to hierarchy problem that challenges this common lore.

Idea

new field

$$V(h) = \frac{1}{2} m_H^2(\phi) h^2 + \ldots = \frac{1}{2}\left(-\Lambda^2 + g\phi\Lambda\right) h^2 + \ldots$$

- Λ^2 not required to cancel
- ϕ changes during cosmological evolution

scanning m_H^2 and stopping at

$$m_H^2(\phi_c) = -\Lambda^2 + g\,\phi_c\,\Lambda \approx m_{EW}^2 \ll \Lambda^2$$

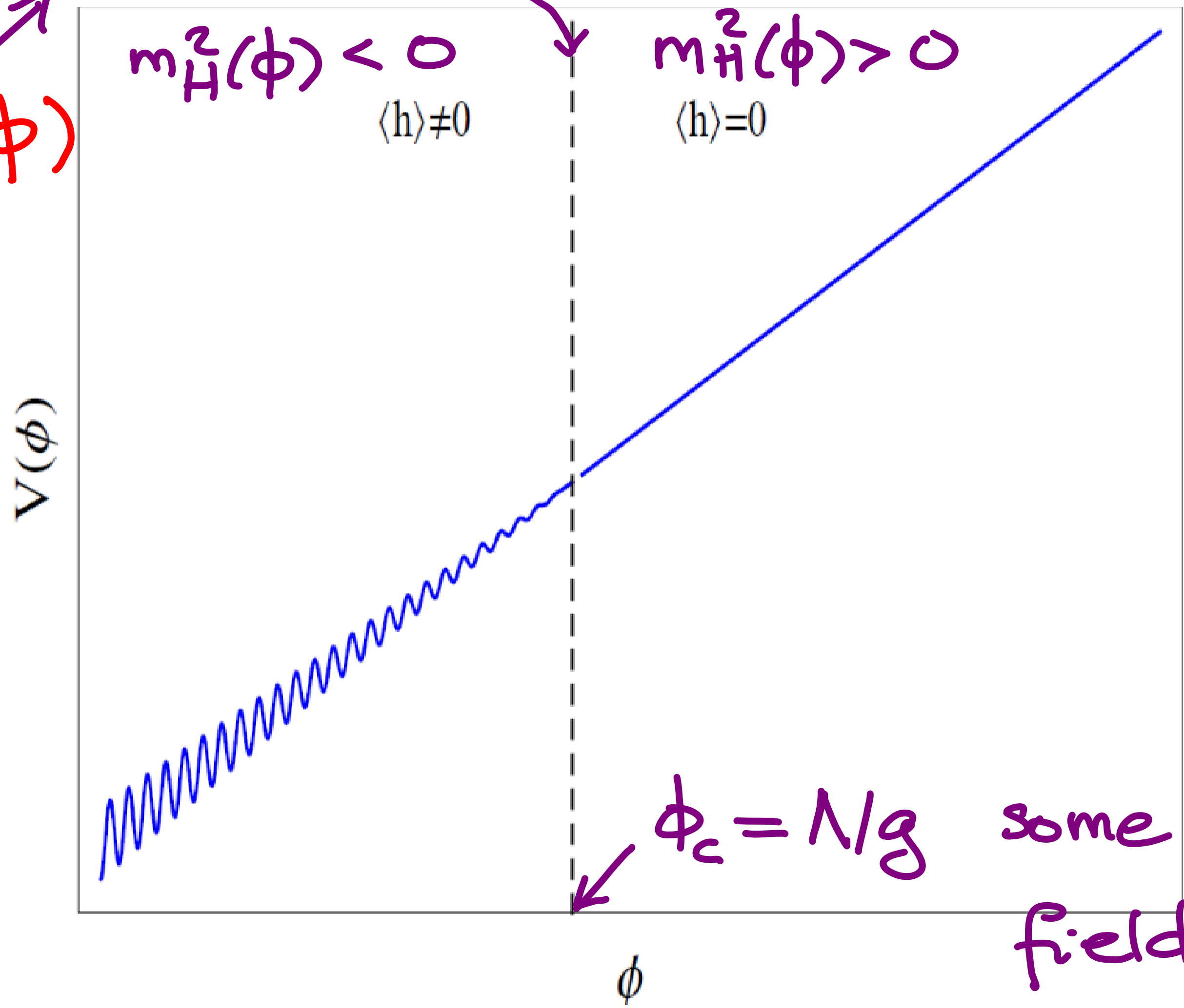
THE RELAXION IDEA

$$V = -\frac{\Lambda^2}{2}\left(1-\frac{g\phi}{\Lambda}\right)h^2 + g\Lambda^3\phi + \epsilon\Lambda_c^4\left(\frac{h}{\Lambda_c}\right)^n \cos\left(\frac{\phi}{f}\right)+\dots$$

THE RELAXION IDEA

$$V = -\frac{\Lambda^2}{2}\left(1-\frac{g\phi}{\Lambda}\right)h^2 + g\Lambda^3\phi + \epsilon\Lambda_c^4\left(\frac{h}{\Lambda_c}\right)^n\cos\left(\frac{\phi}{f}\right)+\dots$$

$m_H^2(\phi)$

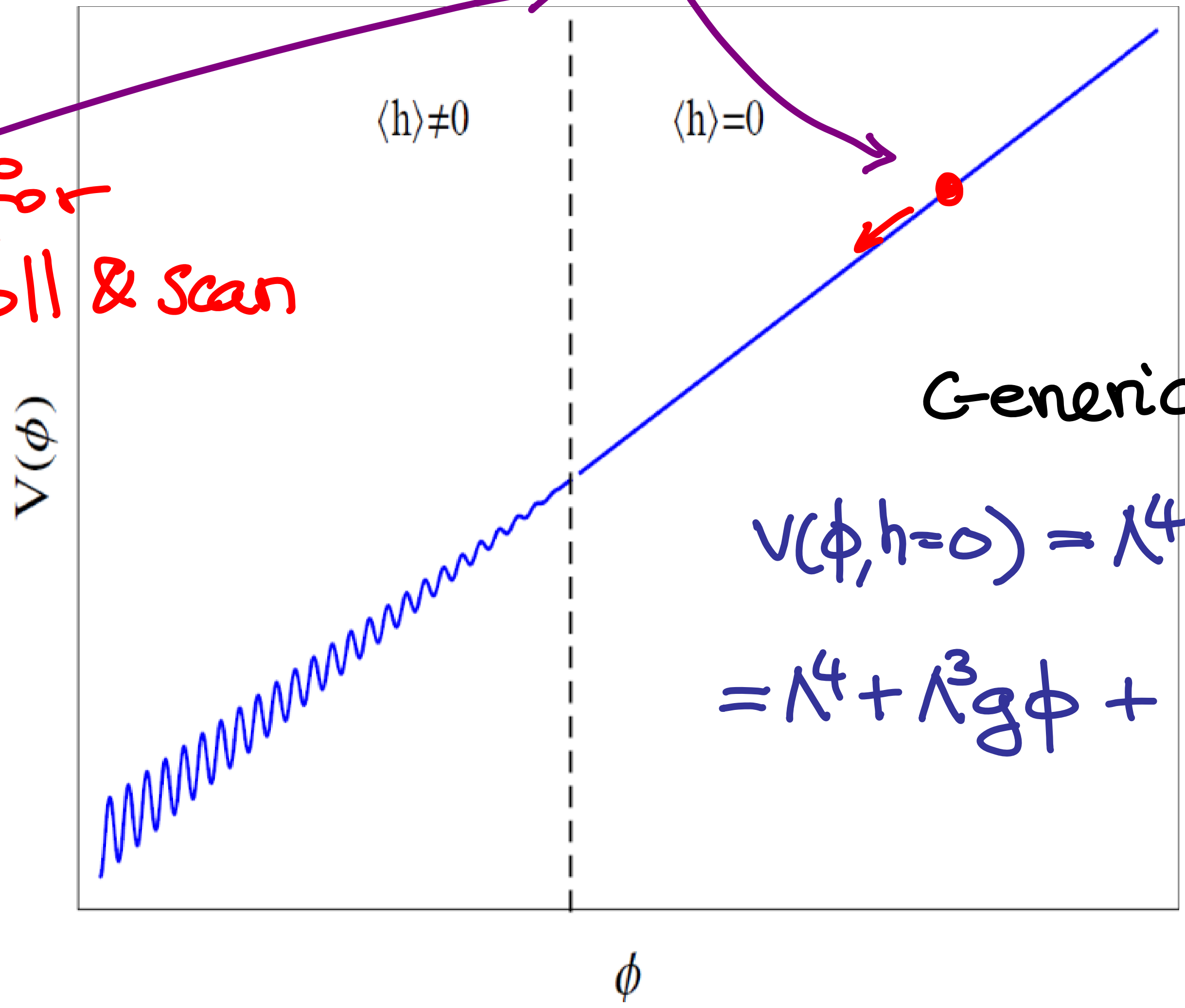


some typical field value

THE RELAXION IDEA

$$V = -\frac{\Lambda^2}{2}\left(1 - \frac{g\phi}{\Lambda}\right)h^2 + g\Lambda^3\phi + \epsilon\Lambda_c^4\left(\frac{h}{\Lambda_c}\right)^n \cos\left(\frac{\phi}{f}\right) + \dots$$

slope for ϕ to roll & scan



Generically

$$V(\phi, h=0) = \Lambda^4 v\left(\frac{g\phi}{\Lambda}\right)$$

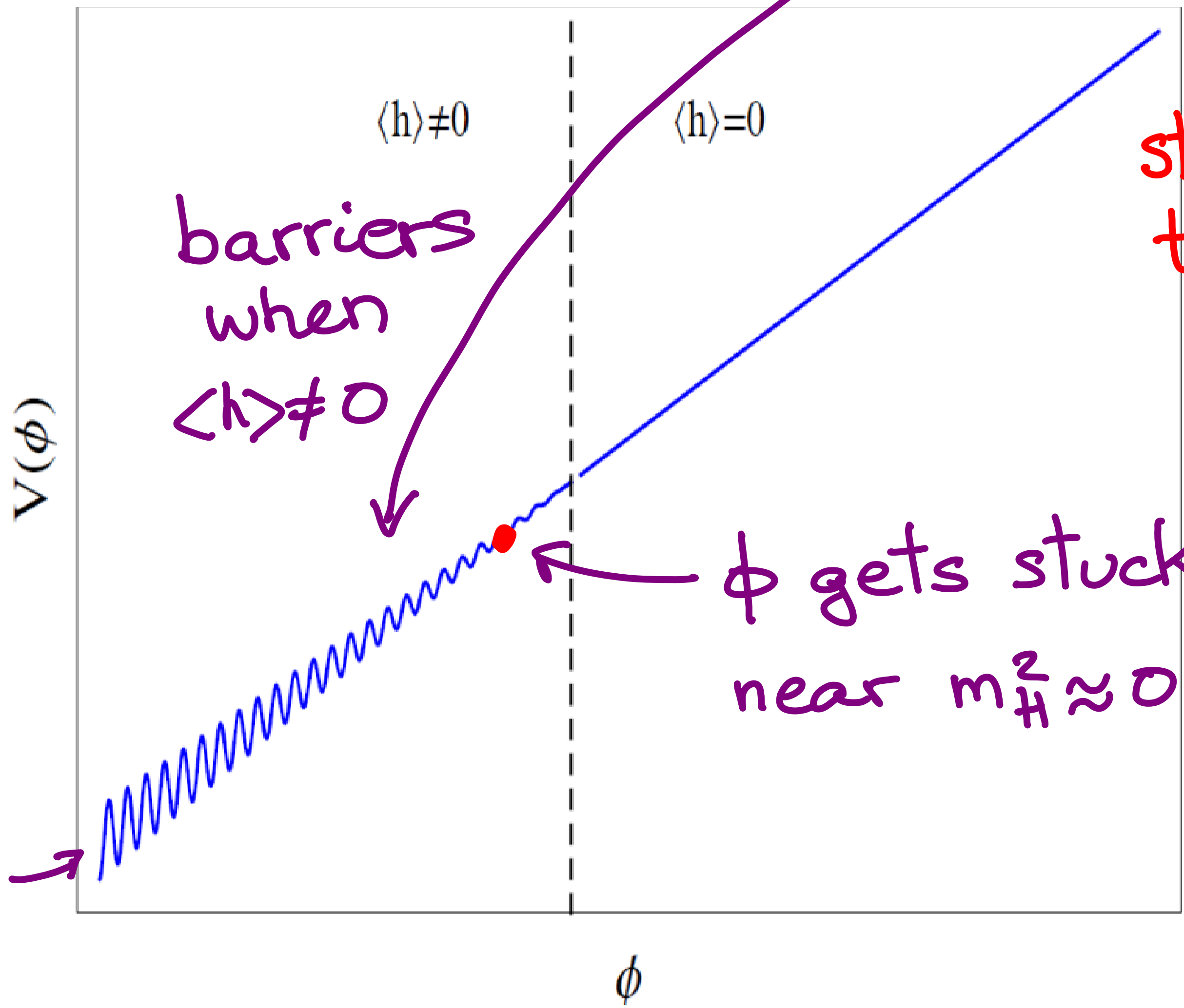
$$= \Lambda^4 + \Lambda^3 g\phi + \Lambda^2 g^2\phi^2 + \dots$$

THE RELAXION IDEA

$$V = -\frac{\Lambda^2}{2}\left(1-\frac{g\phi}{\Lambda}\right)h^2 + g\Lambda^3\phi + \epsilon\Lambda_c^4\left(\frac{h}{\Lambda_c}\right)^n \cos\left(\frac{\phi}{f}\right) + \dots$$

stopping term

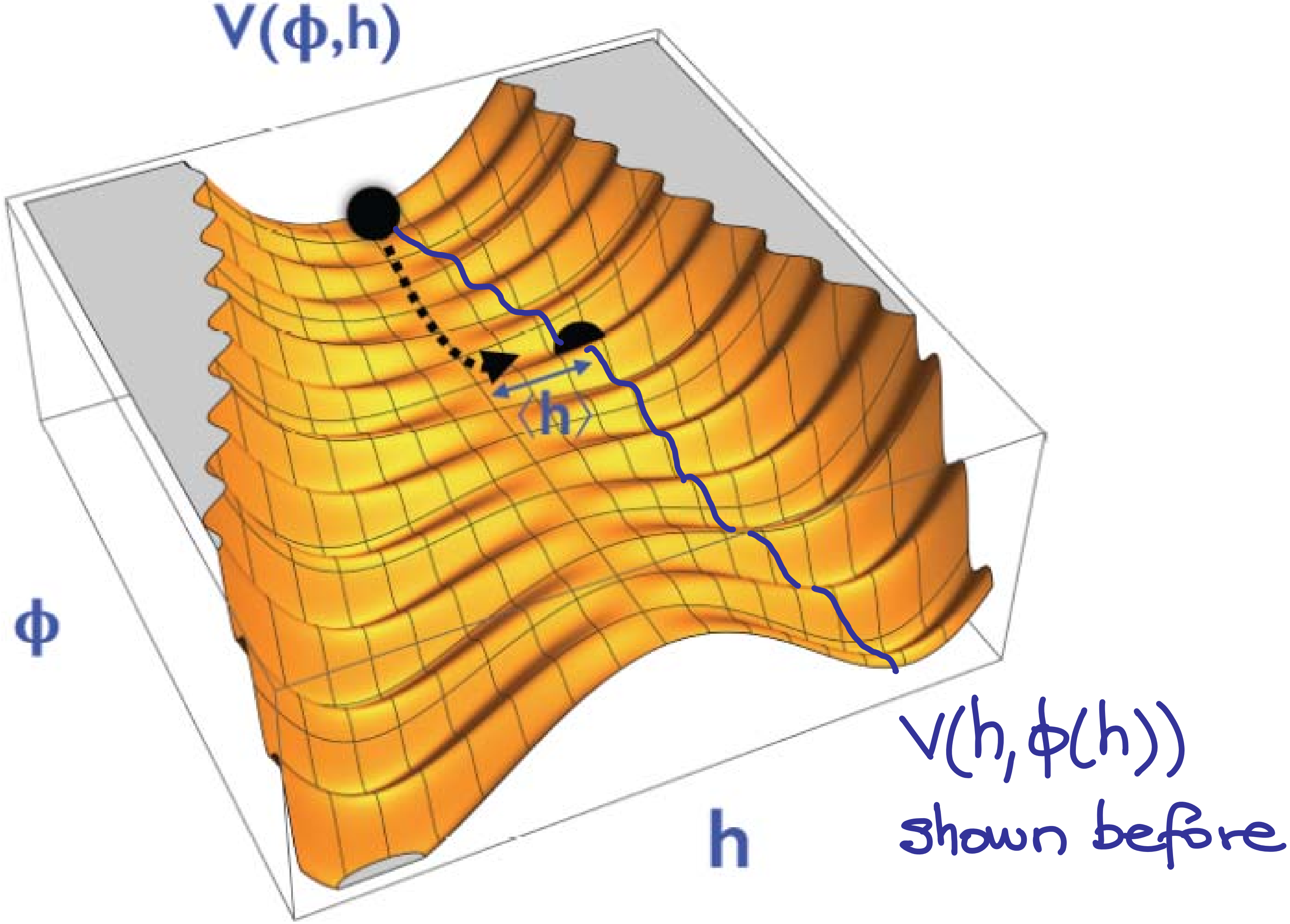
barriers when $\langle h\rangle \neq 0$



$V(\phi, h(\phi))$

ϕ gets stuck near $m_H^2 \approx 0$

2-FIELD POTENTIAL



FRICTION NEEDED

To avoid overshooting the EW range vacua.

Obvious solution : slow-roll during inflation*

$$\ddot{\phi} + 3H_I\dot{\phi} = -\frac{\partial V}{\partial \phi}$$

friction term

Extra inflaton field to provide required N_e.

*Alternative: thermal evolution of V → Hardy

INFLATION SECTOR

Largely unspecified, except

- ϕ subdominant

$$V(\phi) \sim \Lambda^4 < V_I \sim H_I^2 M_P^2$$

- ϕ classical roll dominates

$$(\Delta\phi)_{\text{class}} \sim V'_\phi / H_I^2 > (\Delta\phi)_{\text{quant}} \sim H_I$$

Window for H_I :

$$\frac{\Lambda^2}{M_P} < H_I < (\underbrace{V'_\phi}_{\Lambda^3 g})^{1/3} \quad \Rightarrow \quad g > \left(\frac{\Lambda}{M_P}\right)^3$$

INFLATION SECTOR

Also, inflation long enough for ϕ to scan its typical field range $\sim \frac{\Lambda}{g}$

$$\Delta\phi \sim N_e \cdot \frac{V'_\phi}{H_I^2} \gtrsim \frac{\Lambda}{g}$$

$$\Rightarrow \quad N_e \gtrsim \left(\frac{H_I}{g\Lambda}\right)^2 \geq \left(\frac{\Lambda}{g M_p}\right)^2$$

Usually $N_e \gg 1$ and $\Delta\phi \gg M_p$ needed.

(Assumes constant H_I. Can be much better otherwise $\rightarrow$ Patil, Schwaller)

EW SCALE AS OUTPUT

ϕ stops at $V'=0$

Interplay between ϕ slope and barriers:

$$\Rightarrow \quad \boxed{\langle h \rangle \sim \Lambda_c \left(\frac{g f \Lambda^3}{\epsilon \Lambda_c^4} \right)^{1/n}}$$

EW Scale in terms of fundamental params.

Goal:

get $\langle h \rangle \ll \Lambda$ in a technically natural way

ORIGIN OF THE BARRIERS

How is $V_{br} = \epsilon \Lambda_c^4 \left(\frac{h}{\Lambda_c}\right)^n \cos\left(\frac{\phi}{f}\right)$ generated ?

n = 1 case : GKR model # 1

ϕ QCD axion $\quad \delta\mathcal{L} = \frac{g_s^2}{32\pi^2} \cdot \frac{\phi}{f} \cdot G_{\mu\nu} \tilde{G}^{\mu\nu}$

with $\quad 10^9 \text{ GeV} < f < 10^{12} \text{ GeV}$

(star cooling) (DM abundance)

Axion potential from instanton effects :

$$V(\phi) = (m_u + m_d) \langle q\bar{q} \rangle \cos(\phi/f)$$

$$\Rightarrow \Lambda_c \sim \Lambda_{QCD} , \quad \epsilon \sim y_u$$

PREDICTIONS OF QCD-RELAXION

$\langle h \rangle$ and $\theta_{QCD} = \langle \Delta\phi / f \rangle$: outputs.

$$\langle h \rangle = \frac{g f \Lambda^3}{y_u \Lambda_{QCD}^3} \ll \Lambda \quad \text{for} \quad g \sim \frac{m_u \Lambda_{QCD}^3}{f \Lambda^3} \ll 1$$

Ex: $\Lambda \sim 10^7\,\text{GeV}$ and $f \sim 10^9\,\text{GeV}$ need $g \sim 10^{-35}$

Using the constraint $g > (\Lambda / M_P)^3$ $\Rightarrow$

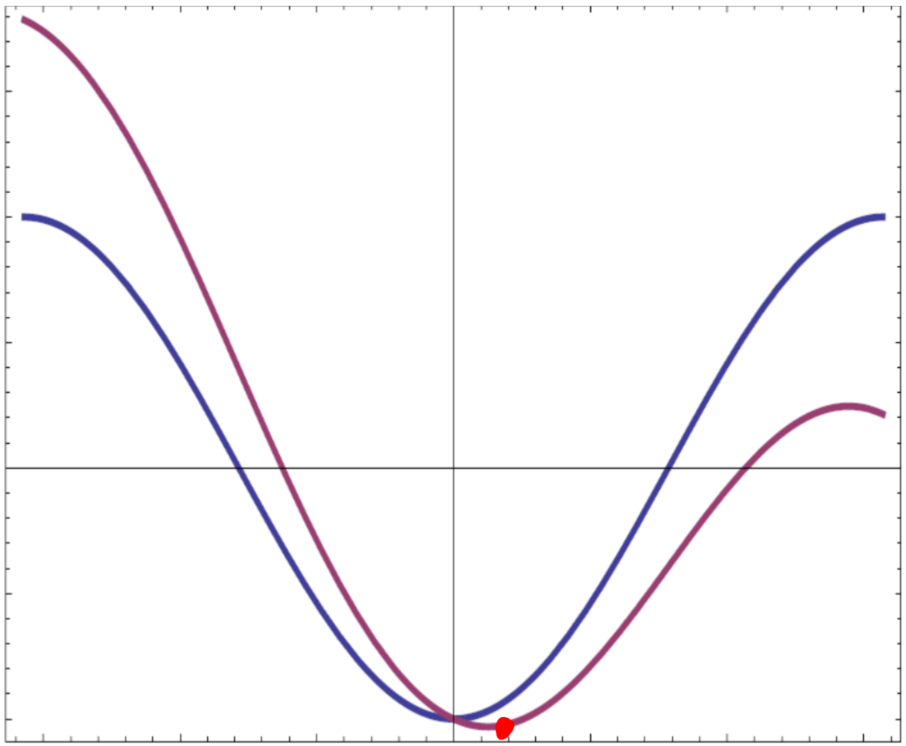
$$\Lambda < 10^7\,\text{GeV} \left(\frac{10^9\,\text{GeV}}{f} \right)^{1/6}$$

PREDICTIONS OF QCD-RELAXION

However

$$\theta_{QCD} \sim O(1) \quad !$$

due to the potential tilt:



$\mapsto \Delta\phi/f = \theta_{QCD}$

BEYOND THE SIMPLE QCD-RELAXION

Ways out :

- Tilt decreases after inflation ($\theta_{QCD} < 10^{-10}$)

  eg. by including $\delta V = \kappa \sigma_I^2 \phi^2$ (σ_I = inflaton)

  $\Rightarrow$ $\Lambda \lesssim 3 \times 10^4\,\mathrm{GeV}$ only

- non-QCD strong gauge sector with

  $\Lambda_c \lesssim \mathrm{TeV}$ and extra fermions at EW scale

GKR model #2

$\Rightarrow$ $\Lambda \lesssim 10^8\,\mathrm{GeV}$

Coincidence problem : why $\Lambda_c \lesssim \mathrm{TeV}$?

SYMMETRIES

Axion Lagrangian

$$\mathcal{L}_a = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{g_s^2}{32\pi^2}\frac{\phi}{f} G_{\mu\nu}\tilde{G}^{\mu\nu}$$

invariant under $\phi \to \phi + c$ (ϕ ~~PQ~~ Goldstone)

Instantons induce $V \sim \epsilon \cos(\phi/f)$: $\phi \to \phi + 2\pi n f$

ϵ : ~~sym~~ spurion

Relaxion couplings $\delta V = \Lambda^3 g \phi + \frac{1}{2} g \phi h^2$

break completely the symmetry

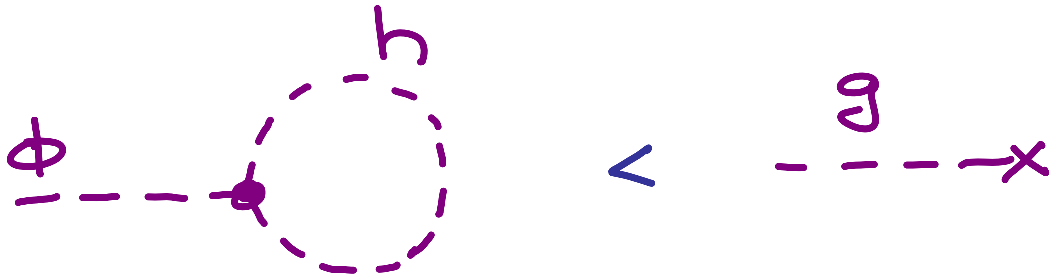
g : ~~sym~~ spurion

$\Rightarrow$ $\epsilon, g \ll 1$ Natural

TECHNICAL NATURALNESS

Besides $g, \epsilon \ll 1$ being natural, also $V(h,\phi)$ should be radiatively natural

E.g



$$g\Lambda \times \frac{\Lambda^2}{16\pi^2} \quad < \quad g\Lambda^3 \quad \checkmark$$

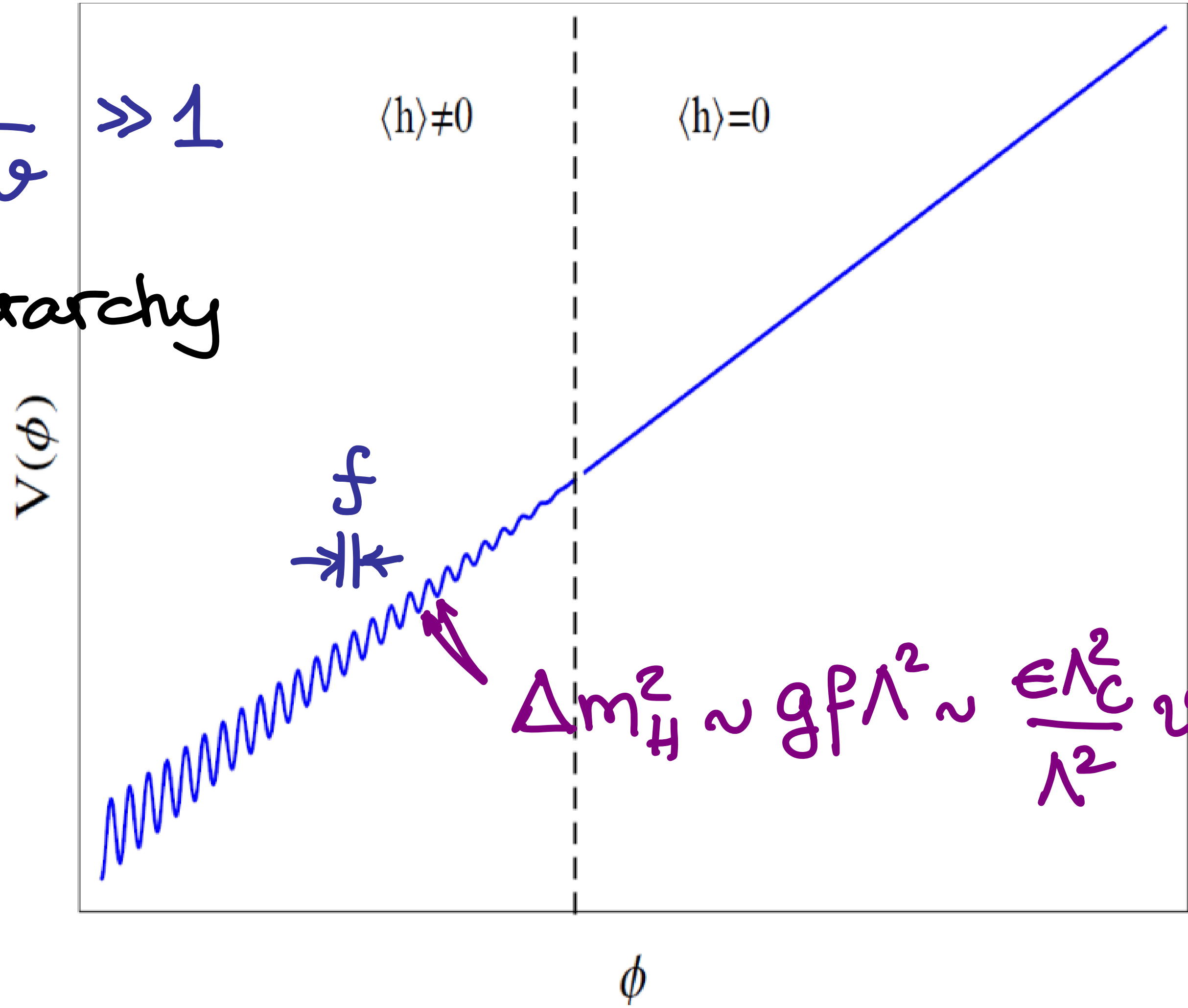
(Explains why we use single spurion g in both terms)

SCALES

$$\frac{\Delta\phi}{f} \gtrsim \frac{\Lambda^4}{\epsilon \Lambda_c^3 v} \gg 1$$

$n = 1$

Is this hierarchy natural ?



f

$$\Delta m_H^2 \sim g f \Lambda^2 \sim \frac{\epsilon \Lambda_c^2}{\Lambda^2} v^2 \ll v^2 \checkmark$$

$$\longleftarrow \Delta\phi \longrightarrow$$

$\sim \Lambda/g \gg \Lambda$ problematic ?

CONCERNS ABOUT $V(\phi)$?

ϕ starts as an angular dof.

PNGBs have compact field range.

$g \neq 0$ makes the field range non compact.

Komargodski et al. stressed (known) concerns about the consistency of this:

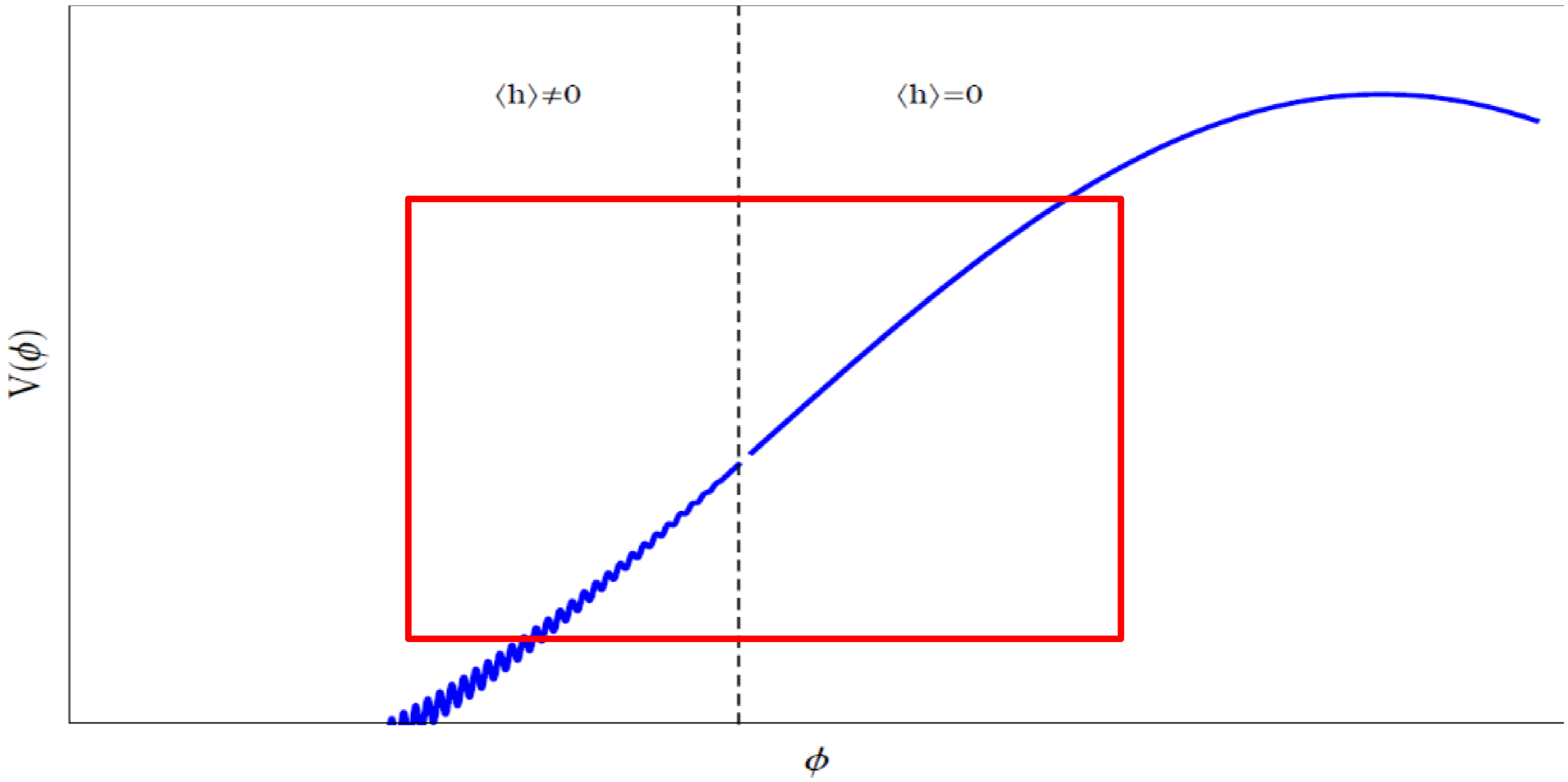
Can't break a gauge symmetry ($\equiv$ redundancy)

$\Rightarrow$ All ϕ operators should be periodic.

CONCERNS ABOUT $V(\phi)$?

In practice, it's enough to have a hierarchy of decay constants: $F = nf \gg f$

$$V \sim A\cos(\phi/F) + B(h)\cos(\phi/f)$$



CONCERNS ABOUT $V(\phi)$?

But, an exponential hierarchy $F \gg f$ does not seem natural ?

Choi, Im + Kaplan, Rattazzi :

QFT examples with N fields, with field-range f leading in the IR to a PNGB (linear comb. of the N fields) with field-range

$$F \sim e^{aN} f \qquad a \sim O(1)$$

Directly relevant for relaxion potentials.

BEYOND GKR

Non-QCD model:

⇨ New physics at TeV (not directly related to natural solution of EW hierarchy)

Our goal →

Can we push Λ higher without leaving new states below TeV ?

Idea: use $\epsilon \Lambda_c^2 h^2 \cos(\phi/f)$ to avoid $\cancel{EXX}$

and try to push $\Lambda_c \to \Lambda$

OUR MODEL (1^{st} TRY)

$$V = \frac{1}{2}\left(-\Lambda^2 + \Lambda g \phi\right) h^2 + \Lambda^3 g \phi + \epsilon \Lambda^2 h^2 \cos(\phi/f)$$

Technically natural ?

$\epsilon \Lambda^4 \cos(\phi/f) \Rightarrow$ high barriers everywhere... (unless $\Lambda < v$)

Solution

Scan also the amplitude of $\cos(\phi/f)$

OUR MODEL: DOUBLE SCANNING

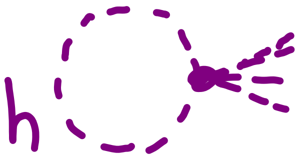
σ new scanner field $\qquad$ g_σ ($\sim g$) new spurion

$$V = (-\Lambda^2 + \Lambda g \phi)|H|^2 + \Lambda^3 g \phi + \Lambda^3 g_\sigma \sigma + A \cos(\phi/f)$$

with $$A \equiv \epsilon \Lambda^4 \left(\beta + c_\phi \frac{g\phi}{\Lambda} - c_\sigma \frac{g_\sigma \sigma}{\Lambda} + \frac{|H|^2}{\Lambda^2} \right)$$

V is now technically natural

Ex.



$\cos(\phi/f)$ $\qquad$ $\epsilon \Lambda^4 \cos(\phi/f)$ ✓

$\cos(\phi/f)$ $\qquad$ $g\phi \cdot \epsilon \Lambda^3 \cos(\phi/f)$ ✓

OUR MODEL: DOUBLE SCANNING

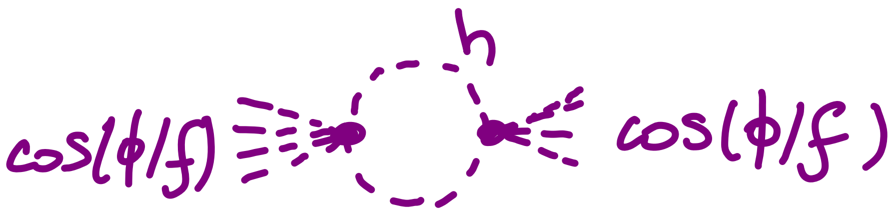
$$V = (-\Lambda^2 + \Lambda g \phi)|H|^2 + \Lambda^3 g \phi + \Lambda^3 g_\sigma \sigma + A \cos(\phi/f)$$

with $\quad A \equiv \epsilon \Lambda^4 \left(\beta + c_\phi \frac{g\phi}{\Lambda} - c_\sigma \frac{g_\sigma \sigma}{\Lambda} + \frac{|H|^2}{\Lambda^2} \right)$

σ new scanner field $\qquad$ g_σ $(\sim g)$ new spurion

V is now technically natural

Ex.



$\epsilon^2 \Lambda^4 \cos^2(\phi/f)$

should be subleading compared to $\epsilon \Lambda^2 h^2 \cos(\phi/f)$

Requires $\quad \epsilon \lesssim v^2/\Lambda^2$

OUR MODEL: DOUBLE SCANNING

σ evolution quite trivial :

$$\sigma(t) = \sigma(0) - g_\sigma \Lambda^3 t / (3 H_I)$$

Take $\sigma(0) \gtrsim \Lambda / g_\sigma$.

⇨ Evolution of ϕ in time-dep. potential

Scanning of $A = \epsilon \Lambda^4 \left(\beta + c_\phi \frac{g\phi}{\Lambda} - c_\sigma \frac{g_\sigma \sigma(t)}{\Lambda} + \frac{|H|^2}{\Lambda^2} \right)$

⇨ there are t-dependent ϕ field values with $A \approx 0$ (ϕ_*)

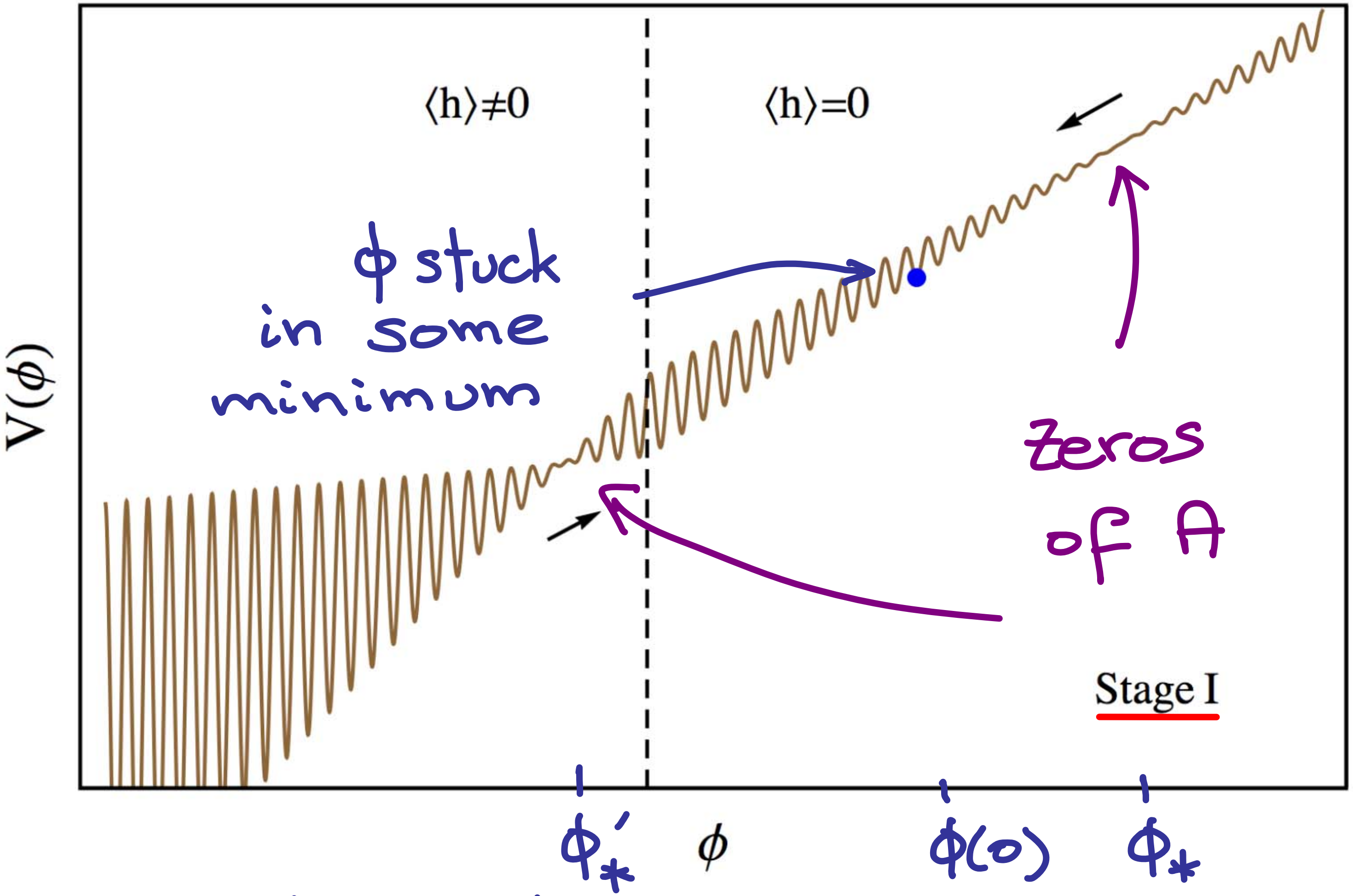
⇨ No barriers

Two branches, depending on $\langle h \rangle$ zero or not.

ϕ EVOLUTION

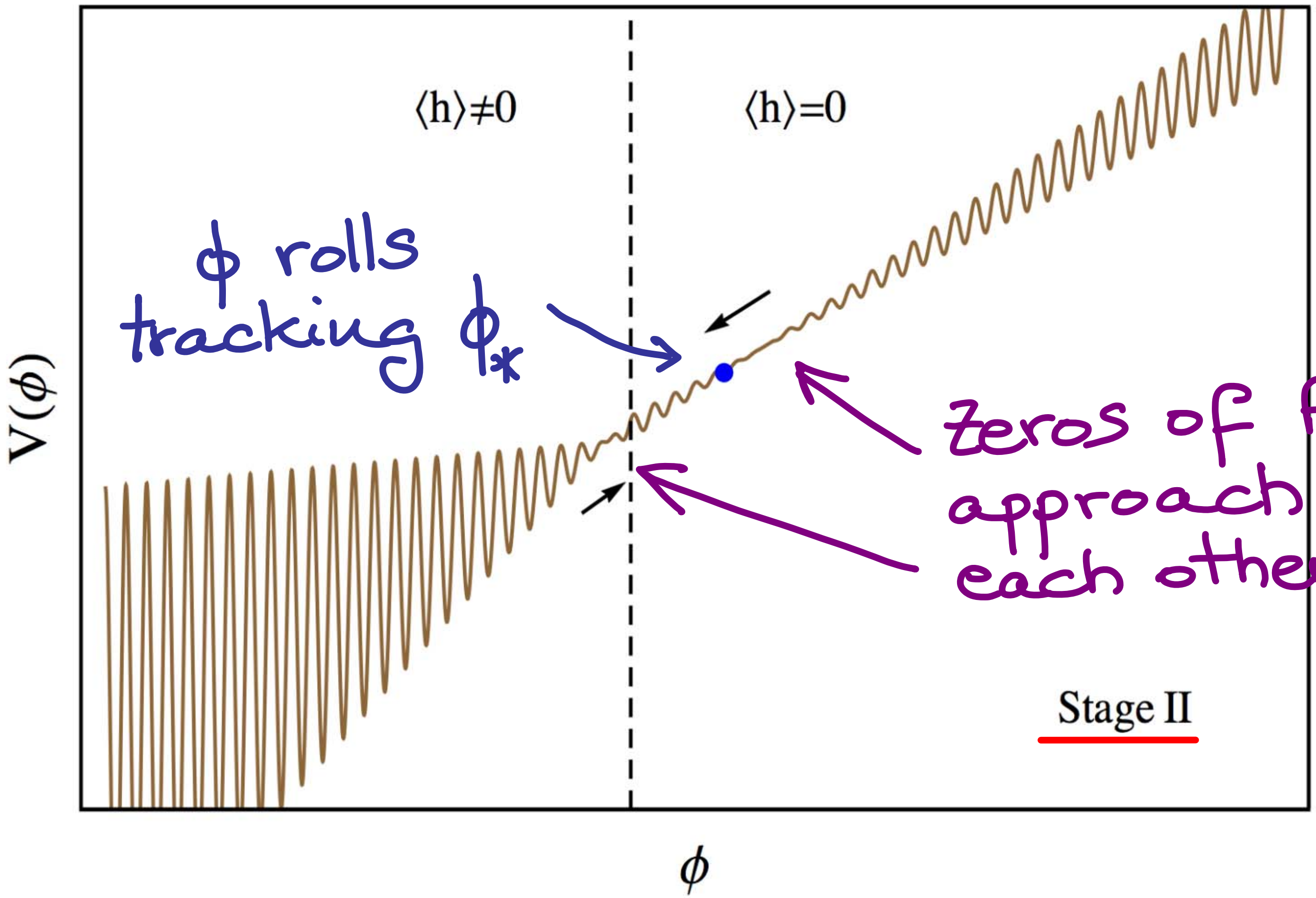
Four stages:

$t=0$



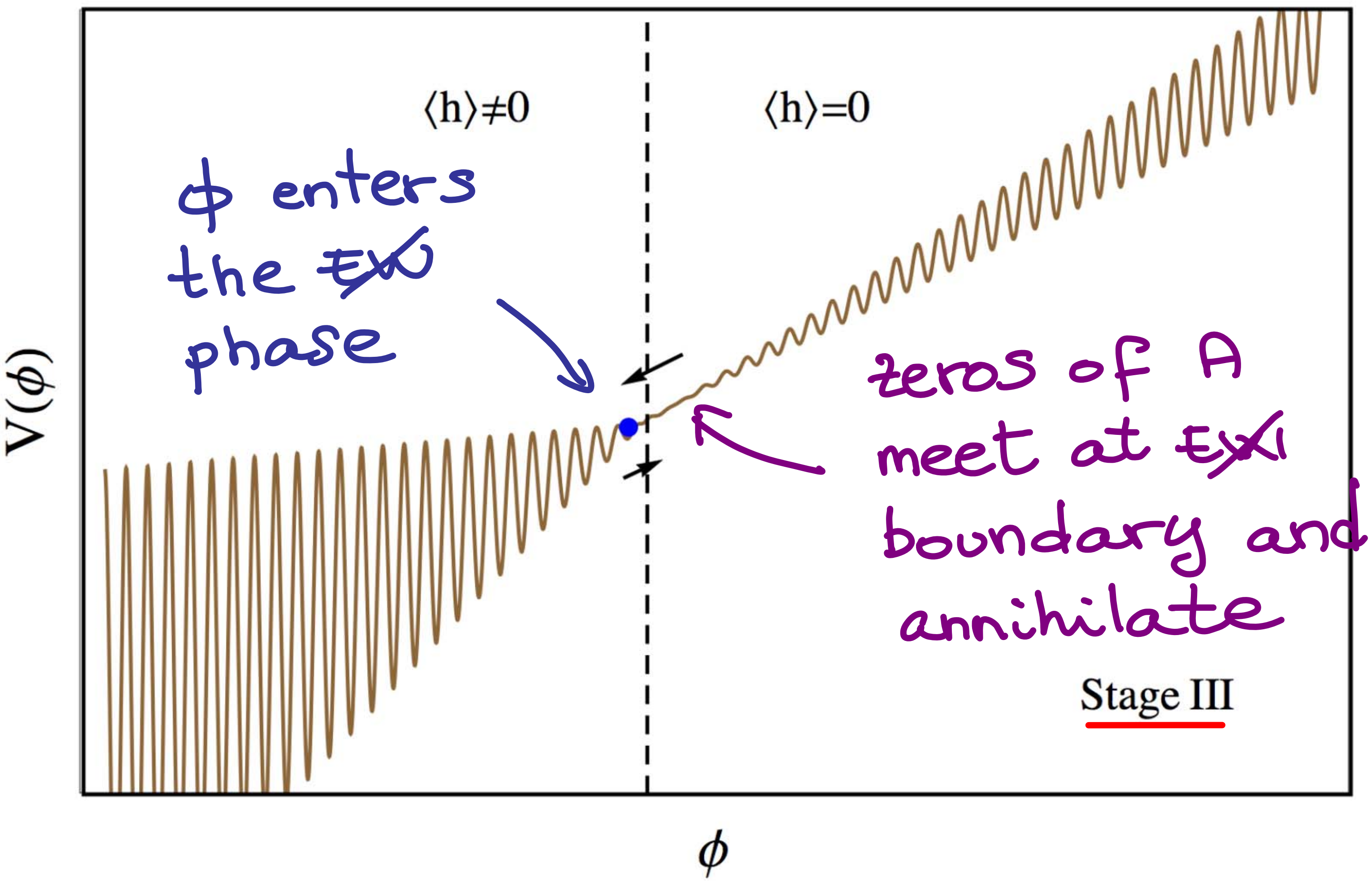
Assumes $\phi_c < \phi(0) < \phi_*$

ϕ EVOLUTION



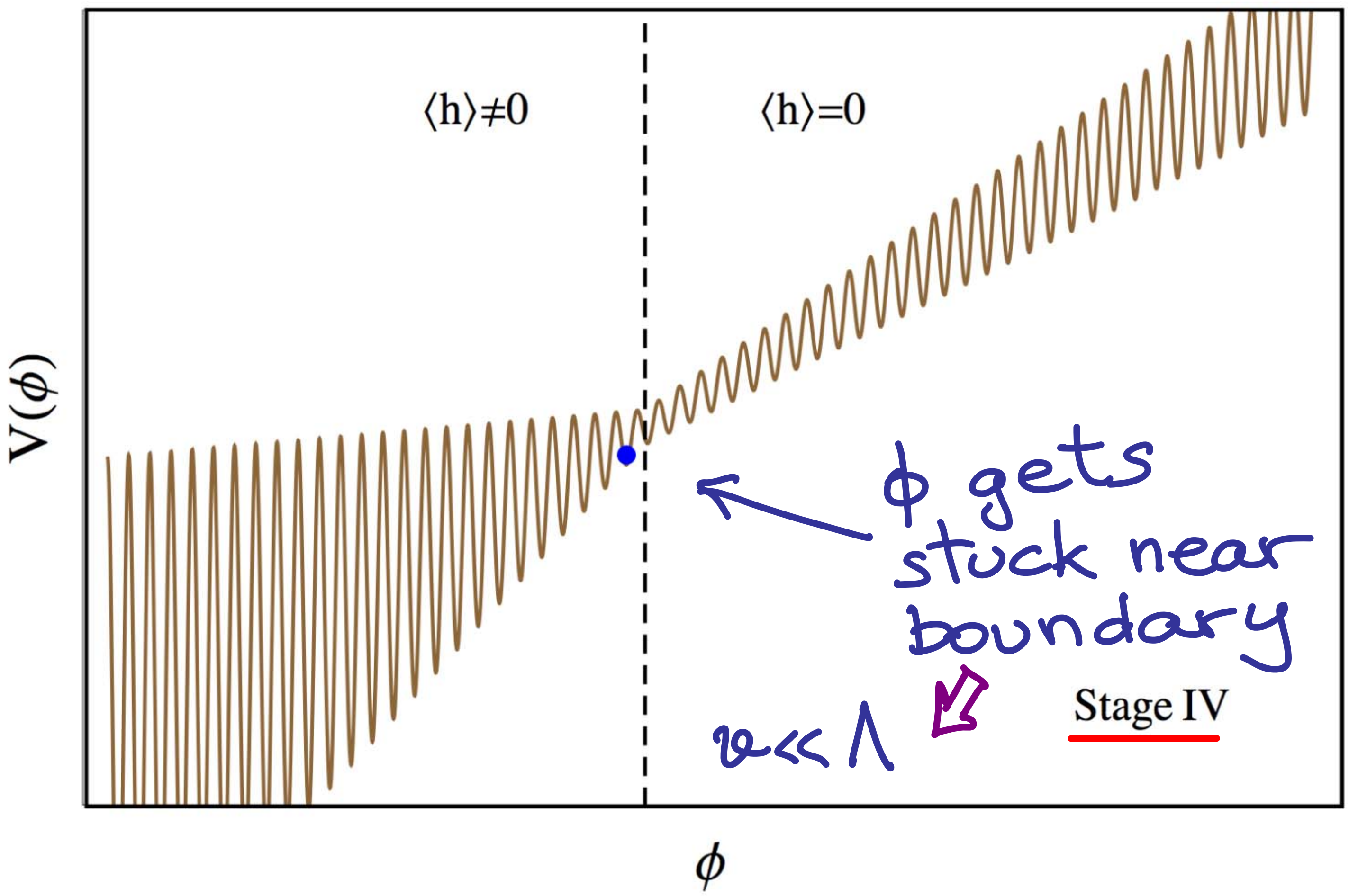
ϕ faster than ϕ_* requires $g \gtrsim g_\sigma$

ϕ EVOLUTION



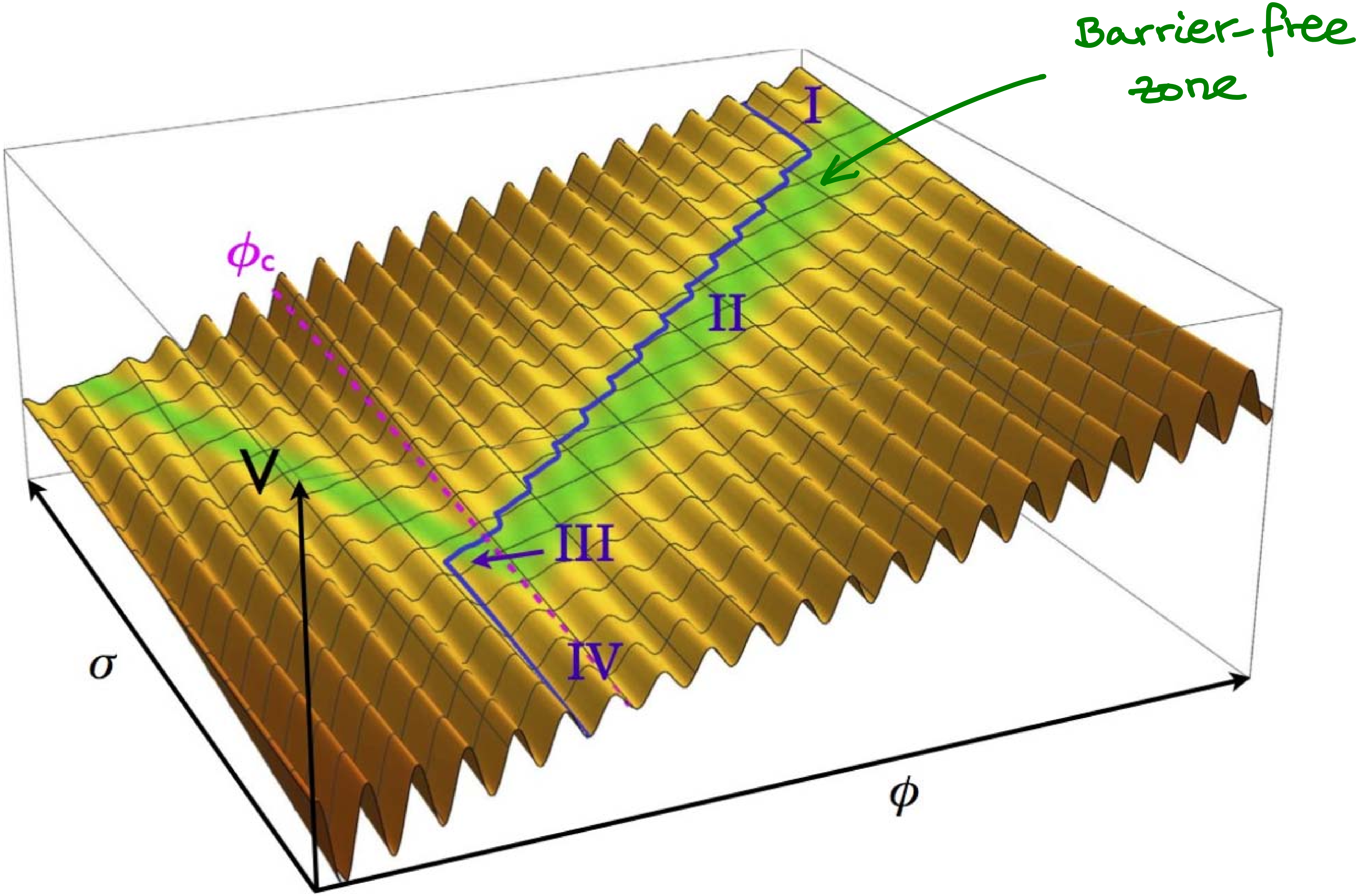
⟨h⟩≠0
⟨h⟩=0
ϕ enters the EW phase
zeros of A meet at EW boundary and annihilate
Stage III
V(ϕ)
ϕ

ϕ EVOLUTION



Movie →

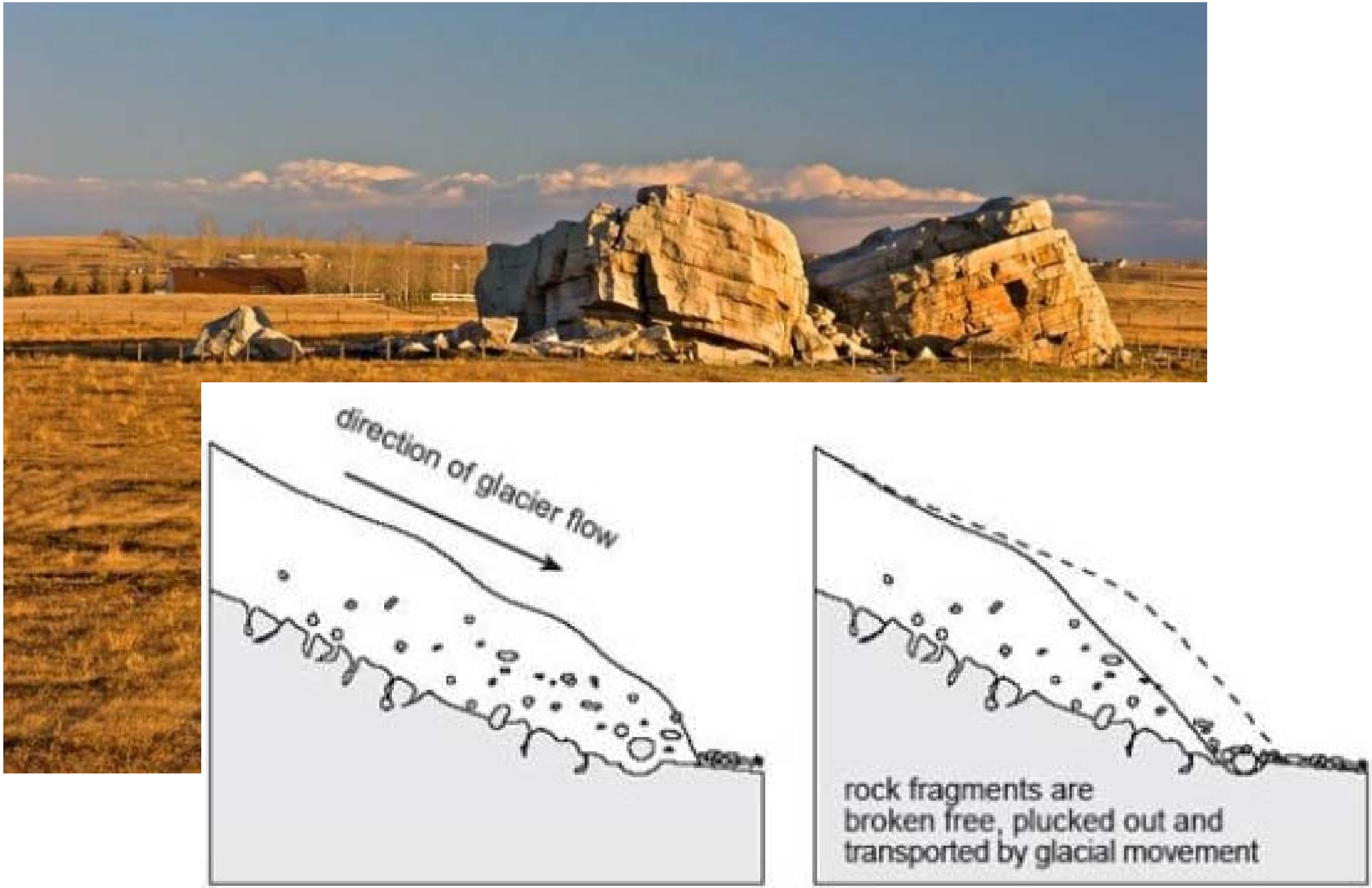
ϕ, σ EVOLUTION



EW SCALE AS COSMOLOGICAL ERRATIC

Okotoks glacial erratic,
Alberta, Canada

EW SCALE AS COSMOLOGICAL ERRATIC



UV ORIGIN OF BARRIERS

Non-QCD sector strong at Λ + fermions L, N

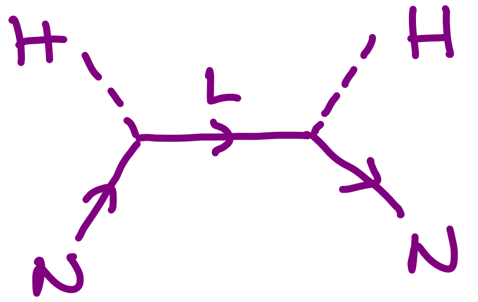
N : Light singlet fermion with $\langle \bar{N} N \rangle = \Lambda^3$

and axion coupling $\phi\, G'_{\mu\nu} \tilde{G}'^{\mu\nu}$

$$\Rightarrow \quad V \sim \Lambda^3 m_N \cos(\phi/f)$$

Just need $\mathcal{L} = \Lambda \bar{L} L + \epsilon(\Lambda + g\phi + g_\sigma \sigma)\bar{N} N + \sqrt{\epsilon}\, \bar{L} H N + \text{h.c.}$

$$m_N = \epsilon\left(\Lambda + g\phi + g_\sigma \sigma - \frac{|H|^2}{\Lambda}\right)$$



PARAMETER CONSTRAINTS

- Natural V

$$\epsilon \lesssim (v/\Lambda)^2$$

- EW Scale as output

$$v^2 \simeq \frac{g\Lambda f}{\epsilon} \qquad \text{(requires } g \ll \epsilon\text{)}$$

- Inflation window

$$\Lambda^2/M_p \lesssim H_I \lesssim g_\sigma^{1/3}\Lambda$$

- Long enough inflation

$$N_e \gtrsim \frac{H_I^2}{g_\sigma^2 \Lambda^2}$$

PARAMETER CONSTRAINTS

The previous constraints give

$$(\Lambda/M_p)^3 \lesssim g_\sigma \lesssim g \lesssim v^4/(f\Lambda^3)$$

and using $f \gtrsim \Lambda$

$$\Lambda \lesssim (v^4 M_p^3)^{1/7} \simeq 2 \times 10^9 \, \mathrm{GeV}$$

Natural model without TeV matter ✓

No coincidence problem ✓

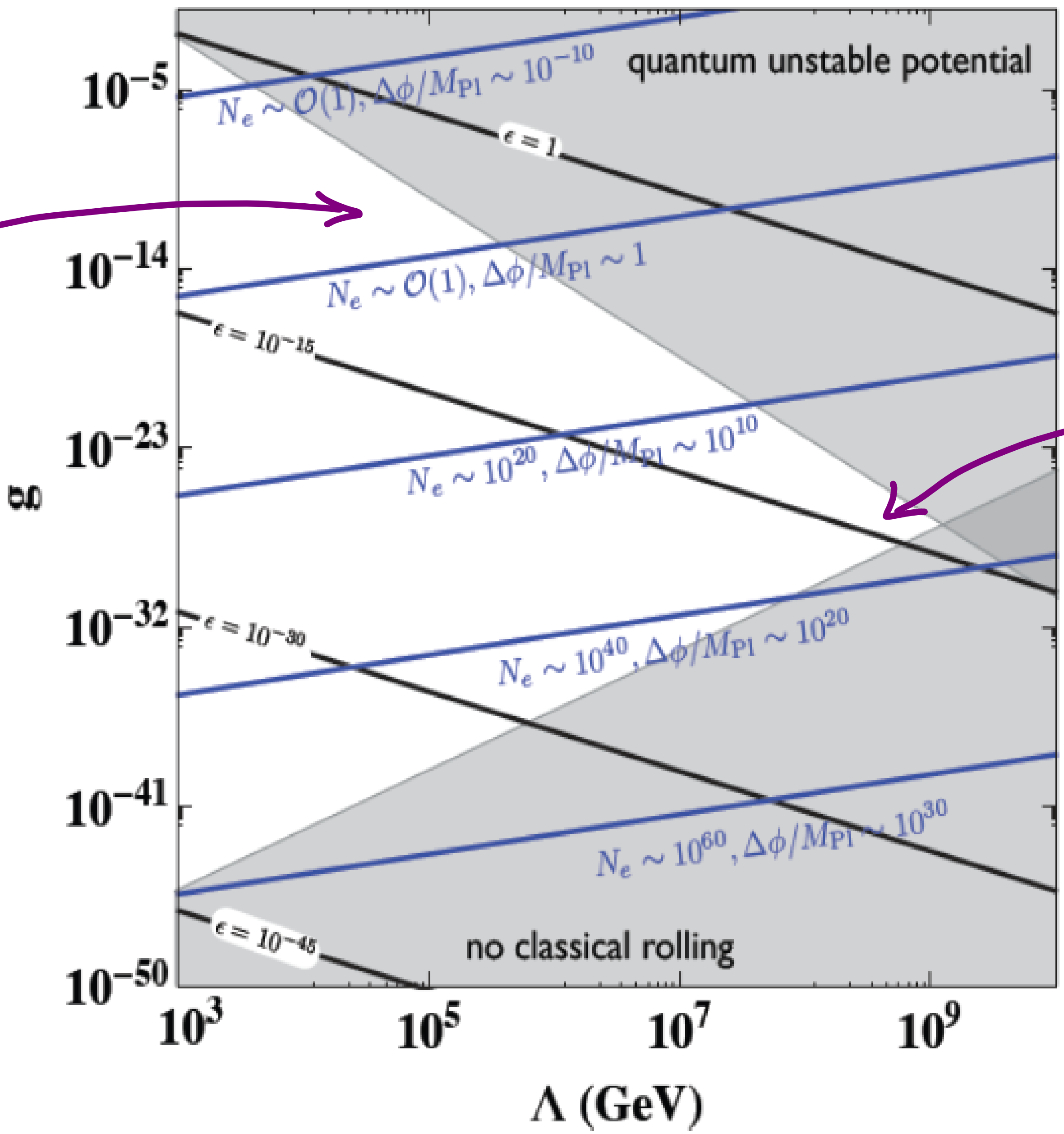
PARAMETER SPACE

$g_\sigma = 0.1 g$

$f = \Lambda$

$N_e, \frac{\Delta\phi}{M_{Pl}} \sim O(1)$

Good for little hierarchy problem.



$N_e, \frac{\Delta\phi}{M_{Pl}}$ exponentially large

SIGNATURES ?

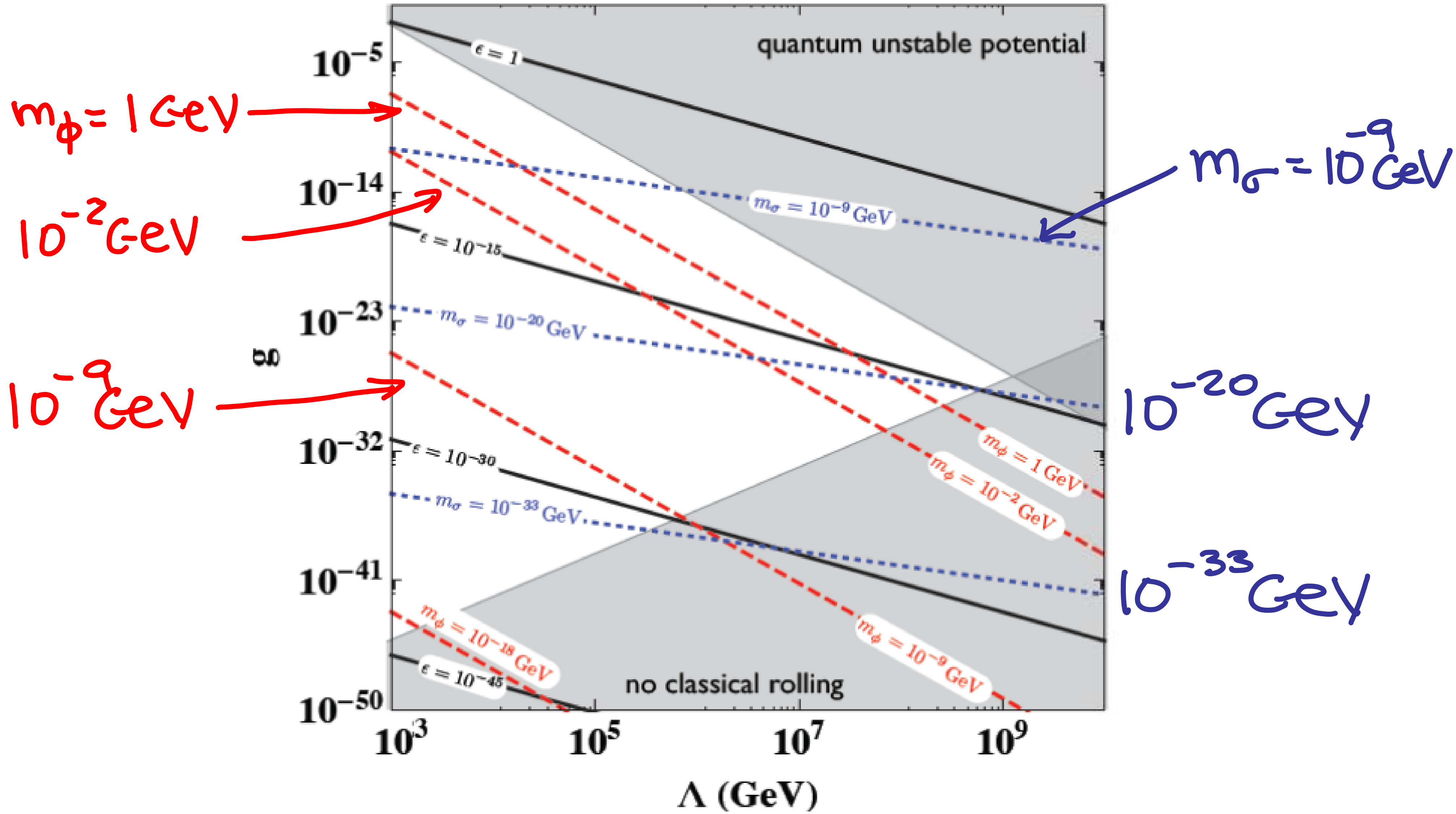
- Below Λ only extra states : ϕ and σ

$$m_\phi^2 \sim \frac{\epsilon \Lambda^4}{f^2} \sim g \frac{\Lambda^5}{f v^2} \lesssim v^2 \qquad \in [10^{-20}, 10^2]\,\mathrm{GeV}$$

$$m_\sigma^2 \sim g_\sigma^2 \Lambda^2 \ll m_\phi^2 \qquad \in [10^{-45}, 10^{-2}]\,\mathrm{GeV}$$

SIGNATURES ?

- Below Λ only extra states : ϕ and σ



SIGNATURES ?

Below Λ only extra states : ϕ and σ

Interactions with SM particles through mixing with the Higgs suppressed by $1/f$ and/or small spurious ϵ, g, g_σ

Ex. $\Lambda \sim 10^{9}$ GeV

$m_\phi \sim 100$ GeV $\qquad \theta_{\phi h} \sim 10^{-21} \qquad \phi^2 h^2 \sim 10^{-14}$

$m_\sigma \sim 10^{-18}$ GeV $\qquad \theta_{\sigma h} \sim 10^{-50}$

No collider signatures

COSMOLOGICAL CONSTRAINTS

From abundances, post-BBN decays, astrophysics...

Decay widths

$$\Gamma_\phi \sim \theta_{\phi h}^2 \, \Gamma_h(m_\phi) \qquad \Gamma_\sigma \sim \theta_{\sigma h}^2 \, \Gamma_h(m_\sigma)$$

can be very small.

Compare with

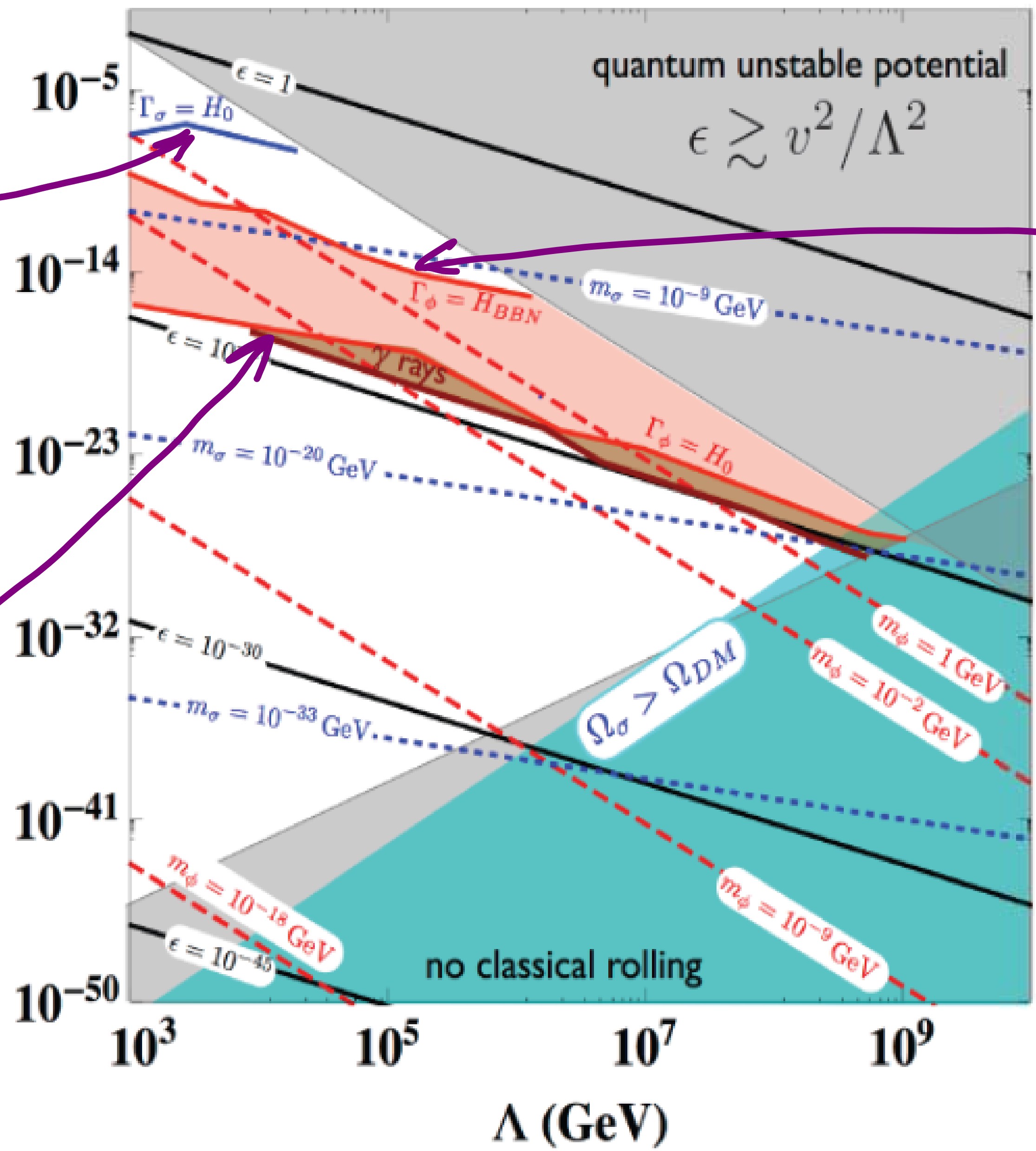
H_0 for cosmological stability

H_{BBN} for potential trouble with BBN

COSMOLOGICAL CONSTRAINTS

σ stable below (DM?)

ϕ stable below



ϕ decays after BBN below

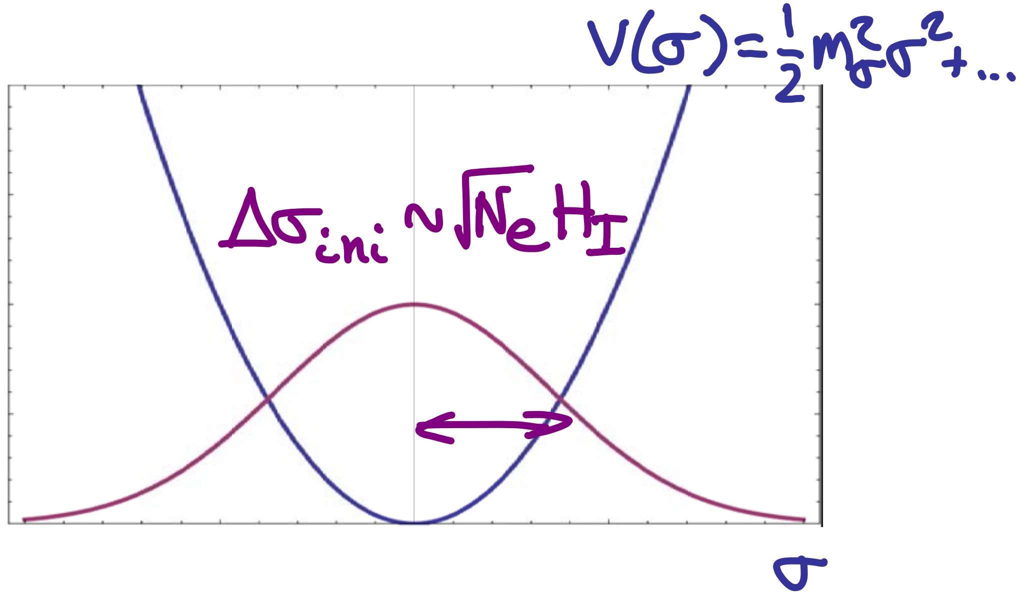
DARK MATTER

σ can be a good DM candidate.

After inflation:

σ typically displaced from its minimum with

$$\rho_{ini}^{\sigma} \sim m_\sigma^2 \Delta\sigma_{ini}^2 \sim H_I^4$$

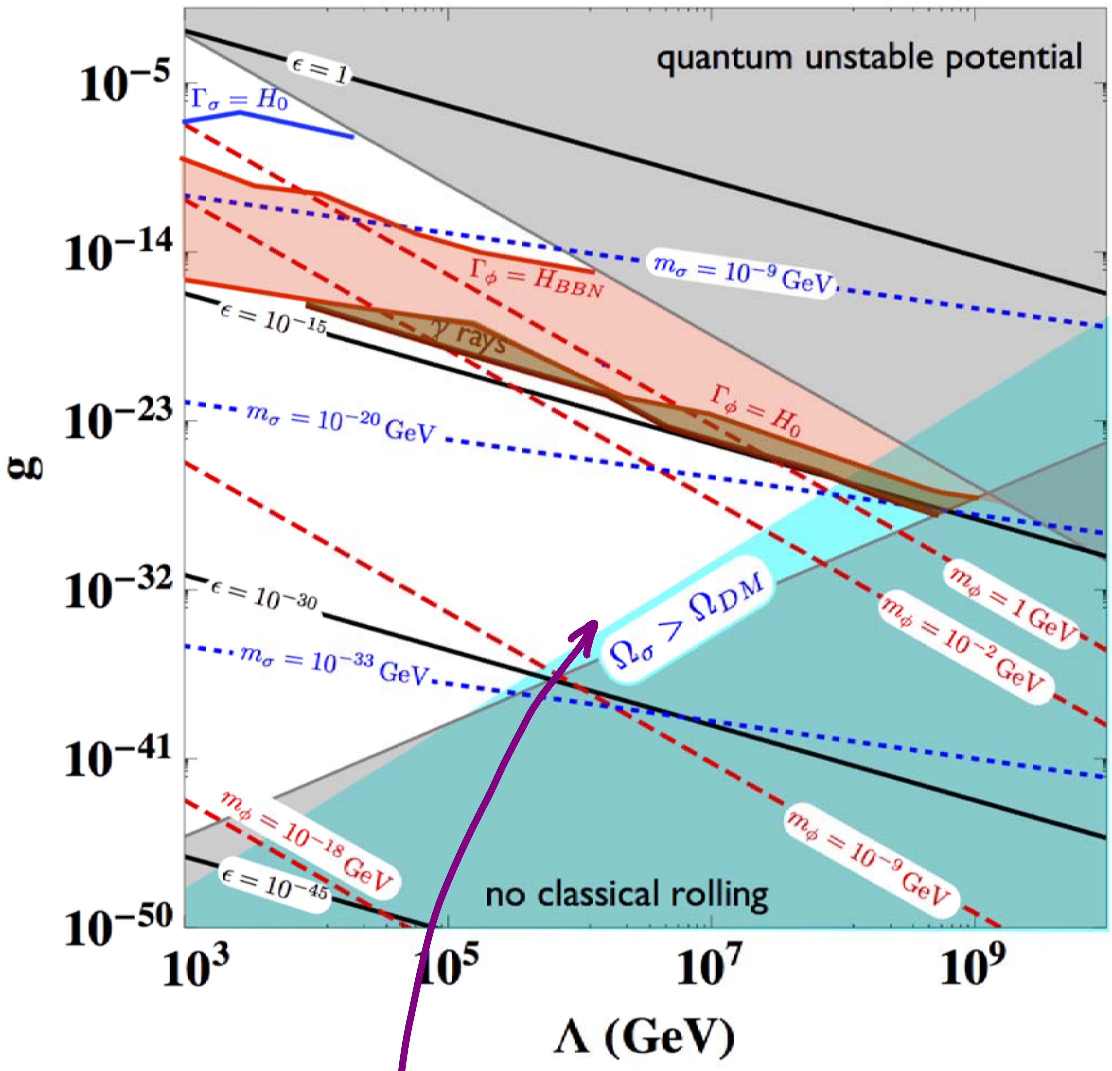


Below $T_{osc,\sigma}^{\sigma} \sim \sqrt{m_\sigma M_P}$ it oscillates with energy density scaling like nonrelativistic matter

$$\rho_\sigma(T) \sim \rho_{ini}^{\sigma} \left(\frac{T}{T_{osc}^{\sigma}}\right)^3 \Rightarrow \Omega_\sigma \gtrsim \left(\frac{4\times10^{-28}}{g_\sigma}\right)^{3/2} \left(\frac{\Lambda}{10^8\,\mathrm{GeV}}\right)^{13/2}$$

(Ω_ϕ always subdominant)

DARK MATTER



σ can be a good DM candidate.

OTHER PROBES

- For $H_0 < \Gamma_\phi < H_{BBN}$, ϕ decays can alter the BBN predictions for light el. abundances (Dedicated analysis required)

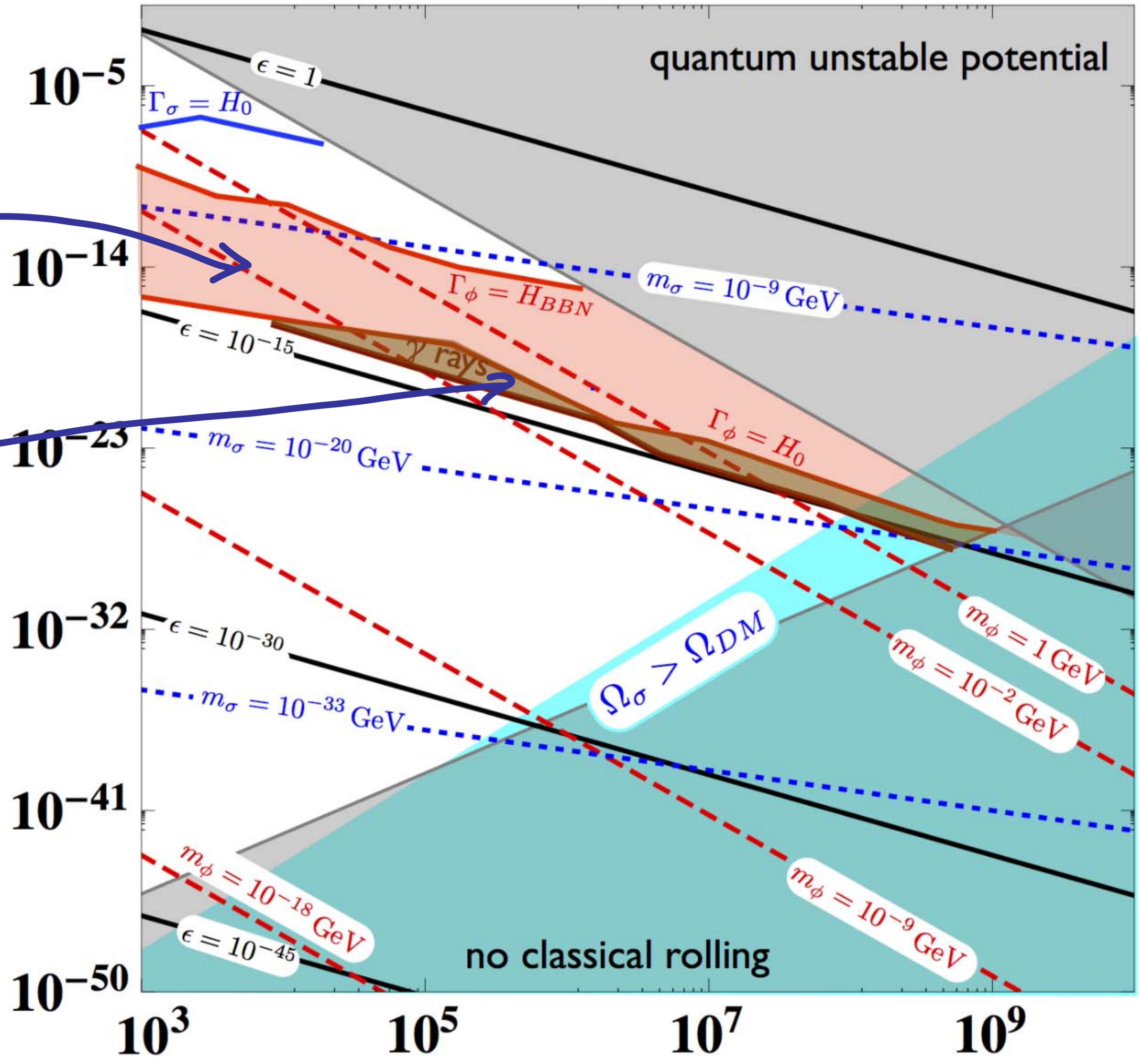
- $\phi \to \gamma\gamma$ decays can distort the diffuse X-ray background. Constraint:

$$\tau_\phi > 10^{27}\,\mathrm{s}\,\frac{\Omega_\phi}{\Omega_{DM}}$$

OTHER PROBES

BBN

γ rays



OTHER PROBES

- σ could be searched by SKA pulsar timing array experiment for $m_\sigma \sim 10^{-23}\,\mathrm{eV}$

  Khmelnitsky, Rubakov '13

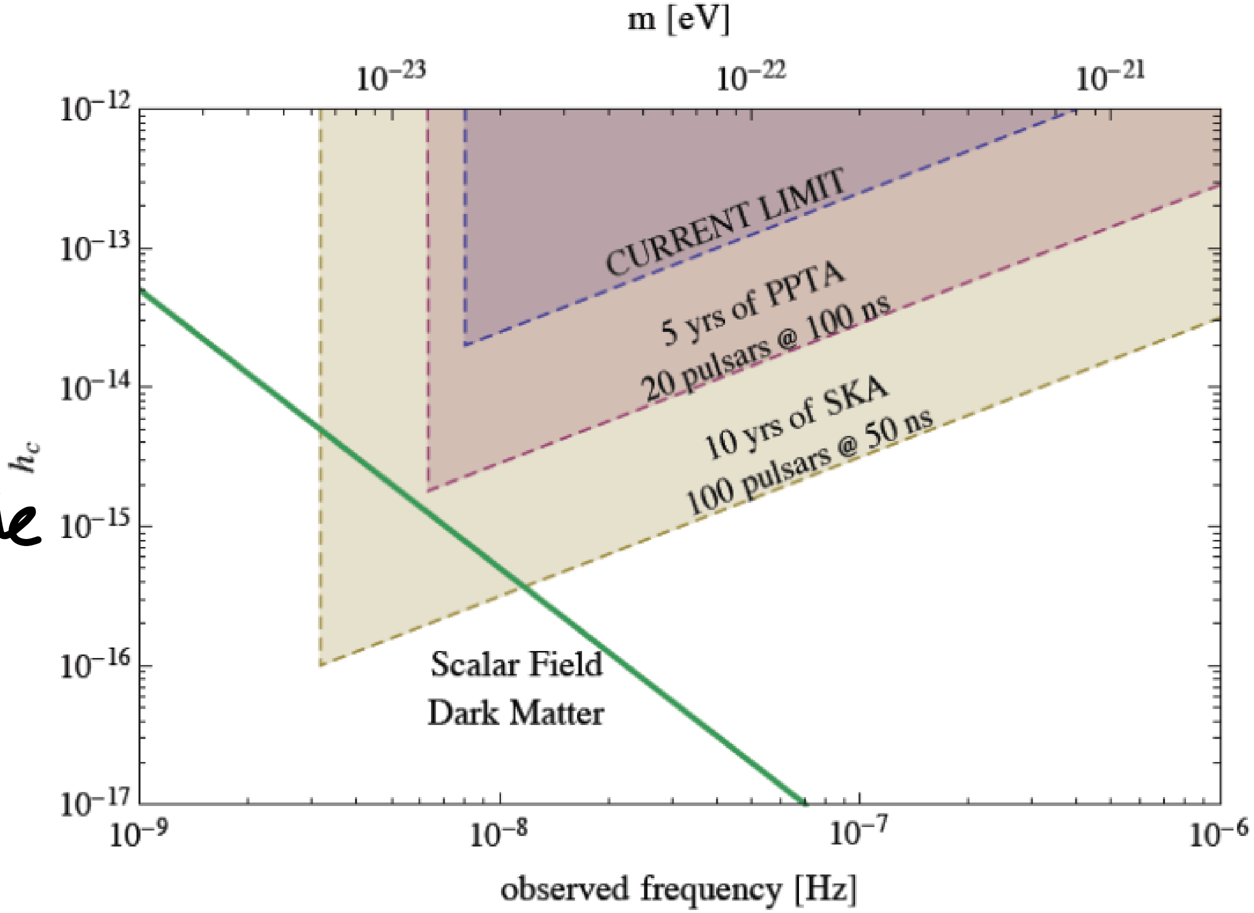
  Oscillating $\sigma \Rightarrow$ oscillating pressure and gravitational potentials of DM halo

  Effect similar to a monochromatic gravitational wave with $\nu \sim m_\sigma \sim 10^{-9}\,\mathrm{Hz}$

OTHER PROBES

- σ could be searched by SKA pulsar timing array experiment for $m_\sigma \sim 10^{-23}$

signal amplitude



Khmelnitsky, Rubakov '13

CONCLUSIONS

New class of solutions to the hierarchy problem: Relaxion

EW Scale from early Universe dynamics

Typical predictions

- (extremely) light axion-like states
  Interesting for DM, astro-ph signals...
- No collider signals guaranteed

Constructed natural model with

$\Lambda \sim 10^9$ GeV

OUTLOOK

- Current models have unpleasant features

$$N_e \gg 1 \qquad \Delta\phi \gg M_P$$

$\Rightarrow$ Room for improvement in inflation sector and model-building.

- Friction: alternatives to inflation?
- Possible to push Λ even higher?
- UV completions?
- Other applications? (~~SUSY~~, c.c.)

...

BACK-UP SLIDES

ϕ EVOLUTION DURING INFLATION

Keep track of fluctuations by solving a Langevin eq.

$$\frac{d\phi}{dN} + \frac{1}{3H_I^2}\frac{dV}{d\phi} = \eta(N)$$

Gaussian random noise

$$\langle \eta(N)\,\eta(N') \rangle = \left(\frac{H_I}{2\pi}\right)^2 \delta(N-N')$$

Equivalent to Fokker-Planck eqn. for $P(\phi,N)$:

$$\frac{\partial P}{\partial N} = \frac{\partial^2}{\partial \phi^2}\left(\frac{H_I^2}{8\pi^2}P\right) + \frac{\partial}{\partial \phi}\left(\frac{V'}{3H_I^2}P\right)$$

quantum jumps

classical roll in V

ϕ EVOLUTION DURING INFLATION

Defining

$$\langle F(\phi)\rangle = \int_{-\infty}^{+\infty} d\phi \, P(\phi,N)\, F(\phi)$$

From Fokker–Planck :

$$\frac{d\langle\phi\rangle}{dN} = -\frac{1}{3H^2}\left\langle \frac{\partial V}{\partial \phi}\right\rangle$$

$$\frac{d\Delta\phi^2}{dN} = \frac{H^2}{4\pi^2} - \frac{2}{3H^2}\left\langle (\phi - \langle\phi\rangle)\frac{\partial V}{\partial\phi}\right\rangle$$

where $\Delta\phi^2 \equiv \langle(\phi - \langle\phi\rangle)^2\rangle$