First Joint ICTP-Trieste/ICTP-SAIFR
School on Particle Physics

Analysis of Collective Flow in Heavy Ion Collisions

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Contents

1 Heavy Ion Collisions

2 Relativistic Hydrodynamics

3 Anisotropic Flow
   - Fourier Expansion
   - Two-Particle Correlation

4 Principal Component Analysis

5 Results
Nuclei collisions help us investigate QCD. The matter in a collision passes through a Quark Gluon Plasma (QGP) phase. The QGP is believed to be one of the early stages of the universe.

From Qin, Guang-You Int. J. Mod. Phys. E24 (2015) no.02, 1530001

Only the final particles produced in the collision are measured. So a model of the intermediate state is needed.
Pb-Pb $\sqrt{s_{\text{NN}}} = 2.76$ TeV
run: 137171, 2010-11-09 00:12:13

From https://cds.cern.ch/record/2032743
Contents

1 Heavy Ion Collisions

2 Relativistic Hydrodynamics

3 Anisotropic Flow
   - Fourier Expansion
   - Two-Particle Correlation

4 Principal Component Analysis

5 Results
To describe the QGP we use the relativistic hydrodynamics equations.

\[ \partial_\mu T^{\mu\nu} = 0 \]  
\[ T^{\mu\nu} = (\epsilon + p) u^\mu u^\nu - pg^{\mu\nu} \]

where \( g^{\mu\nu} \) is the Minkowski metric, \( \epsilon \) is the energy density, \( p \) is the pressure and \( u^\mu \) the four velocity of the fluid.

Using initial conditions for the energy-momentum tensor and an equation of state, we can solve the equations (numerically) and evolve the fluid until it reaches the a freezeout temperature, when the evolution stops and particles are emitted.
Contents

1. Heavy Ion Collisions
2. Relativistic Hydrodynamics
3. Anisotropic Flow
   - Fourier Expansion
   - Two-Particle Correlation
4. Principal Component Analysis
5. Results
Fourier Expansion

The distribution of particles can be expanded in a Fourier series:

\[
\frac{dN}{p_T dp_T d\eta d\phi} = \frac{dN}{2\pi p_T dp_T d\eta} \sum_{n=-\infty}^{\infty} V_n(p_T, \eta) e^{in\phi} \tag{3}
\]

\[
V_n(p_T, \eta) = v_n(p_T, \eta) e^{in\psi_n(p_T, \eta)} = \frac{1}{N(p_T, \eta)} \sum_j e^{in\phi_j} \tag{4}
\]

\(v_n\) is called the anisotropic flow and \(\Psi_n\) the event plane. Here, the sum is over particles in a given interval of \(p_T\) and \(\eta\) in one event.

Also of use, is the Flow Vector:

\[
Q_n(p_T, \eta) \equiv \sum_j e^{in\phi_j} \tag{5}
\]
Two-Particle Correlation

The distribution of particle pairs is determined by the single particle distribution, if non-flow effects are neglected:

\[
\frac{dN_{\text{pairs}}}{d^3p_a d^3p_b} \approx \frac{dN}{d^3p_a} \frac{dN}{d^3p_b}. \tag{6}
\]

It can also be expanded in a Fourier series:

\[
\left\langle \frac{dN_{\text{pairs}}}{d^3p_a d^3p_b} \right\rangle \propto \sum_n V_n \Delta (p_a, p_b) e^{in(\phi_a - \phi_b)}, \tag{7}
\]

\[
V_n \Delta (p_a, p_b) = \langle V_n(p_a) V_n^*(p_b) \rangle = \langle v_n(p_a) v_n(p_b) e^{in(\psi_a - \psi_b)} \rangle. \tag{8}
\]

Here, the brackets indicate an average over events.
Contents

1 Heavy Ion Collisions

2 Relativistic Hydrodynamics

3 Anisotropic Flow
   - Fourier Expansion
   - Two-Particle Correlation

4 Principal Component Analysis

5 Results
Writing $V_{n\Delta}$ in terms of the flow vector $Q_n$, since the event plane is random, $\langle Q_n \rangle$ is zero for $n > 0$ and the two-particle correlation coefficient can be interpreted as a covariance function:

$$\left\langle \left( Q_n(p_a) - \langle Q_n(p_a) \rangle \right) \left( Q_n(p_b) - \langle Q_n(p_b) \rangle \right) \right\rangle$$

$$= \langle Q_n(p_a) Q^*_n(p_b) \rangle = V_{n\Delta}(p_a, p_b)$$

(9)

$V_{n\Delta}$ can be written in terms of its eigenvectors $\Phi_\alpha$:

$$V_{n\Delta}(p_a, p_b) = \langle Q_n(p_a) Q^*_n(p_b) \rangle = \sum_\alpha \lambda_\alpha \Phi_\alpha(p_a) \Phi_\alpha(p_b)$$

(10)
And so the modes can be defined as:

\[ V_n^{(\alpha)}(p) \equiv \sqrt{\lambda_\alpha} \Phi_\alpha(p). \]  

(11)

Since the flow coefficients, \( V_n \), are the variance (noise) of \( Q_n \) they can be expressed by the eigenvectors of the covariance matrix. And if there is only one mode, it must be equal to \( V_n \). So, the first component, with the highest eigenvalue, should be the anisotropic flow and the others must be related to fluctuations. To compare to the usual flow coefficients, the modes are normalized by \( \langle V_0 \rangle \).

\[ u_n^{(\alpha)} = \frac{V_n^{(\alpha)}}{\langle V_0 \rangle} \]  

(12)
Results

First and second components of $v_2$ from simulations and CMS data of Pb+Pb collisions at 2.76TeV with 20-25% centrality.
Results

First and second components of $v_2$ from simulations and CMS data of Pb+Pb collisions at 2.76TeV with 0-5% centrality.
First and second components of $v_3$ from simulations and CMS data of Pb+Pb collisions at 2.76TeV with 20-25% centrality.
Results

First and second components of $v_3$ from simulations and CMS data of Pb+Pb collisions at 2.76TeV with 0-5% centrality.
Bibliography
