1^{st} ICTP-Trieste/ICTP-SAIFR School on Particle Physics: Dark Matter and Particle Physics – Some exercises

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Below are a list of questions, some simple, some involved, that will help illustrate some of the points raised in lectures. The questions approximately follow the order of topics in the lectures, but you should feel free to approach them in any order you wish.

1 Lecture II

Question 1. *Boltzmann Equation in comoving variables* Starting with the standard Boltzmann equation for DM of mass m

$$\frac{dn}{dt} + 3Hn = \langle\sigma v\rangle(n_{eq}^2 - n^2) \tag{1}$$

arrive at the equation in terms of number density per entropy density (number per comoving volume) $Y = n/s$ and $x = m/T$,

$$\frac{dY}{dx} = \frac{x\langle\sigma v\rangle s}{H(m)}(Y_{eq}^2 - Y^2) \tag{2}$$

with $H(m) = 1.67 g_\star^{1/2} m^2/M_{pl}$, the Hubble constant in a radiation dominated era at $T = m$.

Question 2. *Annihilation cross section expanded in x* The thermally averaged annihilation cross section is ofter written as an expansion in relative velocity, since freeze out typically occurs when $v \approx \mathcal{O}(0.1)c$. Show this is

$$\langle\sigma v_{rel}\rangle = a + b\langle v_{rel}^2\rangle + \ldots = a + \frac{6b}{x} + \ldots \tag{3}$$

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Question 3. *Freeze out for the baryons* If the baryons were a thermal relic, what would be their abundance? Estimate the cross section that keeps them in equilibrium and determine the freeze out temperature, by solving $n\langle\sigma v\rangle \approx 3H$. Compare $\eta = n_B/s$ to the observed value of $\eta \sim 6 \times 10^{-10}$. Explain.